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Summary Report of the Group on Single-Particle Nonlinear Dynamics*

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1. Analytical and Semi-analytical Tools

In the past ~ 10 years, our community has developed quite an impressive arsenal of tools, as seen in Table 1.

Table 1. Modern analytical and semi-analytical tools.

Modern tools can easily generate:	with these applications:	Commets
High order one-turn Taylor maps	Fast tracking	
Symplectified one-turn Taylor maps	Fast symplectic tracking	(a)
Lie generator map $e^{-:H_{\rm eff}:}$	nPb fast tracking	(b)
	resonance-base coefficients	(c)
Normal form map $\mathcal{A}e^{-:h:}\mathcal{A}^{-1}$	detuning effects	(d)
	single resonance analysis	
Spectral lines analysis	resonance strengths	(e)
	early prediction of dynamic aperture	(f)

Comments on Table 1:

(a) One example of how to symplectify Taylor maps is the Cremona maps presented in this workshop. [1]

(b) nPb means n-th order Poisson bracket. Tracking using nPb means Taylor expanding the Lie generator up to n-th order in the effective Hamiltonian H_{eff}. Appying the resulting operator to tracking requires evaluating n-th order Poisson brackets. See [2].

(c) By expanding H_{eff} into a resonance base, one obtains the coefficients $C_{n_x n_y n_\delta}$,

$$H_{\text{eff}} = \sum C_{n_x n_y n_\delta} |n_x n_y n_\delta > \tag{1}$$

Although they are not themselves the resonance strengths, the various resonance strengths are directly proportional to them. (Resonance strength would in addition be inversely proportional to the distance of the tune values from the resonance.) A tabulation of all these coefficients, therefore gives an idea of which resonances are potentially problematic for the stability of particle motion. One example of such a tabulation has been made for the LER ring of PEP-II. [2]

(d) The normal form analysis is a powerful analysis tool. It is the nonlinear generalization of the Courant-Snyder analysis for the linear case.

(e) By analyzing the Fourier spectrum of a tracking result (or, conceivably, an experimental rurn-by-turn measurement), one obtains these spectral lines analyses. Each spectral line corresponds to a resonance, while overlapping of spectral lines is an indication of some degree of chaos. Like the analytical methods mentioned above, this analysis contains a wealth of information of the nonlinearities of the accelerator system. Compared with the purely analytical methods, it has the advantage of being non-perturbative. One application of this method to the LHC was presented in this workshop. [3] It was found that the "symmetric" LHC lattice has weaker spectral lines than the "non-symmetric" LHC lattice.

2. Early Prediction of the Dynamic Aperture

For a while now, we have been searching for an effective way to predict the dynamic aperture for 10^{5-7} turns by tracking only 10^{3-4} turns. Progress has continuously been made. Recent ones include the following:

Lyapunov predictor

It is well known that the Lyapunov coefficient $\mathcal{L}$ potentially contains information useful for early prediction of the long term dynamic aperture.

Recently a way to do this quanitatively is proposed. [4] One first track particles for n turns and obtain numerically $\mathcal{L}$ as a function of n up to some modest value of n. For the particle motion to be chaos-free, one wants

$$\mathcal{L}(n) = 0 \tag{2}$$

But by tracking a finite number of turns, one cannot tell whether this condition (2) is fulfilled to an infite accuracy. The accuracy improves of course with increasing n, and the suggested criterion is

$$\mathcal{L}(n) < \frac{1}{n} \ln(An) \tag{3}$$

where A is some parameter independent of n.

The next step of the suggestion is a bold one: it suggests that this parameter A is a universal constant, independent of the details of the nonlinear dynamics of the system under study. In particular, one can obtain its value by tracking any specific map, such as the Henon map, and once obtained, the *same* value can be used for the LHC. This was tested for the LHC, and it seemed to work!

Tune drift predictor

Another quantity that may serve as an early predictor is the instantaneous tune. In this case, one extracts the tune drift $\delta\nu$ as a function of n in a tracking study. Again, we want

$$\delta\nu(n) = 0 \tag{4}$$

but what we can impose for chaos-free condition is [4]

$$\delta\nu < \frac{B}{n} \tag{5}$$

where B is a constant parameter. The next step is to hypothesize that the value of B is universal. Obtaining its value by tracking the Henon map, the same value is used for the LHC, again with apparent success.

