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# Constraints on supersymmetric soft phases from renormalization group relations

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## Abstract

By using relations derived from renormalization group equations (RGEs), we find that strong indirect constraints can be placed on the top squark mixing phase in $A_t$ from the electric dipole moment of the neutron ($d_n$). Since $m_t$ is large, any GUT-scale phase in $A_t$ feeds into other weak scale phases through RGEs, which in turn contribute to $d_n$. Thus $CP$-violating effects due to a weak-scale $A_t$ are strongly constrained. We find that $|\mathrm{Im}A_t^{EW}|$ must be smaller than or of order $|\mathrm{Im}B^{EW}|$, making the electric dipole moment of the top quark unobservably small in most models. Quantitative estimates of the contributions to $d_n$ from $A_u$, $A_d$ and $B$ show that substantial fine-tuning is still required to satisfy the experimental bound on $d_n$. While the low energy phases of the $A$'s are not as strongly constrained as the phase of $B^{EW}$, we note that the phase of a universal $A^{GUT}$ induces large contributions in the phase of $B^{EW}$ through RGEs, and is thus still strongly constrained in most models with squark masses below a TeV.

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## I. INTRODUCTION

Supersymmetry (SUSY) [1] is one of the most compelling extensions of the Standard Model. It is the only known perturbative solution to the naturalness problem [2], it unifies the gauge coupling constants for the observed value of $\sin^2\theta_W$, it allows radiative EW symmetry breaking, and the lightest SUSY partner provides a good dark matter candidate. SUSY models with such features are generally in excellent agreement with experiment, and there is even the possibility that a recent CDF event [3] is of supersymmetric origin [4].

One of the few phenomenological problems associated with SUSY models is their generically large predictions for the electric dipole moment (EDM) of the neutron, $d_n$. Supersymmetric models with universal soft breaking parameters have two physical phases, beyond the CKM and strong phases of the SM, which can be taken to be the triscalar and biscalar soft breaking parameters $A$ and $B$. These phases give a large contribution to $d_n$, of order $10^{-22}(100\,\mathrm{GeV}/M_{susy})^2 e\,cm$, where $M_{susy}$ is a characteristic superpartner mass. The experimental upper bound on $d_n$ is of order $10^{-25} e\,cm$ [5], so that if superpartner masses are near the weak scale, the phases of these complex soft parameters must be fine-tuned to be less than or of order $10^{-2}$–$10^{-3}$ since there is no *a priori* reason for them to be small [6]. If one wants to avoid such a fine-tuning, there are two approaches: suppress $d_n$ with very large squark masses (greater than a TeV) [7], or construct models in which the new SUSY phases naturally vanish [8]. Models with very heavy squarks are unappealing because in such models LSP annihilation is usually suppressed enough so that the relic density is unacceptably large [9]. They also lead to a fine-tuning problem of their own in getting the $Z$ boson mass to come out right in EW symmetry breaking.

It is natural to consider solutions of the second type, and demand that the soft phases are zero by some symmetry. While that would leave only a small CKM contribution to $d_n$ [10–13], and thus avoid any fine-tuning in meeting the experimental bound on $d_n$, it would also mean that there is no non-SM $CP$ violation, which is needed by most schemes for electroweak baryogenesis [14]. Also, such models do not generate signals of non-SM $CP$ violation, such as those involving top squark mixing. There are ways of naturally obtaining small nonzero soft phases which leave sufficient $CP$ violation for baryogenesis [15–18], but these phases would still have to meet the bounds from $d_n$ and would probably be unobservably small in most EW processes—unless the soft terms are not universal.

Recently it has been pointed out that large non-SM $CP$-violating top quark couplings could be probed at high energy colliders [19]. A measurement of a large top quark EDM, for example, would indicate physics beyond the SM, and it is interesting to ask whether SUSY models can yield an observable effect. Several references have attempted to use $CP$ violation from top squark mixing due to the complex parameter $A_t$ to yield large $CP$-violating effects in collider processes involving top quarks [20]. Such papers either explicitly or implicitly assume nonuniversal soft couplings $A_q$ at the GUT scale; otherwise, the phase of $A_t$ would be trivially constrained by $d_n$. We consider whether it is possible to obtain large effects due to the phase of $A_t$ at the EW scale by relaxing the universality of $A$. We will show that due to renormalization group induced effects on other low energy phases, the phase of $A_t$ is strongly constrained by $d_n$, and it is not possible, for most areas of parameter space, to have large $CP$-violating effects due to the imaginary part of $A_t$.

We will assume that no parameters are fine-tuned and thus we will require the phases at the GUT scale to be either identically zero (presumably through some symmetry) or no less than 1/10. If one permits an arbitrary degree of fine-tuning, the whole SUSY $CP$ violation issue becomes moot, and one can derive no constraints on the phase of $A_t$. While one can construct models which give small universal phases, as we said above, the fine-tuning needed to evade the constraints we derive is unlikely to be explained naturally. Our approach in this paper is to assume the reasonable fine-tuning criterion we have just outlined, and ask what it implies about low energy SUSY $CP$-violating phenomenology.

In Sec. II, we review the basics of SUSY $CP$ violation. We present our results derived from RGEs in Sec. III, and impose the neutron EDM constraints on $\mathrm{Im}A_t$ using those results in Sec. IV. In Sec. V we discuss top squark mixing induced $CP$ violating observables in more detail in light of our constraints on the phase of $A_t$, and we give some concluding remarks in Sec. VI. The details from Sec. III are written up in Appendix A, and the full one-loop calculation for the SUSY contribution to the neutron EDM is given in Appendix B.

## II. SUSY $CP$-VIOLATING PHASES

The soft breaking potential in the MSSM is

$$
\begin{aligned}
-\mathcal{L}_{soft} = {} & \frac{1}{2}\left|m_i\right|^2\left|\varphi_i\right|^2 + \frac{1}{2}\sum_{\lambda} M_\lambda \lambda\lambda + \\
& \epsilon_{ij}[A_U \tilde{U}_R^* Y_U \tilde{Q}_L^i]H_u^j + \epsilon_{ij}[A_D \tilde{D}_R^* Y_D^\dagger \tilde{Q}_L^j]H_d^i + \epsilon_{ij}[A_E \tilde{E}_R^* Y_E^\dagger \tilde{L}_L^j]H_d^i + \quad (1) \\
& \epsilon_{ij} B\mu H_u^i H_d^j + h.c. \\
& \quad (2)
\end{aligned}
$$

where we take $A_U = \mathrm{diag}\{A_u, A_c, A_t\}$, $A_D = \mathrm{diag}\{A_d, A_s, A_b\}$, $A_E = \mathrm{diag}\{A_e, A_\mu, A_\tau\}$; $Y_U$, $Y_D$, and $Y_E$ are the Yukawa coupling matrices; $\tilde{Q}$, $\tilde{L}$, $\tilde{U}_R$, $\tilde{D}_R$ and $\tilde{E}_R$ are the squark and slepton fields; $\lambda$ are the gauginos and $\varphi_i$ are the scalars in the theory.

A common simplifying assumption is that this soft Lagrangian arises as the result of a GUT-scale supergravity (SUGRA) model with universal soft triscalar coupling $A$, gaugino mass $M_\lambda = M_{1/2}$, and scalar mass $m_i = m_0$. This provides an explanation for the absence of flavor changing neutral currents which arise from loops with squarks of nondegenerate mass [21]. Such supersymmetric models have only two independent physical $CP$-violating phases beyond the CKM and strong phases of the SM [10] although these phases appear in several different linear combinations in low energy phenomenology [17,22]. We will take the two physical phases to be Arg$A$ and Arg$B$.

It turns out that all $CP$ violating vertices in this model arise through the diagonalization of complex mass matrices [15]. The complex quantities which appear in these matrices are $A_q + \mu^* R_q$ and $\mu$, where $R_q$ is $\tan\beta$ (the ratio of Higgs vacuum expectation values) for $q = d,\ s,\ b$ and $\cot\beta$ for $q = u,\ c,\ t$, and where the phase of $\mu$ is simply equal to the phase of $B^*$ by a redefinition of fields. Thus for $d_n$, which involves only $u$ and $d$ quarks, there are only contributions from three low energy combinations of the two SUSY GUT phases: $\mathrm{Arg}(A_d - \mu\tan\beta)$, $\mathrm{Arg}(A_u - \mu\cot\beta)$, and Arg$\mu$. (In the Appendix B, a complete expression of $d_n$ is given which includes suppressed contributions from phases of the other squark mixings.)

Even with universal boundary conditions, the elements of the matrices $A_U$, $A_D$ and $A_E$ have distinct phases at the EW scale because of renormalization group evolution. We will also relax, in some places, the assumption that their phases started the same at the GUT scale. We assume (for simplicity) that these matrices are diagonal. One possible consequence of this approach is that one could have $d_n \simeq 0$ because Im$A_d$ and Im$A_u \simeq 0$, but other $A_q$, notably $A_t$, could have large phases which lead to observable effects. These include angular correlations and polarizations [20], including effects attributable to the electric dipole moment of the top quark, $d_t$. As discussed in the Introduction, this scenario is strongly constrained by RGE running.

