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The Drell-Yan Process and Factorization in Impact Parameter Space *

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Abstract

The cross section for Drell-Yan pair production in the limit of small x_{target} is derived in the rest frame of the target hadron. Our calculation is based on the fundamental quantity $\sigma(\rho)$, the cross section for the scattering of a $q\bar{q}$-pair with fixed transverse separation ρ off a hadronic target. As in deep inelastic scattering the result can be given in terms of integrals of $\sigma(\rho)$. This is consistent with well known factorization theorems and also relates higher-twist terms in both processes. An analysis of the angular distribution of the produced lepton shows that additional integrals of $\sigma(\rho)$ can be obtained in the Drell-Yan process, which are not measurable in inclusive deep inelastic scattering.

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1 Introduction

The Drell-Yan (DY) process, i.e. the production of massive lepton pairs in hadronic collisions, has remained, together with deep inelastic scattering (DIS), one of the most prominent processes in strong interaction physics. In recent years, in connection with the availability of high energy machines like HERA and the Tevatron, much attention has been devoted to the small-x region of QCD, where parton densities become high and perturbative methods reach their limits.

Although extensive work exists in the field of small-x DIS and related processes, the small-x limit of lepton pair production has received only limited theoretical attention. In our opinion the small-x or high energy region of the DY process, i.e. the region where the lepton pair mass M is much smaller than the available energy $\sqrt{s}$, deserves study for at least two reasons:

First, it is of general theoretical interest to understand the interrelations between the high energy limits of DIS and DY pair production on both nucleon and nuclear targets. Though general factorization theorems are established (see e.g. [1]), it is still worthwhile to develop an intuition for the way they are realized specifically in the small-x region.

Second, the DY process may provide new tools for the experimental investigation of the small-x dynamics in QCD. In particular, lepton pair production in the region $M^2 \ll s$ may be one of the cleanest processes for the study of new phenomena in heavy ion collisions at future colliders.

Recently, a new approach to the DY process has been suggested by Kopeliovich [2], with the aim to understand the observed nuclear shadowing at small x_{target} [3]. It has been observed, that in analogy to DIS, the DY cross section at high energies can be expressed in terms of the scattering cross section of a color-neutral $q\bar{q}$-pair.

In the present investigation, we derive the high energy DY cross section in the target rest frame. The dominant underlying process is the scattering of a parton from the projectile structure function off the target color field. This parton radiates a massive photon, which subsequently decays into a lepton pair. Our treatment of the interaction of the projectile parton with the target hadron makes use of the high energy limit, but it is not restricted to the exchange of a finite number of gluons.

Using the non-perturbative $q\bar{q}$-cross section $\sigma(\rho)$, where ρ is the transverse separation of the pair, a parallel description of DIS and DY pair production is presented in the rather general framework given above. The cross section $\sigma(\rho)$ appears in DIS since the incoming photon splits into a $q\bar{q}$-pair, testing the target field at two transverse positions [4]. Similarly, $\sigma(\rho)$ appears in the DY process due to the interference of amplitudes in which the fast quark of the projectile hits the target at different impact parameters.

Our main focus is on the role of the photon polarization. The interplay of small and large transverse distances, characterized by different values of the parameter ρ, is compared in DIS and the DY process for transverse and longitudinal photons. In addition, the azimuthal angular correlations in the DY process provide a new tool for the investigation of $\sigma(\rho)$, which is not available in inclusive DIS.

The paper is organized as follows: After reviewing the impact parameter description of DIS in Sect. 2, an analogous calculation of the cross section for DY pair production is presented in Sect. 3. In Sect. 4 the angular distribution of the produced lepton is given in terms of integrals of the $q\bar{q}$-cross section $\sigma(\rho)$. Concluding remarks in Sect. 5 are followed by an appendix, which describes the technical details of the calculation.

2 Deep inelastic scattering in the target rest frame

A detailed discussion of small x DIS in the target rest frame and in impact parameter space has been given in [4]. The main non-perturbative input is the scattering cross section $\sigma(\rho)$ of a quark-antiquark pair with fixed transverse separation ρ. In the present section this approach is briefly reviewed and reformulated in a way allowing straightforward generalization to the DY process.

Consider first the scattering of a single energetic quark off an external color field, e.g. the field of a proton (Fig. 1). The complications associated with the color of the quark in the initial and final state can be neglected at this point, since the quark amplitude is only needed as a building block for the scattering of a color-neutral $q\bar{q}$-pair.

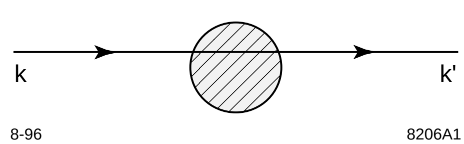

Fig.1 Scattering of a quark off the proton field.

