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Sparticle Spectroscopy and Electroweak Symmetry Breaking with Gauge-Mediated Supersymmetry Breaking

Savas Dimopoulos[a,b], Scott Thomas[c†], James D. Wells[c†]

[a] *Theoretical Physics Division, CERN*
CH-1211 Geneva 23, Switzerland

[b] *Physics Department, Stanford University, Stanford, CA 94305*

[c] *Stanford Linear Accelerator Center, Stanford, CA 94309*

Abstract

The phenomenology associated with gauge-mediated supersymmetry breaking is presented. A renormalization group analysis of the minimal model is performed in which the constraints of radiative electroweak symmetry breaking are imposed. The resulting superpartner and Higgs boson spectra are highly correlated and depend on only a few parameters. Superpartner mass ratios and sum rules are identified which can be tested at future colliders. Some of these relations are logarithmically sensitive to the messenger scale, while others allow gauge-mediation to be distinguished from other schemes for transmitting supersymmetry breaking. Deviations from the minimal model, such as larger messenger representations and additional contributions to Higgs sector masses, can in some circumstances dramatically modify the low energy spectrum. These modifications include a slepton or Higgsino as the lightest standard model superpartner, or exotic mass relations among the scalars and gauginos. The contribution to $b \to s\gamma$ and resulting bound on superpartner masses are also presented for the minimal model. Finally, the unique collider signatures of heavy charged particle production, or decay to the Goldstino within a detector are discussed.

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1 Introduction

Supersymmetry provides an elegant framework in which physics at the electroweak scale can be decoupled from Planck scale physics. The electroweak scale arises dynamically as the effective scale of supersymmetry breaking in the visible sector. The breaking of supersymmetry must be transmitted from a breaking sector to the visible sector through a messenger sector. Most phenomenological studies of low energy supersymmetry implicitly assume that messenger sector interactions are of gravitational strength. The intrinsic scale of supersymmetry breaking is then necessarily of order $\sqrt{F} \sim 10^{11}$ GeV, giving an electroweak scale of $G_F^{-1/2} \sim F/M_p$. While gravitational strength interactions represent a lower limit, it is certainly possible that the messenger scale, M, is anywhere between the Planck and just above the electroweak scale, with supersymmetry broken at an intermediate scale, $G_F^{-1/2} \sim F/M$.

If the messenger scale is well below the Planck scale, it is likely that the usual gauge interactions of the standard model play some role in the messenger sector. This is because standard model gauginos couple at the renormalizable level only through gauge interactions. If the Higgs bosons received masses predominantly from non-gauge interactions in the messenger sector, with only a small contribution from gauge interactions, the standard model gauginos would be unacceptably lighter than the electroweak scale.* It is therefore interesting to consider theories in which the standard model gauge interactions act as messengers of supersymmetry breaking [1, 2, 3]. This mechanism occurs if supersymmetry is realized non-linearly in some sector which transforms under the standard model gauge group. Supersymmetry breaking in the visible sector spectrum then arises as a radiative correction.

In this paper we consider the superpartner spectroscopy and important phenomenological signatures that result from gauge-mediated supersymmetry breaking. Since within this anzatz the gauge interactions transmit supersymmetry breaking, the standard model soft masses arise in proportion to gauge charges squared. This leads to a sizeable hierarchy among the superpartner masses according to gauge quantum numbers. In addition, for a large class of models, there are a number of relations and sum rules among the superpartner masses. Electroweak symmetry breaking is driven by negative radiative corrections to the up-type Higgs mass squared from the large top quark Yukawa coupling and large stop masses [3]. With the constraint of electroweak symmetry breaking the minimal model of gauge-mediation is highly constrained and very predictive, with the

* The argument for light gauginos in the absence of standard model gauge interactions within a messenger sector well below the Planck scale only applies if the gauginos are elementary degrees of freedom. If the standard model gauge group is magnetic at or below the messenger scale the gauginos could in principle receive a large mass from operators suppressed by the confining magnetic scale.

superpartner spectrum depending primarily on two parameters – the overall scale and $\tan\beta$. In addition, there is a logarithmic dependence of various mass relations on the messenger scale.

The precise form of the low lying superpartner spectrum determines the signatures that can be observed at a high energy collider. With gauge-mediated supersymmetry breaking, either a neutralino or slepton is the lightest standard model superpartner. The signature for supersymmetry is then either the traditional missing energy or heavy charged particle production. In a large class of models the general form of the cascade decays to the lightest standard model superpartner is largely fixed by the anzatz of gauge-mediation. In addition, for a low enough supersymmetry breaking scale, the lightest standard model superpartner can decay to its partner plus the Goldstino component of the gravitino within the detector.

In the next subsection the natural lack of flavor changing neutral currents with gauge-mediated supersymmetry breaking is discussed. The minimal model of gauge-mediated supersymmetry breaking (MGM) and its variations are presented in section 2. A renormalization group analysis of the minimal model is performed in section 3, with the constraint of proper radiative electroweak symmetry breaking enforced. Details of the resulting superpartner and Higgs boson spectra are discussed. Mass relations and sum rules are identified which can distinguish gauge mediation from other theories for the soft terms. Some mass relations allow a logarithmically sensitive probe of the messenger scale. In section 4 variations of the minimal model are studied. With larger messenger sector representations the lightest standard model superpartner is naturally a slepton. Alternately, additional sources for Higgs sector masses, can lead in some instances to a Higgsino as the lightest standard model superpartner. The phenomenological consequences of gauge-mediated supersymmetry breaking are given in section 5. The supersymmetric contribution to $\mathrm{Br}(b \to s\gamma)$ in the minimal model, and resulting bound on the overall scale for the superpartners, are quantified. The collider signatures for superpartner production in both the minimal model, and models with larger messenger sector representations, are also detailed. In the latter case, the striking signature of heavy charged particles exiting the detector can result, rather than the traditional missing energy. The signatures resulting from decay of the lightest standard model superpartner to its partner plus the Goldstino are also reviewed. In section 6 we conclude with a few summary remarks and a comment about tuning.

The general expression for scalar and gaugino masses in a large class of models is given in appendix A. A non-minimal model is presented in appendix B which demonstrates an approximate $U(1)_R$ symmetry, and has exotic scalar and gaugino mass relations, even though it may be embedded in a GUT theory. Finally, in appendix C the couplings of the Goldstino component of the gravitino are reviewed. In addition to the general expressions for the decay rate of the lightest standard model superpartner to its partner plus the Goldstino, the severe suppression of the branching ratio to Higgs boson final states in the minimal model is quantified.

1.1 Ultraviolet Insensitivity

Low energy supersymmetry removes power law sensitivity to ultraviolet physics. In four dimensions with $\mathcal{N} = 1$ supersymmetry the parameters of the low energy theory are renormalized however. Infrared physics can therefore be logarithmically sensitive to effects in the ultraviolet. The best example of this is the value of the weak mixing angle at the electroweak scale in supersymmetric grand unified theories [4]. Soft supersymmetry breaking terms in the low energy theory also evolve logarithmically with scale. The soft terms therefore remain "hard" up to the messenger scale at which they are generated. If the messenger sector interactions are of gravitational strength, the soft terms are sensitive to ultraviolet physics all the way to the Planck or compactification scale. In this case patterns within the soft terms might give an indirect window to the Planck scale [5]. However, the soft terms are then also sensitive to flavor violation at all scales. Flavor violation at any scale can then in principle lead to unacceptable flavor violation at the electroweak scale [6]. This is usually avoided by postulating precise relations among squark masses at the high scale, such as universality [4] or proportionality. Such relations, however, do not follow from any unbroken symmetry, and are violated by Yukawa interactions. As a result the relations only hold at a single scale. They are "detuned" under renormalization group evolution, and can be badly violated in extensions of the minimal supersymmetric standard model (MSSM). For example, in grand unified theories, large flavor violations can be induced by running between the Planck and GUT scales [6]. Elaborate flavor symmetries may be imposed to limit flavor violations with a Planck scale messenger sector [7].

Sensitivity to the far ultraviolet is removed in theories with a messenger scale well below the Planck scale. In this case the soft terms are "soft" above the messenger scale. Relations among the soft parameters are then not "detuned" by ultraviolet physics. In particular there can exist a sector which is responsible for the flavor structure of the Yukawa matrix. This can arise from a hierarchy of dynamically generated scales [8], or from flavor symmetries, spontaneously broken by a hierarchy of expectation values [9]. If the messenger sector for supersymmetry breaking is well below the scale at which the Yukawa hierarchies are generated, the soft terms can be insensitive to the flavor sector. Naturally small flavor violation can result without specially designed symmetries.

Gauge-mediated supersymmetry breaking gives an elegant realization of a messenger sector below the Planck scale with "soft" soft terms and very small flavor violation. The direct flavor violation induced in the squark mass matrix at the messenger scale is GIM suppressed compared with flavor conserving squark masses by $\mathcal{O}(m_f^2/M^2)$, where m_f is a fermion mass. The largest flavor violation is generated by renormalization group evolution between the messenger and electroweak scales. This experiences a GIM suppression of $\mathcal{O}(m_f^2/\tilde{m}^2)\ln(M/\tilde{m})$, where $\tilde{m}$ is a squark mass, and is well below current experimental bounds.

If the messenger sector fields transforming under the standard model gauge group have the same quantum numbers as visible sector fields, flavor violating mixing can take place through Yukawa couplings. This generally leads to large flavor violating soft terms [10]. These dangerous mixings are easily avoided by discrete symmetries. For example, in the minimal model discussed in the next section, if the messenger fields are even under R-parity, no mixing occurs. In more realistic models in which the messenger fields are embedded directly in the supersymmetry breaking sector, messenger sector gauge symmetries responsible for supersymmetry breaking forbid flavor violating mixings. The natural lack of flavor violation is a significant advantage of gauge-mediated supersymmetry breaking.

2 The Minimal Model of Gauge-Mediated Supersymmetry Breaking

The standard model gauge interactions act as messengers of supersymmetry breaking if fields within the supersymmetry breaking sector transform under the standard model gauge group. Integrating out the messenger sector fields gives rise to radiatively generated soft terms within the visible sector, as discussed below. The messenger fields should fall into vector representations at the messenger scale in order to obtain a mass well above the electroweak scale. In order not to disturb the successful prediction of gauge coupling unification within the MSSM [4] at lowest order, it is sufficient (although not necessary [11]) that the messenger sector fields transform as complete multiplets of any grand unified gauge group which contains the standard model. If the messenger fields remain elementary degrees of freedom up to the unification scale, the further requirement of perturbative unification may be imposed on the messenger sector. For supersymmetry breaking at a low scale, these constraints allow up to four flavors of $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$, a single $\mathbf{10}+\overline{\mathbf{10}}$ of $SU(5)$, or a single $\mathbf{16}+\overline{\mathbf{16}}$ of $SO(10)$. With the assumptions outlined above, these are the discrete choices for the standard model representations in the messenger sector.

In the following subsection the minimal model of gauge-mediated supersymmetry breaking is defined. In the subsequent subsection variations of the minimal model are introduced.

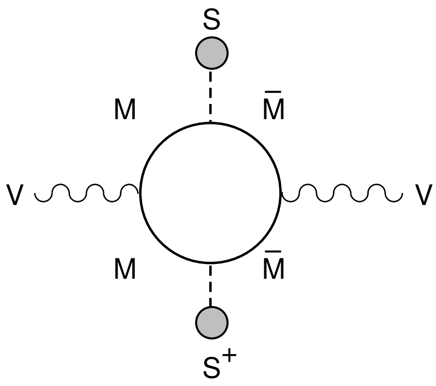

Figure 1: One-loop messenger sector supergraph which gives rise to visible sector gaugino masses.

2.1 The Minimal Model

The minimal model of gauge-mediated supersymmetry breaking (which preserves the successful predictions of perturbative gauge unification) consists of messenger fields which transform as a single flavor of $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$, i.e. there are $SU(2)_L$ doublets ℓ and $\bar{\ell}$, and $SU(3)_C$ triplets q and $\bar{q}$. In order to introduce supersymmetry breaking into the messenger sector, these fields may be coupled to a gauge singlet spurion, S, through the superpotential

$$W = \lambda_2 S\ell\bar{\ell} + \lambda_3 Sq\bar{q} \tag{1}$$

The scalar expectation value of S sets the overall scale for the messenger sector, and the auxiliary component, F, sets the supersymmetry breaking scale. For $F \neq 0$ the messenger spectrum is not supersymmetric,

$$\begin{aligned} m_b &= M\sqrt{1 \pm \frac{\Lambda}{M}} \\ m_f &= M \end{aligned} \tag{2}$$

where $M = \lambda S$ and $\Lambda = F/S$. The parameter Λ/M sets the scale for the fractional splitting between bosons and fermions. Avoiding electroweak and color breaking in the messenger sector requires $M > \Lambda$.

In the models of Ref. [3] the field S is an elementary singlet which couples through a secondary messenger sector to the supersymmetry breaking sector. In more realistic models the messenger fields are embedded directly in the supersymmetry breaking sector. This may be accomplished within a model of dynamical supersymmetry breaking by identifying an unbroken global symmetry with the standard model gauge group. In the present context, the field S should be thought of as a spurion which represents the dynamics which break supersymmetry. The physics discussed in

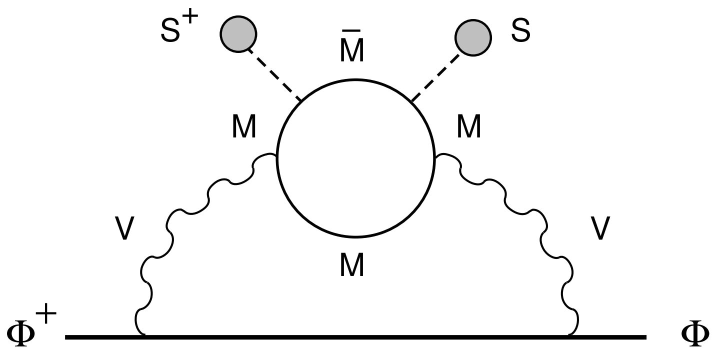

Figure 2: Two-loop messenger sector supergraph which gives rise to visible sector scalar masses. The one-loop subgraph gives rise to visible sector gaugino wave function renormalization. Other graphs related by gauge invariance are not shown.

this paper does not depend on the details of the dynamics represented by the spurion. Because (1) amounts to tree level breaking, the messenger spectrum satisfies the sum rule $\mathcal{S}Tr\ m^2 = 0$. With a dynamical supersymmetry breaking sector, this sum rule need not be satisfied. The precise value of $\mathcal{S}Tr\ m^2$ in the messenger sector, however, does not significantly affect the radiatively generated visible sector soft parameters discussed below.

Integrating out the non-supersymmetric messengers gives rise to effective operators, which lead to supersymmetry breaking in the visible sector. Gaugino masses arise at one-loop from the operator

$$\int d^2\theta \ \ln S\, W^\alpha W_\alpha \ + \ h.c. \tag{3}$$

as shown in Fig. 1. In superspace this operator amounts to a shift of the gauge couplings in the presence of the background spurion. Inserting a single spurion auxiliary component gives a gaugino mass. For $F \ll \lambda S^2$ the gaugino masses are [3]

$$m_{\lambda_i}(M) = \frac{\alpha_i(M)}{4\pi}\,\Lambda \tag{4}$$

where $\Lambda = F/S$ and GUT normalized gauge couplings are assumed ($\alpha_1 = \alpha_2 = \alpha_3$ at the unification scale).† The dominant loop momenta in Fig. 1 are $\mathcal{O}(M)$, so (4) amounts to a boundary condition for the gaugino masses at the messenger scale. Visible sector scalar masses arise at two-loops from the operator

$$\int d^4\theta \ \ln(S^\dagger S)\ \Phi^\dagger e^V \Phi \tag{5}$$

†The standard model normalization of hypercharge is related to the GUT normalization by $\alpha' = (3/5)\alpha_1$.

as shown in Fig. 2. In superspace this operator represents wave function renormalization from the background spurion. Inserting two powers of the auxiliary component of the spurion gives a scalar mass squared. For $F \ll \lambda S^2$ the scalar masses are [3]

$$m^2(M) = 2\Lambda^2 \sum_{i=1}^{3} k_i \left(\frac{\alpha_i(M)}{4\pi}\right)^2 \tag{6}$$

where the sum is over $SU(3) \times SU(2)_L \times U(1)_Y$, with $k_1 = (3/5)(Y/2)^2$ where the hypercharge is normalized as $Q = T_3 + \frac{1}{2}Y$, $k_2 = 3/4$ for $SU(2)_L$ doublets and zero for singlets, and $k_3 = 4/3$ for $SU(3)_C$ triplets and zero for singlets. Again, the dominant loop momenta in Fig. 2 are $\mathcal{O}(M)$, so (6) amounts to a boundary condition for the scalar masses at the messenger scale.

It is interesting to note that for $F \ll \lambda S^2$ the soft masses (4) and (6) are independent of the magnitude of the Yukawa couplings (1). This is because the one-loop graph of Fig. 1 has an infrared divergence, k^{-2}, which is cut off by the messenger mass $M = \lambda S$, thereby cancelling the λF dependence in the numerator. The one-loop subgraph of Fig. (2) has a similar infrared divergence which cancels the λ dependence. For finite $F/(\lambda S^2)$ the corrections to (4) and (6) are small unless M is very close to Λ [11, 12, 13].