## III. RENORMALIZATION GROUP FLOW OF COMPLEX SOFT TERMS

The goal of this section is to demonstrate how a large phase in $A_t$ can feed into other parameters in the theory through renormalization group running. The imaginary part of $A_t$ at the weak scale, $\bar{A}_t^{EW}$, is determined by running $\bar{A}_t^{GUT}$ (and for large $\tan\beta$, $\bar{A}_b^{GUT}$) down to the weak scale via the renormalization group equations (RGEs). (For compactness of notation, we will define $\bar{x} = \mathrm{Im}x$ in the following sections.) We will show that large $\bar{A}_t^{EW}$ induces potentially large values of $\bar{B}^{EW}$ and $\bar{A}_{u,d}^{EW}$, which give an unacceptably large neutron electric dipole moment.

Rather than write RGEs for the whole effective theory, we need only consider a complete subset of them which includes $A_q$ and $B$. The running of these soft terms depends upon the gaugino masses, the top and bottom Yukawas (we ignore tiny effects from the other Yukawa couplings) and the gauge coupling constants $\alpha_a = \lambda_a^2/4\pi$ ($a = 1,\ 2,\ 3$). We define $t = 1/4\pi \ln(Q/M_{GUT})$ and write

$$\frac{dM_a}{dt} = 2b_a\alpha_a M_a \tag{3}$$

$$\frac{dA_t}{dt} = 2c_a\alpha_a M_a + 12\alpha_t A_t + 2\alpha_b A_b \tag{4}$$

$$\frac{dA_{u,c}}{dt} = 2c_a\alpha_a M_a + 6\alpha_t A_t \tag{5}$$

$$\frac{dA_b}{dt} = 2c'_a\alpha_a M_a + 2\alpha_t A_t + 12\alpha_b A_b \tag{6}$$

$$\frac{dA_{d,s}}{dt} = 2c'_a\alpha_a M_a + 6\alpha_b A_b \tag{7}$$

$$\frac{dB}{dt} = 2c'''_a\alpha_a M_a + 6\alpha_t A_t + 6\alpha_b A_b \tag{8}$$

$$\frac{d\alpha_t}{dt} = 2\alpha_t\left(-c_a\alpha_a + 6\alpha_t + \alpha_b\right) \tag{9}$$

$$\frac{d\alpha_b}{dt} = 2\alpha_b\left(-c'_a\alpha_a + \alpha_t + 6\alpha_b\right) \tag{10}$$

$$\frac{d\alpha_a}{dt} = 2b_a\alpha_a^2 \tag{11}$$

where $a$ is summed from 1 to 3, and

$$b_a = \left(\frac{33}{5}, 1, -3\right), \tag{12}$$

$$c_a = \left(\frac{13}{15}, 3, \frac{16}{3}\right), \tag{13}$$

$$c'_a = \left(\frac{7}{15}, 3, \frac{16}{3}\right), \tag{14}$$

$$c'''_a = \left(\frac{3}{5}, 3, 0\right), \tag{15}$$

and the Yukawa coupling constants $\alpha_{t,b} = \lambda_{t,b}^2/4\pi$ are related to the masses by

$$\lambda_t = \frac{g_2}{\sqrt{2}}\frac{m_t}{m_W}\frac{1}{\sin\beta},\ \ \lambda_b = \frac{g_2}{\sqrt{2}}\frac{m_b}{m_W}\frac{1}{\cos\beta}. \tag{16}$$

We note that some references [23] list the $\alpha_t A_t$ coefficient in (5) as 2, but we have confidence that the coefficient is actually 6 [13,24]. Nevertheless our conclusions do not depend qualitatively on this coefficient.

We are mainly interested in the evolution of $\bar{A}_q$ and $\bar{B}$. We can set the phase of the common gaugino mass to zero at the GUT scale by a phase rotation and then $\bar{M}_i = 0$ at all scales. Therefore the RGE for the imaginary parts of the $A_q$ and $B$ can be written without the $M_a$ terms:

$$\frac{d\bar{A}_t}{dt} = 12\alpha_t \bar{A}_t + 2\alpha_b \bar{A}_b, \tag{17}$$

$$\frac{d\bar{A}_b}{dt} = 2\alpha_t \bar{A}_t + 12\alpha_b \bar{A}_b, \tag{18}$$

$$\frac{d\bar{A}_u}{dt} = 6\alpha_t \bar{A}_t, \tag{19}$$

$$\frac{d\bar{A}_d}{dt} = 6\alpha_b \bar{A}_b, \tag{20}$$

$$\frac{d\bar{B}}{dt} = 6\alpha_t \bar{A}_t + 6\alpha_b \bar{A}_b. \tag{21}$$

Using the above RGEs, we can derive the following useful relations:

$$\Delta\bar{B} = \Delta\bar{A}_{u,c} + \Delta\bar{A}_{d,s} = \frac{6}{14}\left(\Delta\bar{A}_t + \Delta\bar{A}_b\right), \tag{22}$$

$$\Delta\bar{A}_{u,c} = \frac{3}{35}\left(6\Delta\bar{A}_t - \Delta\bar{A}_b\right), \tag{23}$$

$$\Delta\bar{A}_{d,s} = \frac{3}{35}\left(6\Delta\bar{A}_b - \Delta\bar{A}_t\right), \tag{24}$$

where $\Delta\bar{B} = \bar{B}^{GUT} - \bar{B}^{EW}$, etc. For small $\tan\beta$, we can neglect $m_b$ so that these relations simplify to

$$\begin{aligned} \Delta\bar{B} &= \Delta\bar{A}_{u,c} = 3\Delta\bar{A}_b = \frac{1}{2}\Delta\bar{A}_t, \\ \Delta\bar{A}_{d,s} &= 0. \end{aligned} \tag{25}$$

Thus, given the GUT values, to obtain the low energy values for the imaginary parts of all the soft terms, one only needs to find $\bar{A}_t^{EW}$ and $\bar{A}_b^{EW}$, and for small $\tan\beta$, we only need the former.

In the small $\tan\beta$ limit ($\alpha_b \simeq 0$), we can use Eq. (17) to obtain the ratio of EW to GUT scale values of the imaginary part of $A_t$:

$$r_t \equiv \bar{A}_t^{EW}/\bar{A}_t^{GUT} = \exp\left[-\int_{t_{EW}}^{t_{GUT}} 12\alpha_t(t)dt\right]. \tag{26}$$

If the top quark were light, the integral in Eq. (26) would be small and $r_t$ would be close to one, but since the top quark is heavy, we find that $r_t$ is well below one. We can use the relations in Eq. (25) and the definition for $r_t$ in (26) to relate the low energy values for the imaginary parts of $A_t$ to $B$ and $A_u$ (for small $\tan\beta$):

$$\bar{A}_t^{EW} = \frac{-2r_t}{1-r_t}\left(\bar{B}^{EW} - \bar{B}^{GUT}\right) = \frac{-2r_t}{1-r_t}\left(\bar{A}_u^{EW} - \bar{A}_u^{GUT}\right). \tag{27}$$

We will make the simplifying assumption that $\bar{A}_u^{GUT}$ and $\bar{B}^{GUT}$ are zero. As we will see in the next section, this is reasonably well justified by our fine-tuning criterion, at least for the phase of $B$.

Next, we must find $r_t$. We obtain a pseudo-analytic solution to Eq. (26) in terms of EW and GUT scale quantities in Eq. (39) of Appendix A, but this is useful only if one has already obtained the GUT values for the $\alpha$'s by numerical integration of the RGEs. While we cannot find a truly analytic solution to Eq. (26), we can place an analytic upper bound on $r_t$ which is sufficient to make our point. We note that the integral in Eq. (26) is simply the area under the curve of the top Yukawa $\alpha_t$ as it runs from the EW scale to the GUT scale. Thus we can place an upper bound on $r_t$ simply by finding a lower bound to that area. In Appendix A, we do this by placing a lower bound on $\alpha_t(t)$ at each $t$, and we obtain

$$r_t \lesssim 1 - 12\alpha_t^{EW}/f_{EW}, \tag{28}$$

which is valid for small $\tan\beta$ so long as $12\alpha_t^{EW} < f_{EW}$. Here $f_{EW} \equiv 2c_a\alpha_a^{EW} \simeq 1.5 + 32/3(\alpha_s^{EW} - .12)$, so, for example, Eq. (28) is valid for $m_t = 175$ if $1.3 < \tan\beta \ll m_t/m_b$ (for smaller $\tan\beta$, $r_t$ gets closer to zero, but does not actually reach it). Thus we have placed an analytic bound on the running of $A_t$ completely in terms of EW quantities. For $\alpha_s(M_Z) = .12$, $\sin\beta \to 1$ (moderate $\tan\beta$) and $m_t = 175$ ($m_t = 160$), we find that $r_t < .43$ ($r_t < .52$), which, from Eq. (27), corresponds to $|\bar{A}_t^{EW}| < 1.5|\bar{B}^{EW}|$ ($|\bar{A}_t^{EW}| < 2.2|\bar{B}^{EW}|$). For small $\tan\beta$, the bound is even stronger, so that for $\tan\beta$ small enough to neglect $m_b$ effects, we obtain

$$|\bar{A}_t^{EW}| < 2.2 \min\left\{|\bar{B}^{EW}|, |\bar{A}_u^{EW}|\right\} \tag{29}$$

and in practice the coefficient is less than 2.