The $\frac{1}{\ln n}$ extrapolation

Another way is to extrapolate the dynamic aperture directly. By tracking, one obtains the dynamic aperture D as a function of n up to some modest value of n. An empirical law is then imposed to extrapolate it to give the value of $D(\infty)$ [5]:

$$D(n) = D(\infty)\left(1 + \frac{b}{\ln n}\right) \tag{6}$$

where b is some fitting parameter. This method has been applied to the LHC.

Long−term lifetime bound

In this method, [6] one first defines a quasi-invariant J, and calculates its maximum change δJ in n_0 turns in the phase space region of interest. Then in order for J to change by an amount ΔJ, it will have to take at least N turns, where

$$N = \frac{\Delta J}{\delta J} n_0 \tag{7}$$

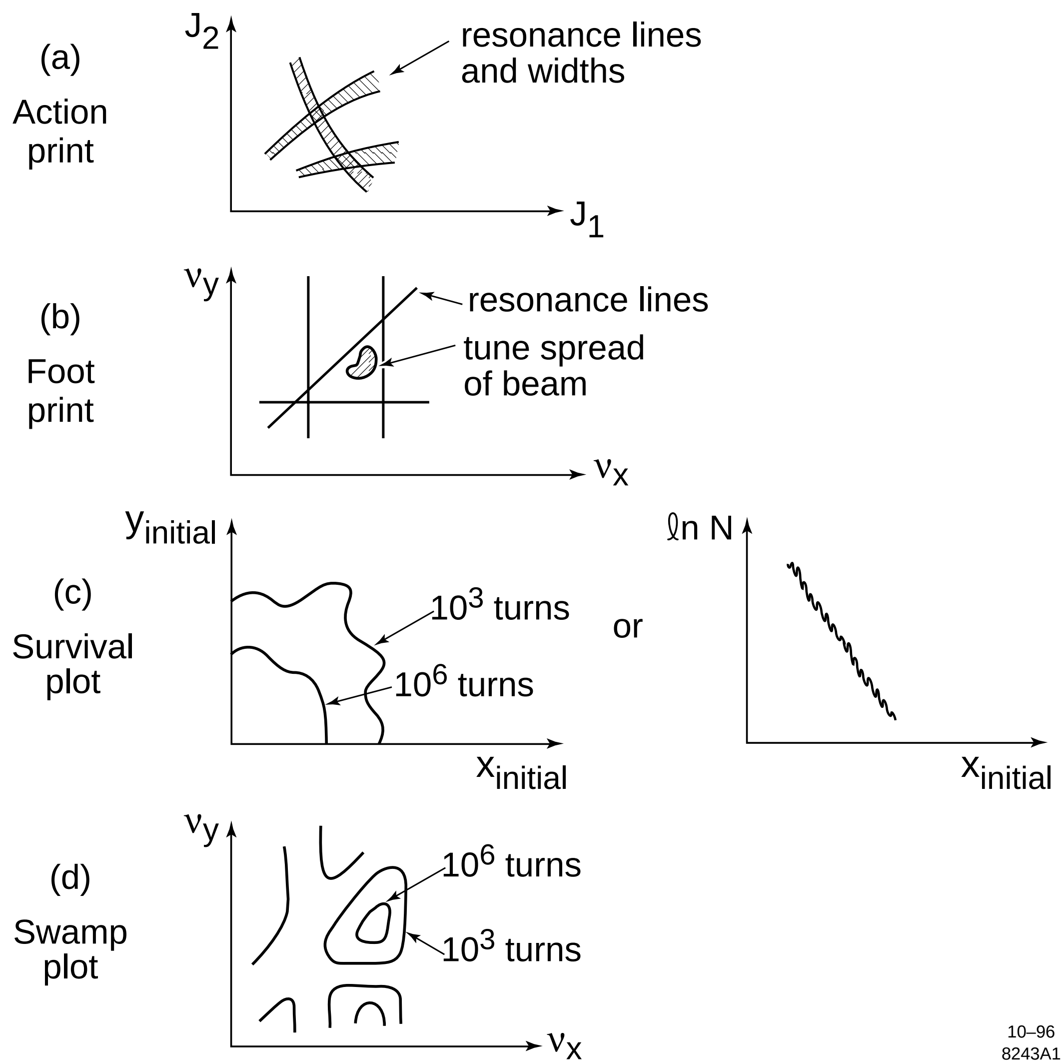
This information gives a particle lifetime bound.