In the high energy limit the soft hadronic field cannot change the energy of the quark significantly. Furthermore, we assume helicity conservation and linear growth of the amplitude with energy. Therefore, introducing an effective vertex $V(k',k)$, the amplitude can be given in the form

$$i2\pi\delta(k_0'-k_0)T_{fi}=\bar{u}_{s'}(k')V(k',k)u_s(k)=i2\pi\delta(k_0'-k_0)\,2k_0\,\delta_{s's}\,\tilde{t}_q(k_\perp'-k_\perp)\,. \quad (1)$$

Here $\tilde{t}_q(p_\perp)$ can be interpreted as the Fourier transform of an impact parameter space amplitude,

$$\tilde{t}_q(p_\perp)=\int d^2x_\perp\, t_q(x_\perp)\, e^{-ip_\perp x_\perp}\,. \quad (2)$$

Note that t is a matrix in color space.

If the interaction of the quark with the color field is treated in the non-Abelian eikonal approximation, t_q is given explicitly by [5]

$$1+it_q(x_\perp)=F(x_\perp)=P\exp\left(-\frac{i}{2}\int_{-\infty}^{\infty}A_-(x_+,x_\perp)dx_+\right)\,. \quad (3)$$

Here $x_\pm = x_0 \pm x_3$ are the light-cone components of x, $A(x_+,x_\perp)$ is the gauge field, and the path ordering P sets the field at smallest x_+ to the rightmost position. The x_--dependence of A is irrelevant as long as it is sufficiently smooth.

However, our analysis in the following does not rely on the specific form of t_q provided by the eikonal approximation Eq. (3).

Consider now the forward elastic scattering of a photon with virtuality Q^2 off an external field which is related to the total cross section via the optical theorem. In the limit of very high photon energy, corresponding to the small-x region, the dominant process is the fluctuation of the photon into a $q\bar{q}$-pair long before the target (see Fig. 2). The quark and antiquark then scatter independently off the external field and recombine far behind the target. The virtualities of the quarks, which are small compared to their energies, do not affect their effective scattering vertices. They enter the calculation only via the explicit quark propagators connected to the photon.

The necessary calculations have been performed many years ago for the Abelian case in light-cone quantization [6] and, more recently, in a covariant approach, treating two gluon exchange in the high energy limit [4].

In the notation of [4] the transverse and longitudinal photon cross sections read

$$\sigma_{T,L}=\int_0^1 d\alpha\int d^2\rho_\perp\sigma(\rho)W_{T,L}(\alpha,\rho)\,, \quad (4)$$

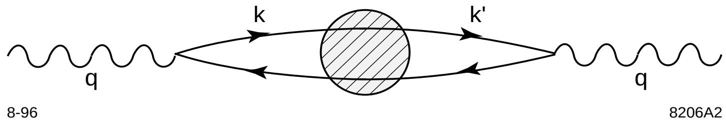

Fig.2 Elastic forward scattering of a virtual photon off the proton field.

where $\rho = |\rho_\perp|$ is the transverse separation of quark and antiquark when they hit the target proton, and α is the longitudinal momentum fraction of the photon carried by the quark. The cross section $\sigma(\rho)$ for the scattering of the $q\bar{q}$-pair is given by

$$\sigma(\rho) = \frac{2}{3}\,\mathrm{Im}\int d^2x_\perp\,\mathrm{tr}\,[it_q(x_\perp)t_{\bar{q}}(x_\perp+\rho_\perp)+t_q(x_\perp)+t_{\bar{q}}(x_\perp+\rho_\perp)]\,. \tag{5}$$

Here $t_q(x_\perp)$ is the quark scattering amplitude in impact parameter space introduced above and $t_{\bar{q}}(x_\perp+\rho_\perp)$ is its antiquark analogue. The last two terms in Eq. (5) correspond to diagrams where only the quark or only the antiquark is scattered.

We denote by $W_{T,L}$ the squares of the light-cone wave functions of a transverse photon and a longitudinal photon with virtuality Q^2. In the case of one massless quark generation with one unit of electric charge they are given by

$$W_T(\alpha,\rho) \;=\; \frac{6\alpha_{\mathrm{em}}}{(2\pi)^2}N^2[\alpha^2+(1-\alpha)^2]K_1^2(N\rho) \tag{6}$$

$$W_L(\alpha,\rho) \;=\; \frac{24\alpha_{\mathrm{em}}}{(2\pi)^2}N^2[\alpha(1-\alpha)]K_0^2(N\rho)\,, \tag{7}$$

where $N^2 = N^2(\alpha,Q^2) \equiv \alpha(1-\alpha)Q^2$ and $K_{0,1}$ are modified Bessel functions. As is illustrated in Fig. 3 the variables α and $1{-}\alpha$ denote the longitudinal momentum fractions of the photon carried by quark and antiquark.

