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Mixing-induced CP Asymmetries in Inclusive B Decays *

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Abstract

We consider CP violating asymmetries that are induced by particle-antiparticle mixing in inclusive channels of neutral B meson decay. Not only are the branching ratios sizable, at the 1% to 50% level, but some of those asymmetries are expected to be large because of substantial CKM phases. The inclusive sum partially dilutes the asymmetries, but the dilution factor is calculable, assuming local quark-hadron duality, and CKM parameters can be reliably extracted. We discuss in detail the determination of $\sin 2\alpha$ from charmless final states in decays of B_d mesons and survey the asymmetries for other inclusive final states. While probably not yet sensitive to standard model predictions, meaningful CP violation studies can be conducted with existing data samples of inclusive neutral B decays.

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1. Introduction

Studying CP asymmetries in B decays promises to reveal crucial information about quark mixing and the nature of CP violation. A rich phenomenology is expected and has been widely discussed in the literature. With the exception of a few 'gold-plated' modes, it is not yet clear which observables will eventually turn out to be the optimal measures of quark mixing parameters. For this reason, and in order to overconstrain the unitarity triangle, it is important to consider different options.

In this letter we propose a class of CP violating asymmetries that occur in partially inclusive neutral B meson decays, that is, in final states specified by their flavor content. Inclusive asymmetries have been investigated earlier, for charged B decays [1] which require a strong interaction phase difference, or for the dilepton asymmetry in semileptonic decays of neutral B mesons [2]. Except for a few scattered studies [3, 4, 5], mixing-induced asymmetries in inclusive nonleptonic neutral B decays have not been analyzed so far, probably because they were thought to be very difficult to measure. In this note we wish to make the point that some inclusive asymmetries are complementary to those in exclusive B decays, and sometimes even theoretically advantageous. The relevant inclusive branching ratios range between 1% to 50% of all neutral B decays. They are orders of magnitude larger than their exclusive counterparts. Thus their experimental feasibility should not be discarded but seriously studied.

Exclusive modes to measure CKM angles are usually favored because of their unique experimental signatures. Theoretically, however, hadronic uncertainties are difficult to quantify, so that clean extractions of CKM angles often imply reliance on flavor symmetries and the ability to measure several modes simultaneously. An example of this type is the determination of $\sin 2\alpha$ from $B_d \to \pi^+\pi^-$ decays [6].

On the other hand, excellent vertex technology, $p/K/\pi$ separation, a reliable heavy flavor decay model and hermiticity of the detector with regard to charged tracks from b-decay distinguishes among the various underlying quark transitions and different b-hadron species, respectively. Such distinctions will enable us to observe the CP asymmetries in inclusive data samples. Constructing the required detectors should be possible as indicated by currently operating or planned devices. The potential rewards are highly promising. Not only would experiments be able to probe inclusive CP asymmetries that are expected to be sizable, but new determinations of $|V_{ub}/V_{cb}|$ may also become feasible. Furthermore, one would be able to flavor-tag almost all b-hadrons [7], which is so crucial for mixing-induced CP violation studies.

As for the inclusive asymmetries, in contrast to those in exclusive decays, many hadronic uncertainties cancel in the sum over all final states of a particular flavor content. This cancellation relies on local parton-hadron duality. The quality of this assumption, as well as the numerical values of remaining hadronic parameters like decay constants, will be tightly constrained by other measurements, so that the accuracy of the theoretical prediction for inclusive CP asymmetries can be cross-checked.

After deriving basic formulae for asymmetries in Sect. 2, we discuss in detail, in Sect. 3, the determination of $\sin 2\alpha$ from the inclusive final state $\overline{u}u\overline{d}d$ (to be precise, no charmed particles and net strangeness zero) and compare it with its exclusive analogue $B_d \to \pi^+\pi^-$. Other inclusive final states can be treated analogously and we summarize the corresponding results in Sect. 4. Sect. 5 concludes and presents an outlook.

2. Inclusive CP asymmetries

The weak interactions mix neutral B mesons with their antiparticles. The time dependence of a state $B(t)$ ($\overline{B}(t)$) that began as a flavor eigenstate B ($\overline{B}$) at $t = 0$, can be written as

$$B(t) = g_+(t)\ B - \frac{q}{p}\ g_-(t)\ \overline{B}, \tag{1}$$

$$\overline{B}(t) = g_+(t)\ \overline{B} - \frac{p}{q}\ g_-(t)\ B. \tag{2}$$

Here

$$\frac{q}{p} = \frac{M_{12}^* - i\Gamma_{12}^*/2}{(\Delta M - i\Delta\Gamma/2)/2} = \frac{M_{12}^*}{|M_{12}|}\left(1 - \frac{1}{2}a\right)\ , \qquad a = \mathrm{Im}\frac{\Gamma_{12}}{M_{12}}, \tag{3}$$

where M_{12} (Γ_{12}) is the off-diagonal element in the mass (decay width) matrix of the $B - \overline{B}$ system ($|B\rangle = |1\rangle$, $|\overline{B}\rangle = |2\rangle$). $\Delta M = M_H - M_L$, $\Delta\Gamma = \Gamma_H - \Gamma_L$ are the differences in mass and decay rate between the mass eigenstates $B_{H,L} = pB \pm q\overline{B}$. Note that the sign convention for $\Delta\Gamma$ is opposite to [8]. We use the CP phase conventions $CP|B\rangle = -|\overline{B}\rangle$ and $CP(\overline{d}b)_{V-A}\ [CP]^{-1} = -(\overline{b}d)_{V-A}$. The second expression for q/p in (3) is valid to first order in the small quantity $\Gamma_{12}/M_{12} = \mathcal{O}(m_b^2/m_t^2)$. The time dependent functions $g_\pm(t)$ are given by

$$g_+(t) = e^{-iMt - \frac{1}{2}\Gamma t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right], \tag{4}$$

$$g_-(t) = e^{-iMt - \frac{1}{2}\Gamma t}\left[\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right]. \tag{5}$$

with M (Γ) the average mass (decay rate) of B_H and B_L.