In Fig. 1, we plot $r_t$ ($= \bar{A}_t^{EW}/\bar{A}_t^{GUT}$) as a function of the top Yukawa coupling for different values of $\alpha_s(M_Z)$ in the limit where effects proportional to $m_b$ can be ignored. For $m_t > 160$ GeV, $\lambda_t$ is always greater than about 0.87 for all values of $\tan\beta$, which means that $r_t$ is always less than .45, in agreement with our analytic bounds. Also plotted are $-\bar{B}^{EW}/\bar{A}_t^{GUT} = (1 - r_t)/2$, and $-\bar{B}^{EW}/\bar{A}_t^{EW} = (1 - r_t)/2r_t$, which is greater than 1 (0.6) for $m_t = 175$ (160). Thus $|\bar{A}_t^{EW}| \lesssim |\bar{B}^{EW}|$, in agreement with our analytic results.

Next we consider moderate $\tan\beta$, where one must take into account the mixing of $\bar{A}_t$ and $\bar{A}_b$ but where $\tan\beta$ is not of order $m_t/m_b$. For $\bar{A}_b^{GUT}/\bar{A}_t^{GUT} > 0$, these effects lower $r_t$, and one can simply use the $m_b = 0$ upper bound on $r_t$ derived above.*

For $\bar{A}_b^{GUT}/\bar{A}_t^{GUT} < 0$ (recall that with universal $A$ this ratio would simply be $+1$), one simply maximizes the positive contribution to $r_t$ from $\bar{A}_b$ to obtain (see Appendix A)

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*There is a subtlety for the case of small positive $\bar{A}_b^{GUT}/\bar{A}_t^{GUT}$ for which there can be a net positive contribution to $r_t$ if $\bar{A}_b$ runs down below zero. However, the maximum effect on the bound is very small.

$$r_t < 1 - 12\alpha_t^{EW}/f_{EW} - \frac{1}{6}\left(\bar{A}_b^{GUT}/\bar{A}_t^{GUT}\right)\frac{\alpha_b^{EW}}{\alpha_t^{EW} - \alpha_b^{EW}}. \tag{30}$$

Note that the last term raises the upper bound on $r_t$, but the effect is small until $\tan\beta$ gets quite close to $m_t/m_b$. For $m_t = 175\,\mathrm{GeV}$, $\bar{A}_b^{GUT} = -\bar{A}_t^{GUT}$, and $\tan\beta = .7m_t/m_b \simeq 35$ (recall that we are evaluating all quantities at the EW scale, so $m_b$ is somewhat lower than the value at $q^2 = m_b^2$), we find the bound $r_t < 0.6$.

Effects due to $m_b$ are evident in Figs. 2-4, which show $\bar{A}_t^{EW}$, $\bar{B}^{EW}$, $\bar{A}_u^{EW}$ and $\bar{A}_d^{EW}$, normalized to $\bar{A}_t^{GUT}$, as a function of $\tan\beta$ for various GUT-scale boundary conditions. In Fig. 2, only the phase of $A_t^{GUT}$ is non-zero, while in Figs. 3 and 4, $\bar{A}_b^{GUT}$ has values of $+\bar{A}_t^{GUT}$ and $-\bar{A}_t^{GUT}$ respectively. In all cases, $r_t$ (the solid curve) remains below 0.35 and has its largest value just below $\tan\beta = m_t/m_b$ for $\bar{A}_b^{GUT}/\bar{A}_t^{GUT} < 0$ (Fig. 4), in agreement with our analytic results. This means that the EW value for the phase of $A_t$ is constrained to be less than about a third, *independent* of constraints from low energy $CP$ violating observables. The magnitude of the imaginary part induced into $\bar{B}^{EW}/\bar{A}_t^{GUT}$ by $\bar{A}_t$ is greater than 0.35 except for large $\tan\beta$ and $\bar{A}_b^{GUT}/\bar{A}_t^{GUT} < 0$. For $\tan\beta = m_t/m_b$ and $\bar{A}_b^{GUT}/\bar{A}_t^{GUT} < 0$, $\bar{B}^{EW}$ actually goes through zero, because $\Delta\bar{B}$ gets equal and opposite contributions from $\Delta\bar{A}_t$ and $\Delta\bar{A}_b$ there. At that point the "t" and "b" RGE coefficients are almost exactly the same at each $t$ (because the Yukawa coupling runnings differ only in a small U(1) coefficient), and the boundary conditions have opposite signs, so that $\bar{A}_t(t) \simeq -\bar{A}_b(t)$ for all $t$. Of course $\bar{A}_u^{EW}$ and $\bar{A}_d^{EW}$ are non-zero because they involve different linear combinations of $\Delta\bar{A}_t$ and $\Delta\bar{A}_b$, so there is still a strong constraint on $\bar{A}_t$ from $d_n$ there.

Finally we note that for large $\tan\beta$, one can place constraints on $\bar{A}_b^{GUT}$ as well, since it can then affect other low energy phases through renormalization group running. For $\tan\beta \sim m_t/m_b$, the constraints are of the same order as on $\bar{A}_t^{GUT}$, while for small $\tan\beta$, $\bar{A}_b^{GUT}$ is unconstrained (though the $\bar{A}_t$ contribution to $\bar{A}_b^{EW}$ for small $\tan\beta$ is constrained to be small and $\bar{A}_b^{EW}$ can be large only if $|\bar{A}_b^{GUT}| \gg |\bar{A}_t^{GUT}|$).

## IV. BOUNDS FROM THE NEUTRON EDM

Now that we have placed an upper bound on the magnitude of $\bar{A}_t$ in terms of $\bar{A}_u$, $\bar{A}_d$, and $\bar{B}$, we need to explore the constraints on the latter three imaginary parts (in low energy observables, we will drop the label EW). As we mentioned in the Introduction, one of the strongest constraints on $CP$ violating phases is the electric dipole moment (EDM) of the neutron, $d_n$. In Appendix B, we write expressions for the full supersymmetric contribution to $d_n$. One sees that all the pieces are proportional to $\bar{A}_u$, $\bar{A}_d$, or $\bar{\mu}$ (except for the negligibly small pieces proportional to $\bar{A}_q$). We can redefine the Higgs fields so that the phase of $\mu$ is just the opposite of the phase of $B$, and thus

$$\bar{\mu} = -\left|\frac{\mu}{B}\right| \bar{B} = \left|\frac{\mu}{B}\right| \left(\frac{1-r_t}{2r_t}\bar{A}_t - \bar{B}^{GUT}\right), \tag{31}$$

where the RHS follows for small $\tan\beta$.

In order to estimate the size of $\bar{\mu}$ we will need an estimate of $|\mu/B|$ in Eq. (31). We can find this ratio by considering the two equations which $\mu$ and $B$ need to satisfy to ensure that EW symmetry breaking occurs and that the $Z$ boson gets the right mass:

$$2B\mu = -(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\sin 2\beta, \tag{32}$$

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}. \tag{33}$$

In the limit that $\tan\beta \to \infty$, we see that the right hand side of Eq. (32) goes to zero, so $B \to 0$, whereas $\mu^2$ is not forced to zero. For $\tan\beta \to 1$, the right hand side of Eq. (33) blows up forcing $\mu$ to take on very large values. When $\mu^2$ dominates Eq. (32) and $\tan\beta = 1$ then we are led to a value of $|B| = |\mu|$. So in both the $\tan\beta \to \infty$ limit and the $\tan\beta \to 1$ limit we find that $|\mu| \geq |B|$. We have run thousands of models numerically [25] which include the one-loop corrections to Eqs. (32) and (33) and found that $|\mu| \gtrsim |B|$ is indeed a good relationship for most of the parameter space. As expected, it is violated most strongly for intermediate values of $\tan\beta$. For example, for $\tan\beta = 10$ we have found a small region of parameter space where $|\mu|/|B|$ is as low as 0.4, although most solutions prefer $|\mu|/|B| > 1$. We will assume that $|\mu|/|B| \gtrsim 1$, and thus the fine-tuning constraint on the phase of $B$ is even stronger than on what we obtain below for the phase of $\mu$.

From Appendix B, we see that $d_n$ can be written in terms of the three imaginary parts,

$$\frac{d_n}{10^{-25} e\, cm} = k_n^{A_u}\frac{\bar{A}_u}{m_0} + k_n^{A_d}\frac{\bar{A}_d}{m_0} + k_n^{\mu}\frac{\bar{\mu}}{m_0} = k_n^{A_u}\frac{\bar{A}_u}{m_0} + k_n^{A_d}\frac{\bar{A}_d}{m_0} - k_n^{\mu}\left|\frac{\mu}{B}\right|\frac{\bar{B}}{m_0}, \tag{34}$$

where we have normalized the RHS by the SUSY mass scale $m_0$, and the LHS by the region of the experimental bound so that the coefficients $k$ are dimensionless. We can rewrite the EW imaginary parts in Eq. (34) using Eq. (25) as

$$\frac{d_n}{10^{-25} e\, cm} = \frac{d_n^{GUT}}{10^{-25} e\, cm} + \frac{1-r_t}{2r_t}\left(-k_n^{A_u} + k_n^{\mu}\left|\frac{\mu}{B}\right|\right)\frac{\bar{A}_t}{m_0}, \tag{35}$$

where $d_n^{GUT}/10^{-25} e\, cm$ is just Eq. (34) with EW values of $\bar{A}_{u,d}$ and $\bar{B}$ replaced by GUT quantities. It vanishes if $\bar{A}_{u,d}^{GUT}$ and $\bar{B}^{GUT}$ are zero. In supergravity models, $|A^{GUT}|$ and $|B^{GUT}|$ are of order $m_0$, so that barring fine-tuned cancellations, the GUT scale phases must be less than order $1/k_n$. If the $k$'s are greater than order 10, then our fine-tuning criterion dictates that we set the GUT phases to zero (presumably protected by some symmetry). Thus we need an estimate of the $k_n's$.

