LIFETIME DIFFERENCE OF B_s MESONS AND ITS IMPLICATIONS [a]

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We discuss the calculation of the width difference $\Delta\Gamma_{B_s}$ between the B_s mass eigenstates to next-to-leading order in the heavy quark expansion. $1/m_b$-corrections are estimated to reduce the leading order result by typically 30%. The error of the present estimate $(\Delta\Gamma/\Gamma)_{B_s} = 0.16^{+0.11}_{-0.09}$ could be substantially improved by pinning down the value of $\langle \bar{B}_s|(\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}|B_s\rangle$ and an accuracy of 10% in $(\Delta\Gamma/\Gamma)_{B_s}$ should eventually be reached. We briefly mention strategies to measure $(\Delta\Gamma/\Gamma)_{B_s}$, and its implications for constraints on ΔM_{B_s}, CKM parameters and the observation of CP violation in untagged B_s samples.

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1 Introduction

Mixing phenomena in neutral B meson systems provide us with an important probe of standard model flavordynamics and its interplay with the strong interaction. As is well-known, non-zero off-diagonal elements of the mixing matrix in the flavor basis $\{|B_s\rangle, |\bar{B}_s\rangle\}$ are generated in second order in the weak interaction through 'box diagrams'. In the B_s system[a] the off-diagonal elements obey the pattern

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right). \qquad (1)$$

The mass and lifetime difference between eigenstates are given by ('H' for 'heavy', 'L' for 'light')

$$\Delta M_{B_s} \equiv M_H - M_L = 2\,|M_{12}|, \qquad (2)$$

$$\Delta\Gamma_{B_s} \equiv \Gamma_L - \Gamma_H = -\frac{2\,\text{Re}\,(M_{12}^*\Gamma_{12})}{|M_{12}|} \approx -2\Gamma_{12}, \qquad (3)$$

up to very small corrections (assuming standard model CP violation). Anticipating the magnitudes of the eigenvalues, we have defined both ΔM_{B_s} and $\Delta\Gamma_{B_s}$ to be positive. Note that the lighter state is CP even and decays more rapidly than the heavier state.

The lifetime difference is an interesting quantity in several respects. Contrary to the neutral kaon system, it is calculable by short-distance methods and directly probes the spectator quark dynamics which generates lifetime differences among all b hadrons. If the mass difference ΔM_{B_s} turns out to be large, the lifetime difference also tends to be large and may well be the

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[a]For B_d mesons there is further CKM suppression and their lifetime difference will not be considered here.

first direct observation of mixing for B_s mesons. If $\Delta\Gamma_{B_s}$ is sizeable, CP violation in the B_s system can be observed without flavor-tagging[1].

The following sections summarize the calculation of Ref.[2] and discuss some of the implications of a non-zero $\Delta\Gamma_{B_s}$.

2 Heavy quark expansion of $\Delta\Gamma_{B_s}$

The mass difference is dominated by the top-quark box diagram, which reduces to a local $\Delta B = 2$ vertex on a momentum scale smaller than M_W. The lifetime difference, on the other hand, is generated by real intermediate states and is not yet local on this scale. But the b quark mass m_b provides an additional short-distance scale that leads to a large energy release (compared to Λ_{QCD}) into the intermediate states. Thus, at typical hadronic scales the decay is again a local process. The lifetime difference can then be treated by the same operator product expansion that applies to the average B_s lifetime and other b hadrons[3].

Summing over all intermediate states, the off-diagonal element Γ_{21} of the decay width matrix is given by

$$\Gamma_{21} = \frac{1}{2M_{B_s}}\langle \bar{B}_s|\,\text{Im}\,i\int d^4x\,T\,\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)|B_s\rangle \qquad (4)$$

with

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}}V_{cb}^*V_{cs}\big(C_1(\mu)(\bar{b}_i c_j)_{V-A}(\bar{c}_j s_i)_{V-A} \\ &\quad + C_2(\mu)(\bar{b}_i c_i)_{V-A}(\bar{c}_j s_j)_{V-A}\big). \end{aligned} \qquad (5)$$

Cabibbo suppressed and penguin operators in $\mathcal{H}_{eff}$ have not been written explicitly. In leading logarithmic approximation, the Wilson coefficients

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are given by $C_{2,1} = (C_+ \pm C_-)/2$, where

$$C_+(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{6/23} C_-(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{-12/23} \quad (6)$$

and μ is of order m_b.