Given the final state f, the asymmetry

$$\mathcal{A}(t) = \frac{\Gamma(B(t) \to f) - \Gamma(\overline{B}(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to \overline{f})} \tag{6}$$

measures CP violation. The time-integrated asymmetry will be denoted by $\mathcal{A}$. Usually f represents an exclusive CP eigenstate as in the familiar cases of $B_d(\overline{B}_d) \to J/\psi K_S$ or $B_d(\overline{B}_d) \to \pi^+\pi^-$. In the following we let f be an inclusive final state, for example all final states with no charm particles and no net strangeness in the decay of B_d ($\overline{B}_d$). This channel is based on the quark level transitions $\overline{b}(d) \to \overline{u}u\overline{d}(d)$ or $b(\overline{d}) \to u\overline{u}d(\overline{d})$. Further examples will be described in Sect. 4. The expressions for the time-dependent decay rates are

$$\Gamma(B(t) \to f) = |g_+|^2\Gamma_{f,11} + \left|\frac{q}{p}\right|^2 |g_-|^2\Gamma_{f,22} - 2\mathrm{Re}\left(\frac{q}{p}\,g_+^* g_- \Gamma_{f,12}\right), \tag{7}$$

$$\Gamma(\overline{B}(t) \to f) = \left|\frac{p}{q}\right|^2 |g_-|^2\Gamma_{f,11} + |g_+|^2\Gamma_{f,22} - 2\mathrm{Re}\left(\frac{p}{q}\,g_+^* g_- \Gamma_{f,21}\right), \tag{8}$$

with $\Gamma_{f,ij} = \sum_k \langle i|f_k\rangle\langle f_k|j\rangle$. The sum runs over all final states f_k that contribute to the partially inclusive channel under consideration. For any final state f, $\Gamma_{f,ji} = \Gamma_{f,ij}^*$. If the summation were extended to include all final states, $\Gamma_{f,ij}$ would coincide with Γ_{ij}, the full decay constant matrix of the $B-\overline{B}$ system. Time-dependent studies determine $|q/p|^2\Gamma_{f,22}/\Gamma_{f,11}$ and

$$\xi \equiv \frac{M_{12}^*}{|M_{12}|}\frac{\Gamma_{f,12}}{\Gamma_{f,11}}. \tag{9}$$

The parameter $|q/p|^2 = 1-a+\mathcal{O}(a^2)$ can be obtained from the dilepton asymmetry (or, more generally, from the asymmetry of tagged neutral B mesons decaying to wrong flavor-specific modes due to $B^0-\overline{B}^0$ mixing). Theory predicts a to be $< 10^{-2}$ $[< 10^{-3}]$ for the B_d $[B_s]$ meson.

In the remainder of this section we restrict ourselves to self-conjugate final states common to both B and $\overline{B}$, $\overline{f}=f$ in (6). The quantity

$$r = \frac{\Gamma_{f,22}}{\Gamma_{f,11}} - 1, \tag{10}$$

parametrizes direct CP violation and should agree with the corresponding inclusive asymmetry in charged B decays up to corrections of $\mathcal{O}(1/m_b^3)$. Using (7), (8) the time-dependent asymmetry (6) is then given by

$$\mathcal{A}(t) = \frac{\mathrm{Im}\,\xi \sin\Delta M t - a\left(\sin^2\frac{\Delta M t}{2} + \sinh^2\frac{\Delta\Gamma t}{4} - \frac{1}{2}\sinh\frac{\Delta\Gamma t}{2}\,\mathrm{Re}\,\xi\right) - \frac{r}{2}\cos\Delta M t}{\left(1+\frac{r}{2}\right)\left(1+2\sinh^2\frac{\Delta\Gamma t}{4}\right) - \sinh\frac{\Delta\Gamma t}{2}\,\mathrm{Re}\,\xi}, \tag{11}$$

which is valid to first order in a and neglecting terms of $\mathcal{O}(a\cdot r)$ and $\mathcal{O}(a\,(\mathrm{Im}\xi)^2)$. The time-integrated asymmetry reads

$$\mathcal{A} = \frac{\frac{x}{1+x^2}\mathrm{Im}\,\xi - a\left(\frac{x^2}{2(1+x^2)} + \frac{y^2}{2(4-y^2)} - \frac{y}{4-y^2}\mathrm{Re}\,\xi\right) - \frac{1}{2}\frac{r}{1+x^2}}{\left(1+\frac{r}{2}\right)\left(1+\frac{y^2}{4-y^2}\right) - \frac{2y}{4-y^2}\mathrm{Re}\,\xi}, \tag{12}$$

where $x = \Delta M/\Gamma$ and $y = \Delta\Gamma/\Gamma$. Neglecting y in the previous two equations is an excellent approximation for B_d mesons where $|y| \lesssim 0.01$. Even for B_s mesons, where $|y| \sim 0.15$ is predicted, $|y\,\mathrm{Re}\xi|/2 < 0.04$. Neglecting y in both cases leads to