Since the gaugino masses arise at one-loop and scalar masses squared at two-loops, superpartners masses are generally the same order for particles with similar gauge charges. If the messenger scale is well below the GUT scale, then $\alpha_3 \gg \alpha_2 > \alpha_1$, so the squarks and gluino receive mass predominantly from $SU(3)_C$ interactions, the left handed sleptons and W-ino from $SU(2)_L$ interactions, and the right handed sleptons and B-ino from $U(1)_Y$ interactions. The gaugino and scalar masses are then related at the messenger scale by $m_3^2 \simeq \frac{3}{8} m_{\tilde{q}}^2$, $m_2^2 \simeq \frac{2}{3} m_{\tilde{l}_L}^2$, and $m_1^2 = \frac{5}{6} m_{\tilde{l}_R}^2$. This also leads to a hierarchy in mass between electroweak and strongly interacting states. The gaugino masses at the messenger scale are in the ratios $m_1 : m_2 : m_3 = \alpha_1 : \alpha_2 : \alpha_3$, while the scalar masses squared are in the approximate ratios $m_{\tilde{q}}^2 : m_{\tilde{l}_L}^2 : m_{\tilde{l}_R}^2 \simeq \frac{4}{3}\alpha_3^2 : \frac{3}{4}\alpha_2^2 : \frac{3}{5}\alpha_1^2$. The masses of particles with different gauge charges are tightly correlated in the minimal model. These correlations are reflected in the constraints of electroweak symmetry breaking on the low energy spectrum, as discussed in section 3.

The parameter $(\alpha/4\pi)\Lambda$ sets the scale for the soft masses. This should be of order the weak scale, implying $\Lambda \sim \mathcal{O}(100\,\text{TeV})$. The messenger scale M is, however, arbitrary in the minimal model, subject to $M > \Lambda$. In models in which the messenger sector is embedded directly in a renormalizable dynamical supersymmetry breaking sector [14], the messenger and effective supersymmetry breaking scales are the same order, $M \sim \Lambda \sim \mathcal{O}(100\,\text{TeV})$, up to small hierarchies from messenger sector Yukawa couplings. This is also true of models with a secondary messenger sector [3]. The messenger scale can, however, be well separated from the supersymmetry breaking scale. This can arise in models with large ratios of dynamical scales. Alternatively with non-renormalizable supersymmetry breaking, which vanishes in the flat space limit, expectation values intermediate between the Planck and supersymmetry breaking scale can develop, leading to $M \gg \Lambda$.

A noteworthy feature of the minimal messenger sector is that it is invariant under charge conjugation and parity, up to electroweak radiative corrections. This has the important effect of enforcing the vanishing of the $U(1)_Y$ Fayet-Iliopoulos D-term at all orders in interactions that involve gauge interactions and messenger fields only. This is crucial since a non-zero $U(1)_Y$ D-term at one-loop would induce soft scalar masses much larger in magnitude than the two-loop contributions (6), and lead to $SU(3)_C$ and $U(1)_Q$ breaking. This vanishing is unfortunately not an automatic feature of models in which the messenger fields also transform under a chiral representation of the gauge group responsible for breaking supersymmetry. In the minimal model a $U(1)_Y$ D-term is generated only by gauge couplings to chiral standard model fields at three loops. The leading log contribution comes from renormalization group evolution and is discussed in section 3.2.1.

The dimensionful parameters within the Higgs sector

$$W = \mu H_u H_d \tag{7}$$

and

$$V = m_{12}^2 H_u H_d \ + \ h.c. \tag{8}$$

do not follow from the anzatz of gauge-mediated supersymmetry breaking. These terms require additional interactions which violate $U(1)_{PQ}$ and $U(1)_{R-PQ}$ symmetries. A number of models have been proposed to generate these terms, including, additional messenger quarks and singlets [3], singlets with an inverted hierarchy [3], and singlets with an approximate global symmetry [15]. In the minimal model the mechanisms for generating the parameters μ and m_{12}^2 are not specified, and they are taken as free parameters at the messenger scale. As discussed below, upon imposing electroweak symmetry breaking, these parameters may be eliminated in favor of $\tan\beta = v_u/v_d$ and m_{Z^0}.

Soft tri-linear A-terms require interactions which violate both $U(1)_R$ and visible sector chiral flavor symmetries. Since the messenger sector does not violate visible sector flavor symmetries, A-terms are not generated at one-loop. However, two-loop contributions involving a visible sector gaugino supermultiplet do give rise to operators of the form

$$\int d^4\theta \ \ln S \ \frac{D^2}{\Box} Q H_u \bar{u} \ + \ h.c. \tag{9}$$

as shown in Fig. 3, and similarly for down-type quarks and leptons. This operator is related by an integration by parts in superspace to a correction to the superpotential in the presence of the

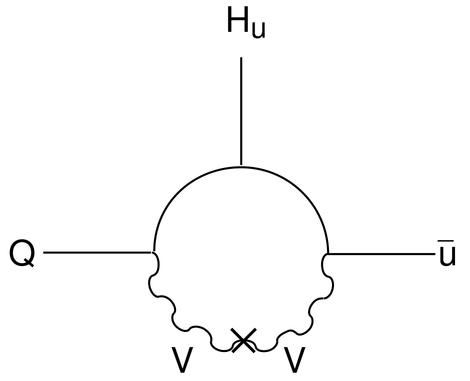

Figure 3: One-loop visible sector supergraph which contains both logarithmic and finite contributions to visible sector A-terms. The cross on the visible sector gaugino line represents the gaugino mass insertion shown in Fig. 1.

background spurion. Inserting a single auxiliary component of the spurion (equivalent to a visible sector gaugino mass in the one-loop subgraph) gives a tri-linear A-term. The momenta in the effective one-loop graph shown in Fig. 3 are equally weighted in logarithmic intervals between the gaugino mass and messenger scale. Over these scales the gaugino mass is "hard." The A-terms therefore effectively vanish at the messenger scale, and are generated from renormalization group evolution below the messenger scale, and finite contributions at the electroweak scale. At the low scale the A-terms have magnitude $A \sim (\alpha/4\pi) m_\lambda \ln(M/m_\lambda)$. Note that A is small compared with the scale of the other soft terms, unless the logarithm is large. As discussed in section 3.2 the A-terms do not have a qualitative effect on the superpartner spectrum unless the messenger scale is large.

The general MSSM has a large number of CP-violating phases beyond those of the standard model. Since here the soft masses are flavor symmetric (up to very small GIM suppressed corrections discussed in section (1.1)), only flavor symmetric phases are relevant. A background charge analysis [16] fixes the basis independent combination of flavor symmetric phases to be $\text{Arg}(m_\lambda \mu (m_{12}^2)^*)$ and $\text{Arg}(A^* m_\lambda)$. Since the A-terms vanish at the messenger scale only the first of these can arise in the soft terms. In the models of [3] the auxiliary component of a single field is the source for all soft terms, giving a correlation among the phases such that $\text{Arg}(m_\lambda \mu (m_{12}^2)^*) = 0 \text{ mod } \pi$. In the minimal model, however, the mechanism for generating μ and m_{12}^2 is not specified, and the phase is arbitrary.

Below the messenger scale the particle content of the minimal model is just that of the MSSM, along with the gravitino discussed in section 5.2 and appendix C. At the messenger scale the boundary conditions for the visible sector soft terms are given by (4) and (6), μ, m_{12}^2, and $A = 0$.

It is important to note that from the low energy point of view the minimal model is just a set of boundary conditions specified at the messenger scale. These boundary conditions may be traded for the electroweak scale parameters

$$(\ \tan\beta \ , \ \Lambda = F/S \ , \ \mathrm{Arg}\,\mu \ , \ \ln M \ ) \tag{10}$$

The most important of these is Λ which sets the overall scale for the superpartner spectrum. Since all the soft masses are related in the minimal model, Λ may be traded for any of these, such as $m_{\tilde{B}}(M)$. It may also be traded for a physical mass, such as $m_{\chi_1^0}$ or $m_{\tilde{l}_L}$. In addition, as discussed in section 3.1 $\tan\beta$ "determines" m_{12} and μ in the low energy theory, and can have important effects on the superpartner spectrum.

2.2 Variations of the Minimal Model

The minimal model represents a highly constrained and very predictive theory for the soft supersymmetry breaking terms. It is therefore interesting to consider how the qualitative features discussed in the remainder of the paper change under deformations away from the minimal model.

The most straightforward generalization is to the other messenger sector representations which are consistent with perturbative unification discussed at the beginning of this section. The expressions for gaugino and scalar masses for a general messenger sector are given in appendix A. The gaugino masses grow like the quadratic index of the messenger sector matter, while the scalar masses grow like the square root of the quadratic index. Models with larger messenger sector representations generally have gauginos which are relatively heavier, compared with the scalars, than the minimal model. This can have important consequences for the standard model superpartner spectrum. In particular, a scalar lepton can be the lightest standard model superpartner (as opposed to the lightest neutralino for the minimal model). This is the case for a range of parameters with two messenger generations of $\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5)$, as discussed in section 4.2.

Another generalization is to introduce multiple spurions with general scalar and auxiliary components, and general Yukawa couplings. Unlike the case with a single spurion, the scalar and fermion mass matrices in the messenger sector are in general not aligned. Such a situation generally arises if the messengers receive masses from a sector not associated with supersymmetry breaking. This can occur in dynamical models with chiral messenger representations which confine to, or are dual to, a low energy theory with vector representations. The messengers can gain a mass at the confinement or duality scale, with supersymmetry broken at a much lower scale [17]. With multiple spurions, the infrared cancelations of the messenger Yukawa couplings, described in the previous subsection for the minimal model, no longer hold. This has the effect of removing the tight correlations in the minimal model between masses of superpartners with different gauge charges. As an example, a model is presented in appendix B with two generations of $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$, and two singlet fields. One of the singlets is responsible for masses in the messenger sector, while the other breaks supersymmetry. Even though the model can be embedded in a GUT theory, it yields a non-minimal spectrum.

Soft scalar masses require supersymmetry breaking, while gaugino masses require, in addition, breaking of $U(1)_R$ symmetry. If $U(1)_R$ symmetry is broken at a lower scale than supersymmetry, the gauginos can be lighter than in the minimal model. This may be represented in the low energy theory by a parameter $\not{R}$ which is the ratio of a gaugino to scalar mass at the messenger scale, relative to that in the minimal model. The general definition of $\not{R}$ in theories with a single spurion is given in appendix A. With multiple spurions $\not{R}<1$ is generally obtained since the messenger scalar and fermion mass matrices do not necessarily align. Since the gauginos have little influence on electroweak symmetry breaking, as discussed in section 4.1, the main effect of $\not{R}<1$ is simply to make the gauginos lighter than in the minimal model. The non-minimal model given in appendix B has, in one limit, an approximate $U(1)_R$ symmetry and light gauginos.

Additional interactions in the messenger sector are required in order to generate μ and m_{12}^2. It is likely that these interactions also contribute to the Higgs boson soft masses.‡ Therefore, even though Higgs bosons and lepton doublets have the same electroweak quantum numbers, their soft masses at the messenger scale can be different. In the low energy theory this may be parameterized by the quantities

$$\begin{aligned}\Delta_+^2 &\equiv m_{H_d}^2 + m_{H_u}^2 - 2m_{\tilde{l}_L}^2 \\ \Delta_-^2 &\equiv m_{H_d}^2 - m_{H_u}^2\end{aligned} \tag{11}$$

where $m_{\tilde{l}_L}^2$ is the gauge-mediated left handed slepton mass, and all masses are evaluated at the messenger scale. In the minimal model $\Delta_\pm^2=0$. Since the Higgs soft masses affect electroweak symmetry breaking, these splittings can potentially have significant effects on the superpartner spectrum. However, as discussed in section 4.3, unless the non-minimal contributions are very large, the general form of the spectrum is largely unaltered.

Finally, the $U(1)_Y$ D-term, D_Y, can have a non-zero expectation value at the messenger scale. This leads to a shift of the soft scalar masses at the messenger scale proportional to hypercharge, Y,

$$\delta m^2(M) = g'Y\ D_Y(M) \tag{12}$$

where $g'=\sqrt{3/5}g_1$ is the $U(1)_Y$ coupling. Note that for the Higgs bosons, this yields a shift

‡We thank Gian Giudice for this important observation.

$\Delta_-^2 = -2g' D_Y(M)$. As discussed in the previous subsection, $D_Y(M) = 0$ in the minimal model as the result of a parity symmetry in the messenger sector. In non-minimal models $D_Y(M)$ need not vanish. In general, an unsupressed $U(1)_Y$ D-term generated at one-loop in the messenger sector destabilizes the electroweak scale, and leads to $SU(3)_C$ and $U(1)_Q$ breaking. However, it is possible for $D_Y(M)$ to be generated at a level which gives rise to soft masses of the same order as the gauge mediated masses. For example, the one-loop contribution to $D_Y(M)$ is suppressed if there is an approximate parity symmetry in the messenger sector, and further suppressed if the messenger couplings are unified at the GUT scale [18]. As another example, if the messengers sector transforms under an additional $U(1)_{\tilde{Y}}$, gauge kinetic mixing between $U(1)_Y$ and $U(1)_{\tilde{Y}}$ couples D_Y and $D_{\tilde{Y}}$ [19]. Renormalization group evolution gives a one-loop mixing contribution $D_Y(M) \simeq \mathcal{O}((g'\tilde{g}/16\pi^2)\ln(M_c/M)D_{\tilde{Y}})$ where M_c is the GUT or compactification scale. If the messengers in this case were embedded directly in a renormalizable dynamical supersymmetry breaking sector, $\tilde{g} D_{\tilde{Y}} \sim (\lambda^2/\tilde{g}^2)F$, where λ is a Yukawa coupling in the dynamical sector. For $\lambda \ll \tilde{g}$ the one-loop contribution to $D_Y(M)$ is then suppressed. With a non-renormalizable model $D_{\tilde{Y}}$ is suppressed by powers of the dynamical scale over the Planck scale.

Given the above variations, we consider the following parameters, in addition to those of the minimal model (10)

$$(\ N \ , \ \ \not{R} \ , \ \Delta_+^2 \ , \ \Delta_-^2 \ , \ D_Y \ ) \tag{13}$$

where N is the number of generations of $\mathbf{5} + \overline{\mathbf{5}}$ in the messenger sector. In the remainder of the paper we consider the minimal case, $N = 1$, $\not{R} = 1$, $\Delta_\pm^2 = 0$, and $D_Y = 0$, and discuss in detail the effects on the superpartner spectrum for $N = 2$, $\not{R} \neq 1$, $\Delta_-^2 \neq 0$, and $D_Y \neq 0$ in section 4.

3 Renormalization Group Analysis

The general features of the superpartner spectrum and resulting phenomenology are determined by the boundary conditions at the messenger scale. Renormalization group evolution of the parameters between the messenger and electroweak scales can have a number of important effects. Electroweak symmetry breaking results from the negative evolution of the up-type Higgs mass squared from the large top quark Yukawa coupling. Imposing electroweak symmetry breaking gives relations among the Higgs sector mass parameters. Details of the sparticle spectrum can also be affected by renormalization. In particular, changes in the splittings among the light states can have an important impact on several collider signatures, as discussed in section 5.2. In addition, general features of the spectrum, such as the splitting between squarks and left handed sleptons, can be logarithmically sensitive to the messenger scale.

In this section the effects of renormalization group evolution on electroweak symmetry breaking and the superpartner spectrum in the minimal model are presented. Gauge couplings, gaugino masses, and third generation Yukawa couplings are evolved at two-loops. Scalar masses, tri-linear A-terms, the μ parameter, and m_{12}^2 are evolved at one-loop. For the scalar masses, the D-term contributions to the β-functions are included (these are sometimes neglected in the literature). Unless stated otherwise, the top quark pole mass is taken to be $m_t^{\rm pole} = 175$ GeV. The renormalization group analysis is similar to that of Ref. [20], being modified for the boundary conditions and arbitrary messenger scale of the minimal model of gauge-mediated supersymmetry breaking.

As discussed in the previous section, the boundary conditions at the high scale are $m_{\lambda_i}, m_i^2, \mu, m_{12}^2$, and sgn μ. The precise scale at which a soft term is defined depends on the messenger field masses. In the minimal model the mass of the messenger doublets and triplets are determined by the messenger Yukawa couplings, $M_{2,3} = \lambda_{2,3} S$, which can in principle differ. Even if $\lambda_2 = \lambda_3$ at the GUT scale these couplings are split under renormalization group evolution [21]. In a full model, the supersymmetry breaking dynamics also in part determine the values of λ_2 and λ_3 through renormalization group evolution. Since we are interested primarily in effects which follow from the anzatz of gauge-mediated supersymmetry breaking, and not on details of the mechanism of supersymmetry breaking, we will neglect any splitting between doublet and triplet masses. This would in fact be the case, up to small gauge coupling corrections, if the messengers are embedded in a supersymmetry breaking sector near a strongly coupled fixed point. All soft terms are therefore assumed to be specified at a single messenger scale, M. The range of allowed M is taken to be $\Lambda \leq M \leq M_{GUT}$. In the minimal model, M precisely equal to Λ is unrealistic since some messenger sector scalars become massless at this point. In what follows the limit $M = \Lambda$ is not to be taken literally in the minimal model, but should be thought of as indicative of a realistic model with a single dynamical scale, and no small parameters [14]. In many of the specific examples given in the following subsections we take $\Lambda = 76, 118, 163$ TeV, with $\Lambda = M$, which gives a B-ino mass at the messenger scale of $m_{\tilde{B}}(M) = 115, 180, 250$ GeV, with the other soft masses related by the boundary conditions (4) and (6).