The heavy quark expansion expresses $\Delta\Gamma_{B_s}$ as a series in local $\Delta B = 2$-operators. In the following we keep $1/m_b$-corrections to the leading term in the expansion. Keeping these terms fixes various ambiguities of the leading order calculation, such as whether the quark mass m_b or meson mass M_{B_s} should be used, and establishes the reliability of the leading order expression obtained in Ref.[4,5]. Compared to the 'exclusive approach' pursued in Ref.[6] that adds the contributions to $\Delta\Gamma_{B_s}$ from individual intermediate states, the inclusive approach is model-independent. The operator product expansion provides a systematic approximation in Λ_{QCD}/m_b, but it relies on the assumption of 'local duality'. The accuracy to which one should expect duality to hold is difficult to quantify, except for models[7] and eventually by comparison with data. We shall assume that duality violations will be less than 10% for $\Delta\Gamma_{B_s}$.

To leading order in the heavy quark expansion, the long distance contributions to $\Delta\Gamma_{B_s}$ are parameterized by the matrix elements of two dimension six operators

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \quad (7)$$

$$Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P} \quad (8)$$

between a $\bar{B}_s$ and B_s state. We write these matrix elements as

$$\langle Q \rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B, \quad (9)$$

$$\langle Q_S \rangle = -\frac{5}{6} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(m_b + m_s)^2} B_S, \quad (10)$$

where f_{B_s} is the B_s decay constant. The 'bag' parameters B and B_S are defined such that $B = B_S = 1$ corresponds to factorization. B also appears in the mass difference, while B_S is specific to $\Delta\Gamma_{B_s}$.

The matrix elements of these operators are not independent of m_b. Their m_b-dependence could be extracted with the help of heavy quark effective theory. There seems to be no gain in doing so, since the number of independent nonperturbative

parameters is not reduced even at leading order in $1/m_b$ and since we work to subleading order in $1/m_b$ even more parameters would appear. The matrix elements of the local $\Delta B = 2$-operators should therefore be computed in 'full' QCD, for instance on the lattice.

Including $1/m_b$-corrections, the width difference is found to be

$$\Delta\Gamma_{B_s} = \frac{G_F^2 m_b^2}{12\pi M_{B_s}}(V_{cb}^* V_{cs})^2 \sqrt{1 - 4z}$$
$$\cdot \left[\left((1-z)K_1 + \frac{1}{2}(1-4z)K_2 \right) \langle Q \rangle \right. \quad (11)$$
$$\left. + (1+2z)(K_1 - K_2)\langle Q_S \rangle + \hat{\delta}_{1/m} + \hat{\delta}_{rem} \right],$$

where $z = m_c^2/m_b^2$ and

$$K_1 = N_c C_1^2 + 2C_1 C_2 \qquad K_2 = C_2^2. \quad (12)$$

The $1/m_b$-corrections are summarized in

$$\hat{\delta}_{1/m} = (1+2z)\Big[K_1\left(-2\langle R_1 \rangle - 2\langle R_2 \rangle \right)$$
$$+ K_2\left(\langle R_0 \rangle - 2\langle \tilde{R}_1 \rangle - 2\langle \tilde{R}_2 \rangle \right) \Big]$$
$$- \frac{12z^2}{1-4z}\Big[K_1\left(\langle R_2 \rangle + 2\langle R_3 \rangle \right)$$
$$+ K_2\left(\langle \tilde{R}_2 \rangle + 2\langle \tilde{R}_3 \rangle \right) \Big]. \quad (13)$$

The operators R_i and $\tilde{R}_i$ involve derivatives on quark fields or are proportional to the strange quark mass m_s, which we count as Λ_{QCD}. For instance,

$$R_1 = \frac{m_s}{m_b}(\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S+P}, \quad (14)$$

$$R_2 = \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\rho D^\rho s_i)_{V-A}(\bar{b}_j s_j)_{V-A}. \quad (15)$$