$$\mathcal{A}(t) = \frac{2}{2+r}\left[\mathrm{Im}\,\xi\sin\Delta M t - a\sin^2\frac{\Delta M t}{2} - \frac{r}{2}\cos\Delta M t\right] \qquad t \ll \frac{1}{\Delta\Gamma}, \tag{13}$$

$$\mathcal{A} = \frac{x}{1+x^2}\;\frac{2}{2+r}\left[\mathrm{Im}\,\xi - \frac{x}{2}a - \frac{1}{2x}r\right], \tag{14}$$

where (13) does not apply to $t \gtrsim 1/\Delta\Gamma$, which can be relevant for B_s mesons. The second and third terms in square brackets could be measured separately, from the dilepton asymmetry and direct CP violation in charged B decays, respectively. The first term is specific to the partially inclusive final state for neutral B mesons and it is the one of interest here. For the charmless final state, it is much larger than the other two, as will be seen below. Let us also mention that for an exclusive decay $\xi = (M_{12}^*\langle f|\overline{B}\rangle)/(|M_{12}|\langle f|B\rangle)$ and $1+r=|\xi|^2$, and the well-known expressions for asymmetries in exclusive decays are recovered from (13), (14).

Finally we note that due to $B-\overline{B}$ mixing a time dependent CP asymmetry persists, even when all final states are summed over. This asymmetry allows a direct determination of a since in that case $r=0$, $\mathrm{Im}\xi = xa/2$ and $\mathrm{Re}\xi = y/2$ in (11), (13).

3. $\sin 2\alpha$ **from charmless inclusive** B_d **decay**

In this section we compute $\mathrm{Im}\,\xi$ that enters the asymmetries (13), (14) for the inclusive final state f with no charmed particles and net strangeness zero ($C = S = 0$ with the additional constraint of no $\overline{c}c$ pairs). We then discuss the determination of $\sin 2\alpha$ from this channel. The quantity $\Gamma_{f,12}$ is given by

$$\Gamma_{f,12} = \frac{1}{2M_B} \sum\!\!\!\!\!\!\!\int_k \, (2\pi)^4 \delta^{(4)}(p_B - p_{f_k}) \, \langle B|\mathcal{H}^\dagger_{eff}|f_k\rangle\langle f_k|\mathcal{H}_{eff}|\overline{B}\rangle. \tag{15}$$

To lowest order in the strong interaction, the above final state is uniquely associated with the $b \to u\overline{u}d$ transition (and its hermitian conjugate) in the weak effective Hamiltonian $\mathcal{H}_{eff}$, up to penguin-penguin interference contributions, which do not contribute to $\mathrm{Im}\,\xi$, see below. Thus, f is mainly $\overline{u}u\overline{d}d$ (plus light quark pairs). We may use completeness of the intermediate states to write (the approximate relation will be explained shortly)

$$\Gamma_{f,12} \approx \frac{1}{2M_B} \langle B| \int d^4x \, \mathcal{H}^{f\dagger}_{eff}(x) \mathcal{H}^f_{eff}(0)|\overline{B}\rangle. \tag{16}$$

The optical theorem further implies

$$\Gamma_{f,12} \approx \frac{1}{2M_B} \langle B|\mathrm{Im}\; i \int d^4x \, T \, \mathcal{H}^f_{eff}(x) \mathcal{H}^f_{eff}(0)|\overline{B}\rangle, \tag{17}$$

where the relevant effective hamiltonian reads

$$\mathcal{H}^f_{eff} = \frac{G_F}{\sqrt{2}} \left[\lambda_u \left(C_1 Q^u_1 + C_2 Q^u_2\right) - \lambda_t \sum_{i=3}^{6} C_i Q_i\right] + \text{h.c.} \tag{18}$$

Here $\lambda_i = V^*_{id} V_{ib}$. The operators Q^u_1, Q^u_2 denote 'current-current' operators with flavor content $(\overline{d}b)(\overline{u}u)$ and Q_i, $i = 3,..,6$, denote 'penguin' operators of the same flavor content. The detailed expressions as well as the Wilson coefficient functions can be found in [9].

The right hand side of (17) is related to the forward scattering amplitude which can be expanded in the heavy quark mass, following the methods reviewed in [10]. Assuming only local duality, this procedure allows us to go beyond the purely partonic prediction, which is recovered as the leading term in the expansion. On the other hand, the identification of $\Gamma_{f,12}$ with the r.h.s. of (17) is only approximate for the final state with no charmed particles and therefore, strictly speaking, there is no heavy quark expansion for the asymmetry. The identification would be exact, if the final state were $C = S = 0$, including $\overline{c}c$ pairs. Higher order QCD effects mix the $C = S = 0$ final states without and with charmed particles (which, in our definition, include charmonia). First, a gluon can be radiated in a $b \to u\overline{u}d$ transition and split into a $\overline{c}c$. This is a very small correction, because it requires a highly virtual gluon. Second, the $\overline{c}c$ pair, created in the $b \to \overline{c}cd$ transition, can recombine and turn into a $\overline{u}u$. While the leading logarithmic contribution from this process is included in the $b \to d\overline{u}u$ penguin operators above, the constant terms are not. Since we only work to leading logarithmic accuracy, it is consistent to neglect this mixing. In the following we shall assume that both effects are indeed small and treat (17) as an equality.

Note that the problem just discussed does not exist for $C = \pm 1$ final states reached through the $b \to c$ or $b \to \overline{c}$ (plus light quarks and perhaps a $\overline{c}c$ pair) transition, in which case a heavy quark expansion is literally possible. We discuss this case in Sect. 4.