The renormalization of the $\overline{\rm DR}$ mass parameters according to the one- and two-loop β functions described above for the minimal model with $M = \Lambda = 76$ TeV, $m_{\tilde{B}}(M) = 115$ GeV, $\tan\beta = 3$, and sgn$(\mu) = +1$, is shown in Fig. 4. As can be seen, renormalization has an effect on the mass parameters. In the following subsections the constraints imposed by electroweak symmetry breaking, and the form of the low energy spectrum resulting from renormalization group evolution are discussed.

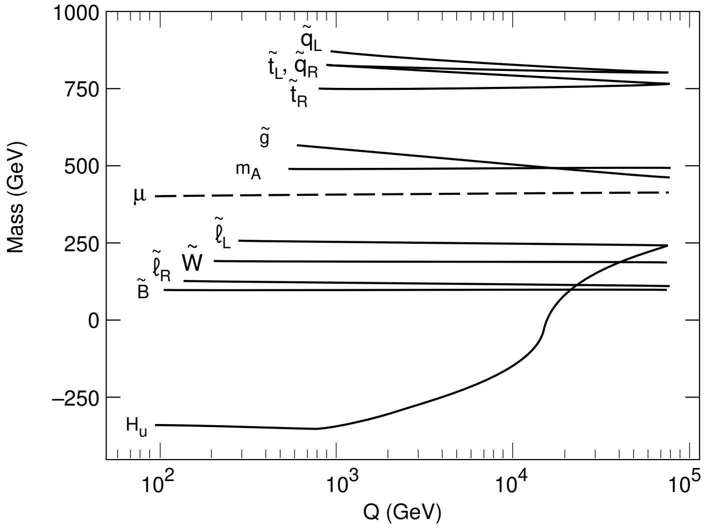

Figure 4: Renormalization group evolution of the $\overline{\mathrm{DR}}$ mass parameters with MGM boundary conditions. The messenger scale is $M = 76\,\mathrm{TeV}$, $m_{\tilde{B}}(M) = 115\,\mathrm{GeV}$, $\tan\beta = 3$, $\mathrm{sgn}(\mu) = +1$, and $m_t^{\mathrm{pole}} = 175$ GeV. The masses are plotted as $m \equiv \mathrm{sgn}(m)(|m|)^{1/2}$.

3.1 Electroweak Symmetry Breaking

The most significant effect of renormalization group evolution is the negative contribution to the up-type Higgs boson mass squared from the large top quark Yukawa coupling. As can be seen in Fig. 4 this leads to a negative mass squared for H_u. The β-function for $m_{H_u}^2$ is dominated by the heavy stop mass

$$\frac{dm_{H_u}^2}{dt} \simeq \frac{1}{16\pi^2}(6h_t^2(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2) + \cdots) \tag{14}$$

where $t = \ln(Q)$, with a small $\mathcal{O}(g_2^2 m_2^2/h_t^2 m_{\tilde{t}}^2)$ correction from gauge interactions. For the parameters given in Fig. 4 the full β-function for $m_{H_u}^2$ is approximately constant above the stop thresholds, differing by less than 1% between M and $m_{\tilde{t}}$. The evolution of $m_{H_u}^2$ is therefore approximately linear in this region (the non-linear feature in Fig. 4 is a square-root singularity because the quantity plotted is $\mathrm{sgn}(m_{H_u})\sqrt{|m_{H_u}|^2}$). It is worth noting that for $\tan\beta$ not too large, and M not too much larger than Λ, $m_{H_u}^2(m_{\tilde{t}})$ is then well approximated by

$$m_{H_u}^2(m_{\tilde{t}}) \simeq m_{H_u}^2(M) - \frac{3}{8\pi^2}h_t^2(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\ln(M/m_{\tilde{t}}) \tag{15}$$

(although throughout we use numerical integration of the full renormalization group equations). For the parameters of Fig. 4 the approximation (15) differs from the full numerical integration by 2% (using the messenger scale values of h_t, $m_{\tilde{t}_L}$, and $m_{\tilde{t}_R}$). The magnitude of the small positive gauge contribution can be seen below the stop thresholds in Fig. 4.

The negative value of $m_{H_u}^2$ leads to electroweak symmetry breaking in the low energy theory. The mechanism of radiative symmetry breaking [3] is similar to that for high scale supersymmetry breaking with universal boundary conditions. With high scale supersymmetry breaking, $m_{H_u}^2 < 0$ develops because of the large logarithm. Here $m_{H_u}^2 < 0$ results not because the logarithm is large, but because the stop masses are large [3]. Notice in Fig. 4 that $m_{H_u}^2$ turns negative in less than a decade of running. The negative stop correction effectively amounts to an $\mathcal{O}((\alpha_3/4\pi)^2(h_t/4\pi)^2\ln(M/m_{\tilde{t}}))$ three-loop contribution which is larger than the $\mathcal{O}((\alpha_2/4\pi)^2)$ two-loop contribution [3]. Naturally large stop masses which lead automatically to radiative electroweak symmetry breaking are one of the nice features of low scale gauge-mediated supersymmetry breaking.

Imposing electroweak symmetry breaking gives relations among the Higgs sector mass parameters. In the approach taken here we solve for the electroweak scale values of μ and m_{12}^2 in terms of $\tan\beta$ and m_{Z^0} using the minimization conditions

$$|\mu|^2 + \frac{m_{Z^0}^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d) - (m_{H_u}^2 + \Sigma_u)\tan^2\beta}{\tan^2\beta - 1} \tag{16}$$

$$\sin 2\beta = \frac{-2m_{12}^2}{(m_{H_u}^2 + \Sigma_u) + (m_{H_d}^2 + \Sigma_d) + 2|\mu|^2} \tag{17}$$

where $\Sigma_{u,d}$ represent finite one-loop corrections from gauge interactions and top and bottom Yukawas [22]. These corrections are necessary to reduce substantially the scale dependence of the minimization conditions. In order to minimize the stop contributions to the finite corrections, the renormalization scale is taken to be the geometric mean of the stop masses, $Q^2 = m_{\tilde{t}_1}m_{\tilde{t}_2}$. The finite corrections to the one-loop effective potential make a non-negligible contribution to (16) and (17). For example, with the parameters of Fig. 4, minimization of the tree level conditions with the renormalization scheme given above gives $\mu = 360$ GeV, while inclusion of the finite corrections results in $\mu = 395$ GeV. The minimization conditions (16) and (17) depend on the value of the top quark mass, m_t, mainly through the running of $m_{H_u}^2$, and also through the finite corrections. For the parameters of Fig. 4, a top mass range in the range $m_t = 175 \pm 15$ GeV gives a μ parameter in the range $\mu = 395 \pm 50$ GeV.

The correlation between μ and m_{12}^2 can be obtained from (16) and (17) in terms of $\tan\beta$ and m_{Z^0}. The relation between the W-ino mass, m_2, and μ (evaluated at the renormalization scale) imposed by electroweak symmetry breaking in the minimal model is shown in Fig. 5 for $\tan\beta = 2, 3, 5, 10$, and 30, with $\Lambda = M$ and $\text{sgn}(\mu) = +1$. The actual correlation is of course between the Higgs sector mass parameters. The parameter m_2 is plotted just as a representative mass of states transforming under $SU(2)_L$, and of the overall scale of the soft masses (the gaugino masses directly affect electroweak symmetry breaking only through very small higher order corrections to renormalized Higgs sector

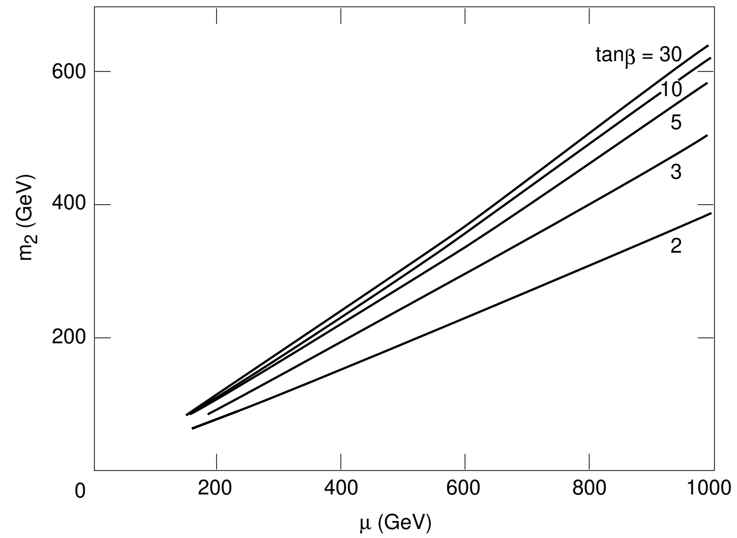

Figure 5: The relation between m_2 and $|\mu|$ imposed by electroweak symmetry breaking with MGM boundary conditions for $\tan\beta = 2, 3, 5, 10, 30$, and $\Lambda = M$.

parameters). The μ parameter typically lies in the range $\frac{3}{2}m_2 \lesssim |\mu| \lesssim 3m_2$ or $m_{\tilde{l}_L} \lesssim |\mu| \lesssim 2m_{\tilde{l}_L}$, depending on the precise values of $\tan\beta$ and $\ln M$.

The correlation between μ and the overall scale of the soft masses arises because the stop masses set the scale for $m_{H_u}^2$ at the electroweak scale, and therefore the depth of the Higgs potential. For $\tan\beta \gg 1$ the conditions (16) reduce to $|\mu|^2 \simeq -m_{H_u}^2 - \frac{1}{2}m_{Z^0}^2$. In this limit, for $|\mu|^2 \gg m_{Z^0}^2$, $|\mu| \simeq (-m_{H_u}^2)^{1/2}$, with the small difference determining the electroweak scale. At moderate $\tan\beta$ the corrections to this approximation increase μ for a fixed overall scale. At fixed $\tan\beta$, and M not too far above Λ, $m_{H_u}^2$ at the renormalization scale is approximately a linear function of the overall scale Λ, as can be seen from Eq. (15). The very slight non-linearity in Fig. 5 arises from $\ln M$ dependence, and $\mathcal{O}(m_{Z^0}^2/\mu^2)$ effects in the minimization conditions. The limit $|\mu|^2, |m_{H_u}|^2 \gg m_{Z^0}^2$ of course represents a tuning among the Higgs potential parameters in order to obtain proper electroweak symmetry breaking.

The ratio m_{12}/μ at the renormalization scale is plotted in Fig. 6 for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV and $\Lambda = M$. Again, the tight correlation, approximately independent of the overall scale, arises because all soft terms are related to a single scale, Λ. The small splitting between the three cases shown in Fig. 6 arises from $\ln M$ dependence, and $\mathcal{O}(m_{Z^0}^2/\mu^2)$ effects in the minimization conditions. Ignoring corrections from the bottom Yukawa, $m_{12}/\mu \to 0$ for $\tan\beta \gg 1$. The saturation of m_{12}/μ at large $\tan\beta$ is due to bottom Yukawa effects in the renormalization group evolution and finite corrections to $m_{H_d}^2$. Any theory for the origin of μ and m_{12}^2 with minimal gauge mediation, and only the MSSM degrees of freedom at low energy, would have to reproduce (or at least be compatible

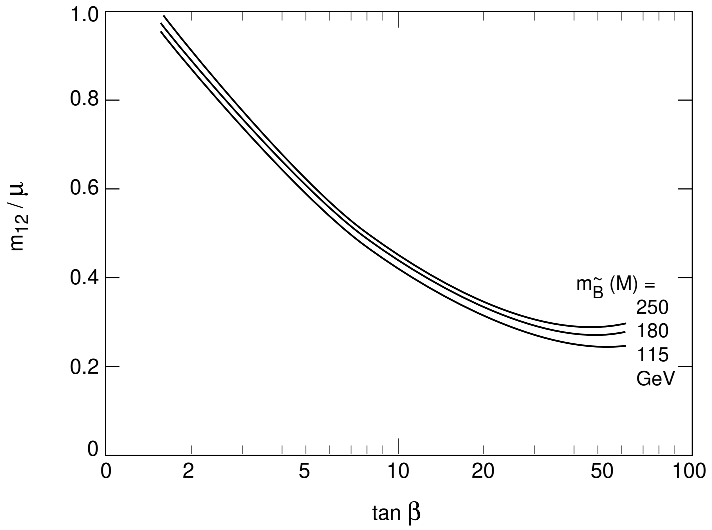

Figure 6: The ratio m_{12}/μ as a function of $\tan\beta$ imposed by electroweak symmetry breaking for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV and $\Lambda = M$.

with) the relation given in Fig. 6. Note that all the low scale Higgs sector mass parameters are quite similar in magnitude over essentially all the parameter space of the MGM.

3.2 Sparticle Spectroscopy

The gross features of the superpartner spectrum are determined by the boundary conditions at the messenger scale. Renormalization group evolution can modify these somewhat, and electroweak symmetry breaking imposes relations which are reflected in the spectrum. Mixing and D-term contributions also shift some of the states slightly. The physical spectrum resulting from the renormalized parameters given in Fig. 4 is presented in Table 1. In the following subsections the spectroscopy of the electroweak states, strongly interacting states, and Higgs bosons are discussed. We also consider the dependence of the spectrum on the messenger scale, and discuss quantitative relations among the superpartner masses which test the hypothesis of gauge-mediation and can be sensitive to the messenger scale.

3.2.1 Electroweak States

The physical charginos, $\chi_i^\pm$, and neutralinos, χ_i^0, are mixtures of the electroweak gauginos and Higgsinos. As discussed in the previous subsection, imposing electroweak symmetry breaking with MGM boundary conditions implies $\frac{3}{2}m_2 \lesssim |\mu| \lesssim 3m_2$ over all the allowed parameter space. With this inequality, in the limit $\mu^2 - m_2^2 \gg m_{W^\pm}^2$ the lightest chargino is mostly gaugino. Likewise, in

Particle	Mass (GeV)
$\tilde{u}_L, \tilde{d}_L$	869, 871
$\tilde{u}_R, \tilde{d}_R$	834, 832
$\tilde{t}_1, \tilde{t}_2$	765, 860
$\tilde{g}$	642
$A^0, H^0, H^\pm$	506, 510, 516
$\chi_3^0, \chi_2^\pm, \chi_4^0$	404, 426, 429
$\tilde{\nu}_L, \tilde{l}_L$	260, 270
$\chi_1^\pm, \chi_2^0$	174, 175
$\tilde{l}_R$	137
h^0	104
χ_1^0	95

Table 1: Superpartner physical spectrum for the parameters given in Fig. 4.

the limit $\mu^2 - m_1^2 \gg m_{Z^0}^2$, the lightest two neutralinos are mostly gaugino. Under renormalization group evolution both m_1 and m_2 are slightly decreased. For the parameters of Fig. 4 this amounts to a -15 GeV shift for the B-ino and a -10 GeV shift for the W-ino. At the electroweak scale $m_2 \simeq 2m_1$. The lightest neutralino is therefore mostly B-ino. For example, with the parameters given in Fig. 4, the χ_1^0 eigenvectors are $N_{1\tilde{B}} = 0.98$, $N_{1\tilde{W}} = -0.09$, $N_{1d} = 0.14$, and $N_{1u} = -0.07$. Expanding in $m_{Z^0}^2/(\mu^2 - m_1^2)$, the χ_1^0 mass is given by [23]

$$m_{\chi_1^0} \simeq m_1 - \frac{m_{Z^0}^2 \sin^2\theta_W \ (m_1 + \mu \sin 2\beta)}{|\mu|^2 - m_1^2}. \tag{18}$$

Note that the shift in the physical χ_1^0 mass relative to the B-ino mass parameter m_1 depends on $\mathrm{sgn}(\mu)$. For the parameters in Fig. 4 with $\mathrm{sgn}(\mu) = +1$ this amounts to a -5 GeV shift. Except for very large $\tan\beta$ discussed below, χ_1^0 is the lightest standard model superpartner.

The lightest chargino and second lightest neutralino are mostly W-ino, and form an approximately degenerate triplet of $SU(2)_L$, $(\chi_1^+, \chi_2^0, \chi_1^-)$, as can be seen in Table 1. This approximate triplet is very degenerate, with splittings arising only at $\mathcal{O}(m_{Z^0}^4/\mu^4)$. Expanding in $m_{W^\pm}^2/(\mu^2 - m_2^2)$, the triplet mass is given by [23]

$$m_{\chi_2^0, \chi_1^\pm} \simeq m_2 - \frac{m_{W^\pm}^2 (m_2 + \mu \sin 2\beta)}{|\mu|^2 - m_2^2}. \tag{19}$$

Again, the shift in the physical mass relative to the W-ino mass parameter m_2 is anticorrelated with $\mathrm{sgn}(\mu)$. For the parameters of Fig. 4 this amounts to a -19 GeV shift.

The heavier chargino and two heaviest neutralinos are mostly Higgsino, and form an approximately degenerate singlet and triplet of $SU(2)_L$, all with mass set by the μ parameter. This is also apparent in Table 1, where χ_3^0 is the singlet, and $(\chi_2^+, \chi_4^0, \chi_2^-)$ form the triplet. The splitting between the singlet and triplet is $\mathcal{O}(m_{Z^0}^2/\mu^2)$ while the splitting among the triplets are $\mathcal{O}(m_{Z^0}^4/\mu^4)$. All these splittings may be verified by an effective operator analysis which shows that splittings within a triplet require four Higgs insertions, while splitting between a singlet and triplet requires only two.