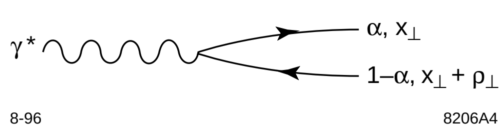

Fig.3 Light-cone wave function of the virtual photon in the mixed representation.

Notice that the color factor 1/3 in Eq. (5) is compensated by a factor 3 in Eqs. (6),(7), so that $\sigma(\rho)$ is the cross section for one color-neutral $q\bar{q}$-pair and the color summation is included in the definition of $W_{T,L}$.

Consider now the region of a relatively soft quark, $\alpha < \Lambda^2/Q^2$, with a hadronic scale $\Lambda \ll Q$. This region, where $N^2 \simeq \alpha Q^2 = a^2$, corresponding to Bjorken's aligned jet model [7] (see also [8]), gives a higher-twist contribution to σ_L and a leading-twist contribution to σ_T,

$$\sigma_{T,\bar{q}} = \frac{6\alpha_{\rm em}}{(2\pi)^2 Q^2} \int_0^{\Lambda^2} da^2 \int d^2\rho_\perp a^2 K_1^2(a\rho)\sigma(\rho) \,. \tag{8}$$

A possible interpretation of DIS in this kinematical region is the splitting of the photon into a fast, on-shell antiquark and a soft quark, which is not far off-shell. In a frame where the proton is fast, the latter one corresponds to an incoming antiquark described by a scaling antiquark distribution $\bar{q}(x)$. We thus denote the cross section by $\sigma_{T,\bar{q}}$. Using the standard formula for the contribution of the antiquark structure to the transverse cross section,

$$\sigma_{T,\bar{q}} = \frac{(2\pi)^2\alpha_{\rm em}}{Q^2} x\bar{q}(x) \,, \tag{9}$$

the antiquark distribution can then be given in terms of the $q\bar{q}$-cross section,

$$x\bar{q}(x) = \frac{6}{(2\pi)^4} \int_0^{\Lambda^2} da^2 \int d^2\rho_\perp a^2 K_1^2(a\rho)\sigma(\rho) \,. \tag{10}$$

This formula will be reproduced below from the impact parameter space description of DY pair production at small $x_{\rm target}$, in agreement with factorization.

The above discussion in terms of scatterings off an external field can be generalized to the case of a realistic hadron target by summing appropriately over all contributing field configurations. Such an approach has already been used for the treatment of DIS in [9] and for the treatment of diffraction in [10].

3 Drell-Yan Process in the target rest frame

In this section the DY pair production cross section at small $x_{\rm target}$ will be calculated in the target rest frame. Such an approach has recently been suggested by Kopeliovich in the context of nuclear shadowing [2].

Consider the kinematical region where the mass of the produced lepton pair is large compared to the hadronic scale, but much smaller than the hadronic center of mass energy, $\Lambda^2 \ll M^2 \ll s$. Furthermore, let the longitudinal momentum fraction x_F of the projectile hadron carried by the DY pair be large, but not too close to 1. We assume here that the last condition allows us to neglect higher-twist contributions from spectator partons in the projectile [11].

In the parton model the above process is described as the fusion of a projectile quark with momentum fraction $x \approx x_F$ and a target antiquark with momentum fraction $x_{\mathrm{target}} \approx M^2/sx_F \ll 1$. (Here and below we neglect the antiquark distribution of the projectile at the relevant values of x_F.)

However, a different physical picture of this process is appropriate in the target rest frame: A large-x quark of the projectile scatters off the gluonic field of the target and radiates a massive photon, which subsequently decays into leptons (compare [12]). The two relevant diagrams, corresponding to the photon being radiated before or after the interaction with the target, are shown in Fig. 4. Diagrams where the quark interacts with the target both before and after the photon vertex are suppressed in the high energy limit [13]. Note that in the above approach no antiquark distribution of the target has to be introduced. Instead, its effect is produced by the target color field.

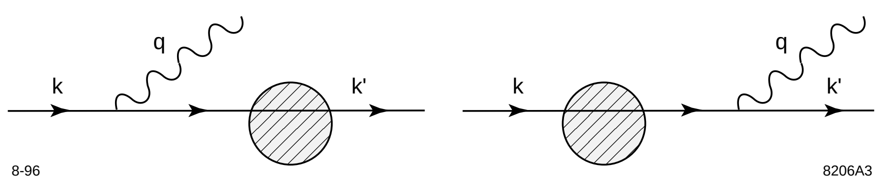

Fig.4 Production of a massive photon by a quark scattering off the target field. A quark with momentum k interacts with an external field producing a photon with momentum q and an outgoing quark with momentum k'.