This method is based on a rigorous derivation. In practical applications, one should note that (a) one should use a rather accurate expression for J, otherwise the lifetime bound obtained, although rigorously valid, would be too pessimistic, and (b) this method is most useful in the weakly chaotic region, perhaps up to 80% of the dynamic aperture.

Applied to the LHC, it was found that for a particle with $J_1 = 1.5 \times 10^{-7}$ m and $J_2 = 1.2 \times 10^{-7}$ m, and when the tunes are close to the (6,1) resonance, the particle lifetime is at least 10^7 turns. [6]

3. How are the Results Commonly Presented

Figure 1 is a sketch of the ways results are often presented in recent nonlinear dynamics studies. Figure 1(a) to (d) are referred to as action print [7], footprint, survival plot, and swamp plot [2], respectively. For Fig.1(c), we show two variations of the survival plot, one for a 2-D system, the other for a 1-D system. One application of the 1-D variation was mentioned in Eq.(6). Most likely, Figs.1(a) and (b) are used to exhibit some analytical calculation, while Figs.1(c) and (d) are used to show tracking results.



Various presentations of nonlinear dynamics effects.

4. Do We Understand the Mechanism of the Dynamic Aperture

We have made great progress in developing the tools to confidently calculate many important nonlinear dynamics quantities, but unfortunately the quantity which we started out to study, the dynamic aperture, is not one of them. In his plenary talk, Oide surveyed three mechanisms for a particle to escape from confinement: overlapping resonances, Arnold diffusion, and modulated diffusion (riple). [8] On the other hand, although much progress has been made to identify the possible mechanisms for the dynamic aperture, exactly which mechanism applies under which conditions is a very difficult question and remains largely unanswered. “We are talking about sophisticated things but our knowledge is very limited.” [9]

To be more specific, in tracking studies, we routinely produce the short-term dynamic aperture, but the long-term dynamic aperture is still not readily obtained. A brute force tracking for one study case of the LHC, for example, takes 24 hrs. Tracking with one-turn nonlinear maps is being studied, and is not without uncertainties.

We have studied nonlinear dynamics by accelerator experiments. The results are extremely useful – for example, we learned that ripple effects are of critical importance in real accelerators (more on this later) – but again not all experimental results have been fully understood.

An intriguing question was raised by Oide during the workshop: When we optimize the operation of an accelerator using tracking studies as guidance, we often optimize the *short-term* dynamic aperture. To optimize is the *long-term* dynamic aperture, the question arises whether the short-term and the long-term dynamic apertures are in fact correlated.

The basic idea of early predictor such as that of Eq.(6) assumes 100% correlation betweent he short-term and long-term dynamic apertures. However, it is also conceivable that, under some circumstances, the short-term and long-term dynamic apertures do not even have the same physical mechanism, thus can not be 100% correlated. In such cases, the effort to optimize the short-term dynamic aperture would be barking at the wrong tree!

A discussion within the working group generated a tentative consensus: there seems to be some evidence [10] that the long-term dynamic aperture

(10^7 turns) is indeed closely correlated to the medium-term dynamic aperture (10^5 turns), but is sometimes poorly correlated to the short-term dynamic aperture (10^3 turns). (Obviously such an observation is of phenomenological value and can not be rigorous.)

A more general question raised by Ritson is in a similar spirit: We often optimize the accelerator design with simplified accelerator models (for example, 2-D, 4-D instead of 6-D). This leads to the question whether we are optimizing using the relevant criteria.

5. Ripple Effect

As mentioned earlier, one of the key results learned recently when confronting theory with experiment has been that ripple effects are critically important to determine the dynamic aperture accurately. One set of experiments that demonstrates this very convincingly was that performed at HERA. [11] By compensating the detected ripples in the horizontal and vertical tunes, the observed beam loss rate can be noticeably reduced.

The importance of ripple was first pointed out by tracking studies and a recent study for the LHC was presented to the group. [12] The experience accumulated up to now by comparing the tracking results (tracking with ripples included) and the experimental observations in the SPS and HERA indicates that (a) ripples must be included if one wants to determine the dynamic aperture in an operating storage ring, and (b) the predicted dynamic aperture taking into account of ripples is still 20% larger than that observed.