In Figs. 5a, 5b, and 5c, we plot the values for $k_n^{A_u}$, $k_n^{A_d}$, and $k_n^{\mu}$ respectively in many different models as a function of squark mass, and as a function of $\tan\beta$ in Figs. 5d, 5e, and 5f. We see that $k_n^{A_u}$ and $k_n^{A_d}$ are fairly flat functions of $\tan\beta$, whereas $-k_n^{\mu}$ increases with $\tan\beta$ due to the $\mu\tan\beta$ terms in the expression for $d_n$. We also see that most models give $k_n^{A_u} > 2$ (0.8), $k_n^{A_u} > 7$ (3), and $|k_n^{\mu}| > 100(40)$, for squark masses below 500 GeV (1 TeV), so that order one phases in all the SUSY complex quantities usually give a neutron EDM which is of order 100 (40) times the experimental bound. We note that these are substantially larger contributions (and thus stronger constraints) than claimed by the recent work of Falk and Olive [26], though this is probably due to the fact that they use very heavy squark masses in an effort to find the smallest fine-tuning of phases consistent with cosmology. While one can argue whether or not the bounds on the phases of $A_{u,d}$ represent a fine-tuning, the bound on the phase of $\mu$ (and thus $\bar{B}^{EW}$, which comes from $\bar{B}^{GUT}$ and $\bar{A}_t^{GUT}$) certainly does. Thus, by our fine-tuning criterion, the phases of $B^{GUT}$ and $A_t^{GUT}$ should be zero. We note that in the case of *universal* $A$ it is *irrelevant* whether or not the low energy phases of $A_u$ and $A_d$ are strongly constrained, since the phase of the universal $A^{GUT}$ makes a large contribution to the low energy value of $\bar{\mu}$ (since $\bar{A}_t^{GUT} = \bar{A}^{GUT}$).

To give an idea of what level of neutron EDM one expects with different initial assumptions, we plot in Fig. 6 $d_n/10^{-25}e$ cm with universal $|A^{GUT}|$ for three cases: (a,d) $\mathrm{Arg}A_t^{GUT} = \mathrm{Arg}A_b^{GUT} = 0.1$ and all other phases zero, (b,e) $\mathrm{Arg}A_t^{GUT} = -\mathrm{Arg}A_b^{GUT} = 0.1$ and all other phases zero, and (c,f) universal phases $\mathrm{Arg}A^{GUT} = \mathrm{Arg}B^{GUT} = 0.1$. As one can see, even with phases of order 0.1, most models have an absolute value for $d_n/10^{-25}e\,cm$ greater than one, inconsistent with the experimental bounds.

As can be gathered by the spread of points in the scatter plots and the number of parameters involved, the results depend on one's model assumptions. For example, if one requires $\tan\beta$ to be small (say because of $b$-$\tau$ unification), and the squarks are allowed to be very heavy, then there is very little fine-tuning needed for the current experimental bound on $d_n$. On the other hand, if SUSY is detected at LEP 2 or TeV 33, then even the smallest $\tan\beta$ models would require fine-tuning.

In minimal supergravity models the natural scale for the $A$ terms is $m_0$. In Fig. 7 we have plotted $d_n/10^{-25}e{\cdot}$cm versus $ImA_t^{EW}/m_0$ to succinctly demonstrate how quickly the EDM rises when $ImA_t^{EW} \neq 0$. To construct this plot we chose a random phase for $A_t$ at the GUT scale, forced all other phases equal to zero at that scale, and then ran all the parameters down to the weak scale. A sharp drop in $d_n$ occurs at $ImA_t^{EW}/m_0 \simeq 0$ because $\mathrm{Im}A_t^{GUT}$ can be small there and thus induces only small phases into the other low energy soft parameters. Models with $d_n$ around $10^{-25}e{\cdot}$cm at $ImA_t^{EW}/m_0 \simeq 0$ occur for low $\tan\beta$ where $\mathrm{Im}A_t^{GUT} \gg \mathrm{Im}A_t^{EW}$ but where $d_n$ otherwise tends to be smaller. This means that most models with $d_n$ below the experimental bound in Fig. 6 also have a small EW value for $\mathrm{Im}A_t$, and thus from Fig. 7 we can place a stronger constraint on $\mathrm{Im}A_t^{EW}$ than we obtained on $\mathrm{Im}A_t^{GUT}$: $ImA_t^{EW}/m_0 \lesssim 1/20$.

Thus we conclude that models with universal GUT-scale phases of the soft parameters, and models in which only $A_t^{GUT}$ has a non-zero phase, have difficulty meeting the bounds from $d_n$ and our fine-tuning criterion. Models with non-zero $\bar{A}_c^{GUT}$, $\bar{A}_s^{GUT}$, or $\bar{A}_l^{GUT}$ can meet the constraint from $d_n$ without fine-tuning, as can those with non-zero $\bar{A}_b^{GUT}$ for small $\tan\beta$. For the remainder of the paper, we will for simplicity set all the GUT-scale phases to zero except for that of $A_t$. Even though our fine-tuning criterion implies that $\bar{A}_t^{GUT}$ should be zero, we find it useful to ask what effects one would have if one allows that fine-tuning.

## V. THE TOP QUARK EDM

Now that the top quark has finally been discovered, one can envision some nice experiments which measure properties of this known particle. Future colliders, such as the NLC, can provide many precision measurements of the production cross-section and decay properties of the top quark. It is possible that signatures of new physics could arise out of such a study. One property of the top quark which has received much attention [19] is the possibility of measuring its EDM by looking at the decay distributions of the $t\bar{t}$ pairs. (Other CP-violating observables are possible, such as those arising from $t \to bW$ decays, but we will make our point only with the top EDM.) It is generally estimated that the top quark EDM ($d_t$) can be measured to values as low as $\mathcal{O}(10^{-18})\, e\,\mathrm{cm}$ [19]. Given the constraints which we derived above, we ask if the minimal supersymmetric standard model can yield a value for $d_t$ this large.

In the context of supersymmetry, it has been proposed [20] that a large $d_t$ is possible if the phase of $A_t^{EW}$ is of order one. But in Sec. III, we showed that $\bar{A}_t^{EW}$ is constrained to be smaller than or of order the phases which contribute to $d_n$. The EDM of the top is thus constrained to be less than a constant times the neutron EDM:

$$\frac{d_t}{d_n} \lesssim \xi \frac{m_t}{m_d} \frac{\mathrm{det} M_{\tilde{q}}^2}{\mathrm{det} M_{\tilde{t}}^2}, \tag{36}$$

where $\mathrm{det} M_{\tilde{q}}^2 = m_{\tilde{q}_1}^2 m_{\tilde{q}_2}^2$ is the determinant of the (down) squark mass-squared matrix, and the value of $\xi$ depends upon many different SUSY parameters, but is generically of order 1. Normalizing $d_n$ to the experimental bound, we see that

$$d_t \lesssim \xi \frac{\mathrm{det} M_{\tilde{q}}^2}{\mathrm{det} M_{\tilde{t}}^2} \frac{d_n^{expt}}{10^{-25}\, e\,\mathrm{cm}}\, 2 \times 10^{-21}\, e\,\mathrm{cm}. \tag{37}$$

In addition to this constraint, we recall that the phase of $A_t$ at the EW scale must be less than about 1/3, just from the RGE suppression factor $r_t$. Thus, as long as $\det M_{\tilde{d}} \simeq \det M_{\tilde{t}}$, we expect $d_t$ to fall about three orders of magnitude below detectability at proposed future high energy colliders.

We can turn this analysis around. If a large top quark EDM is discovered, can it be explained in the MSSM? One possibility is that a conspiracy occurs between *several* large phases in the theory to render $d_n$ below experimental limits, yet produce a $d_t$ detectable at high energy colliders. This is equivalent to saying that all the $\mathcal{O}(1)$ coefficients which we absorbed into the parameter $\xi$ in Eq. (37) actually conspire to give $\xi \gtrsim 10^3$. As we argued in the Introduction, we would not view this as a likely explanation.

Another possibility to consider is that the top squarks are much lighter than the other squarks. For $d_t$ to be observable, we would need the determinants in Eq. (36) to have a ratio $\gtrsim 10^3$. This is possible, but it too would require some fine-tuning. The large top-quark-induced running of the $\tilde{t}_R$ goes in the right direction—the lightest top squark mass eigenvalue tends to be smaller than the other quarks. However, $\tilde{t}_2$ generally tracks fairly well with the other squarks, $\tilde{q}_L$, and thus, we estimate that

$$\frac{\det M^2_{\tilde{d}}}{\det M^2_{\tilde{t}}} \lesssim \frac{m^2_{\tilde{d}}}{m^2_{\tilde{t}_1}}, \tag{38}$$

which means that we would need $m_{\tilde{t}_1} \lesssim m_{\tilde{d}}/\sqrt{1000}$ to yield an observable $d_t$. If experiment determines that $m_{\tilde{t}_1} > 80\,\mathrm{GeV}$ then this condition would imply that the superpartners of the light quarks are above $2.5\,\mathrm{TeV}$. This is essentially the heavy squark "solution" to the $CP$ violation problem we mentioned in the Introduction, with an additional fine-tuning implied by the small ratio $m_{\tilde{t}_1}/m_{\tilde{d}}$.