The right handed sleptons are lighter than the other scalars since they only couple to the messenger sector through $U(1)_Y$ interactions. The low scale value of $m_{\tilde{l}_R}$ is shifted by a number of effects. First, renormalization due to gaugino interactions increases $m_{\tilde{l}_R}$ in proportion to the gaugino mass. In addition, the radiatively generated $U(1)_Y$ D-term, proportional to $S \equiv \frac{1}{2}\mathrm{Tr}(Y\tilde{m}^2)$ where Tr is over all scalars, also contributes to renormalization of scalar masses [24]. With gauge-mediated boundary conditions, $S = 0$ at the messenger scale as the result of anomaly cancelation, $\mathrm{Tr}(Y\{T^a, T^b\}) = 0$, where T^a is any standard model gauge generator. The β-function for S is homogeneous and very small in magnitude (below the messenger scale and above all sparticle thresholds $\beta_S = (66/20\pi)\alpha_1 S$ [25]) so $S \simeq 0$ in the absence of scalar thresholds. The largest contribution comes below the squark thresholds. The "image" squarks make a large negative contribution to S in this range. Although not visible with the resolution in Fig. 4, the slope of $m_{\tilde{l}_R}(Q)$ has a kink at the squark thresholds from this effect. Finally, the classical $U(1)_Y$ D-term also increases the physical mass in the presence of electroweak symmetry breaking, $\tilde{m}^2_{\tilde{l}_R} = m^2_{\tilde{l}_R} - \sin^2\theta_W \cos 2\beta m^2_{Z^0}$, where $\cos 2\beta < 0$. For the parameters of Fig. 4 the gauge and $U(1)_Y$ D-term contributions to renormalization, and the classical $U(1)_Y$ D-term, contribute a positive shift to $\tilde{m}_{\tilde{l}_R}$ of +2, +3, and +6 GeV respectively. As discussed in section 5.2, the sum of all these small shifts, and the $m_{\chi_1^0}$ shift (18), can have an important effect on signatures at hadron colliders.

The left handed sleptons receive mass from both $SU(2)_L$ and $U(1)_Y$ interactions. Under renormalization $m_{\tilde{l}_L}$ is increased slightly by gaugino interactions. The most important shift is the splitting between $m_{\tilde{l}_L}$ and $m_{\tilde{\nu}_L}$ arising from $SU(2)_L$ classical D-terms in the presence of electroweak symmetry breaking, $m^2_{\tilde{l}_L} - m^2_{\tilde{\nu}_L} = -m^2_{W^\pm}\cos 2\beta$. For the parameters of Fig. 4 this amounts to a 10 GeV splitting between $\tilde{l}_L$ and $\tilde{\nu}_L$.

Because of the larger Yukawa coupling, the $\tilde{\tau}$ slepton masses receive other contributions, beyond those for $\tilde{e}$ and $\tilde{\mu}$ discussed above. The τ Yukawa gives a negative contribution to the renormalization group evolution of $m_{\tilde{\tau}_L}$ and $m_{\tilde{\tau}_R}$. In addition the left and right handed $\tilde{\tau}$ are mixed in the presence

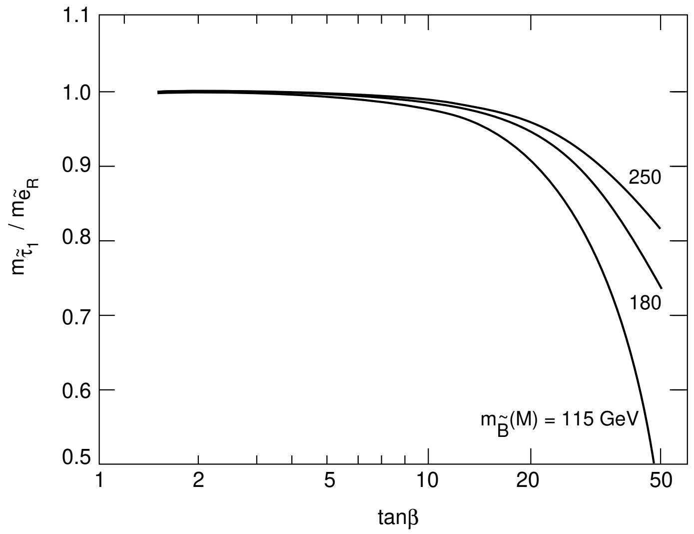

Figure 7: The ratio $m_{\tilde{\tau}_R}/m_{\tilde{e}_R}$ as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV, and $\Lambda = M$.

of electroweak symmetry breaking

$$m^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{\tilde{\tau}_L} + m^2_\tau + \Delta_{\tilde{\tau}_L} & m_\tau(A_{\tilde{\tau}} - \mu\tan\beta) \\ m_\tau(A_{\tilde{\tau}} - \mu\tan\beta) & m^2_{\tilde{\tau}_R} + m^2_\tau + \Delta_{\tilde{\tau}_R} \end{pmatrix} \tag{20}$$

where $\Delta_{\tilde{\tau}_L} = (-\frac{1}{2} + \sin^2\theta_W)m^2_{Z^0}\cos 2\beta$ and $\Delta_{\tilde{\tau}_R} = -\sin^2\theta_W\cos 2\beta$ are classical D-term contributions. As discussed in section 2.1, $A_{\tilde{\tau}}$ is only generated by renormalization group evolution in proportional to m_2. It is therefore small, and does not contribute significantly to the mixing terms. For the parameters of Fig. 4 $A_{\tilde{\tau}} \simeq -25$ GeV, and remains small for all $\tan\beta$. For large $\tan\beta$ the τ Yukawa coupling becomes large, and the mixing terms cause level repulsion which lowers the $\tilde{\tau}_1$ mass below $\tilde{l}_R$. The ratio $m_{\tilde{\tau}_R}/m_{\tilde{e}_R}$ is shown in Fig. 7 as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV, and $\Lambda = M$. For large $\tan\beta$ the $\tilde{\tau}_1$ can be significantly lighter than $\tilde{e}_R$ and $\tilde{\mu}_R$. The negative contributions to $m_{\tilde{\tau}_1}$ in this regime comes partially from renormalization group evolution, but mostly from mixing. For example, with $m_{\tilde{B}}(M) = 115$ GeV, $\Lambda = M$, and $\tan\beta = 40$, the Yukawa renormalization and mixing contributions to $m_{\tilde{\tau}_1}$ amount to -10 GeV and -36 GeV shifts with respect to $m_{\tilde{e}_R}$. For these parameters $m_{\tilde{e}_R} = 137$ GeV and $m_{\tilde{\tau}_1} = 91$ GeV. As the messenger scale is increased the relative contribution to the mass shift due to renormalization group evolution increases.

The negative shift of the lightest $\tilde{\tau}$ can even be large enough to make $\tilde{\tau}_1$ the lightest standard model superpartner. The ratio $m_{\tilde{\tau}_1}/m_{\chi^0_1}$ is plotted as a function of $\tan\beta$ in Fig. 8. For $m_{\tilde{B}}(M) = 115$ GeV and $\Lambda = M$ the $\tilde{\tau}_1$ becomes lighter than χ^0_1 at $\tan\beta = 38 \pm 4$ for $m_t^{\rm pole} = 175 \pm 15$ GeV. The negative shift from mixing is an $\mathcal{O}(m_\tau\mu\tan\beta/(m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R}))$ correction to the lightest eigenvalue, and

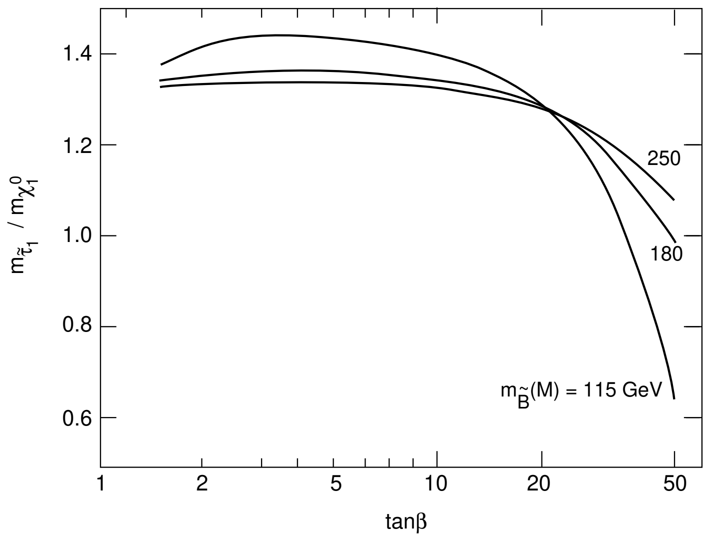

Figure 8: The ratio $m_{\tilde{\tau}_1}/m_{\chi_1^0}$ as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV. For large $\tan\beta$ the $\tilde{\tau}_1$ becomes the lightest standard model superpartner.

therefore becomes smaller as the overall scale of the soft masses is increased, as can be seen in Fig. 8. In addition, $\tan\beta$ is bounded from above if the b quark Yukawa coupling remains perturbative up to a large scale. We find that for $m_{\tilde{B}}(M) \gtrsim 200$ GeV the $\tilde{\tau}_1$ is never lighter than χ_1^0 in the minimal model without h_b becoming non-perturbative somewhere below the GUT scale. The slight decrease in $m_{\tilde{\tau}_1}/m_{\chi_1^0}$ which can be seen in Fig. 8 at small $\tan\beta$ is due partly to the increase in μ which decreases the χ_1^0 eigenvalue for $\mathrm{sgn}(\mu) = +1$, and partly to classical $U(1)_Y$ D-terms which increase $m_{\tilde{e}_R}$. Both these effects are less important for a larger overall scale. The negative shift of $\tilde{\tau}_1$ relative to the other sleptons and χ_1^0 at large $\tan\beta$ can have important implications for collider signatures, as discussed in section 5.2.

The τ Yukawa also gives a negative shift in the mass of the $\nu_{\tilde{\tau}}$ sneutrino under renormalization group evolution. The magnitude of this shift is however smaller than for that of $\tilde{\tau}_R$ by $\mathcal{O}(m^2_{\tilde{l}_R}/m^2_{\tilde{l}_L})$. For example, with $m_{\tilde{B}}(M) = 115$ GeV, $\Lambda = M$, and $\tan\beta = 40$, the Yukawa renormalization contribution amounts to a -2 GeV shift in $m_{\nu_{\tilde{\tau}}}$ with respect to $m_{\nu_{\tilde{e}}}$.

3.2.2 Strongly Interacting States

The gluino and squarks receive mass predominantly from $SU(3)_C$ interactions with the messenger sector. The gluino mass increases under renormalization in proportion to α_3 at lowest order, $m_3 = m_3(M)(\alpha_3(m_3)/\alpha_3(M))$. The physical pole mass of the gluino is related to the renormalized $\overline{\mathrm{DR}}$ mass by finite corrections [26]. With very heavy squarks, the general expression for the finite corrections given in Ref. [26] reduces to

$$m_{\tilde{g}}^{\rm pole} \simeq m_3 \left[1 + \frac{\alpha_3}{4\pi} \left(15 + 12 I(r) \right) \right] \tag{21}$$

where m_3 and α_3 are the renormalized $\overline{\rm DR}$ parameters evaluated at the scale m_3. The first term in the inner parenthesis is from QCD corrections, and the second from squark-quark couplings, where the loop function is $I(r) = \frac{1}{2}\ln r + \frac{1}{2}(r-1)^2 \ln(1-r^{-1}) + \frac{1}{2}r - 1$ for $r \geq 1$ where $r = m_{\tilde{q}}^2/m_3^2$. For $r \gg 1$ $I(r) \to \frac{1}{2}\ln r$, $I(2) = 0$, and $I(1) = -\frac{1}{2}$. The largest corrections to (21) are $\mathcal{O}((m_t^2/m_3^2)\alpha_3/4\pi)$. In the minimal model $r \lesssim \frac{8}{3}$, which happens to be near the zero of $I(r)$. For example, with the parameters of Fig. 4, $I(r) \simeq 0.035$. The corrections to the gluino pole mass are then dominated by QCD. For the parameters of Fig. 4 the finite corrections amount to a +70 GeV contribution to the physical mass.

The squark masses receive a small increase under renormalization in proportion to the gluino mass at lowest order. Since the gluino and squark masses are related within gauge mediation by $m_3^2(M) \simeq \frac{3}{8} m_{\tilde{q}}^2(M)$, this can be written as a multiplicative shift of the squark masses by integrating the one-loop β-function

$$m_{\tilde{q}} \simeq m_{\tilde{q}}(M) \left[1 + \frac{\not{R}^2}{3} \left(\frac{\alpha_3^2(m_{\tilde{q}})}{\alpha_3^2(M)} - 1 \right) \right]^{1/2} \tag{22}$$

where $\not{R}$ is the gaugino masses parameter defined in section 2.2. The $\mathcal{O}((m_i^2/m_3^2)(\alpha_i^2(m_{\tilde{q}})/\alpha_i^2(M) - 1))$, $i = 1, 2$, renormalization group corrections to (22) are quite small since α_2 runs very slowly, and α_1 is small. For the minimal model with $\not{R} = 1$ and $\Lambda = M$ renormalization amounts to a 7% upward shift in the squark masses. For a messenger scale not too far above Λ, the squark masses are determined mainly by $\alpha_3(M)$.

The left and right handed squarks are split mainly by $SU(2)_L$ interactions with the messenger sector at $\mathcal{O}(m_{\tilde{l}_L}^2/m_{\tilde{q}}^2)$. The much smaller splitting between up and down type left handed squarks is due to classical $SU(2)_L$ D-terms at $\mathcal{O}(m_{W^{\pm}}^2/m_{\tilde{q}}^2)$. Finally, the small splitting between up and down right handed sleptons is from classical $U(1)_Y$ D-terms at $\mathcal{O}(m_{Z^0}^2/m_{\tilde{q}}^2)$ and $U(1)_Y$ interactions with the messenger sector at $\mathcal{O}(m_{\tilde{l}_R}^2/m_{\tilde{q}}^2)$. The magnitude of these small splittings can be seen in Table 1. The gluino is lighter than all the first and second generation squarks for any messenger scale in the minimal model.

The stop squarks receive additional contributions because of the large top quark Yukawa. For a messenger scale not too much larger than Λ, the positive renormalization group contribution from gauge interactions is largely offset by a negative contribution from the top Yukawa. In addition the

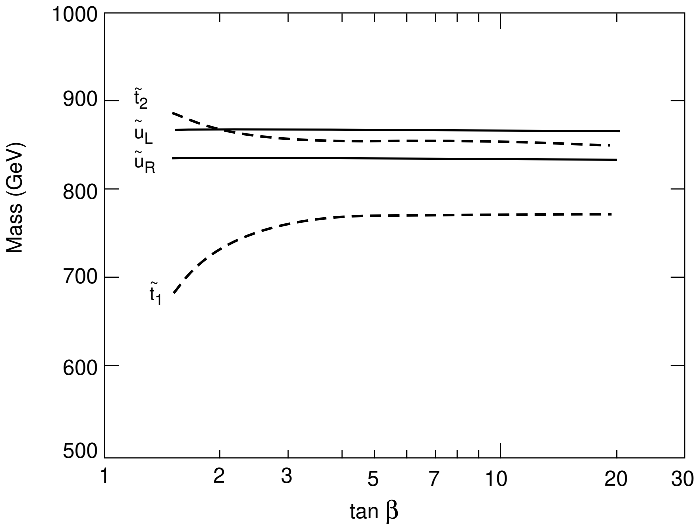

Figure 9: Squark spectrum as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115$ GeV and $\Lambda = M$.

left and right handed stops are mixed in the presence of electroweak symmetry breaking

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + \Delta_{\tilde{t}_L} & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix} \tag{23}$$

where $\Delta_{\tilde{t}_L} = (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta m_{Z^0}^2$ and $\Delta_{\tilde{t}_R} = \frac{2}{3}\sin^2\theta_W \cos 2\beta m_{Z^0}^2$ are classical D-term contributions. The radiatively generated A-terms for squarks are somewhat larger than for the electroweak scalars because of the larger gluino mass. For the parameters of Fig. 4 $A_{\tilde{t}} \simeq -250$ GeV, and does not vary significantly over all $\tan\beta$. At large $\tan\beta$ mixing induces only an $\mathcal{O}(m_t A_t/m_{\tilde{t}}^2)$ correction to the lightest eigenvalue. Because of the large squark masses, A-terms therefore do not contribute significantly to mixing. At small $\tan\beta$ the top Yukawa and μ become large, and $\tilde{t}_1$ can be pushed down. The squark spectrum is shown in Fig. 9 as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115$ GeV and $\Lambda = M$. In contrast to the $\tilde{\tau}$, most of the negative shift in the stop masses relative to the first two generations comes from renormalization group evolution (except for small $\tan\beta$). In Fig. 9, for large $\tan\beta$, the renormalization and mixing contributions to $m_{\tilde{t}_1}$ are -50 GeV and -5 GeV respectively. Because of the large overall squark masses, and relatively small μ and A-terms, a light stop is never obtained in the MGM parameter space.