It is not yet understood why there is a stubborn 20% discrepancy (not to mention why it seems always to have the undesirable sign). Furthermore, when comparing tracking with experiment, there are significant beam dynamics details which are not understood. In particular, Schmidt pointed out that, in one of the comparisons made on the SPS, the tracking results clearly indicated a significant dependence of the dynamic aperture on the tune ripple frequency (between 40 Hz and 180 Hz), while experimentally such a dependence was not observed.

Attempts to understand the ripple effect better must be continued. One such attempt indicates there are three operating regimes: [13]

(a) When the ripple is slow and amplitude of ripple is small, the phase space distorts adiabatically. Particle motion is basically bounded.

(b) When the riple amplitude is increased, phase space islands move in such a way to transport particles from the beam core to other resonance islands with larger amplitudes. Particles get lost by a slow diffusion process.

(c) When the ripple amplitude increases further, the islands transport particles all the way to the separatrix, particles get lost by a rapid diffusion.

Yan pointed out that synchro-betatron resonances can be viewed as a ripple effect. A practical question followed as to whether it would be better to have a large synchrotron tune ν_s or a small one. In the language of the tune footprint [see Fig.1(b)], large ν_s can be viewed as adding new resonance lines, while small ν_s can be viewed as the widening of each of the existing resonance lines. There was a consensus among the group that small ν_s should be more preferable, except that nonlinear dynamics are not the only criteria to determine the choice of ν_s. Other effects to be considered include engineering considerations [14] and the collective effects [15].

6. Beam-beam Effect

Although not a topic systematically covered in the group, the beam-beam effect inevitably came up on several occassions during the presentations and discussions. Biagini reported on an empirical formula of Bassetti's for the maximum vertical beam-beam tune shift limit ξ_y for 14 e^+e^- colliders. [16] The accuracy of this empirical formula is quite intriguing.

Also, there was an interesting work on the study of synchrotron and synchro-betatron resonances driven by the beam-beam forces in a monochromatized collider. [17] It was found, for example, that the beam-beam induced tune shifts are

$$\Delta\nu_y = \xi_y, \qquad \Delta\nu_s = -\xi_y \frac{\eta_y^2}{\beta_y \beta_s} \tag{8}$$

where η_y, β_y, β_s are the dispersion, vertical β-function, and longitudinal β-function at the interaction point. To avoid the synchrotron resonance, one could consider a lattice with negative momentum compaction.

7. Other Topics

There are several very interesting and important topics presented to the working group but are not covered in this summary:

Isochronous ring nonlinear dynamics [18]
How to detect the source of nonlinearity in a beam line [19]
Sorting of magnets [20]
Fokker-Planck description of nonlinear dyanmics [21, 22, 23]
Residue symplectification to reduce chaos [24]

8. Summary

1. We have developed a power arsenal of modern analysis tools, which accurately predict a large number of beam dynamics quantities.

2. However, the dynamic aperture – the quantity we are after – remains elusive. We need to pinpoint its mechanism. We need to predict it more accurately and with more confidence.

3. Ripple effects are critical. More and more studies are emphasizing the ripple effects out of necessity. Important details are still not understood when confronted with experiments. Tools mentioned in item 1 above need to pay more attention to ripples.

Acknowledgments

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[7] E. Todesco, application to the LHC, presentation to the working group.

[8] K. Oide, plenary talk, this conference.

[9] Direct quote from K. Hirata, this conference.

[10] F. Schmidt, presentation to the working group.

[11] O. Bruening, presentation to the working group.

[12] W. Fischer and F. Schmidt, presentation to the working group.

[13] A. Bazzani, presentation to the working group.

[14] D. Ritson, comments in the working group.

[15] S. Berg, comments in the working group.

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[17] K. Hirata, S. Petracca, presentation to the working group.

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[19] Y. Yan, presentation to the working group.

[20] R. Bartolini, presentation to the working group.

[21] K. Hirata, presentation to the working group.

[22] G. Turchetti, presentation to the working group.

[23] J. Ellison, presentation to the working group.

[24] W. Wan, presentation to the working group.