Finally, one could appeal to differences between $d_t$ and $d_d$ due to effects proportional to $m_t^2/v^2$, which are negligible in $d_d$. To achieve $\xi$ of order $10^3$, one again needs a fine-tuned conspiracy of couplings.

Thus we conclude that if a large $d_t$ were found, one would probably have to look beyond the MSSM for an explanation.

## VI. CONCLUDING REMARKS

It has long been noted that the phases of soft supersymmetric parameters generically lead to an unacceptably large neutron EDM. This fine-tuning problem has slowly become less vexing as the theoretical expectations for the squark masses have risen faster than the experimental bound on the neutron EDM has fallen. Nevertheless, for squark masses below about a TeV, we showed in Sec. V that the phase of $B$ and universal phase of $A$ do not meet the fine-tuning criterion set forth in the Introduction (see Fig 6c). Certainly, if supersymmetry is discovered at LEP 2 or TeV 33, a fundamental explanation for the absence of a neutron EDM would be needed, and any scheme for baryogenesis at the EW scale would require that mechanism to leave small effective low energy phases in the soft terms [15–18].

From the phenomenological point of view, it is tempting to postulate that the soft phases are not universal—that the EW phase of $A_t$ is large, while the other phases which directly contribute to the neutron EDM are small. This would allow interesting signatures of supersymmetric $CP$ violation to be visible in top quark physics at future colliders. But we have demonstrated by using the renormalization group equations that the imaginary part of $A_t$ must be less than twice the imaginary part of $B$, and $A_t$-induced CP-violating observables such as the top EDM are thus expected to be unobservably small in almost all minimal SUSY models.

These constraints are particularly important for models of EW baryogenesis which rely upon the phase of the stop LR mixing parameter, $A_t - \mu \tan\beta$, to generate enough $CP$ violation for baryogenesis. Such models must also have sufficiently small $|A_t - \mu \tan\beta|$ to ensure that the phase transition is first order [27]. There has also been a recent attempt to explain the observed $CP$ violation in the neutral kaon system with zero CKM phase and non-zero off-diagonal phases in the general $A$ matrices [28]. If the universal diagonal $A$ parameter has a large phase at the GUT scale, it will, as we noted above, give a large contribution to $d_n$ through a renormalization group induced phase in $\mu$, as well as from a direct contribution. One could evade such bounds by insisting that the off-diagonal components of the $A$ matrices have a large phase, while the phases of the diagonal $A$'s and of $B$ vanish. Although this hypothesis can probably be technically consistent with our fine-tuning criterion (phases either zero or large), this scenario strikes us as unnatural.

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## VII. APPENDIX A

In this appendix, we provide the details related to our analytic results of Sec. III. It is interesting to note that we can use the RGEs for the top Yukawa and gauge coupling constants in Eqs. (9) and (11) to write a pseudo-analytic solution to $r_t = \mathrm{Im}A_t^{EW}/\mathrm{Im}A_t^{GUT}$. The integral in Eq. (26) can be rewritten as $\ln(\alpha_t^{EW}/\alpha_t^{GUT}) - \sum_a (c_a/b_a)\ln(\alpha_a^{GUT}/\alpha_a^{EW})$, which allows us to write a pseudo-analytic $r_t$ in terms of EW and GUT scale quantities (the latter of which cannot be found analytically):

$$r_t = \frac{\alpha_t^{EW}}{\alpha_t^{GUT}} \Pi_{a=1}^{3} \left( \frac{\alpha_a^{EW}}{\alpha_a^{GUT}} \right)^{c_a/b_a} . \tag{39}$$

To place an analytic upper bound on $r_t$, we must place a lower bound on the area $\int_{t_{EW}}^{t_{GUT}} 12\alpha_t(t)dt$. We will need the $\alpha_b = 0$ limit of the running of the top Yukawa coupling in (9),

$$\frac{d\alpha_t}{dt} = -f(t)\alpha_t + 12\alpha_t^2, \tag{40}$$

where $f(t) = 2c_a\alpha_a$. While this cannot be solved analytically, we note that $\alpha_3(t)$ runs down with energy and one can show that $f(t)$ will be at its maximum value at the EW scale. Thus if we take $f(t)$ to the constant $f_{EW}$, we will minimize the running of $\alpha_t$, and Eq (40) can be solved analytically to yield the bound

$$\alpha_t(t) > \left( \frac{12}{f_{EW}} + \left( \frac{1}{\alpha_t^{EW}} - \frac{12}{f_{EW}} \right) e^{f_{EW}(t-t_{EW})} \right)^{-1} , \tag{41}$$

which is valid for $12\alpha_t^{EW} < f_{EW}$ (larger $\alpha_t^{EW}$ allows the bound on $\alpha_t(t)$ to reach infinity for $t < t_{GUT}$ and thus makes the bound useless), which corresponds to $\tan\beta > 1.3$ for $m_t = 175$. If we replace $\alpha_t(t)$ in the integral above by the RHS of Eq. (41), we can find an analytic solution for the lower bound on the area which, for the relevant range of $f_{EW}$ and $t_{EW}$, can be approximated by

$$-f_{EW}t_{EW} - \ln\left[ \frac{12\alpha_t^{EW}}{f_{EW}} + \left( 1 - \frac{12\alpha_t^{EW}}{f_{EW}} \right) e^{-f_{EW}t_{EW}} \right] \simeq -\ln\left[ 1 - \frac{12\alpha_t^{EW}}{f_{EW}} \right], \tag{42}$$

which yields Eq. (28) directly.

For moderate $\tan\beta$, we need to include $m_b$ effects which mix $\bar{A}_t$ with $\bar{A}_b$, and $\alpha_t$ with $\alpha_b$. The coupled differential equations (17) and (18) can be solved analytically only if the coefficients, which here are proportional to $\alpha_t$ and $\alpha_b$, are constants. To obtain bounds on the running of $\bar{A}_t$ and $\bar{A}_b$, we can break up the range of energy from $t_{EW}$ to $t_{GUT}$ into small regions where the coefficients are effectively constant, and iteratively evolve from the GUT scale down to the weak scale. At each energy $t_j$, the value for $\bar{A}_t$ is given by

$$\begin{aligned} \bar{A}_t\,(t_{j+1}) \simeq{} & \bar{A}_t(t_j) \exp\left(-12\alpha_t(t_j)\delta t\right) \\ & -\frac{1}{6}\bar{A}_b(t_j) \left\{ \frac{\alpha_b(t_j)}{\alpha_t(t_j) - \alpha_b(t_j)} \left[\exp\left(-12\alpha_b(t_j)\delta t\right) - \exp\left(-12\alpha_t(t_j)\delta t\right)\right] \right\}, \end{aligned} \tag{43}$$

provided that $T \equiv \alpha_b/(\alpha_t - \alpha_b)$ is not large. Here $\delta t = t_j - t_{j+1}$, which is positive. Iterating Eq. (43) gives a complicated expression with terms proportional to each of the $T(t_j)$'s. However, each of these terms is positive, so that taking $T(t_j)$ to its maximum value maximizes the size of the quantity in { }'s in Eq. (43), which is what we need for the case

$\bar{A}_b^{GUT}/\bar{A}_t^{GUT} < 0$. Once we take $T(t_j) \to T_{max}$, many terms cancel, and we are left with (taking $\delta t \to 0$) the upper limit

$$\begin{aligned}\bar{A}_t^{EW} < {} & \bar{A}_t^{GUT} \exp\left(-\int_{t_{EW}}^{t^{GUT}} 12\alpha_t(t)dt\right) \\ & -\frac{1}{6}\bar{A}_b^{GUT}\left(\frac{\alpha_b}{\alpha_t-\alpha_b}\right)_{max}\left[\exp\left(-\int_{t_{EW}}^{t^{GUT}} 12\alpha_b(t)dt\right) - \exp\left(-\int_{t_{EW}}^{t^{GUT}} 12\alpha_t(t)dt\right)\right]. \end{aligned} \tag{44}$$

One can show analytically that $T(t)$ reaches its maximum value at the lowest energy of the range, and thus we can replace $T_{max}$ by $\alpha_b^{EW}/(\alpha_t^{EW} - \alpha_b^{EW})$. To obtain a simpler bound, one can reduce the [ ]'s in Eq. (44) to 1 by taking a lower bound on $\alpha_b(t)$ to be zero and an upper bound on $\alpha_t(t)$ to be infinity. Finally one uses the $m_b \simeq 0$ bound on $r_t$ obtained in Eq. (28) for the first term in Eq. (44) to obtain the upper bound on $r_t$ in Eq. (30).

## VIII. APPENDIX B

In this Appendix we present analytic expressions for the full one-loop SUSY contribution to the neutron electric dipole moment, $d_n$. The gluino [11] and chargino [7] contributions appear in the literature. While an expression for the neutralino contribution is given by Kizukuri and Oshimo [7], it is written in terms of $4 \times 4$ complex unitary matrices which must be determined numerically. Below we give an expression for this neutralino contribution solely in terms of the mass matrices (and other MSSM parameters), and a useful approximation to that expression, which do not require calculating complex unitary matrices.