3.2.3 Higgs Bosons

The qualitative features of the Higgs boson spectrum are determined by the pseudo-scalar mass $m_{A^0}^2 = 2m_{12}^2/\sin 2\beta$. The pseudo-scalar mass is shown in Fig. 10 as a function $m_{\chi_1^0}$, for $\tan\beta = 2, 3, 5, 30$, and $\Lambda = M$. The lightest neutralino mass is plotted in Fig. 10 as representative of

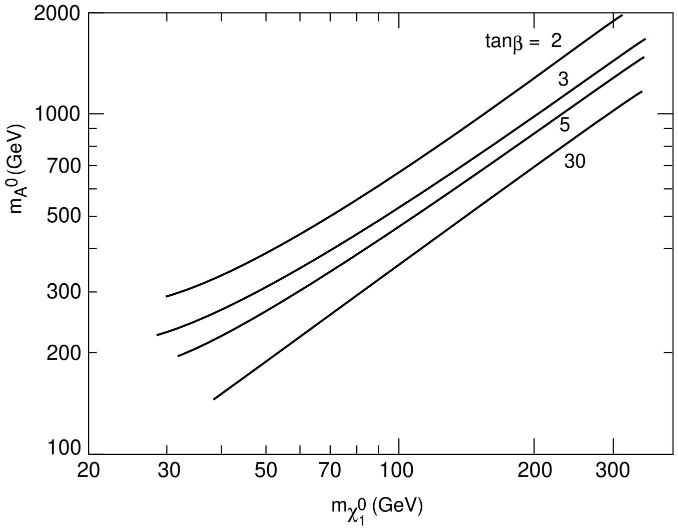

Figure 10: The pseudo-scalar Higgs mass, m_{A^0}, as a function of the lightest neutralino mass, $m_{\chi_1^0}$, for $\tan\beta = 2, 3, 5, 30$, and $\Lambda = M$.

the overall scale of the superpartner spectrum. Using the minimization conditions (16) and (17) the pseudo-scalar mass may be written, for $\tan\beta \gg 1$, as $m_{A^0}^2 \simeq |\mu|^2 + (m_{H_d}^2 + \Sigma_d) - \frac{1}{2}m_{Z^0}^2$. For moderate values of $\tan\beta$ this gives the inequality $m_{A^0} \gtrsim |\mu|$ over all the allowed parameter space. Since electroweak symmetry breaking implies $3m_1 \lesssim |\mu| \lesssim 6m_1$, $m_{A^0} \gg m_{\chi_1^0}$ in this range. For small $\tan\beta$ the corrections from (16) to this approximate relation make m_{A^0} even larger. For $\tan\beta \gtrsim 35$ the negative contribution of the bottom Yukawa to the renormalization group evolution of $m_{H_d}^2$, and finite corrections, allow $m_{A^0} \lesssim |\mu|$. Also note that since μ is determined by the overall scale of the superpartner masses, m_{A^0} scales linearly with $m_{\chi_1^0}$ for $|\mu|^2 \gg m_{Z^0}^2$. This scaling persists for moderate values of $\tan\beta$, as can be seen in Fig. 10. The non-linear behavior at small m_{A^0} is due to $\mathcal{O}(m_{Z^0}^2/\mu^2)$ contributions to the mass.

Over essentially all the allowed parameter space $m_{A^0} \gg m_{Z^0}$. In this case the Higgs decoupling limit is reached in which A^0, H^0 and $H^\pm$ form an approximately degenerate complex doublet of $SU(2)_L$, with fractional splittings of $\mathcal{O}(m_{Z^0}^2/m_{A^0}^2)$. This limit is apparent in Table 1. Since $m_{A^0} \gtrsim |\mu|$ over most of the parameter space, the heavy Higgs bosons are heavier than the Higgsinos, except for $\tan\beta$ very large.

In the decoupling limit the light Higgs, h^0, remains light with couplings approaching standard model values. The radiative corrections to m_{h^0} are sizeable [27] since the stop squarks are so heavy with MGM boundary conditions [28]. The physical h^0 mass is shown in Fig. 11 as a function of $m_{\chi_1^0}$ for $\tan\beta = 2, 3, 5, 30$, and $\Lambda = M$. Since $m_{\tilde{t}_{1,2}}, m_{A^0} \gg m_{Z^0}$ and stop mixings are small, as discussed in section 3.2.2, m_{h^0} is well approximated by the leading log correction to the tree level

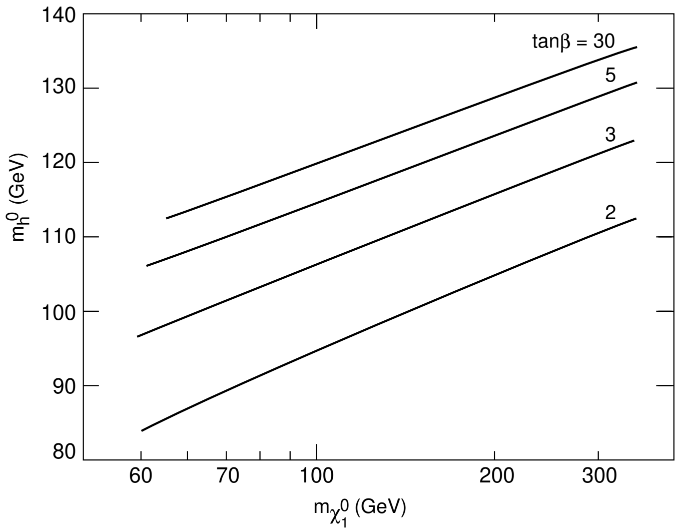

Figure 11: The lightest Higgs mass, m_{h^0}, as a function of the lightest neutralino mass, $m_{\chi_1^0}$, for $\tan\beta = 2, 3, 5, 30$, and $\Lambda = M$.

mass in the decoupling limit

$$m_{h^0}^2 \simeq \cos^2 2\beta m_{Z^0}^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right). \tag{24}$$

For moderate values of $\tan\beta$ (24) overestimates the full one-loop mass shown in Fig. 11 by $4-5$ GeV. The $\tan\beta$ dependence of m_{h^0} in Fig. 11 comes mainly from the tree level contribution. In general, the one-loop corrections are largely independent of $\tan\beta$ and depend mainly on the overall scale for the superpartners through the stop masses. This is apparent from the log dependence of m_{h^0} on $m_{\chi_1^0}$ in Fig. 11, and in the approximation (24). Note that for $m_{\chi_1^0} < 100$ GeV, $m_{h^0} \lesssim 120$ GeV.

3.2.4 Messenger Scale Dependence

Much of the spectroscopy discussed above assumed a low messenger scale $M \sim \Lambda \sim \mathcal{O}(100\,\text{TeV})$. However, in principle M can be anywhere between Λ and M_{GUT}. The physical spectrum for $m_{\tilde{B}}(M) = 115$ GeV and $\tan\beta = 3$ is shown in Fig. 12 as a function of the messenger scale. The scalar masses are sensitive to the gauge couplings at the messenger scale. For a fixed B-ino mass at the messenger scale (proportional to $\alpha_1(M)$), the squarks become lighter as the messenger scale is increased because α_3/α_1 is smaller at higher scales. Conversely the right handed sleptons become heavier as M is increased because of the larger contribution from renormalization group evolution. The gauge coupling α_2 increases more slowly than α_1 as the scale is increased. For fixed $m_{\tilde{B}}(M)$, as in Fig. 12, the left handed slepton masses therefore become smaller as the mes-

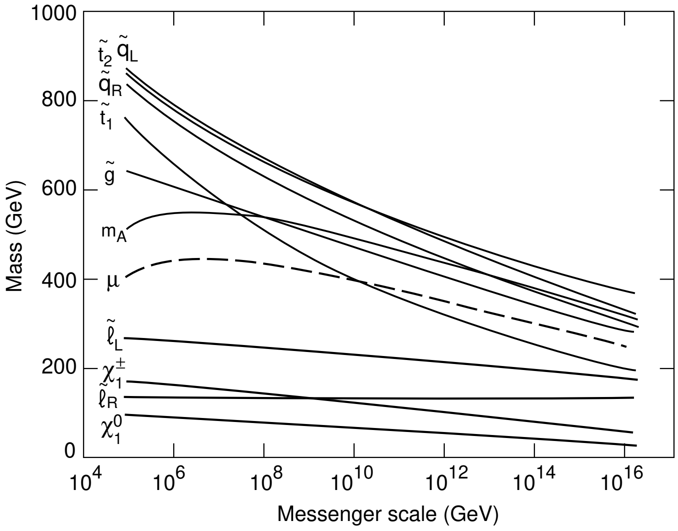

Figure 12: The physical spectrum as a function of the messenger scale for $m_{\tilde{B}}(M) = 115$ GeV and $\tan\beta = 3$.

senger scale is increased. The sensitive dependence of the squark masses on $\alpha_3(M)$ provides a logarithmically sensitive probe of the messenger scale, as discussed in the next section.

For larger messenger scales the spread among the superpartner masses becomes smaller. This is simply because all soft masses are proportional to gauge couplings squared, and the gauge couplings converge at larger scales. The boundary conditions for the scalar masses with $M = M_{GUT}$ satisfy the relations $m^2_{\tilde{e}_R} : m^2_{\tilde{e}_L} : m^2_{\tilde{Q}_L} : m^2_{\tilde{u}_R} : m^2_{\tilde{d}_R} = (3/5) : (9/10) : 3 : (8/5) : (21/15)$. These do not satisfy GUT relations because only $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions are included. If the full $SU(5)$ gauge interactions are included $m^2_{\bar{\mathbf{5}}} : m^2_{\mathbf{10}} = 2 : 3$ where $\tilde{e}_L, \tilde{d}_R \in \bar{\mathbf{5}}$ and $\tilde{e}_R, \tilde{Q}_L, \tilde{u}_R \in \overline{\mathbf{10}}$. Of course, for a messenger scale this large, gravitational effects are also important.

For a messenger scale slightly above Λ, $m^2_{H_u}$ is driven to more negative values by the top Yukawa under renormalization group evolution. Obtaining correct electroweak symmetry breaking therefore requires larger values of μ and m^2_{12}. This can be seen in Fig. 12 as an increase in μ and m_{A^0} for $M \gtrsim \Lambda$. For larger messengers scales the increase in the magnitude of $m^2_{H_u}$ from more running is eventually offset by the smaller stop masses. This can be seen in Fig. 12 as a decrease in μ and m_{A^0} for $M \gtrsim 10^7$ GeV. The spectra as a function of the messenger scale for different values of $\tan\beta$ are essentially identical to Fig. 12 aside from μ, m_{A^0}, and $m_{\tilde{t}}$. This is because $\tan\beta$ only affects directly the Higgs sector parameters, which in turn influence the mass of the other states only through two-loop corrections (except for the third generation scalars discussed in sections 3.2.1 and 3.2.2).

3.2.5 Relations Among the Superpartner Masses

The minimal model of gauge-mediated supersymmetry breaking represents a very constrained theory of the soft terms. In this section we present some quantitative relations among the superpartner masses. These can be used to distinguish the MGM from other theories of the soft terms and within the MGM can be logarithmically sensitive to the messenger scale.

The gaugino masses at the messenger scale are in proportion to the gauge couplings squared, $m_1 : m_2 : m_3 = \alpha_1 : \alpha_2 : \alpha_3$. Since $\alpha_i m_{\lambda_i}^{-1}$ is a renormalization group invariant at one loop, this relation is preserved to lowest order at the electroweak scale, where m_{λ_i} are the $\overline{DR}$ masses. The MGM therefore yields, in leading log approximation, the same ratios of gaugino masses as high scale supersymmetry breaking with universal gaugino boundary conditions. "Gaugino unification" is a generic feature of any weakly coupled gauge-mediated messenger sector which forms a representation of any GUT group and which has a single spurion. The gaugino mass ratios are independent of $\not{R}$. However, as discussed in section 2.2, with multiple sources of supersymmetry breaking and/or messenger fermion masses the gaugino masses can be sensitive to messenger Yukawa couplings. "Gaugino unification" therefore does *not* follow just from the anzatz of gauge-mediation, even for messenger sectors which can be embedded in a GUT theory. An example of such a messenger sector is given in appendix B. Of course, a messenger sector which forms an incomplete GUT multiplet (and modifies gauge coupling unification unification at one-loop under renormalization group evolution) does not in general yield "gaugino unification" [29].

With gauge-mediated supersymmetry breaking the scalar and gaugino masses are related at the messenger scale. For a messenger sector well below the GUT scale, $\alpha_3 \gg \alpha_2 > \alpha_1$, so the most important scalar-gaugino correlations are between squarks and gluino, left handed sleptons and W-ino, and right handed sleptons and B-ino. The ratios are of course proportional to $\not{R}$ which determines the overall scale of the gaugino masses at the messenger scale, and are modified by renormalization group evolution to the low scale. Ratios of the $\overline{\rm DR}$ masses $m_3/m_{\tilde{q}_R}$, $m_2/m_{\tilde{l}_L}$, and $m_1/m_{\tilde{l}_R}$ in the minimal model are shown in Fig. 13 as a function of the messenger scale. As discussed in the next section, these ratios can be altered with non-minimal messenger sectors.

In the minimal model, with $\not{R} = 1$, a measurement of any ratio m_{λ_i}/m gives a logarithmically sensitive measurement of the messenger scale. Because of the larger magnitude of the $U(1)_Y$ gauge β-function the ratio $m_{\tilde{l}_R}/m_1$ is most sensitive to the messenger scale. Notice also that $m_3/m_{\tilde{q}}$ is larger for a larger messenger scale, while $m_{\tilde{l}_L}/m_2$ and $m_{\tilde{l}_R}/m_1$ decrease with the messenger scale. Because of this disparate sensitivity, within the anzatz of minimal gauge-mediation, both $\not{R}$ and $\ln M$ could be extracted from a precision measurement of all three ratios.

For $\not{R} \leq 1$ the ratio of scalar mass to associated gaugino mass is always ≥ 1 for any messenger

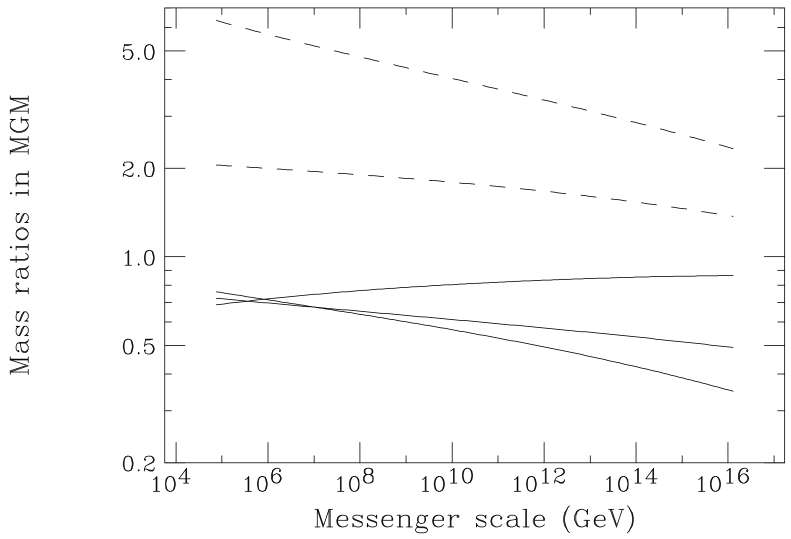

Figure 13: Ratios of $\overline{\rm DR}$ mass parameters with MGM boundary conditions as a function of the messenger scale: $m_{\tilde{q}_R}/m_{\tilde{l}_R}$ (upper dashed line), $m_{\tilde{l}_L}/m_{\tilde{l}_R}$ (lower dashed line), $m_3/m_{\tilde{q}_R}$ (upper solid line), $m_2/m_{\tilde{l}_L}$ (middle solid line), and $m_1/m_{\tilde{l}_R}$ (lower solid line).

scale. Observation of a first or second generation scalar lighter than the associated gaugino is therefore a signal for $R\!\!\!/ > 1$. As discussed in section 4.2, $R\!\!\!/ > 1$ is actually possible with larger messenger sector representations. In fact, as discussed in appendix A in models with a single spurion in the messenger sector, $R\!\!\!/$ is senstive to the index of the messenger sector matter. Additional matter which transforms under the standard model gauge group between the electroweak and messenger scales would of course modify these relations slightly through renormalization group evolution contributions.

Ratios of scalar masses at the messenger scale are related by ratios of gauge couplings squared. These ratios are reflected in the low energy spectrum. In particular, since $\alpha_3 \gg \alpha_1$ if the messenger scale is well below the GUT scale, the ratio $m_{\tilde{q}}/m_{\tilde{l}_R}$ is sizeable. Ratios of the $\overline{\rm DR}$ masses $m_{\tilde{q}_R}/m_{\tilde{l}_R}$ and $m_{\tilde{l}_L}/m_{\tilde{l}_R}$ are shown in Fig. 13 as a function of the messenger scale. For $\Lambda = M$, $m_{\tilde{q}_R}/m_{\tilde{l}_R} \simeq 6.3$. Notice that $m_{\tilde{q}_R}/m_{\tilde{l}_R}$ is smaller for larger messenger scales, and is fairly sensitive to $\ln M$. This is because α_3 decreases rapidly at larger scales, while α_1 increases. This sensitivity allows an indirect measure of $\ln M$. The ratio $m_{\tilde{l}_L}/m_{\tilde{l}_R}$ is also fairly sizeable but not as sensitive to the messenger scale. For $\Lambda = M$, $m_{\tilde{l}_L}/m_{\tilde{l}_R} \simeq 2.1$. It is important to note that with $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge-mediated supersymmetry breaking, any parity and charge conjugate invariant messenger sector which forms a representation of any GUT group and which has a single spurion yields, at leading order, the same scalar mass ratios as in the minimal model. These mass ratios therefore represent a fairly generic feature of minimal gauge-mediation.