After these general remarks, let us return to the calculation of $\mathrm{Im}\,\xi$. Combining (17) and (18), we obtain contributions from $Q_{1,2}$ interfering with themselves and from penguin operators interfering with $Q_{1,2}$. The penguin-penguin interference has CKM phase $\lambda_t^2/|\lambda_t|^2 = \exp(2i\beta)$, which is cancelled by the mixing phase $M_{12}^*/|M_{12}| = \exp(-2i\beta)$. Therefore it does not contribute to $\mathrm{Im}\,\xi$. It is now straightforward to deduce $\Gamma_{f,12}$ from the results of [8], where Γ_{12} in the $B_s - \overline{B}_s$ system has been computed[†].

The total inclusive decay rate of a B_d meson into the $\overline{u}u\overline{d}d$ final state, $\Gamma_{f,11}$ is given by

$$\Gamma_{f,11} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{ud}^* V_{ub}|^2 \,(K_1 + 3K_2)\,, \tag{19}$$

where

$$K_1 = 3C_1^2 + 2C_1C_2 \approx -0.39 \qquad K_2 = C_2^2 \approx 1.25. \tag{20}$$

Here we have neglected the small penguin contributions. We also omitted the known next-to-leading order radiative corrections since these are not available for $\Gamma_{f,12}$. For further use, we also define the combinations

$$K_1' = 2(3C_1C_3 + C_1C_4 + C_2C_3) \approx 0.023 \qquad K_2' = 2C_2C_4 \approx -0.063, \tag{21}$$

related to the interference of penguin operators with $Q_{1,2}$. The numerical values of the coefficients K_i, K_i' quoted refer to evaluation of the Wilson coefficients at the scale $m_b = 4.8\,\mathrm{GeV}$ with $\Lambda_{QCD} = 200\,\mathrm{MeV}$. Next we recall that, within the phase conventions we are using, the mixing phase for the B_d system reads $M_{12}^*/|M_{12}| = \exp(-2i\beta)$ and $\lambda_u/\lambda_u^* = \exp(-2i\gamma)$, where α, β, γ are the standard angles of the unitarity triangle. Putting everything together we get the result

$$\begin{aligned}\mathrm{Im}\,\xi \;=\; & -8\pi^2\,\frac{f_B^2 M_B}{m_b^3}\,\sin 2\alpha\left[1 + \frac{4}{3}\frac{2K_1+K_2}{K_1+3K_2}\,(B-1) + \frac{5}{3}\frac{K_2-K_1}{K_1+3K_2}\,(B_S-1)\right. \\ & \left. -\frac{1}{3}\left(\frac{M_B^2}{m_b^2}-1\right) + \frac{\sin\alpha\sin(\alpha+\beta)}{\sin\beta\sin 2\alpha}\left(\frac{4}{3}\frac{2K_1'+K_2'}{K_1+3K_2}\,B + \frac{5}{3}\frac{K_2'-K_1'}{K_1+3K_2}\,B_S\right)\right].\end{aligned} \tag{22}$$

Here f_B is the B_d meson decay constant in the normalization in which $f_\pi = 131\,\mathrm{MeV}$ and $M_B = 5.28\,\mathrm{GeV}$ is the B_d meson mass. The bag factors are defined in terms of hadronic matrix elements as

$$\langle B|(\overline{d}_i b_i)_{V-A}(\overline{d}_j b_j)_{V-A}|\overline{B}\rangle \;=\; \frac{8}{3} f_B^2 M_B^2\, B, \tag{23}$$

$$\langle B|(\overline{d}_i b_i)_{S+P}(\overline{d}_j b_j)_{S+P}|\overline{B}\rangle \;=\; -\frac{5}{3} f_B^2 M_B^2 \frac{M_B^2}{m_b^2}\, B_S. \tag{24}$$

$B = B_S = 1$ corresponds to factorization (at the scale m_b). Notice that in (22), the scale-dependent coefficients $K_{1,2}$ enter only in the terms that deviate from the factorization limit.

[†]Eq. (14) and Appendix A of [8]. For $b \to u\bar{u}d$ and B_d the CKM elements have to be adjusted in an obvious way and $z = m_c^2/m_b^2$ and m_s are set to zero.

This property, apparently an accident that would not persist beyond the leading logarithmic approximation, also holds for the subleading terms in the heavy quark expansion, proportional to $(M_B^2/m_b^2-1)\sim\Lambda_{QCD}/m_b$. In evaluating the hadronic matrix elements in these subleading terms we have employed factorization at the scale m_b. More details regarding their treatment can be found in [8]. Retaining the complete $\mathcal{O}(\Lambda_{QCD}/m_b)$ corrections has the advantage of avoiding various troublesome ambiguities. For instance, working at leading order in the heavy quark expansion it is not clear whether to use the bag parameters and the decay constant in full QCD or those in the static limit, which differ from the former by terms of $\mathcal{O}(\Lambda_{QCD}/m_b)$. Similarly the numerically important distinction between the b-quark and the B_d-meson mass cannot be made at leading order. These problems are absent in (22), where decay constant and bag parameters have to be taken in full QCD, leading to an expression that is complete to next-to-leading order in the heavy quark expansion. Numerically, we find for $m_b = 4.8\,\mathrm{GeV}$

$$\begin{aligned}\mathrm{Im}\,\xi &= -0.12\,\sin 2\alpha\left(\frac{f_B}{180\,\mathrm{MeV}}\right)^2\Big[1+0.19(B-1)+0.81(B_S-1)-0.07\\ &\quad -0.05\,\frac{\sin\alpha\sin(\alpha+\beta)}{\sin\beta\sin 2\alpha}\Big],\end{aligned} \tag{25}$$

where we set $B=B_S=1$ in the penguin contribution. With $x_d=0.73$, we see from (14) that the time-integrated asymmetry is of order 5% times $\sin 2\alpha$.