The complete set can be found in Ref.[2]. Operators with gluon fields contribute only at order $(\Lambda_{QCD}/m_b)^2$. Since the matrix elements of the R_i, $\tilde{R}_i$ are $1/m_b$-suppressed compared to those of Q and Q_S, we estimate them in the factorization approximation, assuming factorization at a scale of order m_b (A smaller scale would be preferable, but would require us to calculate the anomalous dimension matrix.). Then all matrix elements can be expressed in terms of quark masses and the B_s mass and decay constant. No new nonperturbative

Table 1: Dependence of a, b and c on the b-quark mass (in GeV) and renormalization scale for fixed values of all other short-distance parameters. The last column gives $(\Delta\Gamma/\Gamma)_{B_s}$ for $B = B_S = 1$ (at given μ), $f_{B_s} = 210\,\text{MeV}$.

m_b	μ	a	b	c	$(\Delta\Gamma/\Gamma)_{B_s}$
4.8	m_b	0.009	0.211	−0.065	0.155
4.6	m_b	0.015	0.239	−0.096	0.158
5.0	m_b	0.004	0.187	−0.039	0.151
4.8	$2m_b$	0.017	0.181	−0.058	0.140
4.8	$m_b/2$	0.006	0.251	−0.076	0.181

parameters enter at order $1/m_b$ in this approximation.

The term $\hat{\delta}_{rem}$ denotes the contributions from Cabibbo-suppressed decay modes and penguin operators. They can be estimated[2] to be below $\pm 3\%$ and about -5%, respectively, relative to the leading order contribution. We neglect this term in the following numerical analysis.

3 Numerical estimate

It is useful to separate the dependence on the long-distance parameters f_{B_s}, B and B_S and write $(\Delta\Gamma/\Gamma)_{B_s}$ as

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left[aB + bB_S + c\right]\left(\frac{f_{B_s}}{210\,\text{MeV}}\right)^2 , \quad (16)$$

where c incorporates the explicit $1/m_b$-corrections. In the numerical analysis, we express Γ_{B_s} as the theoretical value of the semileptonic width divided by the semileptonic branching ratio. The following parameters are kept fixed: $m_b - m_c = 3.4\,\text{GeV}$, $m_s = 200\,\text{MeV}$, $\Lambda_{LO}^{(5)} = 200\,\text{MeV}$, $M_{B_s} = 5.37\,\text{GeV}$, $B(B_s \to Xe\nu) = 10.4\%$. Then a, b and c depend only on m_b and the renormalization scale μ. For some values of m_b and μ, the coefficients a, b, c are listed in Tab. 1. For a central choice of parameters, which we take as $m_b = 4.8\,\text{GeV}$, $\mu = m_b$, $B = B_S = 1$ and $f_{B_s} = 210\,\text{MeV}$, we obtain $(\Delta\Gamma/\Gamma)_{B_s} = 0.220 - 0.065 = 0.155$, where the leading term and the $1/m_b$-correction are separately quoted. We note that the $V - A$ 'bag' parameter B has a very small coefficient and is

practically negligible. The $1/m_b$-corrections are not small and decrease the prediction for $\Delta\Gamma_{B_s}$ by about 30%.

The largest theoretical uncertainties arise from the decay constant f_{B_s} and the second 'bag' parameter B_S. In the large-N_c limit, one has $B_S = 6/5$, while estimating B_S by keeping the logarithmic dependence on m_b (but not $1/m_b$-corrections as required here for consistency) and assuming factorization at the scale $1\,\text{GeV}$ gives[5] $B_S = 0.88$. B_S has never been studied by either QCD sum rules or lattice methods. In order to estimate the range of allowed $\Delta\Gamma_{B_s}$ conservatively, we vary $B_S = 1 \pm 0.3$, $f_{B_s} = (210 \pm 30)\,\text{MeV}$ and obtain

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.16^{+0.11}_{-0.09}. \quad (17)$$

This estimate could be drastically improved with improved knowledge of B_S and f_{B_s}.