To find the neutron EDM, we first calculate the EDM of the up and down quarks ($d_q$) from one loop diagrams with photons attached to either ($a$) an internal boson or ($b$) an internal fermion line. Then the neutron EDM is related to the quark EDM's in the Naive Quark Model by $d_n = (4d_d - d_u)/3$, though recent work has argued that this expression overestimates $d_n$ if the strange quark carries a large fraction of the neutron and proton spin [29]. The Feynman integrals associated with ($a$) and ($b$) are [30]:

$$I^a(x) = \frac{1}{(1-x)^2}\left[-\frac{3}{2} + \frac{x}{2} - \frac{\ln x}{1-x}\right],\ I^b(x) = \frac{1}{(1-x)^2}\left[\frac{1}{2} + \frac{x}{2} + \frac{x\ln x}{1-x}\right]. \tag{45}$$

As we mentioned earlier, all SUSY $CP$ violating effects arise from diagonalizing complex mass matrices [15]. Gluino loops contribute to the quark EDM $d_q$ through the complex phase in the left-right mixing elements for up and down squarks:

$$d_q(\tilde{g}) = \frac{-2}{3\pi}Q_q e\alpha_s \frac{m_q m_{\tilde{g}}\mathrm{Im}(A_q - \mu R_q)}{m_{\tilde{q}_0}^4} I^b\Big(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}_0}^2}\Big). \tag{46}$$

We have averaged over the nearly degenerate squark mass eigenstates for simplicity: $m^2_{\tilde{q}_0} = m_{\tilde{q}_1} m_{\tilde{q}_2}$ and $I(x_0) = (I(x_1) + I(x_2))/2$. Here $Q_q e$ is the charge of quark $q$, $R_q = \tan\beta$ ($\cot\beta$) for $q = d$ ($u$), and $m_{\tilde{g}}$ is the gluino mass. (Note that we use Im$z$ for the imaginary part of $z$ in this appendix because it is clearer than $\bar{z}$ in more complicated expressions.)

The chargino contribution is proportional to the imaginary part of products of elements of the matrices $U$ and $V$ which diagonalize the chargino mass matrix. It turns out that one can write those products directly in terms of the elements of the chargino mass matrix, so that the chargino contribution to $d_q$ can be written

$$d_q(\tilde{\chi}^+) = \frac{e}{(4\pi)^2}\Bigg\{ g^2 R_q \frac{m_q m_{\tilde{W}} \mathrm{Im}\mu}{m^4_{\tilde{q}'_0}} \frac{[\omega I^a(y_1) + Q_{q'} I^b(y_1)] - [\omega I^a(y_2) + Q_{q'} I^b(y_2)]}{y_1 - y_2} + \frac{m_q \mathrm{Re}\mu}{\sin\beta\cos\beta}[\omega I^a(y_0) + Q_{q'} I^b(y_0)] \sum_{r=q'_i}^{dsb \text{ or } uct} \frac{\mathrm{Im}(A_r - \mu R_{q'})}{m^4_{\tilde{r}_0}} \frac{m_r^2 |V_{qr}|^2}{v^2} \Bigg\}, \tag{47}$$

where $m_{\tilde{W}}$ is the wino mass, $\omega = +1$ ($-1$) for $q = d$ ($u$), $y_1 = m^2_{\tilde{\chi}1}/m^2_{\tilde{q}'_0}$, and $I(y_0) = (I(y_1) + I(y_2))/2$. The primed quantities refer to the SU(2) partner, so if $q = d$, then $q' = u$ and $r$ is summed over the set $\{u, c, t\}$. Previous expressions for $d_q(\tilde{\chi}^+)$ have neglected the squark mixing piece, which is the second term in Eq. (47). This piece is suppressed relative to the other contributions by $m_r^2 |V_{qr}|^2/v^2$, which is less than $10^{-4}$ for $q = u$ or $d$ (but it can affect the EDM's of other quarks), though it is interesting that there is a (tiny) *direct* contribution to $d_n$ from Im$A_t$.

The neutralino contribution,

$$d_q(\tilde{\chi}^0) = \frac{-Q_q e}{(4\pi)^2} \frac{1}{m^2_{\tilde{q}_0}} \frac{m_q}{v_q} \Bigg\{ \sum_{h=1}^{1,2} (a^q_{Lh} - a^q_{Rh}) \mathrm{Im}\Phi_{h\hat{q}} + \frac{\mathrm{Re}(A_q + \mu R_q) v_q}{m^2_{\tilde{q}_0}} \sum_{h,l}^{1,2} a^q_{Lh} a^q_{Rl} \mathrm{Im}\Phi_{hl} - \frac{\mathrm{Im}(A_q - \mu R_q) v_q}{m^2_{\tilde{q}_0}} \sum_{h,l}^{1,2} a^q_{Lh} a^q_{Rl} \mathrm{Re}\Phi_{hl} \Bigg\}, \tag{48}$$

arises from the 4x4 complex neutralino mass matrix. The index $\hat{q} = 3$ (4) for $q = d$ ($u$). Recall [31] that the "1" and "2" weak eigenstates are gauginos, and the "3" and "4" weak eigenstates are higgsinos which couple to down and up quarks respectively. Thus "34" and "43" terms are absent, which will allow us to simplify expressions involving the neutralino mass matrix, since that is the position of the complex coefficient $\mu$. We have dropped terms of order $m_f^2/v^2$ relative to the others. The gauge coefficients $a_{Li}$ are:

$$a^q_{L1} = \sqrt{2} g \tan\theta_w \, (Q_q - T^q_{3L}) = \sqrt{2} g \tan\theta_w / 6, \tag{49}$$

$$a^q_{L2} = \sqrt{2} g T^q_{3L}, \tag{50}$$

and the $a_{Ri}$ are the same as the $a_{Li}$ with $T_{3L} \to T_{3R} = 0$. The neutralino phases appear through a 4x4 matrix

$$\Phi_{hl} = \sum_{i=1}^{4} U^T_{hi} \hat{M}_{ii} I^b_{ii} U_{il}, \tag{51}$$

where $U$ diagonalizes the neutralino mass matrix $M$, and $\hat{M}$ is the diagonal result. Here $I^b_{ij} = I^b(x_i)\delta_{ij}$, *i.e.* $I^b_{ij}$ is the diagonal matrix of Feynman integrals for the corresponding mass eigenvalues in $\hat{M}$. In the limit that the $I^b(x_i)$ are equal, the real part of $\Phi_{hl}$ can simply be written

$$\mathrm{Re}\Phi_{hl} \simeq I^b(x_0)\mathrm{Re}M_{hl}, \tag{52}$$

where $I^b(x_0) = \sum_i^4 I^b(x_i)/4$. The imaginary part of $\Phi_{hl}$ is more difficult because it vanishes in the limit of degenerate neutralino masses (except for the irrelevant "34" and "43" terms). We know that $\mathrm{Im}\mu$ is the only complex coefficient in the neutralino mass matrix $M$, so we can write

$$\mathrm{Im}\Phi_{hl} \simeq \Omega_{hl}\mathrm{Im}\mu, \tag{53}$$

where $\Omega_{hl}$ is a real matrix to be determined. This allows us to see that $d_q(\tilde{\chi}^0)$ is proportional to $\mathrm{Im}\mu$ and $\mathrm{Im}(A_q - \mu R_q)$, just as the other contributions are. It turns out that $\mathrm{Im}\Phi_{hl}$ is proportional to $\mathrm{Im}(MM^*M)_{hl}$, $\mathrm{Im}(MM^*MM^*M)_{hl}$, and $\mathrm{Im}(MM^*MM^*MM^*M)_{hl}$ (except for the "34" and "43" pieces). To extract the $\mathrm{Im}\mu$ dependence, we ignore all terms of higher order in $\mathrm{Im}\mu/|\mu|$, which is a valid approximation for the phases allowed by the experimental bound on $d_n$. Then these products (for $(h,l) \neq (3,4),\ (4,3)$) simplify as follows:

$$\mathrm{Im}(MM^*M)_{hl} \simeq \mathrm{Im}\mu \sum_{p=0}^{2}(-1)^p(M_R^p P M_R^{2-p})_{hl}, \tag{54}$$

$$\mathrm{Im}(MM^*MM^*M)_{hl} \simeq \mathrm{Im}\mu \sum_{p=0}^{4}(-1)^p(M_R^p P M_R^{4-p})_{hl}, \tag{55}$$

$$\mathrm{Im}(MM^*MM^*MM^*M)_{hl} \simeq \mathrm{Im}\mu \sum_{p=0}^{6}(-1)^p(M_R^p P M_R^{6-p})_{hl}, \tag{56}$$

where $M_R = \mathrm{Re}M$ and $P$ is a matrix with $-1$ in the 34 and 43 positions and 0 everywhere else (so that $\mathrm{Im}(\mu P) = \mathrm{Im}M$). After some calculation, we obtain an expression for the imaginary part of the complex matrix $\Phi$:

$$\begin{aligned}\mathrm{Im}\Phi_{hl} \simeq \Omega_{hl}\mathrm{Im}\mu \simeq \frac{3}{2}\mathrm{Im}\mu \sum_{s=1}^{4}\Big(I(x_s) - \sum_{j}^{\neq s} I(x_j)\Big) \times \\ \Bigg[\sum_{i,j,k}^{\neq s} \epsilon_{ijk}\hat{M}_i^4\hat{M}_j^6 \sum_{p=0}^{2}(-1)^p(M_R^p P M_R^{2-p})_{hl} - \sum_{i,j,k}^{\neq s} \epsilon_{ijk}\hat{M}_i^2\hat{M}_j^6 \sum_{p=0}^{4}(-1)^p(M_R^p P M_R^{4-p})_{hl} + \\ \sum_{i,j,k}^{\neq s} \epsilon_{ijk}\hat{M}_i^2\hat{M}_j^4 \sum_{p=0}^{6}(-1)^p(M_R^p P M_R^{6-p})_{hl}\Bigg] \times\end{aligned} \tag{57}$$