When the charm and up quarks are degenerate in mass, all CP asymmetries must vanish in the CKM model, while $\mathcal{A}$ in (14) does not for the charmless final state considered. There is no contradiction, because if $m_c=m_u$, the asymmetry is no longer an observable, since the charmless final state can not be experimentally distinguished from a charmed final state. Summing also over final states with charm, the asymmetry vanishes. Note that for charged B decays, even when $m_c\neq m_u$, the asymmetry for the inclusive $C=0$ final state (and for the $C=\pm 1$ final states, see Sect. 4) must vanish by CPT conservation. No such constraint exists for neutral B decays due to $B-\overline{B}$ mixing.

Returning to (14), we note that the terms involving the direct CP asymmetry r and the dilepton asymmetry a do not exceed several permille, as follows from the estimates of r for the $\overline{u}u\overline{d}d$, $\overline{d}d\overline{d}d$, $\overline{s}s\overline{d}d$ final states in [1] and a in [2]. Thus, unless $\sin 2\alpha$ is small, we may approximate $\mathcal{A}\approx \mathrm{Im}\,\xi\cdot x_d/(1+x_d^2)$. The penguin contribution to $\mathrm{Im}\,\xi$ enters (22), (25) with small coefficient, but becomes enhanced if β is close to its current lower limit, $\beta\approx 0.18$. If in addition $\alpha\approx\pi/2$, the penguin contribution can be sizable and dominate the asymmetry. However, since the penguin term is calculable in the inclusive approach, it can be corrected for. Note that the value of β that is needed for this purpose is related to the CP asymmetry in the clean process $B_d(\overline{B}_d)\to J/\psi K_S$ and will be available when the inclusive asymmetry is measured.

The conventional method for the determination of $\sin 2\alpha$ makes use of the CP asymmetry in the exclusive decay $B_d(\overline{B}_d)\to\pi^+\pi^-$. However, the asymmetry coefficient (the factor multiplying the oscillation term $\sin\Delta M t$) is not just $\sin 2\alpha$ but [11]

$$\sin 2\alpha - 2\left|\frac{A_2}{A_1}\right|\cos 2\alpha\cos(\delta_1-\delta_2)\sin(\phi_1-\phi_2) \tag{26}$$

where A_i, δ_i and ϕ_i are the amplitude, the strong phase and the weak phase, respectively, of the tree ($i=1$) and the penguin contribution ($i=2$) for $B_d\to\pi^+\pi^-$. The strong phases and

Final state	Transition	d	CKM factor	Remarks
$C=0,\, S=0$ no charm	$\overline{b}\to\overline{u}u\overline{d}$ $b\to u\overline{u}d$	-0.11	$\sin 2\alpha$	(i)
$C=0,\, S=0$ with charm	$\overline{b}\to\overline{c}c\overline{d}$ $b\to c\overline{c}d$	-0.41	$-\sin 2\beta$	(ii)
$C=-1,\, S=0$	$\overline{b}\to\overline{c}u\overline{d}$ $b\to u\overline{c}d$	-0.21	$\lambda\,\lvert V_{ub}/V_{cb}\rvert\,\sin(\alpha-\beta)$	(iii)
$C=1,\, S=0$	$\overline{b}\to\overline{u}c\overline{d}$ $b\to c\overline{u}d$	-0.21	$\lambda\,\lvert V_{ub}/V_{cb}\rvert\,\sin(\alpha-\beta)$	(iii)

Table 1: $\mathrm{Im}\xi \equiv d\cdot$(CKM factor) entering CP asymmetries in inclusive B_d decay. The dilution factor d is obtained with $m_b = 4.8\,\mathrm{GeV}$, $m_c = 1.4\ \mathrm{GeV}$, renormalization scale equal to m_b, $f_B = 180\,\mathrm{MeV}$, B=B_S=1. Remarks: (i) Penguin contribution should be taken into account, see Sect. 3. (ii) Penguin contribution enters with CKM combination $\sin(\alpha+\beta)\sin\beta/\sin\alpha$ and remains small. (iii) d does not include $1/m_b$ corrections.

$|A_2/A_1|$ are unknown and $\cos(\delta_1-\delta_2)$ could be one, in which case the penguin contribution remains invisible in the time dependence. In [12] it is argued that due to the penguin effects in $B_d\to\pi^+\pi^-$ the asymmetry, which is supposed to measure $\sin 2\alpha$, could be 0.4 even if $\sin 2\alpha\approx 0$. The situation is less problematic for larger values of $\sin 2\alpha$. It is possible to eliminate the penguin contribution by an isospin analysis [6], which requires the rates of $B^+\to\pi^+\pi^0$ and $B_d\to\pi^0\pi^0$ and their CP conjugates as additional input. The measurement of $B_d\to\pi^0\pi^0$ is experimentally extremely challenging, in particular in view of the very small branching fraction, which has been estimated to be below 10^{-6} [13].