The sizeable hierarchy which arises in gauge-mediated supersymmetry breaking between scalar masses of particles with different gauge charges generally does not arise with universal boundary conditions with a large overall scalar mass. With gravity mediated supersymmetry breaking and universal boundary conditions the largest hierarchy results for the no-scale boundary condition $m_0 = 0$. In this case the scalar masses are "gaugino-dominated," being generated in proportion to the gaugino masses under renormalization group evolution. The scalar mass ratios turn out to be just slightly smaller than the maximum gauge-mediated ratios. With no-scale boundary conditions at M_{GUT}, $m_{\tilde{q}_R}/m_{\tilde{e}_R} \simeq 5.6$ and $m_{\tilde{l}_L}/m_{\tilde{l}_R} \simeq 1.9$. However, the scalars in this case are just slightly lighter than the associated gauginos, in contrast to the MGM with $\not{R} = 1$, in which they are heavier. It is interesting to note, however, that for $M \sim 1000$ TeV and $N = 2$ or $\not{R} \simeq \sqrt{2}$, gauge mediation coincidentally gives almost identical mass ratios as high scale supersymmetry breaking with the no-scale boundary condition at the GUT scale.

With gauge-mediation, scalar masses at the messenger scale receive contributions proportional to gauge couplings squared. Splitting among squarks with different gauge charges can therefore be related to right and left handed slepton masses (cf. Eq. 6). This can be quantified in the form of sum rules which involve various linear combinations of all the first generation scalar masses squared [11]. The splitting due to $U(1)_Y$ interactions with the messenger sector can be quantified by $\text{Tr}(Ym^2)$, where Tr is over first generation sleptons and squarks. As discussed in section 3.2.1 this quantity vanishes with gauge-mediated boundary conditions as the result of anomaly cancelation. It is therefore interesting to consider the low scale quantity

$$M_Y^2 = \frac{1}{2}\left(m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2\right) - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_L}^2 - \frac{1}{2}\left(m_{\tilde{e}_L}^2 + m_{\tilde{\nu}_L}^2\right) + m_{\tilde{e}_R}^2 + \frac{10}{3}\sin^2\theta_W \cos 2\beta m_{Z^0}^2 \tag{25}$$

where the the sum of the m^2 terms is $\frac{1}{2}\text{Tr}(Ym^2)$ over the first generation, and the $\mathcal{O}(m_{Z^0}^2)$ term is a correction for classical $U(1)_Y$ D-terms. The contribution of the gaugino masses to M_Y^2 under renormalization group evolution cancels at one-loop. So this quantity is independent of the gaugino spectrum. In addition, the β-function for $\text{Tr}(Ym^2)$ is homogeneous [24] and independent of the Yukawa couplings at one-loop, even though the individual masses are affected. So if $M_Y^2 = 0$ at the messenger scale, it is not generated above scalar thresholds. It only receives very small contributions below the squark thresholds of $\mathcal{O}((\alpha_1/4\pi)m_{\tilde{q}}^2 \ln(m_{\tilde{q}}/m_{\tilde{l}}))$. The relation $M_Y^2 \simeq 0$ tests the assumption that splittings within the squark and slepton spectrum are related to $U(1)_Y$ quantum numbers. The quantity M_Y^2 also vanishes in any model in which soft scalar masses are univeral within GUT multiplets. This is because $\text{Tr}Y = 0$ over any GUT multiplet. Within the anzatz of gauge-mediation, a violation of $M_Y^2 \simeq 0$ can result from a number of sources. First, the messengers might not transform under $U(1)_Y$. In this case the B-ino should also be very light. Second, a large $U(1)_Y$ D-term can be generated radiatively if the messenger sector is not parity and charge conjugate invariant. Finally, the squarks and/or sleptons might transform under additional gauge interactions which couple with the messenger sector so that $\mathrm{Tr}(Ym^2)$ does not vanish over any generation. This implies the existence of additional electroweak scale matter in order to cancel the $\mathrm{Tr}(Y\{T^a, T^b\})$ anomaly, where T^a is a generator of the extra gauge interactions.

Unfortunately, sum rules which involve near cancelation among squark and slepton masses squared, such as $M_Y^2 = 0$, if in fact satisfied, are often not particularly useful experimentally. This is because the squark masses are split only at $\mathcal{O}(m_{\tilde{l}}^2/m_{\tilde{q}}^2)$ by $SU(2)_L$ and $U(1)_Y$ interactions with the messenger sector, and at $\mathcal{O}(m_{Z^0}^2/m_{\tilde{q}}^2)$ from classical $SU(2)_L$ and $U(1)_Y$ D-terms. Testing such sum rules therefore requires, in general, measurements of squark masses at the sub-GeV level, as can be determined from the masses given in Table 1. It is more useful to consider sum rules, such as the ones given below, which isolate the dominant splitting arising from $SU(2)_L$ interactions, and are only violated by $U(1)_Y$ interactions. These violations are typically smaller than the experimental resolution. The sum rules may then be tested with somewhat less precise determinations of squark masses.

The near degeneracy among squarks may be quantified by the splitting between right handed squarks

$$\Delta_{\tilde{q}_R}^2 = m_{\tilde{u}_R}^2 - m_{\tilde{d}_R}^2. \tag{26}$$

Ignoring $U(1)_Y$ interactions, this quantity is a renormalization group invariant. It receives non-zero contributions at $\mathcal{O}(m_{\tilde{e}_R}^2/m_{\tilde{q}}^2)$ from $U(1)_Y$ interactions with the messenger sector and renormalization group contributions from the B-ino mass, and $\mathcal{O}(m_{Z^0}^2/m_{\tilde{q}}^2)$ from classical $U(1)_Y$ D-terms at the low scale. Numerically $\Delta_{\tilde{q}_R}^2/(m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2) \simeq 0$ to better than 0.3% with MGM boundary conditions. The near degeneracy between right handed squarks is a necessary condition if squarks receive mass mainly from $SU(3)_C$ interactions. The quantity $\Delta_{\tilde{q}_R}^2$ also vanishes to the same order with universal boundary conditions, but need not even approximately vanish in theories in which the soft masses are only universal within GUT multiplets.

An experimentally more interesting measure which quantifies the splitting between left and right handed squarks is

$$M_{L-R}^2 = m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - \left(m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2\right) - \left(m_{\tilde{l}_L}^2 + m_{\tilde{\nu}_L}^2\right) \tag{27}$$

This quantity is also a renormalization group invariant ignoring $U(1)_Y$ interactions. It formally vanishes at the same order as (26). Numerically $M_{L-R}^2/(m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2) \simeq 0$ to better than 1% with

MGM boundary conditions. Without the left handed slepton contribution, $M^2_{L-R}/(m^2_{\tilde{u}_R}+m^2_{\tilde{d}_R})\simeq 0$ can be violated by up to 10%. This relation tests the assumption that the splitting between the left and right handed squarks is due mainly to $SU(2)_L$ interactions within the messenger sector. The splitting is therefore correlated with the left handed slepton masses, which receive masses predominantly from the same source.

If the squarks and sleptons receive mass predominantly from gauge interactions with the messenger sector, the masses depend only on gauge quantum numbers, and are independent of generation up to very small $\mathcal{O}(m_f^2/M^2)$ corrections at the messenger scale, where m_f is the partner fermion mass. However, third generation masses are modified by Yukawa contributions under renormalization group evolution and mixing. Mixing effects can be eliminated by considering the quantity $\mathrm{Tr}(m^2_{LR})$ where m^2_{LR} is the left-right scalar mass squared matrix. In addition, it is possible to choose linear combinations of masses which are independent of Yukawa couplings under renormalization group evolution at one-loop, $m^2_{\tilde{u}_L}+m^2_{\tilde{u}_R}-3m^2_{\tilde{d}_L}$, and similarly for sleptons [23]. The quantities

$$M^2_{\tilde{t}-\tilde{q}}=m^2_{\tilde{t}_1}+m^2_{\tilde{t}_2}-2m^2_t-3m^2_{\tilde{b}_2}-\left(m^2_{\tilde{u}_L}+m^2_{\tilde{u}_R}-3m^2_{\tilde{d}_L}\right) \tag{28}$$

$$M^2_{\tilde{\tau}-\tilde{e}}=m^2_{\tilde{\tau}_1}+m^2_{\tilde{\tau}_2}-3m^2_{\tilde{\nu}_\tau}-\left(m^2_{\tilde{e}_L}+m^2_{\tilde{e}_R}-3m^2_{\tilde{\nu}_e}\right) \tag{29}$$

only receive contributions at two-loops under renormalization, and in the case of $M^2_{\tilde{t}-\tilde{q}}$ from $\tilde{b}$ mixing effects which are negligible unless $\tan\beta$ is very large. The relations $M^2_{\tilde{t}-\tilde{q}}\simeq 0$ and $M^2_{\tilde{\tau}-\tilde{e}}\simeq 0$ test the assumption that scalars with different gauge quantum numbers have a flavor independent mass at the messenger scale. They vanish in any theory of the soft terms with flavor independent masses at the messenger scale, but need not vanish in theories in which alignment of the squark mass matrices with the quark masses is responsible for the lack of supersymmetric contributions to flavor changing neutral currents. Within the anzatz of gauge-mediation, violations of these relations would imply additional flavor dependent interactions with the messenger sector.

If the quantities (28) and (29) are satisfied, implying the masses are generation independent at the messenger scale, it is possible to extract the Yukawa contribution to the renormalization group evolution. The quantities

$$M^2_{h_t}=m^2_{\tilde{t}_1}+m^2_{\tilde{t}_2}-2m^2_t-\left(m^2_{\tilde{u}_L}+m^2_{\tilde{u}_R}\right) \tag{30}$$

$$M^2_{h_\tau}=m^2_{\tilde{\tau}_1}+m^2_{\tilde{\tau}_2}-\left(m^2_{\tilde{e}_L}+m^2_{\tilde{e}_R}\right) \tag{31}$$

are independent of third generation mixing effects. Under renormalization group evolution $M^2_{h_t}$ receives an $\mathcal{O}((h_t/4\pi)^2 m^2_{\tilde{t}}\ln(M/m_{\tilde{t}}))$ negative contribution from the top Yukawa. For moderate values of $\tan\beta$ this amounts to a 14% deviation from $M^2_{h_t}/(m^2_{\tilde{t}_1}+m^2_{\tilde{t}_1})=0$ for $M=\Lambda$ and grows to 29% for $M = 10^5\Lambda$. Given an independent measure of $\tan\beta$ to fix the value of h_t, this quantity gives an indirect probe of $\ln M$. Unfortunately it requires a fairly precise measurement of the squark and stop masses, but is complimentary to the $\ln M$ dependence of the mass ratios of scalars with different gauge charges discussed above. The quantity $M^2_{h_\tau}$ is only significant if $\tan\beta$ is very large. If this is the case, the splitting between $\tilde{\nu}_\tau$ and $\tilde{\nu}_e$, $\Delta^2_{\tilde{\nu}_\tau - \tilde{\nu}_e} = m^2_{\tilde{\nu}_\tau} - m^2_{\tilde{\nu}_e}$, gives an independent check of the renormalization contribution through the relation

$$M^2_{h_\tau} = 3\Delta^2_{\tilde{\nu}_\tau - \tilde{\nu}_e} \tag{32}$$

4 Variations of the Minimal Model

The results of the renormalization group analysis given above are for the minimal model of gauge-mediated supersymmetry breaking. In this section we discuss how variations of the minimal model affect the form of the superpartner spectrum and the constraints imposed by electroweak symmetry breaking.

4.1 Approximate $U(1)_R$ Symmetry

Soft scalar masses require supersymmetry breaking, while gaugino masses require both supersymmetry and $U(1)_R$ breaking, as discussed in section 2.2. It is therefore possible that the scale for gaugino masses is somewhat different than that for the scalar masses, as quantified by the parameter $\not{R}$. An example of a messenger sector with an approximate $U(1)_R$ symmetry is given in appendix B. The gaugino masses affect the scalar masses only through renormalization group evolution. For $\not{R} < 1$ the small positive contribution to scalar masses from gaugino masses is slightly reduced. The scalar mass relations discussed in section 3.2.5 are not affected by this renormalization and so are not altered. The main effect of $\not{R} < 1$ is simply to lower the overall scale for the gauginos relative to the scalars. This also does not affect the relation among gaugino masses.

4.2 Multiple Messenger Generations

The minimal model contains a single messenger generation of $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$. This can be extended to any vector representation of the standard model gauge group. Such generalizations may be parameterized by the equivalent number of $\mathbf{5}+\overline{\mathbf{5}}$ messenger generations, $N = C_3$, where C_3 is defined in Appendix A. For a $\mathbf{10}+\overline{\mathbf{10}}$ of $SU(5)$ $N = 3$. From the general expressions given in appendix A for gaugino and scalar masses, it is apparent that gaugino masses grow like N while scalar masses grow like $\sqrt{N}$ [42]. This corresponds roughly to the gaugino mass parameter $\not{R} = \sqrt{N}$.

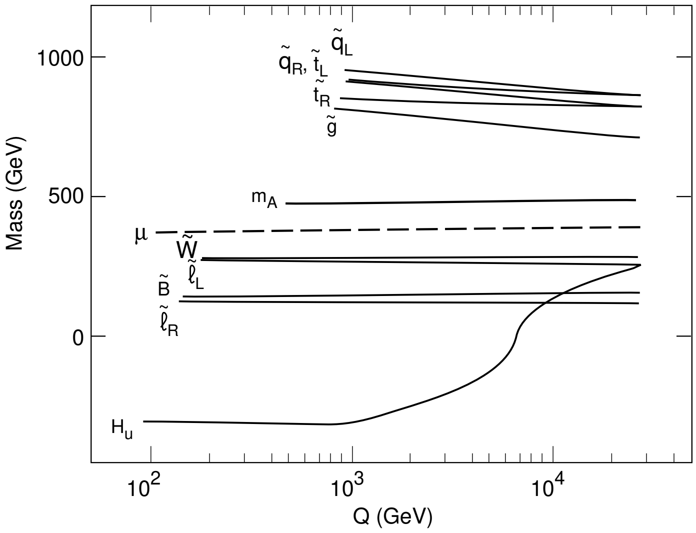

Figure 14: Renormalization group evolution of the $\overline{\rm DR}$ mass parameters with boundary conditions of the two generation messenger sector. The messenger scale is $M = 54$ TeV, $\Lambda = M$, $m_{\tilde{B}}(M) = 163$ GeV, and $\tan\beta = 3$.

Messenger sectors with larger matter representations therefore result in gauginos which are heavier relative to the scalars than in the minimal model.

The renormalization group evolution of the $\overline{\rm DR}$ parameters for $N = 2$ with a messenger scale of $M = 54$ TeV, $\Lambda = M$, $m_{\tilde{B}}(M) = 163$ GeV, and $\tan\beta = 3$ is shown in Fig. 14. The renormalization group contribution to the scalar masses proportional to the gaugino masses is slightly larger than for $N = 1$. Notice that at the low scale the renormalized right handed slepton masses are slightly smaller than the B-ino mass. The physical slepton masses, however, receive a positive contribution from the classical $U(1)_Y$ D-term, while the physical χ_1^0 mass receives a negative contribution from mixing with the Higgsinos for sgn$(\mu) = +1$. With $N = 2$ and the messenger scale not too far above Λ, the $\tilde{l}_R$ and χ_1^0 are therefore very close in mass. For the parameters of Fig. 14 $m_{\chi_1^0} = 138$ GeV and $m_{\tilde{l}_R} = 140$ GeV, so that χ_1^0 remains the lightest standard model superpartner. The D-term and Higgsino mixing contributions become smaller for a larger overall scale. For $M = 60$ TeV, $\Lambda = M$, and $\tan\beta = 3$, the χ_1^0 and $\tilde{l}_R$ masses cross at $m_{\chi_1^0} = m_{\tilde{l}_R} \simeq 153$ GeV. Since the B-ino mass decreases while the right handed slepton masses increase under renormalization, $m_{\tilde{e}_R} > m_{\chi_1^0}$ for a messenger scale well above Λ. The near degeneracy of $\tilde{l}_R$ and χ_1^0 is just a coincidence of the numerical boundary conditions for $N = 2$ and sgn$(\mu) = +1$. For messenger sectors with $N \geq 3$ a right handed slepton is naturally the lightest standard model superpartner.

The heavier gauginos which result for $N \geq 2$ only slightly modify electroweak symmetry breaking through a larger positive renormalization group contribution to the Higgs soft masses, and finite corrections at the low scale. The negative contribution to the $\tilde{\tau}_1$ mass relative to $\tilde{e}_R$ and $\tilde{\mu}_R$ from mixing and Yukawa contributions to renormalization are therefore also only slightly modified. For a given physical scalar mass at the low scale, the ratio $m_{\tilde{\tau}_1}/m_{\tilde{e}_R}$ is very similar to the $N=1$ case. For $N \geq 3$, and the regions of the $N=2$ parameter space in which $m_{\tilde{l}_R} < m_{\chi_1^0}$, the $\tilde{\tau}_1$ is the lightest standard model superpartner. As discussed in section 5.2, collider signatures for these cases are much different than for the MGM with $N=1$ with χ_1^0 as the lightest standard model superpartner.

4.3 Additional Soft Terms in the Higgs Sector

The Higgs sector parameters μ and m_{12}^2 require additional interactions with the messenger sector beyond the standard model gauge interactions. In the minimal model the precise form of these interactions is not specified, and μ and m_{12}^2 are taken as free parameters. The additional interactions which couple to the Higgs sector are likely to contribute to the Higgs soft masses $m_{H_u}^2$ and $m_{H_d}^2$, and split these from the left handed sleptons, $m_{\tilde{l}_L}^2$. Splittings of $\mathcal{O}(1)$ are not unreasonable since the additional interactions must generate μ and m_{12}^2 of the same order. The Higgs splitting may be parameterized by $\Delta_\pm^2$, defined in Eq. (11) of section 2.2.