$$\left[\sum_{i,j,k}^{\neq s}\epsilon_{ijk}\hat{M}_i^4\hat{M}_j^6\Big(3\hat{M}_s^2-\sum_n^{\neq s}\hat{M}_n^2\Big)-\sum_{i,j,k}^{\neq s}\epsilon_{ijk}\hat{M}_i^2\hat{M}_j^6\Big(3\hat{M}_s^4-\sum_n^{\neq s}\hat{M}_n^4\Big)+\right.$$
$$\left.\sum_{i,j,k}^{\neq s}\epsilon_{ijk}\hat{M}_i^2\hat{M}_j^4\Big(3\hat{M}_s^6-\sum_n^{\neq s}\hat{M}_n^6\Big)\right]^{-1},$$

where $\sum_j^{\neq s}$ means sum over the three members of the set $\{1,2,3,4\}-\{s\}$. Here $\hat{M}_j$ are the mass eigenvalues of $\hat{M}$ (*i.e.* the four physical neutralino masses).

The expression above is completely analytic and exact except for the approximation we made in dropping higher order terms in $\text{Im}\mu/|\mu|$, but it has so many terms that it is not that useful. Let us find an approximation to this expression using the information about the neutralino mass eigenstates, namely that they are fairly close together and the heaviest neutralino is lighter than the squarks ($x_4 \ll 1$) in most SUSY models. This means that we can take a simple linear fit to the Feynman integral by evaluating $I^b(x)$ at the lowest and highest values of $x$:

$$I^b(x) \simeq K_0 + K_1 x \simeq I^b(x_1) + S_{41}(x - x_1), \tag{58}$$

where $S_{41}$ is the slope

$$S_{41} = \frac{I^b(x_4) - I^b(x_1)}{x_4 - x_1}, \tag{59}$$

and $x_j = m_{\tilde{\chi}_j^0}/m_{\tilde{q}0}$. Thus $K_1 = S_{41}$ and $K_0 = I^b(x_1) - S_{41}x_1$. Note that this approximation gives *exact* values for $x_1$ and $x_4$, and is only off for $x_2$ and $x_3$—a rough estimate is that the approximation is correct to about 5%.

If we plug Eq. (58) into $\text{Im}\Phi_{hl}$ in Eq. (51), we see that the $K_0$ piece vanishes (except for the "34" and "43" pieces), and we obtain

$$\text{Im}\Phi_{hl} \simeq S_{41}\frac{\text{Im}(MM^*M)_{hl}}{m_{\tilde{q}0}^2} \simeq \frac{S_{41}}{m_{\tilde{q}0}^2}\text{Im}\mu\sum_{p=0}^{2}(-1)^p(M_R^p P M_R^{2-p})_{hl}. \tag{60}$$

The neutralino contribution to $d_q$ is found by plugging Eq. (52) for $\text{Re}\Phi_{hl}$ and Eq. (57) or Eq. (60) for $\text{Im}\Phi_{hl}$ into (48).

Finally, we want to relate the expressions for the three SUSY contributions to the quark EDM in Eqs. (46), (47), and (48) in terms of the coefficients $k_n$ from Section IV. Using the Naive Quark Model, $d_n = 4/3d_d - 1/3d_u$, we can write $k_n^x = 4/3k_d^x - 1/3k_u^x$ and

$$k_q^x = \frac{d_q^x}{10^{-25}e\,cm}\frac{m_0}{\text{Im}x}, \tag{61}$$

where $x = A_u$, $A_d$, or $\mu$, and $d_q^x$ is the contribution to $d_q$ from complex quantity $x$. We can see that if we neglect the tiny second term of $d_q(\tilde{\chi}^+)$ in Eq. (47), then $k_n^{A_u}$ ($k_n^{A_d}$) gets contributions only from the gluino and neutralino contributions to $d_u$ ($d_d$), whereas $k_n^\mu$ gets contributions from all three of the SUSY contributions to $d_u$ and $d_d$.

FIGURES

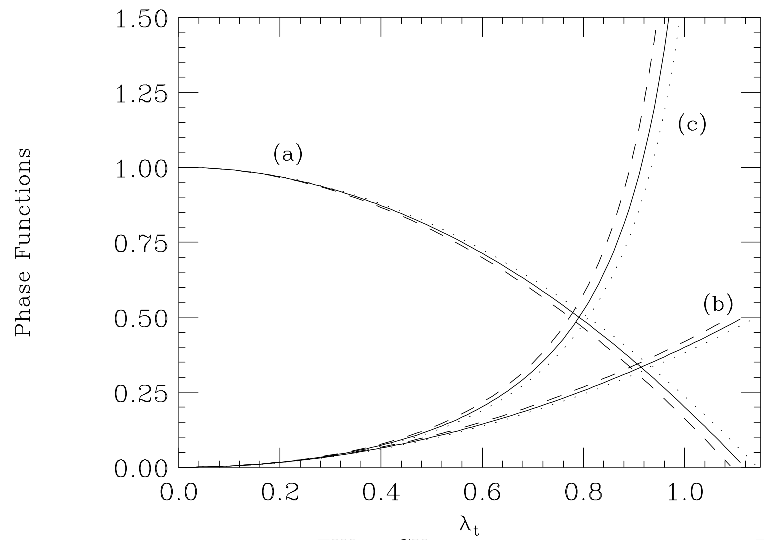


FIG. 1. Plot of the ratios (a) $r_t = \bar{A}_t^{EW}/\bar{A}_t^{GUT}$, (b) $(1-r_t)/2 = -\bar{B}^{EW}/\bar{A}_t^{GUT}$, and (c) $(1-r_t)/2r_t = -\bar{B}^{EW}/\bar{A}_t^{EW}$ versus the top quark Yukawa coupling for $\alpha_s(M_Z) = 0.118 \pm 0.006$.

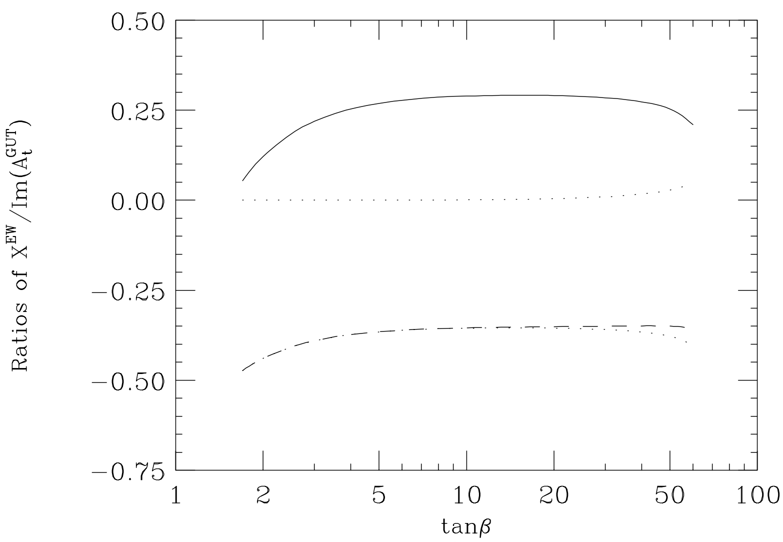


FIG. 2. The ratios of imaginary parts to $\mathrm{Im}A_t^{GUT}$ versus $\tan\beta$ with $\mathrm{Im}A_t^{GUT} \neq 0$ and $\mathrm{Im}A_b^{GUT} = 0$. The solid line is $\mathrm{Im}A_t^{EW}/\mathrm{Im}A_t^{GUT}$; the dashed line is $\mathrm{Im}B^{EW}/\mathrm{Im}A_t^{GUT}$; and the upper (lower) dotted line is $\mathrm{Im}A_d^{EW}/\mathrm{Im}A_t^{GUT}$ ($\mathrm{Im}A_u^{EW}/\mathrm{Im}A_t^{GUT}$).

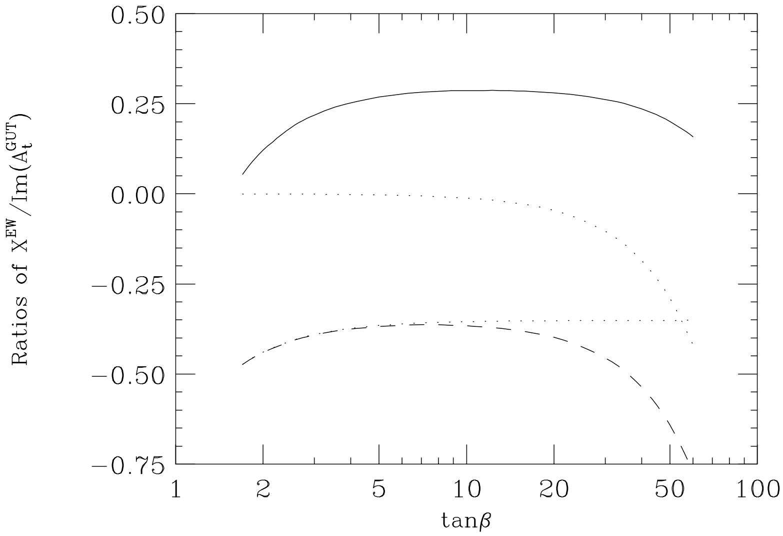


FIG. 3. Same as Fig. 2 with $\mathrm{Im}A_b^{GUT} = \mathrm{Im}A_t^{GUT}$.