In contrast, the B_d branching ratio governed by the inclusive $b\to u\overline{u}d$ transitions is at the 1% level, orders of magnitude larger than the exclusive case. The inclusive CP asymmetry we are proposing thus offers an alternative route towards measuring $\sin 2\alpha$. Unlike the exclusive case it is not contaminated by the presence of unknown strong interaction phases and the penguin contribution can be quantified. In addition, it has the potential of becoming an accurately known quantity as the knowledge of f_B, B and B_S improves, either through improvements in lattice gauge theory, or additional measurements of mixing parameters. We also emphasize that the assumption of local duality that underlies the theoretical prediction can be checked by measuring the lifetime difference of the B_s mass eigenstates. If local duality works for Γ_{12} generated by the $b\to c\overline{c}s$ transition, it will work only better for $b\to u\overline{u}d$. These advantages are somewhat compensated by the dilution of the asymmetry incurred by summing over many final states as well as the experimental challenge of performing an inclusive measurement. But even if the situation turns out to be more favorable for $B\to\pi\pi$, the inclusive measurement would provide a useful independent cross-check.

4. Inclusive CP asymmetries driven by other quark transitions

In this section we discuss the inclusive final states specified by $S=0$ and (a) $C=0$ with charmed particles (as opposed to Sect. 3), (b) $C=-1$ and (c) $C=1$ in decays of both B_d and B_s mesons. The branching fraction for the first channel is about 1% [20%] for B_d

Final state	Transition	d	CKM factor	Remarks
$C=0$, $S=0$ with charm	$\overline{b} \to \overline{c}c\overline{s}$ $b \to c\overline{c}s$	-0.51	$2\lambda^2\eta$	(i)
$C=-1$, $S=0$	$\overline{b} \to \overline{c}u\overline{s}$ $b \to u\overline{c}s$	-0.28	$-\eta$	(ii)
$C=1$, $S=0$	$\overline{b} \to \overline{u}c\overline{s}$ $b \to c\overline{u}s$	-0.28	$-\eta$	(ii)

Table 2: $\mathrm{Im}\xi \equiv d\cdot$(CKM factor) entering CP asymmetries in inclusive B_s decay. The dilution factor d is obtained with $m_b = 4.8\,\mathrm{GeV}$, $m_c = 1.4$ GeV, renormalization scale equal to m_b, $f_{B_s} = 210\,\mathrm{MeV}$, B=B_S=1. Remarks: (i) The penguin contribution has approximately the same CKM phase. (ii) d does not include $1/m_b$ corrections.

$[B_s]$, while (b) and (c) taken together comprise about 50% [3%] of all B_d $[B_s]$ decays. We emphasize that a significant fraction of time-evolved B_d's ($\sim$ 10%) are seen in channel (c) due to $B_d - \overline{B}_d$ mixing. This channel could be overlooked if one focussed on the tiny unmixed B_d rate governed by $\overline{b} \to \bar{u}c\bar{d}$.

We write

$$\mathrm{Im}\,\xi = d \cdot \mathrm{CKM\ factor}, \tag{27}$$

and list both factors in Tab. 1 for B_d and Tab. 2 for B_s. The asymmetry is then obtained from (11) – (14) (in the case of CP self-conjugate final states, for the general case see below). In order to obtain the 'dilution factor' d for other values of the bag factors B and B_S than those used in the tables, the given values can be multiplied by B_S for a rough estimate. For a different choice of decay constant f_B, we recall that d depends quadratically on f_B. The definition of the Wolfenstein parameters λ, ρ, η can be found, e.g., in [9].

Let us turn to case (a). For B_d decay, the asymmetry measures $\sin 2\beta$. The dilution factor is larger than for the charmless final state, mainly because the total width $\Gamma_{f,11}$ is phase space suppressed for the $b \to c\bar{c}d$ transition. We find a sizable $\mathrm{Im}\,\xi \approx 0.41 \sin 2\beta$, and with (14) a significant asymmetry $\mathcal{A} \approx 0.20 \sin 2\beta$. The penguin contribution is below 5% and has been neglected. The quantity r is expected to be numerically small [1] and vanishes identically in leading log approximation. The same final state for B_s decays normally involves an $s\bar{s}$ pair and leads to a Cabibbo-suppressed interference term $\mathrm{Im}\,\xi \approx -\lambda^2\eta$.

Cases (b) and (c) require some generalization of Sect. 2, because the final state is not self-conjugate, $f \neq \overline{f}$ in (6). To be definite consider case (b). The decay $B_d(t) \to f$ is related to the $\overline{b} \to \overline{c}u\overline{d}$ and $b \to u\bar{c}d$ transitions in the weak effective hamiltonian, not including the hermitian conjugates. We call this piece $\mathcal{H}^f_{eff}$. The 2×2 mixing matrix that corresponds to this decay is denoted by $\Gamma_{f,ij}$ and determined by the matrix elements of $\mathcal{H}^{f\dagger}_{eff}(x)\mathcal{H}^f_{eff}(0)$ according to (16). The decay $\overline{B}_d(t) \to \overline{f}$ is governed by the $b \to c\overline{u}d$ and $\overline{b} \to \overline{u}c\overline{d}$ transitions and the corresponding mixing matrix $\Gamma_{\overline{f},ij}$ is related to $\mathcal{H}^f_{eff}(x)\mathcal{H}^{f\dagger}_{eff}(0)$. Since $\mathcal{H}^f_{eff}$ is not hermitian, Γ_f and $\Gamma_{\overline{f}}$ are not equal, but both matrices are hermitian.