In the high energy limit, i.e. $q_0, k_0, k_0' \gg M^2$, the corresponding cross section, including the decay of the photon into the lepton pair, reads ($e^2 = 4\pi\alpha_{\mathrm{em}}$)

$$\frac{d\hat{\sigma}}{dx_F dM^2} = \frac{e^2}{72(2\pi)^3} \cdot \frac{1}{x_F k_0 k_0' M^2} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{d^2 k_\perp'}{(2\pi)^2} |T|^2 \,. \tag{11}$$

Here T is the amplitude for the production of the virtual photon, given by the sum of the two diagrams in Fig. 4,

$$i2\pi\delta(q_0+k_0'-k_0)T_\lambda = e\bar{u}_{s'}(k') \left[V(k', k-q)\frac{i}{\not{k} - \not{q}}\not{\epsilon}_\lambda(q) + \not{\epsilon}_\lambda(q)\frac{i}{\not{k}' + \not{q}}V(k'+q, k)\right] u_s(k)\,. \tag{12}$$

The matrix V is the effective quark scattering vertex introduced in the previous section, and $\epsilon(q)$ is the polarization vector of the produced photon, accessible via the lepton angular distribution. Averaging over s and summation over s' and λ is understood in Eq. (11).

When the cross section is explicitly calculated, the quark scattering amplitudes $t_q(x_\perp)$ implicit in V combine in a way very similar to the case of DIS. Therefore, the final result can be expressed in terms of the $q\bar{q}$-cross section introduced in the previous section [2]. This cross section arises from the interference of the two diagrams in Fig. 4. To understand the parallelism of the DY process and DIS, observe that in the DY cross section the product of two quark amplitudes tests the external field at two different transverse positions. In DIS this corresponds to the quark-antiquark pair wave function of the virtual photon, which tests the external field at two transverse positions as well. The details of this calculation are presented in the appendix.

To make the analogy to DIS more apparent, the cross sections for the production of the lepton pair via transversely and longitudinally polarized photons are given separately:

$$\frac{d\sigma_{T,L}}{dx_F dM^2} = \frac{\alpha_{\rm em}}{9(2\pi)M^2} \int\limits_0^{(1-x_F)/x_F} d\alpha \int d^2\rho_\perp \frac{q\Big(x_F(1+\alpha)\Big)}{(1+\alpha)^2} \sigma(\rho) W^{DY}_{T,L}(\alpha,\rho)\,. \tag{13}$$

Here $q(x)$ is the quark distribution of the projectile, $\alpha = k_0'/q_0$ is the ratio of energies or longitudinal momenta of outgoing quark and photon, and $W^{DY}_{T,L}$ are the analogues of the squares of the photon wave functions defined for DIS in the previous section,[1]

$$W_T^{DY}(\alpha,\rho) = \frac{12\alpha_{\rm em}}{(2\pi)^2} N^2[\alpha^2 + (1+\alpha)^2] K_1^2(N\rho) \tag{14}$$

$$W_L^{DY}(\alpha,\rho) = \frac{24\alpha_{\rm em}}{(2\pi)^2} N^2[\alpha(1+\alpha)] K_0^2(N\rho)\,. \tag{15}$$

As in the DIS case a subsidiary variable $N^2 = \alpha(1+\alpha)M^2$ has been introduced.

We have defined the polarization of the massive photon in the u-channel frame. In this frame the photon is at rest and the z-axis, defining the longitudinal polarization vector, is antiparallel to the momentum of the target hadron. Since the polarizations are invariant with respect to boosts along the z-axis, one could also say that the longitudinal polarization is defined by the direction of the photon momentum, in a frame where photon and target hadron momenta are antiparallel. This last definition makes it obvious that the polarizations in the u-channel frame of DY pair production are analogous to the standard polarization choice in DIS, defining σ_T and σ_L.

[1] Our formula for the transverse polarization is similar but not identical to the result given in [2].

Note, that Eqs. (14),(15) can be obtained from their analogues in DIS, Eqs. (6),(7), by the substitutions $Q^2 \to M^2$ and $1-\alpha \to 1+\alpha$. The last substitution reflects the fact that the longitudinal parton momenta in units of the photon momentum are α and $1-\alpha$ in DIS, as opposed to α and $1+\alpha$ in DY pair production. Since the transverse polarizations are summed rather than averaged in the DY process, an additional factor of 2 appears in Eq. (14) as compared to Eq. (6).

Consider now the region of a relatively soft outgoing quark, $\alpha < \Lambda^2/M^2$, with a hadronic scale $\Lambda \ll M$. In analogy to the DIS case, this region gives a higher-twist contribution for longitudinal polarization and a leading-twist contribution for transverse polarization:

$$\frac{d\sigma_{T,\bar{q}}}{dx_F dM^2} = \frac{\alpha_{\rm em}^2 q(x_F)}{6\pi^3 M^4} \int_0^{\Lambda^2} da^2 \int d^2\rho_\perp a^2 K_1^2(a\rho)\sigma(\rho)\,. \tag{16}$$

Here, assuming a sufficiently smooth behavior of $q(x)$, terms suppressed by powers of Λ/M have been dropped.