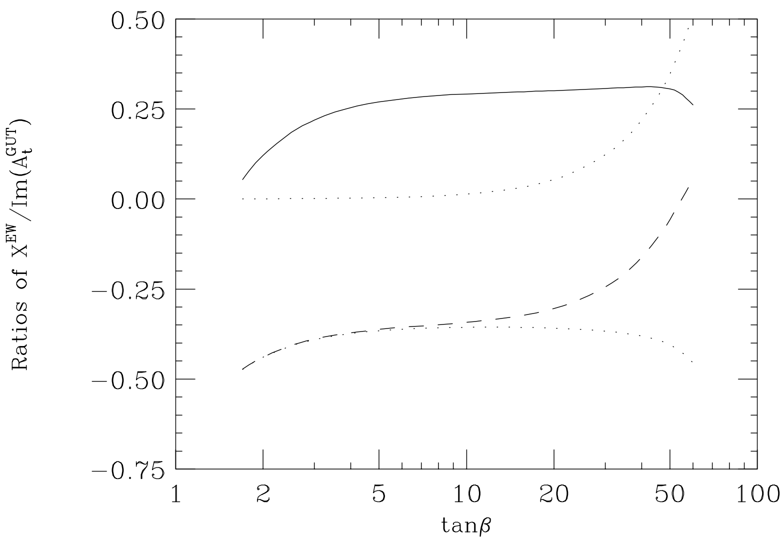


FIG. 4. Same as Fig. 2 with $\mathrm{Im}A_b^{GUT} = -\mathrm{Im}A_t^{GUT}$.

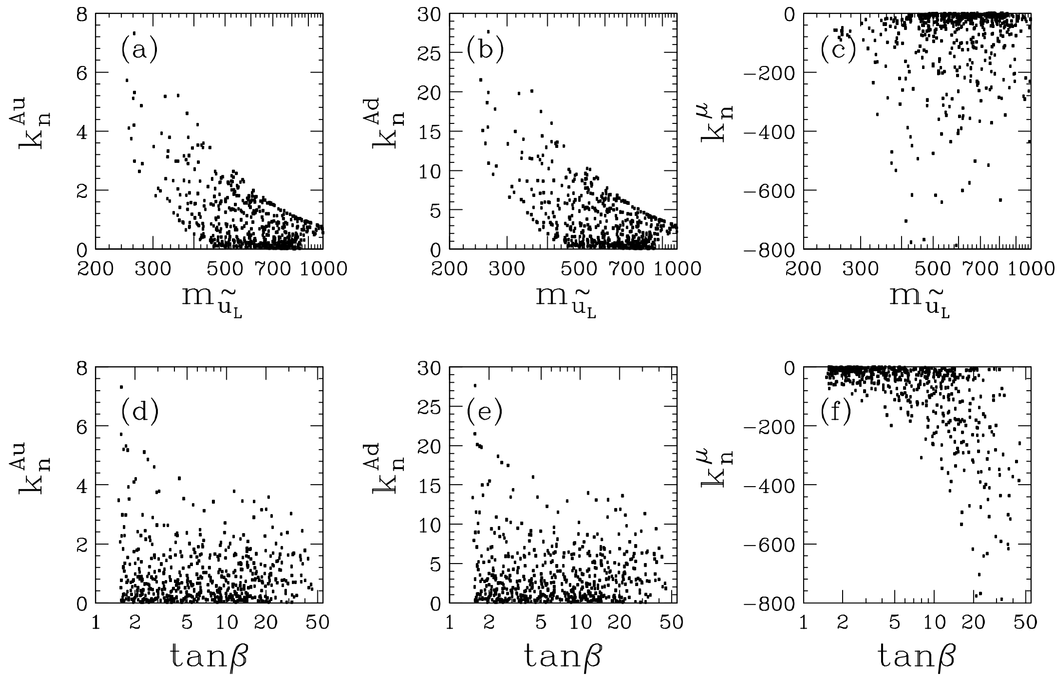


FIG. 5. Scatter plot of (a and d) $k_n^{A_u}$, (b and e) $k_n^{A_d}$, and (c and f) $k_n^{\mu}$ versus squark mass and versus $\tan\beta$. Each point represents a solution of the supersymmetric parameter space with universal scalar and gaugino mass terms which is within other experimental limits.

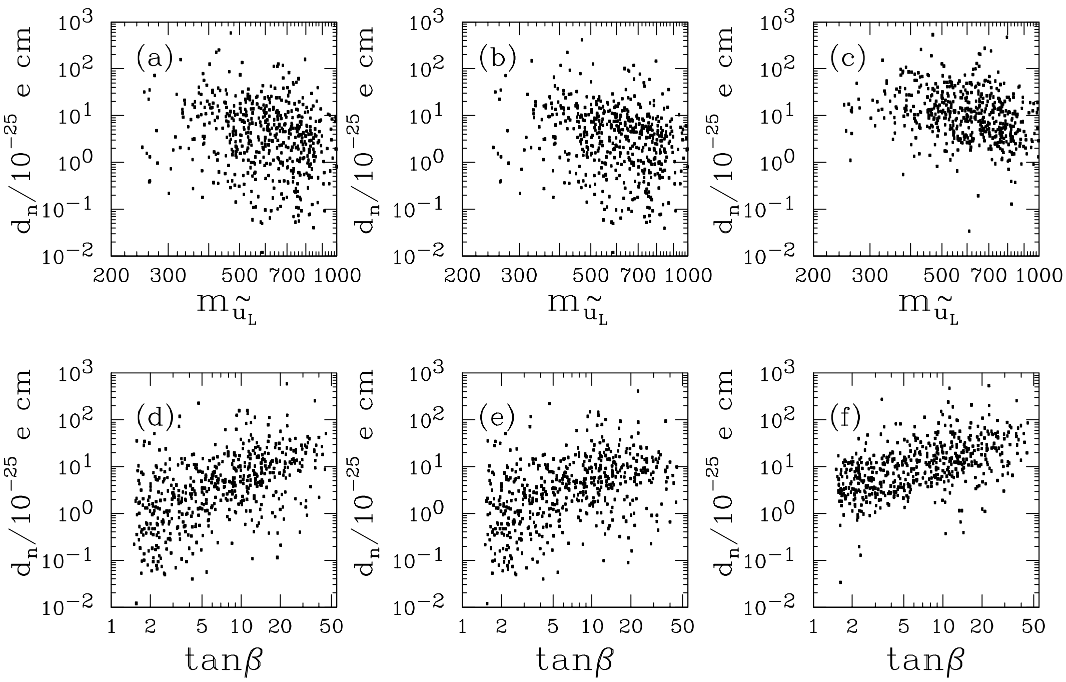

FIG. 6. Scatter plot of $d_n/10^{-25}e{\cdot}$cm versus squark mass and versus $\tan\beta$ for (a and d) $ArgA_t^{GUT} = ArgA_b^{GUT} = 0.1$ with all other phases zero, (b and e) $ArgA_t^{GUT} = -ArgA_b^{GUT} = 0.1$ with all other phases zero, and for (c and f) universal phases $ArgA^{GUT} = ArgB^{GUT} = 0.1$. Each point represents a solution of the supersymmetric parameter space with universal scalar and gaugino mass terms which is within other experimental limits.

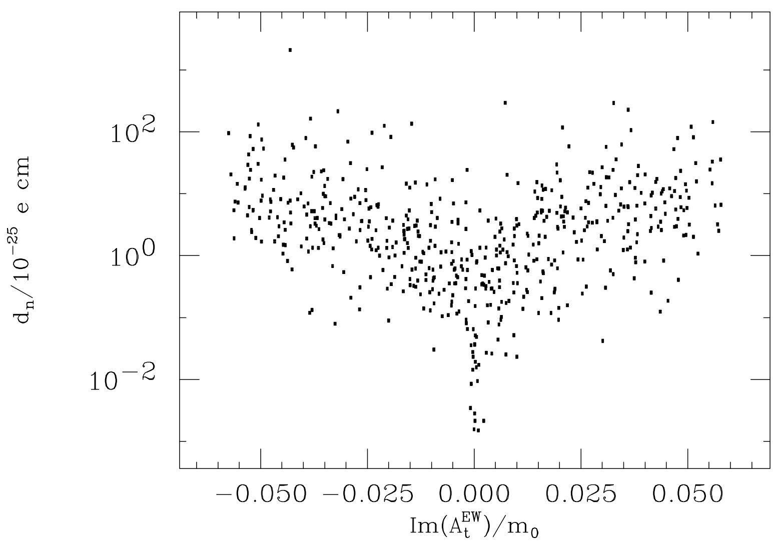


FIG. 7. Scatter plot of $d_n/10^{-25}e{\cdot}$cm versus $ImA_t^{EW}/m_0$. Each point represents a solution of the supersymmetric parameter space with universal scalar and gaugino mass terms which is within other experimental limits.

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