It is possible that other scalars also receive additional contributions to soft masses. The right handed sleptons receive a gauge-mediated mass only from $U(1)_Y$ coupling, and are therefore most susceptible to a shift in mass from additional interactions. Right handed sleptons represent the potentially most sensitive probe for such interactions [30]. Note that additional messenger sector interactions do not modify at lowest order the relations among gaugino masses. Since additional interactions *must* arise in the Higgs sector, we focus in this section on the effect of additional contributions to the Higgs soft masses on electroweak symmetry breaking and the superpartner spectrum. We also consider the possibility that m_{12}^2 is generated entirely from renormalization group evolution [31].

Additional contributions to Higgs sector masses can in principle have large effects on electroweak symmetry breaking. With the Higgs bosons split from the left handed sleptons the minimization condition (16) is modified to

$$|\mu|^2 + \frac{m_Z^2}{2} = \frac{(m_{H_d,0}^2 + \Sigma_d) + (m_{H_u,0}^2 + \Sigma_u)\tan^2\beta}{\tan^2\beta - 1} - \frac{\Delta_+^2}{2} + \frac{\Delta_-^2}{2}\left(\frac{\tan\beta^2 + 1}{\tan\beta^2 - 1}\right) \qquad (33)$$

where all quantities are evaluated at the minimization scale, and $m_{H_u,0}^2$ and $m_{H_d,0}^2$ are the gauge-mediated contributions to the soft masses. The relation between μ at the minimization scale and $\Delta_- \equiv \mathrm{sgn}(\Delta_-^2(M))\sqrt{|\Delta_-^2(M)|}$ at the messenger scale is shown in Fig. 15 for $\Delta_+(M) = 0$, $m_{\tilde{B}}(M) = 115, 180, 250,$ GeV, $\tan\beta = 3$, and $\Lambda = M$. For moderate values of Δ_-, the additional

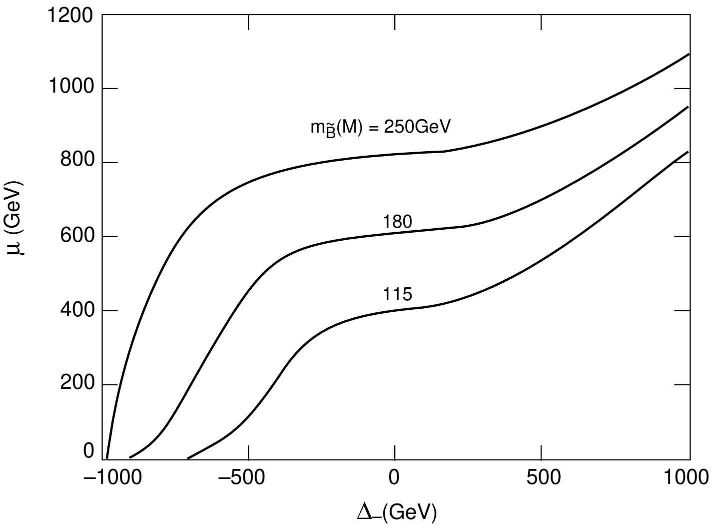

Figure 15: The relation between the low scale $|\mu|$ parameter and $\Delta_- \equiv \text{sgn}(\Delta_-^2(M))(|\Delta_-^2(M)|)^{1/2}$ at the messenger scale imposed by electroweak symmetry breaking for $\Delta_+(M) = 0$, $m_{\tilde{B}}(M) = 115, 180, 250$, GeV $\tan\beta = 3$, and $\Lambda = M$.

Higgs splittings contribute in quadrature with the gauge-mediated contributions, and only give $\mathcal{O}(\Delta_-^2/\mu^2)$ corrections to the minimization condition (33). This is the origin of the shallow plateau in Fig. 15 along which μ does not significantly vary. The plateau extends over the range $|\Delta_-^2| \lesssim |m_{H_u,0}^2|$ at the messenger scale. For $\tan\beta \gg 1$ the minimization condition (33) becomes $|\mu|^2 \simeq -m_{H_u}^2 + \frac{1}{2}(\Delta_-^2 - \Delta_+^2 - m_{Z^0}^2)$. For very large $(\Delta_-^2 - \Delta_+^2)$ this reduces to $\sqrt{2}|\mu| \simeq (\Delta_-^2 - \Delta_+^2)^{1/2}$. This linear correlation between μ and Δ_- for Δ_- large and $\Delta_+ = 0$ is apparent in Fig. 15. The non-linear behavior at small Δ_- arises from $\mathcal{O}(m_{Z^0}^2/\mu^2)$ contributions to the minimization condition (33).

The physical correlation between μ and $\Delta_\pm$ is easily understood in terms of $m_{H_u}^2$ at the messenger scale. For $\Delta_+ = 0$ and $\Delta_- > 0$, $m_{H_u}^2$ is more negative than in the minimal model, leading to a deeper minimum in the Higgs potential. In fact, for the $m_{\tilde{B}}(M) = 115$ GeV case shown in Fig. 15, $m_{H_u}^2 < 0$ already at the messenger scale for $\Delta_- \gtrsim 260$ GeV. Obtaining correct electroweak symmetry breaking for $\Delta_- > 0$ therefore requires a larger value of $|\mu|$, as can be seen in Fig. 15. Conversely, for $\Delta_+ = 0$ and $\Delta_- < 0$, $m_{H_u}^2$ is less negative than in the minimal model, leading to a more shallow minimum in the Higgs potential. Obtaining correct electroweak symmetry breaking in this limit therefore requires a smaller value of $|\mu|$, as can also be seen in Fig. 15. Eventually, for Δ_- very negative, $m_{H_u}^2$ at the messenger scale is large enough that the negative renormalization group evolution from the top Yukawa is insufficient to drive electroweak symmetry breaking. In Fig. 15 this corresponds to $|\mu| < 0$.

With $\Delta_- = 0$ and $\Delta_+ > 0$ both $m_{H_u}^2$ and $m_{H_d}^2$ are larger at the messenger scale than in the minimal model, leading to a more shallow minimum in the Higgs potential. This results in smaller values of μ, and conversely larger values of μ for $\Delta_+ < 0$. Again, there is only a significant effect for $|\Delta_+^2| \gtrsim |m_{H_u}^2|$.

The pseudo-scalar Higgs mass also depends on additional contributions to the Higgs soft masses, $m_{A^0}^2 = 2|\mu|^2 + (m_{H_u,0}^2 + \Sigma_u) + (m_{H_d,0}^2 + \Sigma_d) + \Delta_+^2$. For large $\tan\beta$ the minimization condition (33) gives $m_{A^0}^2 \simeq -(m_{H_u,0}^2 + \Sigma_u) + (m_{H_d,0}^2 + \Sigma_d) + \Delta_-^2$. Again, for $|\Delta_-^2| \lesssim |m_{H_u,0}^2|$ the pseudo-scalar mass is only slightly affected, but can be altered significantly for Δ_- very large in magnitude. Notice that m_{A^0} is independent of Δ_+ in this limit. This is because in the contribution to $m_{A^0}^2$ the change in $|\mu|^2$ induced by Δ_+^2 is cancelled by a compensating change in $m_{H_d}^2$. This approximate independence of m_{A^0} on Δ_+ persists for moderate values of $\tan\beta$. For example, for the parameters of Fig. 15 with $m_{\tilde{B}}(M) = 115$ GeV and $\Delta_- = 0$, m_{A^0} only varies between 485 GeV and 525 GeV for -500 GeV $< \Delta_+ < 500$ GeV, while μ varies from 510 GeV to 230 GeV over the same range.

The additional contributions to the Higgs soft masses can, if large enough, change the form of the superpartner spectrum. The charginos and neutralinos are affected mainly through the value of μ implied by electroweak symmetry breaking. For very large $|\mu|$, the approximately degenerate singlet χ_3^0 and triplet $(\chi_2^+, \chi_4^0, \chi_2^-)$ discussed in section 3.2.1 are mostly Higgsino, and have mass μ. For $\mu \lesssim m_2$ the charginos and neutralinos are a general mixture of gaugino and Higgsino. A value of μ in this range, as evidenced by a sizeable Higgsino component of χ_1^0, χ_2^0, or $\chi_1^\pm$, or a light χ_3^0 or $\chi_2^\pm$, would be strong evidence for deviations from the minimal model in the Higgs sector.

The heavy Higgs masses are determined by m_{A^0}. Since $m_{A^0}^2$ is roughly independent of Δ_+^2, while $|\mu|$ is sensitive to $(\Delta_-^2 - \Delta_+^2)$, the relative shift between the Higgsinos and heavy Higgses is sensitive to the individual splittings of $m_{H_u}^2$ and $m_{H_d}^2$ from the left handed sleptons, $m_{\tilde{l}_L}^2$. Within the MGM, given an independent measure of $\tan\beta$ (such as from left handed slepton - sneutrino splitting, $m_{\tilde{l}_L}^2 - m_{\nu_L}^2 = -m_{W^\pm}^2 \cos 2\beta$) the mass of the Higgsinos and heavy Higgses therefore provides an indirect probe for additional contributions to the Higgs soft masses.

Non-minimal contributions to Higgs soft masses can also affect the other scalar masses through renormalization group evolution. The largest effect comes from the radiative contribution to the $U(1)_Y$ D-term, which is generated in proportion to $S = \frac{1}{2}\mathrm{Tr}(Ym^2)$. In the minimal model the Higgs contribution to S vanishes at the messenger scale because the Higgses are degenerate and have opposite hypercharge. For $\Delta_- > 0$ they are no longer degenerate and give a negative contribution to S. This increases the magnitude of the contribution in the minimal model from running below the squark thresholds. To illustrate the effect of the Higgs contribution to S on the scalar masses, $m_{\tilde{l}_R}/m_{\chi_1^0}$ is shown in Fig. 16 as a function of Δ_- at the messenger scale for $m_{\tilde{B}}(M) = 115, 180, 250\,\mathrm{GeV}$, $\tan\beta = 3$, $\mathrm{sgn}(\mu) = +1$, and $\Lambda = M$. For Δ_- very large and positive

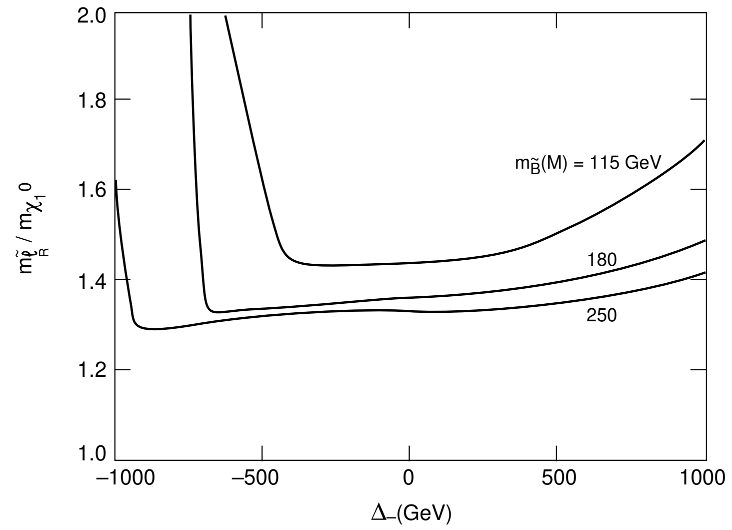

Figure 16: The ratio $m_{\tilde{l}_R}/m_{\chi_1^0}$ as a function of Δ_- at the messenger scale for $m_{\tilde{B}}(M) = 115, 180, 250\,\mathrm{GeV}$, $\tan\beta = 3$, $\mathrm{sgn}(\mu) = +1$, and $\Lambda = M$.

the radiatively generated $U(1)_Y$ D-term contribution to right handed slepton masses increases the ratio $m_{\tilde{l}_R}/m_{\chi_1^0}$. For Δ_- very negative, the rapid increase in $m_{\tilde{l}_R}/m_{\chi_1^0}$ occurs because $|\mu|$ is so small that χ_1^0 becomes mostly Higgsino with mass μ.

All these modifications of the form of the superpartner spectrum are significant only if the Higgs bosons receive additional contributions to the soft masses which are roughly larger in magnitude than the gauge-mediated contribution.

The μ parameter is renormalized multiplicatively while m_{12}^2 receives renormalization group contributions proportional to μm_λ, where m_λ is the B-ino or W-ino mass. As suggested in Ref. [31], it is therefore interesting to investigate the possibility that m_{12}^2 is generated only radiatively below the messenger scale, with the boundary condition $m_{12}^2(M) = 0$. Most models of the Higgs sector interactions actually suffer from $m_{12}^2 \gg \mu^2$ [3, 15], but $m_{12}^2(M) = 0$ represents a potentially interesting, and highly constrained subspace of the MGM. In order to illustrate what constraints this boundary condition implies, the relation between $m_{12}(M)$ and $\tan\beta$ imposed by electroweak symmetry breaking is shown in Fig. 17 for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV and $\Lambda = M$. The non-linear feature at $m_{12}(M) \simeq 0$ is a square root singularity since the β-function for m_{12}^2 is an implicit function of $\tan\beta$ only through the slow dependence of μ on $\tan\beta$. The value of $\tan\beta$ for which $m_{12}(M) = 0$ is almost entirely independent of the overall scale of the superpartners. This is because to lowest order the minimization condition (17) fixes m_{12}^2 at the low scale to be a homogeneous function of the overall superpartner scale (up to $\ln(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2)$ finite corrections) $m_{12}^2 \simeq f(\alpha_i, \tan\beta)(\alpha/4\pi)^2\Lambda^2$. If

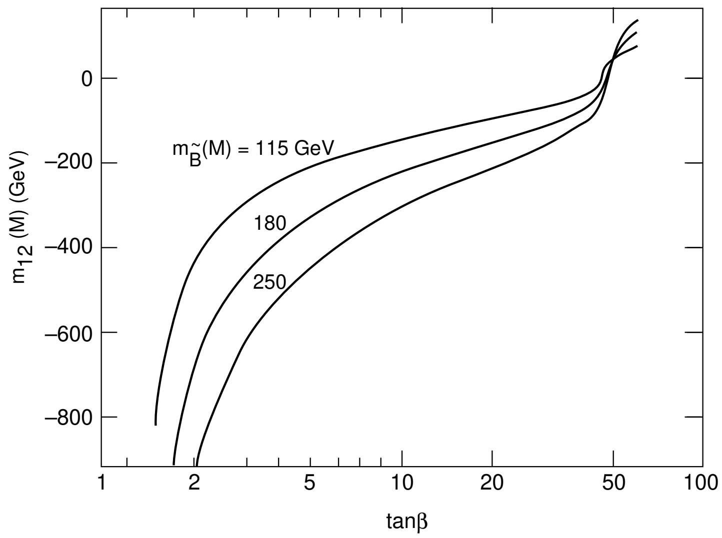

Figure 17: The relation between $m_{12}(M)$ and $\tan\beta$ imposed by electroweak symmetry breaking for $m_{\tilde{B}}(M) = 115, 180, 250$ GeV, and $\Lambda = M$.

m_{12} vanishes at any scale, then the function f vanishes at that scale, thereby determining $\tan\beta$. For $m_{12}(M) = 0$ and $\Lambda = M$ we find $\tan\beta \simeq 46$.

With the boundary condition $m_{12}(M) = 0$, the resulting large value of $\tan\beta$ is natural. This is because $m_{12}^2(Q)$ at the minimization scale, Q, is small. With the parameters given above $m_{12}(Q) \simeq -80$ GeV. For $m_{12}(Q) \to 0$, H_d does not participate in electroweak symmetry breaking, and $\tan\beta \to \infty$. As discussed in section 3.2.1, at large $\tan\beta$, $m_{\tilde{\tau}_1}$ receives a large negative contribution from the τ Yukawa due to renormalization group evolution and mixing. For the values of $\tan\beta$ given above we find $m_{\tilde{\tau}_1} \lesssim m_{\chi_1^0}$. It is important to note that for such large values of $\tan\beta$, physical quantities, such as $m_{\tilde{\tau}_1}/m_{\chi_1^0}$, depend sensitively on the precise value of the b Yukawa through renormalization group and finite contributions to the Higgs potential.

4.4 $U(1)_Y$ D-term

The $U(1)_Y$ D-term can be non-vanishing at the messenger scale, as discussed in section 2.2. This gives an additional contribution to the soft scalar masses proportional to the $U(1)_Y$ coupling, as given in Eq. (12). This splits m_{H_u} and m_{H_d}, and has the same affect on electroweak symmetry breaking as Δ_- discussed in the previous subsection. The right handed sleptons have the smallest gauge-mediated contribution to soft masses, and are therefore most susceptible to $D_Y(M)$. The biggest effect on the scalar spectrum is therefore a modification of the splitting between left and right handed sleptons. This splitting can have an important impact on the relative rates of $p\bar{p} \to$

$l^+l^-\gamma\gamma + \not{E}_T$ and $p\bar{p} \to l^\pm\gamma\gamma + \not{E}_T$ at hadron colliders [30] as compared with the minimal model discussed in section 5.2.1.