The above kinematical region corresponds to the contribution from the antiquark distribution of the target as calculated in the parton model at leading order,

$$\frac{d\sigma_{T,\bar{q}}}{dx_F dM^2} = \frac{4\pi\alpha_{\rm em}^2}{9M^4} q(x_F) \cdot x_t \bar{q}(x_t)\,. \tag{17}$$

By comparing this formula with Eq. (16), an expression for $x\bar{q}(x)$ can be derived which is identical to Eq. (10) obtained in the case of DIS. This, of course, was to be expected in view of the factorization theorems (see e.g. [1]) relating DIS and the DY process.

So far the target hadron has been treated simply as a given external color field. As already pointed out in the last section, a more realistic model has to include an appropriate summation over all contributing field configurations. These field configurations, together with the produced lepton pair and the projectile remnant, form the final state of the scattering process. If we assume that at some stage before hadronization the target field is separated from the rest of the final state, the inclusiveness of the process translates into a summation over all field configurations in the cross section. This corresponds exactly to the discussion of the previous section, where a summation over all field configurations of the target had to be performed for the cross section of DIS.

4 Angular distributions

In DY pair production the transverse and longitudinal photon polarizations can be distinguished by measuring the angle between the direction of the decay lepton and the z-axis. However, more information can be obtained by considering the azimuthal angle as well. In particular, additional integrals involving the $q\bar{q}$-cross section $\sigma(\rho)$ are provided by the angular correlations.

As explained in the last section the u-channel frame is most suitable for an analysis along the lines of small-x DIS. We work with a right-handed coordinate system, the z-axis being antiparallel to $\vec{p}_t$ and the y-axis parallel to $\vec{p}_p \times \vec{p}_t$, where $\vec{p}_p$ and $\vec{p}_t$ are the projectile and target momenta in the photon rest frame (see e.g. [14]).

The direction of the produced lepton is characterized by the standard polar and azimuthal angles θ and ϕ. To obtain the complete angular dependence of the cross section, interference terms between different photon polarizations have to be considered in equations analogous to (11) and (12). The obtained contributions are multiplied by typical angle dependent functions obtained from the leptonic tensor (for details see e.g. [15]).

In general, the angular dependence can be given in the form

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \sim 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi \,. \tag{18}$$

The cross section will be presented after integration over the transverse momentum of the pair. The dependence on $q_\perp^2$ can be recovered from the formulae in the appendix, where some details of the calculation are given.

In compact notation the results of our impact parameter space calculation of the DY cross section read

$$\frac{d\sigma}{dx_F dM^2 d\Omega} = \frac{\alpha_{\rm em}^2}{2(2\pi)^4 M^2} \int_0^{(1-x_F)/x_F} d\alpha \int d^2 r_\perp \frac{q\Big(x_F(1+\alpha)\Big)}{(1+\alpha)^2} \sigma(r/N) \sum_i f_i(\alpha, r) h_i(\theta,\phi)\,, \tag{19}$$

where $i \in \{T,\, L,\, TT,\, LT\}$ labels the contributions of transverse and longitudinal polarizations and of the transverse-transverse and longitudinal-transverse interference terms. Note, that in contrast to Eq. (13) here the integration is over the dimensionless variable $r_\perp = N\rho_\perp$. The angular dependence is given by the functions

$$h_T(\theta,\phi) = 1 + \cos^2\theta \quad , \qquad h_{TT}(\theta,\phi) = \sin^2\theta \cos 2\phi \,, \tag{20}$$

$$h_L(\theta,\phi) = 1 - \cos^2\theta \quad , \qquad h_{LT}(\theta,\phi) = \sin 2\theta \cos\phi \,. \tag{21}$$

Finally, the α- and r-dependent coefficients read

$$
\begin{aligned}
f_T(\alpha, r) &= [\alpha^2 + (1+\alpha)^2] K_1^2(r) && (22)\\
f_L(\alpha, r) &= 4\alpha(1+\alpha) K_0^2(r) && (23)\\
f_{TT}(\alpha, r) &= \alpha(1+\alpha)\Big[r^{-1}K_1''(r) + r^{-2}K_1'(r) - r^{-3}K_1(r) - 2K_1^2(r)\Big] && (24)\\
f_{LT}(\alpha, r) &= r^{-1}(1+2\alpha)\sqrt{\alpha(1+\alpha)}\Big[K_0(r)\Big(rA(r) - 1\Big) && (25)\\
&\quad -K_1(r)\Big((2r)^{-1} - rA'(r)\Big) - K_1'(r)/2\Big] .
\end{aligned}
$$