The final states with $|C| = 1, S = 0$ are governed by single CKM combinations. Consequently $\Gamma_{f,11} = \Gamma_{\overline{f},22}$, $\Gamma_{f,22} = \Gamma_{\overline{f},11}$. Furthermore, the off-diagonal elements of Γ_f and $\Gamma_{\overline{f}}$ coincide within the leading logarithmic approximation. Using these relations, we derive the asymmetries

$$\mathcal{A}(t) = \frac{2\,\mathrm{Im}\left((M_{12}^{*}\Gamma_{f,12})/|M_{12}|\right)\,\sin\Delta Mt - a\Gamma_{f,22}(1-\cos\Delta Mt)}{(1+\cos\Delta Mt)\Gamma_{f,11}+(1-\cos\Delta Mt)\Gamma_{f,22}} \qquad t \ll \frac{1}{\Delta\Gamma}, \tag{28}$$

$$\mathcal{A} = \frac{2x\,\mathrm{Im}\left(\frac{M_{12}^{*}}{|M_{12}|}\Gamma_{f,12}\right) - x^2 a\Gamma_{f,22}}{(2+x^2)\Gamma_{f,11}+x^2\Gamma_{f,22}}. \tag{29}$$

When one sums over cases (b) and (c), the final state $C=\pm1$ is self-conjugate and (13), (14) apply. Although no new information is obtained from this final state, it has the experimental advantage that the charge of the single charm quark need not be determined. Establishing that one, and only one, charmed hadron exists in the final state is sufficient.

Now for the final state with $C=-1$, $\Gamma_{f,11} \gg \Gamma_{f,22}$, because B_d decays through $\bar{b}\to\bar{c}u\bar{d}$, while $\overline{B}_d$ decays through the CKM suppressed channel $b\to u\bar{c}d$. For the final state with $C=1$, the situation is just opposite. Thus, we get

$$\mathcal{A} = \frac{2x}{2+x^2}\,\mathrm{Im}\,\xi \qquad C=-1, \tag{30}$$

$$\mathcal{A} = \frac{2}{x}\left[\mathrm{Im}\,\xi - \frac{x}{2}a\right] \qquad C=1, \tag{31}$$

$$\mathcal{A} = \frac{x}{1+x^2}\left[2\mathrm{Im}\,\xi - \frac{x}{2}a\right] \qquad |C|=1. \tag{32}$$

Here $\mathrm{Im}\,\xi$ is defined as $\mathrm{Im}\,\xi = \mathrm{Im}(M_{12}^{*}\Gamma_{f,12}/(|M_{12}|\Gamma_{f,11}))$ where $f=u\bar{c}d\bar{d}$. It is identical for all cases and can be read off from Tab. 1. Note that since here the CP conjugate of f is not yet included in the definition of the final state, an explicit factor of 2 appears in front of $\mathrm{Im}\,\xi$ in (32) which is absent in (14). With $|V_{ub}/V_{cb}|=0.08$, $\lambda=0.22$, we estimate $\mathrm{Im}\,\xi \approx 0.004\,\sin(\beta-\alpha)$. Assuming that the term involving a can still be neglected, we see that for $x=x_d=0.73$ the asymmetry is about five times larger for the final state with $C=1$ compared to $C=-1$, but still smaller than about 1%. Since β will be known from $B_d\to J/\psi K$, the angle α or, equivalently γ can be extracted.

We remark that the ratio $|V_{ub}/V_{cb}|$ that enters this asymmetry can in principle be obtained from the same measurement of the inclusive $b\to c\bar{u}q$ and $b\to u\bar{c}q$ (q=d,s) transitions, because

$$\frac{\Gamma_{f,22}}{\Gamma_{f,11}} = \frac{\Gamma(B_q^0\to\bar{f})}{\Gamma(B_q^0\to f)} = \left|\frac{V_{ub}V_{cq}}{V_{cb}V_{uq}}\right|^2. \tag{33}$$

While with neutral B mesons the CKM extraction requires time-dependent studies, it can also be performed using a ratio of inclusive time-integrated rates of $B^{\pm}$ decays. Since in leading order the phase space functions coincide for the $b\to c\bar{u}q$ and $b\to u\bar{c}q$ transitions, the calculable corrections to the above ratio arise only at order α_s and $1/m_b^3$. (This is in contrast to the determination of $|V_{ub}/V_{cb}|$ from the ratio of inclusive semileptonic rates $\Gamma(b\to u\ell\overline{\nu})/\Gamma(b\to c\ell\overline{\nu})$, where mass effects do not cancel at tree level.)

Turning to B_s mesons, we wish to consider several scenarios, because the $B_s-\overline{B}_s$ mixing parameter x_s is very large: (1) The $(\Delta m)_{B_s}t$ - oscillations can be resolved; (2) Although the $(\Delta m)_{B_s}t$ - oscillations cannot be resolved, the two exponentials $\exp(-\Gamma_H t)$ and $\exp(-\Gamma_L t)$ can be distinguished; (3) Only time-integrated measurements (with or without a cutoff) can be performed. Scenario (1) would be ideal, and would allow us to determine the relevant CKM parameters (phases and ratio of magnitudes) from flavor-tagged time-dependent studies. The CKM model predicts those time-dependent CP asymmetries to be of $\mathcal{O}(10\%)$ for inclusive transitions governed by $b \to c\overline{u}s/u\overline{c}s$ (Tab. 2). For B_s, $\Gamma_{f,22}$ is not CKM suppressed compared to $\Gamma_{f,11}$ and should therefore not be neglected. The quantity $\mathrm{Im}\,\xi$ shown in Tab. 2 is defined by $\mathrm{Im}\,\xi = \mathrm{Im}(M_{12}^{*}\Gamma_{f,12}/(|M_{12}|\Gamma_{f,11}))$ with $f = u\bar{c}s\bar{s}$. Detailed expressions for the asymmetries can be derived from (7), (8). If $(\Delta m)_{B_s}t$ - oscillations cannot be resolved, one could still extract CKM information from time-dependences of untagged data samples [scenario (2)] [14]. If only time-integrated measurements can be performed the asymmetries become very small, inversely proportional to x_s. In this case, the asymmetries could still provide constraints on either x_s or CKM parameters.