4 Measuring $\Delta\Gamma_{B_s}$

In principle, both Γ_L and Γ_H can be measured by following the time-dependence of flavor-specific modes[1], such as $\bar{B}_s \to D_s l\nu$, given by

$$e^{-\Gamma_H t} + e^{-\Gamma_L t}. \quad (18)$$

In practice, this is a tough measurement. Alternatively, since the average B_s lifetime is predicted[2] to be equal to the B_d lifetime within 1%, it is sufficient to measure either Γ_L or Γ_H.

The two-body decay $B_s \to D_s^+ D_s^-$ has a pure CP even final state and measures Γ_L. Since D^0 and $D^\pm$ do not decay into ϕ as often as D_s, the $\phi\phi X$ final state tags a B_s-enriched B meson sample, whose decay distribution informs us about Γ_L.

A cleaner channel is $B_s \to J/\psi\phi$, which has both CP even and CP odd contributions. These could be disentangled by studying the angular correlations[8]. In practice, this might not be necessary, as the CP even contribution is expected[6] to be dominant by more than an order of magnitude. In any case, the inequality

$$\Gamma_L \geq 1/\tau(B_s \to J/\psi\phi) \quad (19)$$

holds. CDF[9] has fully reconstructed 58 $B_s \to J/\psi\phi$ decays from run Ia+Ib and determined $\tau(B_s \to J/\psi\phi) = 1.34^{+0.23}_{-0.19} \pm 0.05\,\text{ps}$. Together

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with $\tau(B_d) = 1.54 \pm 0.04$ ps, assuming equal average B_d and B_s lifetimes, this yields

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} \geq 0.3 \pm 0.4, \qquad (20)$$

which still fails to be significant.

In the Tevatron run II, as well as at HERA-B, one expects $10^3 - 10^4$ reconstructed $J/\psi\phi$, which will give a precise measurement of $\Delta\Gamma_{B_s}$.

5 Implications of non-zero $\Delta\Gamma_{B_s}$

5.1 CKM elements

Once $\Delta\Gamma_{B_s}$ is measured (possibly before ΔM_{B_s} is measured!), an alternative route to obtain the mass difference could use this measurement combined with the theoretical prediction for $(\Delta M/\Delta\Gamma)_{B_s}$ [1,10]. The decay constant f_{B_s} drops out in this ratio, as well as the dependence on CKM elements, since $|(V_{cb}V_{cs})/(V_{ts}V_{tb})|^2 = 1 \pm 0.03$ by CKM unitarity. However, the dependence on long-distance matrix elements does not cancel even at leading order in $1/m_b$ and the prediction depends on the ratio of 'bag' parameters B_S/B, which is not very well-known presently. We obtain $\Delta\Gamma/\Delta M = (5.6 \pm 2.6) \cdot 10^{-3}$, where the largest error (± 2.3) arises from varying B_S/B between 0.7 and 1.3.

When lattice measurements yield an accurate value of B_S/B as well as control over the $SU(3)$ flavor-symmetry breaking in Bf_B^2, the above indirect determination of ΔM_{B_s} in conjunction with the measured mass difference in the B_d system provides an alternative way of determining the CKM ratio $|V_{ts}/V_{td}|$, especially if the latter is around its largest currently allowed value. In contrast, the ratio $\Gamma(B \to K^*\gamma)/\Gamma(B \to \{\varrho,\omega\}\gamma)$ is best suited for extracting small $|V_{ts}/V_{td}|$ ratios, provided the long distance effects can be sufficiently well understood.

5.2 CP violation

The existence of a non-zero $\Delta\Gamma_{B_s}$ allows the observation of mixing-induced CP asymmetries without tagging the initial B_s or $\bar{B}_s$ [1,11]. These measurements are difficult, but the gain in statistics, when tagging is obviated, makes them worthwhile to be considered. The mass difference drops out in

the time dependence of untagged samples, which is given by

$$A_+(e^{-\Gamma_L t} + e^{-\Gamma_H t}) + A_-(e^{-\Gamma_L t} - e^{-\Gamma_H t}). \quad (21)$$

A_- carries CKM phase information even in the absence of direct CP violation.

In combination with an analysis of angular distributions, a measurement of the CKM angle γ from exclusive B_s decays governed by the $\bar{b} \to c\bar{c}s$ or $\bar{b} \to \bar{c}u\bar{s}$ transition can be considered [11].

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