Here the first two functions give the transverse and longitudinal contributions of the last section. The function A is defined by the following definite integral, that can be expressed through the difference of the modified Bessel function I_0 and the modified Struve function $\mathbf{L}_0$ [17],

$$
A(r) = \int_0^\infty \frac{dt \sin rt}{\sqrt{1+t^2}} = \frac{\pi}{2}\Big(I_0(r) - \mathbf{L}_0(r)\Big) . \tag{26}
$$

As discussed in the previous section the integral involving f_T receives a contribution from large ρ. In Eq. (19) this is most easily seen by recalling that $\sigma(\rho) \sim \rho^2$ at small ρ. Replacing $\sigma(r/N)$ with the model form r^2/N^2 results in a divergent α-integration. This shows the sensitivity to the large ρ-behavior of $\sigma(\rho)$. In DY pair production on nuclei this sensitivity will show up as leading-twist shadowing, since configurations with large cross section are absorbed at the surface. This is analogous to the leading-twist shadowing in DIS [16].

In contrast to the integral of f_T, the integrals involving f_L, f_{TT} and f_{LT} are dominated by the region of small ρ at leading twist. To see this, notice that replacing $\sigma(r/N)$ with r^2/N^2 in Eq. (19) results in finite α-integrations for f_L, f_{TT} and f_{LT}. This leading-twist contribution corresponds to the effect of the gluon distribution of the target. Integrations involving higher powers of r are sensitive to large transverse distances, but they are suppressed by powers of M. This corresponds to the fact that in the leading order (and leading-twist) parton model these angular coefficients vanish.

The above discussion shows that in the longitudinal contribution and in the interference terms, shadowing appears only at higher twist or at higher order in α_S. While higher-twist terms are suppressed by Λ^2/M^2 in the longitudinal cross section and in the transverse-transverse interference term, they are only suppressed by Λ/M in the longitudinal-transverse interference. This results from the weaker suppression of f_{LT} at small α (see Eq. (25)).

The presented formulae contain contributions from all transverse sizes of the effective $q\bar{q}$-pair interacting with the target gluonic field, thus including all higher-twist corrections from this particular source. Our analysis also gives a simple and intuitive derivation of the dominant QCD-corrections at small x, associated with the gluon distribution of the target.

5 Conclusions

A detailed calculation of the DY cross section, including its angular dependence, has been performed in the target rest frame in the limit of high energies and small x_{target}. The close similarity with the impact parameter description of DIS has been established for transverse and longitudinal photon polarizations and the availability of additional angular observables in the DY process has been demonstrated.

As is well known, in the small-x limit DIS can be calculated from the elastic scattering of the quark-antiquark component of the virtual photon wave function off the hadronic target. The DIS cross section is given by a convolution of the photon wave function with the $q\bar{q}$-cross section $\sigma(\rho)$. This picture holds even when the interaction of each of the quarks with the target is completely non-perturbative.

The cross section for DY pair production via transversely and longitudinally polarized massive photons can be given as a convolution of the above $q\bar{q}$-cross section with analogues of the transverse and longitudinal photon wave functions. These functions depend on the photon momentum fractions carried by the quarks and on the photon virtuality in exactly the same way as in DIS. For this analogy to hold polarization by polarization the DY process has to be analyzed in the u-channel frame. This frame corresponds to the $\gamma^* p$-frame of small-x DIS, since it uses photon and target hadron momenta for the definition of the z-axis.

As in the DIS case, the transverse photon contribution is sensitive to large distances in impact parameter space. It receives a leading-twist contribution from large ρ, which corresponds to the effect of a non-perturbative antiquark distribution in the target. Our approach includes, beyond this leading-twist contribution and the α_S-correction from small ρ, all higher-twist terms associated with different transverse distances inside the target. The universal function $\sigma(\rho)$ relates these contributions directly to the analogous terms in DIS.

In addition to the transverse-longitudinal analysis, which can be performed using the polar angle of the produced lepton, the azimuthal angle allows the investigation of interference terms of different polarizations. These terms involve convolutions of $\sigma(\rho)$ with new functions, not available in DIS.

Our analysis shows that rather detailed information about the function $\sigma(\rho)$ can be obtained from a sufficiently precise measurement of angular correlations in the DY process at small x_{target}. Even more could be learned from a measurement of the nuclear dependence of these angular correlations. Using the Glauber approach to nuclear shadowing, this type of measurement would provide additional information about the functional dependence of the $q\bar{q}$-cross section on ρ. We expect that future measurements of the DY process will help to disentangle the interplay of small and large transverse distances in small-x physics.

Several aspects of the presented approach require further study:

The leading-twist part of our calculation combines the standard $q\bar{q}$-annihilation cross section with the α_S-corrections associated with the target gluon density. It is certainly necessary to include other α_S-corrections systematically into our approach. For example, corrections associated with the radiation of a gluon off the projectile quark can be treated in the impact parameter space by methods developed in [10].