5. Conclusion and Outlook

One cannot overemphasize the importance of experimentally distinguishing among the various inclusive b-quark transitions. Once shown to be experimentally feasible, then (a) almost all b-hadrons could be flavor-tagged; (b) inclusive direct CP violating effects in charged B mesons and/or b-baryons could be probed; (c) new determinations of $|V_{ub}/V_{cb}|$ may become possible; (d) mixing-induced inclusive CP asymmetries, predicted to be sizable, could be searched for.

Both time-dependent and time-integrated studies can discover inclusive, mixing-induced CP violation. Whenever possible, time-dependent measurements should be pursued. While the experimental hurdles can be daunting, the advantages are obvious. Compared to exclusive transitions, inclusive modes have huge branching fractions, ranging between 1% to 50%, with optimal asymmetries between $\mathcal{O}(1\%)$ and $\mathcal{O}(20\%)$. The sum of modes governed by a b-quark transition dilutes the CP violating effects. However, this dilution factor is calculable, up to B meson matrix elements of local operators. These remaining hadronic parameters are already being calculated on the lattice and will be available more accurately in the future, either from improved lattice determinations or from other measurements of quark mixing parameters.

In particular, the asymmetry in B_d meson decay into charmless final states with no net strangeness provides a determination of $\sin 2\alpha$, that, compared to the determination from exclusive modes, does not suffer from uncontrolled penguin contributions, or the requirement to measure modes with tiny branching fractions. The inclusive asymmetry in B_d decays driven by $b \to c\bar{c}d$ is large and leads to an independent constraint on $\sin 2\beta$. The total inclusive time-dependent asymmetry, using all neutral B decays, is non-zero in general and measures a.

Single charmed final states, into which essentially half of all B_d mesons decay, are predicted to show CP asymmetries of up to 1%. CP violating effects of $\mathcal{O}(1\%)$ and $\mathcal{O}(10\%)$ are predicted for B_s decays governed by $b \to c\bar{c}s$ and $b \to u\bar{c}s/c\overline{u}s$, respectively, where both channels are a measure of η.

Meaningful results could already be obtained from existing data samples collected at LEP, SLC and Fermilab. CP violation is demonstrated if the number of flavor-tagged events differs from its CP-conjugated counterpart. (Here events denote specific neutral B decay topologies optimally weighted by additional information.) While flavor-tagging is automatically accomplished at polarized Z factories [15], it causes some statistical loss at unpolarized Z or $\Upsilon(4S)$ factories and at hadron accelerators. We are eager to learn about the CP information already contained within existing data. In the future, improved technology and dedicated experiments should then allow to probe in detail interesting aspects of flavor physics with inclusive B decays.

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References

[1] M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett. **43** (1979) 242; J.-M. Gérard and W.-S. Hou, Phys. Rev. **D43** (1991) 2909; H. Simma, G. Eilam and D. Wyler, Nucl. Phys. **B352** (1991) 367; R. Fleischer, Z. Phys. **C58** (1993) 483

[2] A. Pais and S.B. Treiman, Phys. Rev. **D12** (1975) 2744; T. Altomari, L. Wolfenstein and J.D. Bjorken, Phys. Rev. **D 37** (1988) 1860; M. Lusignoli, Z. Phys. **C41** (1989) 645

[3] I. Dunietz and R.G. Sachs, Phys. Rev. **D37** (1988) 3186; [(E) ibid. **D39** (1989) 3515]

[4] R. Aleksan et al., Phys. Lett. **B317** (1993) 173; Z.Phys. **C67** (1995) 251; Phys. Lett. **B356** (1995) 95

[5] A. Datta, E.A. Paschos and Y.L. Wu, Nucl. Phys. **B311** (1988) 35

[6] M. Gronau and D. London, Phys. Rev. Lett. **65** (1990) 3381

[7] I. Dunietz, FERMILAB-PUB-94/163-T, hep-ph/9409355

[8] M. Beneke, G. Buchalla and I. Dunietz, FERMILAB-PUB-96/095-T, hep-ph/9605259, *to appear in* Phys. Rev. **D54**

[9] G. Buchalla, A.J. Buras and M.E. Lautenbacher, "Weak Decays Beyond Leading Logarithms", FERMILAB-PUB-95/305-T, hep-ph/9512380, *to appear in* Rev. Mod. Phys.

[10] I. Bigi *et al.*, *in* B Decays, second edition, ed. S. Stone (World Scientific, Singapore, 1994)

[11] M. Gronau, Phys. Rev. Lett. **63** (1989) 1451

[12] M. Gronau, Phys. Lett. **B300** (1993) 163

[13] G. Kramer and W.F. Palmer, Phys. Rev. **D52** (1995) 6411

[14] I. Dunietz, Phys. Rev. **D52** (1995) 3048

[15] W.B. Atwood, I. Dunietz and P. Grosse-Wiesmann, Phys. Lett. **B216** (1989) 227; W.B. Atwood, I. Dunietz, P. Grosse-Wiesmann, S. Matsuda and A.I. Sanda, Phys. Lett. **B232** (1989) 533