Furthermore, higher-twist contributions from sources not considered here should be carefully analyzed. At small x_{target}, corresponding to large x_F, higher-twist corrections from comoving projectile partons are potentially important [11].

Finally, to go beyond the classical field model, we have argued that the summation over all field configurations of the target is identical in DIS and the DY process. It would be highly desirable, to derive this statement in the framework of QCD and to specify the type of expected corrections.

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Appendix

Some details of the calculations leading to the results of Sections 3 and 4 are presented below.

In analogy to Eq. (11) the angular distribution of the lepton in the DY process is given by

$$\frac{d\hat{\sigma}}{dx_F dM^2 d\Omega} = \frac{e^2}{192(2\pi)^4} \cdot \frac{1}{x_F k_0 k'_0 M^4} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \Big(T_\lambda T^*_{\lambda'} \Big) L^{\mu\nu} \epsilon^\lambda_\mu \epsilon^{\lambda' *}_\nu \,, \tag{27}$$

where the polarization sum is understood. The leptonic tensor $L^{\mu\nu}$ is contracted with the photon polarization vectors

$$\epsilon_\pm = (0, 1, \pm i, 0)\,, \quad \epsilon_0 = (0, 0, 0, 1)\,, \tag{28}$$

defined in the u-channel frame, which has been specified in Sect. 4. This expression gives the functions $h_i(\theta, \phi)$, introduced in Eqs. (20),(21).

The amplitudes T_λ are most conveniently calculated in the target rest frame, in a system where $q_\perp = 0$. This corresponds to the u-channel frame, boosted appropriately along its z-axis. Of course now the amplitude is a function of $k_\perp$ ($k_\perp \neq 0$), and the $q_\perp$-integration in Eq. (27) has to be replaced by a $k_\perp$-integration,

$$\int \frac{d^2 q_\perp}{q_0^2} \to \int \frac{d^2 k_\perp^2}{k_0^2}\,, \tag{29}$$

leading to

$$\begin{aligned} \frac{d\hat{\sigma}}{dx_F dM^2 d\Omega} &= \frac{e^2}{96(2\pi)^4} \cdot \frac{q_0^2}{x_F k_0^3 k'_0 M^4} \int \frac{d^2 k_\perp d^2 k'_\perp}{(2\pi)^4} \Bigg[\frac{h_T}{2} \Big(|T_+|^2 + |T_-|^2 \Big) + h_L |T_0|^2 \\ & - \frac{h_{TT}}{2} \Big(T_+ T_-^* + T_- T_+^* \Big) - \frac{h_{LT}}{2\sqrt{2}} \Big(T_0 T_+^* + T_0 T_-^* + T_+ T_0^* + T_- T_0^* \Big) \Bigg] . \end{aligned} \tag{30}$$

Now the amplitudes have to be calculated explicitly. In the high energy approximation the fermion propagators appearing in Eq. (12) can be treated as follows,

$$\frac{1}{\not p} \approx \frac{\sum_r u_r(p) \bar{u}_r(p)}{p^2}\,. \tag{31}$$

Here $u(p) \equiv u(p_+, \bar{p}_-, p_\perp)$, with $\bar{p}_- \equiv p_\perp^2 / p_+$, for off-shell momentum p. This approximation, which has been used in the above form in [10], corresponds to dropping the instantaneous terms in light-cone quantization [6].

In our frame with $q_\perp = 0$ the resulting spinor products in the high energy approximation are given by

$$\bar{u}(k-q)\not{\epsilon}_\lambda u(k) \equiv g_\lambda(k_\perp, q_0, \alpha)\,, \quad \bar{u}(k')\not{\epsilon}_\lambda u(k'+q) \equiv g_\lambda(k'_\perp, q_0, \alpha)\,, \tag{32}$$

where the helicity dependence has been suppressed. By choosing some spinor representation explicit formulae are easily obtained.

Using the definition of $\tilde{t}_q$ in Eq. (1), the following expression for the product of two amplitudes can now be given,

$$T_\lambda T^*_{\lambda'} = \Big[2eq_0\alpha(1{+}\alpha)\Big]^2 \Big|\tilde{t}_q(k'_\perp{-}k_\perp)\Big|^2 \left(\frac{g_\lambda(k_\perp)}{k_\perp^2{+}N^2} - \frac{g_\lambda(k'_\perp)}{k'^2_\perp{+}N^2}\right)\left(\frac{g_{\lambda'}(k_\perp)}{k_\perp^2{+}N^2} - \frac{g_{\lambda'}(k'_\perp)}{k'^2_\perp{+}N^2}\right)^*. \tag{33}$$

Here the q_0- and α-dependence of the function g has been suppressed. To relate this formula to the $q\bar{q}$-cross section of Sect. 2, observe that in the high energy limit the quark and antiquark scattering amplitudes are dominated by gluon exchange. This implies the relation $t_{\bar{q}}(x_\perp) = -t_q^\dagger(x_\perp)$. Note in particular, that this relation is respected by the eikonal approximation Eq. (3). From the definition of $\sigma(\rho)$ in Eq. (5) one now derives

$$|\tilde{t}_q(k'_\perp{-}k_\perp)|^2 = -\frac{3}{2}\int d^2\rho_\perp \sigma(\rho)\, e^{i\rho_\perp(k'-k)_\perp} + 2(2\pi)^2\delta^2(k'_\perp{-}k_\perp)\,\mathrm{Im}\,\tilde{t}_q(0)\,. \tag{34}$$

When inserted into Eq. (33) the second term on the r.h.s. of Eq. (34) does not contribute, so that the product $T_\lambda T^*_{\lambda'}$ can indeed be expressed through the the $q\bar{q}$-cross section $\sigma(\rho)$.

Two remarks have to be made concerning the treatment of the photon polarization vectors in Eqs. (32),(33):

First, recall that Eq. (33) holds in the target rest frame with z-axis parallel to $\vec{q}$. Therefore, the transverse polarizations are the same as in the u-channel frame (see Eq. (28)). However, before the $k_\perp$-integration in Eq. (30) can be performed, the $k_\perp$-dependence of the orientation of x- and y-axis has to be explicitly introduced. This is most easily done by assuming k to be exactly parallel to the projectile momentum and writing $\hat{e}_x = k_\perp/|k_\perp|$ (see the definition of the u-channel frame in Sect. 4).

Second, the boost from the u-channel frame to the target rest frame transforms the longitudinal polarization vector to

$$\epsilon_0 = \frac{q}{M} - \frac{M}{q_0}(1,\vec{0})\,. \tag{35}$$

However, taking advantage of gauge invariance, the first term can be dropped. This significantly simplifies the evaluation of the corresponding spinor products in Eq. (32).

It is now straightforward to choose an explicit spinor representation, to evaluate Eq. (32), and to combine Eqs. (34),(33) and (30). Performing the $k_\perp$ and $k'_\perp$-integrations, the result stated in Eq. (19) is obtained.

References

[1] J.C. Collins, D.E. Soper, and G. Sterman, in *Perturbative Quantum Chromodynamics*, ed. by A.H. Mueller (World Scientific, 1989)

[2] B. Kopeliovich, Proc. of the *XXIII Int. Workshop on Gross Properties of Nuclei and Nucl. Excitations* (Hirschegg, 1995)

[3] E772 Collab., D.M. Alde et al., Phys. Rev. Lett. 64 (1990) 2479

[4] N.N. Nikolaev and B.G. Zakharov, Z. Phys. C49 (1991) 607

[5] O. Nachtmann, Ann. Phys. 209 (1991) 436

[6] J.D. Bjorken, J.B. Kogut, and D.E. Soper, Phys. Rev. D3 (1971) 1382

[7] J.D. Bjorken, talk at the *Conf. on Fund. Int. of Elementary Particles* (Moscow, 1995) SLAC-PUB-7096;
J.D. Bjorken and J.B. Kogut, Phys. Rev. D8 (1973) 1341

[8] P.V. Landshoff, J.C. Polkinghorne, and R.D. Short, Nucl. Phys. B28 (1971) 225

[9] I. Balitsky, Nucl. Phys. B463 (1996) 99

[10] W. Buchmüller, M.F. McDermott, and A. Hebecker, preprint SLAC-PUB-7204 and DESY 96-126, hep-ph/9607290

[11] E.L. Berger and S.J. Brodsky , Phys. Rev. Lett. 42 (1979) 940;
S.J. Brodsky, P. Hoyer, A.H. Mueller, and W.-K. Tang, Nucl. Phys. B369 (1992) 519;
A. Brandenburg, S.J. Brodsky, V.V. Khoze, and D. Müller, Phys. Rev. Lett. 73 (1994) 939

[12] S.M. Berman, D.J. Levy, and T.L. Neff, Phys. Rev. Lett. 23 (1969) 1363

[13] S.J. Brodsky and P. Hoyer, Phys. Lett. B298 (1993) 165

[14] NA10 Collab., S. Falciano et al., Z. Phys. C31 (1986) 513

[15] E. Mirkes and J. Ohnemus, Phys. Rev. D51 (1995) 4891;
E. Mirkes, Nucl. Phys. B387 (1992) 3

[16] L. Frankfurt and M. Strikman, Phys. Rep. 160 (1988) 235

[17] I.S. Gradshteyn and I.M. Ryzhik, *Tables of Integrals, Series, and Products; fifth ed.* (Academic Press, 1994)