5 Phenomenological Consequences

Since the parameter space of the MGM is so constrained it is interesting to investigate what phenomenological consequences follow. In the first subsection below we discuss virtual effects, with emphasis on the constraints within the MGM from $b \to s\gamma$. In the second subsection we discuss the collider signatures associated with the gauge-mediated supersymmetry breaking. These can differ significantly from the standard MSSM with R-parity conservation and high scale supersymmetry breaking. This is because first, the lightest standard model superpartner can decay within the detector to its partner plus the Goldstino, and second, the lightest standard model superpartner can be either either χ_1^0 or $\tilde{l}_R^\pm$.

5.1 Virtual Effects

Supersymmetric theories can be probed indirectly by virtual effects on low energy, high precision, processes [32]. Among these are precision electroweak measurements, electric dipole moments, and flavor changing neutral currents. In the minimal model of gauge-mediation, supersymmetric corrections to electroweak observables are unobservably small since the charginos, left handed sleptons, and squarks are too heavy. Likewise, the effect on $R_b = \Gamma(Z^0 \to b\bar{b})/\Gamma(Z^0 \to \text{had})$ is tiny since the Higgsinos and both stops are heavy. Electric dipole moments can arise from the single CP-violating phase in the soft terms, discussed in section 2.1. The dominant contributions to the dipole moments of atoms with paired or unpaired electrons, and the neutron, come from one-loop chargino processes, just as with high scale supersymmetry breaking. The bounds on the phase are therefore comparable to those in the standard MSSM, $\text{Arg}(m_\lambda \mu (m_{12}^2)^*) \lesssim 10^{-2}$ [33, 34]. It is important to note that in some schemes for generating the Higgs sector parameters μ and m_{12}^2, the soft terms are CP conserving [3], in which case electric dipole moments are unobservably small. This is also true for the boundary condition $m_{12}^2(M) = 0$ since $(m_\lambda \mu (m_{12}^2)^*)$ vanishes in this case.

Contributions to flavor changing neutral currents can come from two sources in supersymmetric theories. The first is from flavor violation in the squark or slepton sectors. As discussed in section 1.1 this source for flavor violation is naturally small with gauge-mediated supersymmetry breaking. The second source is from second order electroweak virtual processes which are sensitive to flavor violation in the quark Yukawa couplings. At present the most sensitive probe for contributions of this type beyond those of the standard model is $b \to s\gamma$. In a supersymmetric theory one-loop

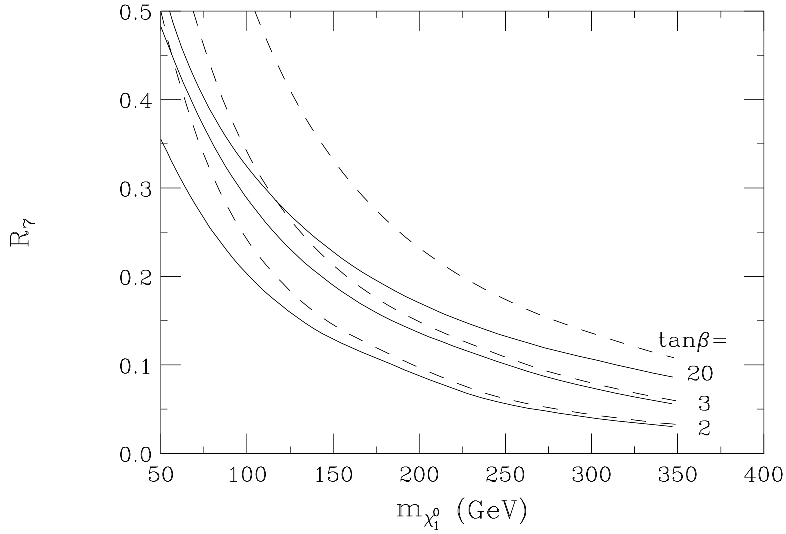

Figure 18: The parameter $R_7 \equiv C_7^{\rm MSSM}(m_{W^\pm})/C_7^{\rm SM}(m_{W^\pm}) - 1$ as a function of the lightest neutralino mass, $m_{\chi_1^0}$, for $\tan\beta = 2, 3, 20$, and $\Lambda = M$. The solid lines are for $\mu > 0$ and the dashed lines for $\mu < 0$.

$\chi^\pm - \tilde{t}$ and $H^\pm - t$ contributions can compete with the standard model $W^\pm - t$ one-loop effect. The standard model effect is dominated by the transition magnetic dipole operator which arises from the electromagnetic penguin, and the tree level charged current operator, which contributes under renormalization group evolution. The dominant supersymmetric contributions are through the transition dipole operator. It is therefore convenient to parameterize the supersymmetric contributions as

$$R_7 \equiv \frac{C_7^{\rm MSSM}(m_{W^\pm})}{C_7^{\rm SM}(m_{W^\pm})} - 1 \tag{34}$$

where $C_7(m_{W^\pm})$ is the coefficient of the dipole operator at a renormalization scale $m_{W^\pm}$, and $C_7^{\rm MSSM}(m_{W^\pm})$ contains the entire MSSM contributions (including the $W^\pm - t$ loop). In the limit of decoupling the supersymmetric states and heavy Higgs bosons $R_7 = 0$. The parameter R_7 is shown in Fig. 18 as a function of the lightest neutralino mass, $m_{\chi_1^0}$, for both signs of μ, $\tan\beta = 2, 3, 20$, and $\Lambda = M$ [35]. The χ_1^0 mass is plotted in Fig. 18 as representative of the overall scale of the superpartner masses. The dominant contribution comes from the $H^\pm - t$ loop which adds constructively to the standard model $W^\pm - t$ loop. The $\chi^\pm - \tilde{t}$ loop gives a destructive contribution which is smaller in magnitude because the stops are so heavy. The sgn μ dependence of R_7 results from this small destructive contribution mainly because the Higgsino component of the lightest chargino is larger(smaller) for sgn $\mu = +(-)$ (cf. Eq. 19). The $\chi^\pm - \tilde{t}$ loop amounts to roughly a $-15(5)\%$ contribution compared with the $H^\pm - t$ loop for sgn $\mu = +(-)$. The non-standard model contribution to R_7 decreases for small $\tan\beta$ since $m_{H^\pm} \simeq m_{A^0}$ increases in this region.

In order to relate R_7 to $\mathrm{Br}(b \to s\gamma)$ the dipole and tree level charged current operators must be evolved down to the scale m_b. Using the results of Ref. [36], which include the leading QCD contributions to the anomalous dimension matrix, we find

$$\frac{\mathrm{Br}^{\mathrm{MSSM}}(b \to s\gamma)}{\mathrm{Br}^{\mathrm{SM}}(b \to s\gamma)} \simeq |1 + 0.45\ R_7(m_{W^\pm})|^2. \tag{35}$$

for $m_t^{\mathrm{pole}} = 175$ GeV. For this top mass $\mathrm{Br}^{\mathrm{SM}}(b \to s\gamma) \simeq (3.25 \pm 0.5) \times 10^{-4}$ where the uncertainties are estimates of the theoretical uncertainty coming mainly from $\alpha_s(m_b)$ and renormalization scale dependence [37]. Using the "lower" theoretical value and the 95% CL experimental upper limit of $\mathrm{Br}(b \to s\gamma) < 4.2 \times 10^{-4}$ from the CLEO measurement [38], we find $R_7 < 0.5$.[§] This bound assumes that the non-standard model effects arise predominantly in the dipole operator, and are constructive with the standard model contribution. In the MGM for $\mu > 0$, $\tan\beta = 3$, and $\Lambda = M$, this bound corresponds to $m_{\chi_1^0} \gtrsim 45$ GeV, or a charged Higgs mass of $m_{H^\pm} \gtrsim 300$ GeV.

The present experimental limit does not severely constrain the parameter space of the MGM. This follows from the fact that the charged Higgs is very heavy over most of the allowed parameter space. Except for very large $\tan\beta$ $m_{H^\pm} \gtrsim |\mu|$, and imposing electroweak symmetry breaking implies $3m_{\chi_1^0} \lesssim |\mu| \lesssim 6m_{\chi_1^0}$, as discussed in sections 3.1 and 3.2.3. For example, with the parameters of Table 1 $m_{H^\pm} \simeq 5.4m_{\chi_1^0}$. Note that since the stops are never light in the minimal model there is no region of the parameter space for which the $\chi_1^\pm - \tilde{t}$ loop can cancel the $H^\pm - t$ loop.

Precise measurements of $\mathrm{Br}(b \to s\gamma)$ at future B-factories, and improved calculations of the anomalous dimension matrix and finite contributions at the scale m_b, will improve the uncertainty in R_7 to ± 0.1 [40]. Within the MGM, even for $\tan\beta = 2$ and $\mu > 0$, a measurement of $\mathrm{Br}(b \to s\gamma)$ consistent with the standard model would give a bound on the charged Higgs mass of $m_{H^\pm} \gtrsim 1200$ GeV, or equivalently an indirect bound on the chargino mass of $\chi_1^\pm$ mass of $m_{\chi_1^\pm} \gtrsim 350$ GeV. Such an indirect bound on the chargino mass is more stringent than the direct bound that could be obtained at the main injector upgrade at the Tevatron [30], and significantly better than the direct bound that will be available at LEP II.

5.2 Collider Signatures

Direct searches for superpartner production at high energy colliders represent the best probe for supersymmetry. Most searches assume that R-parity is conserved and that the lightest standard model superpartner is a stable neutralino. Pair production of supersymmetric states then takes place

[§] This is somewhat more conservative than the bound of $R_7 < 0.2$ suggested in Ref. [39].

through gauge or gaugino interactions, with cascade decays to pairs of neutralinos. The neutralinos escape the detector leading to the classic signature of missing energy. With gauge-mediated supersymmetry breaking the collider signatures can be much different in some circumstances. First, for a messenger scale well below the Planck scale, the gravitino is naturally the lightest supersymmetric particle. If the supersymmetry breaking scale is below a few 1000 TeV, the lightest standard model superpartner can decay to its partner plus the Goldstino component of the gravitino inside the detector [41, 42]. The Goldstino, and associated decay rates, are discussed in appendix C. Second, as discussed in sections 3.2.1 and 4.2 it is possible that the lightest standard model superpartner is a slepton [41, 42]. If the supersymmetry breaking scale is larger than a few 1000 TeV, the signature for supersymmetry is then a pair of heavy charged particles plowing through the detector, rather than missing energy.

The form of the superpartner spectrum has an important impact on what discovery modes are available at a collider. With gauge-mediation, all the strongly interacting states, including the stops, are generally too heavy to be relevant to discovery in the near future. In addition, the constraints of electroweak symmetry breaking imply that the heavy Higgs bosons and mostly Higgsino singlet χ_3^0 and triplet $(\chi_2^+, \chi_4^0, \chi_2^-)$ are also too heavy. The mostly B-ino χ_1^0, mostly W-ino triplet $(\chi_1^+, \chi_2^0, \chi_1^-)$, right handed sleptons $\tilde{l}_R^{\pm}$, and lightest Higgs boson, h^0, are the accessible light states.

In this section we discuss the collider signatures of gauge-mediated supersymmetry breaking associated with the electroweak supersymmetric states. In the next two subsections the signatures associated with either a neutralino or slepton as the lightest standard model superpartner are presented.

5.2.1 Missing Energy Signatures

The minimal model has a conserved R-parity by assumption. At moderate $\tan\beta$, χ_1^0 is the lightest standard model superpartner. If decay to the Goldstino takes place well outside the detector the classic signature of missing energy results. However, the form of the low lying spectrum largely dictates the modes which can be observed. The lightest charged states are the right handed sleptons, $\tilde{l}_R^{\pm}$. At an e^+e^- collider the most relevant mode is then $e^+e^- \to \tilde{l}_R^+\tilde{l}_R^-$ with $\tilde{l}_R^{\pm} \to l^{\pm}\chi_1^0$. For small $\tan\beta$ all the the sleptons are essentially degenerate so the rates to each lepton flavor should be essentially identical. For large $\tan\beta$ the $\tilde{\tau}_1$ can become measurably lighter than $\tilde{e}_R$ and $\tilde{\mu}_R$ (cf. Fig. 7). If sleptons receive masses at the messenger scale only from standard model gauge interactions, the only source for splitting of τ_1 from $\tilde{e}_R$ and $\tilde{\mu}_R$ is the τ Yukawa in renormalization group evolution and mixing. As discussed in section 3.2.1 the largest effect is from $\tilde{\tau}_L - \tilde{\tau}_R$ mixing proportional to $\tan\beta$. A precision measurement of $m_{\tilde{\tau}_1}$ therefore provides an indirect probe of whether $\tan\beta$ is

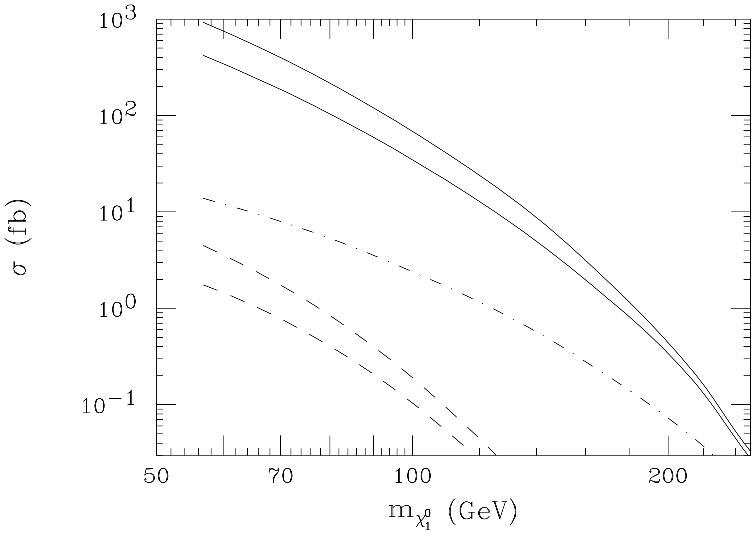

Figure 19: Production cross sections (fb) for $p\bar{p}$ initial state to the final states $\chi_1^{\pm}\chi_2^0$ (upper solid line), $\chi_1^+\chi_1^-$ (lower solid line), $\tilde{l}_R^+\tilde{l}_R^-$ (dot-dashed line), $\tilde{\nu}_L\tilde{l}_L^{\pm}$ (upper dashed line), and $\tilde{l}_L^+\tilde{l}_L^-$ (lower dashed line). Lepton flavors are not summed. The center of mass energy is 2 TeV, sgn(μ) = +1, and $\Lambda = M$.

large or not.

At a hadron collider both the mass and gauge quantum numbers determine the production rate for supersymmetric states. The production cross sections for electroweak states in $p\bar{p}$ collisions at $\sqrt{s}$ = 2 TeV (appropriate for the main injector upgrade at the Tevatron) are shown in Fig. 19 as a function of $m_{\chi_1^0}$ for MGM boundary conditions with $\Lambda = M$ and sgn$(\mu) = +1$. The largest cross section is for pairs of the mostly W-ino $SU(2)_L$ triplet $(\chi_1^+, \chi_2^0, \chi_1^-)$ through off-shell $W^{\pm *}$ and Z^{0*}. Pair production of $\tilde{l}_R^+\tilde{l}_R^-$ is relatively suppressed even though $m_{\tilde{l}_R} < m_{\chi_1^{\pm}}$ because scalar production suffers a β^3 suppression near threshold, and the right handed sleptons couple only through $U(1)_Y$ interactions via off-shell γ^* and Z^{0*}. However, as the overall scale of the superpartner masses is increased $\tilde{l}_R^+\tilde{l}_R^-$ production becomes relatively more important as can be seen in Fig. 19. This is because production of the more massive $\chi_1^{\pm}\chi_1^0$ and $\chi_1^+\chi_1^-$ is reduced by the rapidly falling parton distribution functions. Pair production of $\tilde{l}_L^+\tilde{l}_L^-$, $\tilde{l}_L^{\pm}\tilde{\nu}_L$, and $\tilde{\nu}_L\tilde{\nu}_L$ through off-shell γ^*, Z^{0*}, and $W^{\pm *}$ is suppressed relative to $\tilde{l}_R^+\tilde{l}_R^-$ by the larger left handed slepton masses.

The renormalization group and classical $U(1)_Y$ D-term contributions which slightly increase $m_{\tilde{l}_R}$, and the renormalization group contribution which decreases $m_{\chi_1^0}$, have an impact on the relative importance $\tilde{l}_R^+\tilde{l}_R^-$ production. These effects, along with the radiatively generated $U(1)_Y$ D-term, "improve" the kinematics of the leptons arising from $\tilde{l}_R^{\pm} \to l^{\pm}\chi_1^0$ since $m_{\tilde{l}_R} - m_{\chi_1^0}$ is increased [31]. However, the overall rate is simultaneously reduced to a fairly insignificant level [30]. For example, with $\text{sgn}(\mu) = +1$ an overall scale which would give an average of one $\tilde{l}_R^+ \tilde{l}_R^-$ event in 100 pb^{-1} of integrated luminosity, would result in over 80 chargino events. As discussed in section 3.2.1, the shift in the triplet $(\chi_1^+, \chi_2^0, \chi_1^-)$ mass from mixing with the Higgsinos is anti-correlated with $\text{sgn}(\mu)$. For $\text{sgn}(\mu) = -1$ the splitting between the right handed sleptons and triplet is larger, thereby reducing slightly chargino production. For example, with $\text{sgn}(\mu) = -1$, a single $\tilde{l}_R^+ \tilde{l}_R^-$ event in 100 pb^{-1} of integrated luminosity, would result in 30 chargino events. The relative rate of the $\tilde{l}_R^+ \tilde{l}_R^-$ initial state is increased in the minimal model for $\not{R} > 1$. However, as discussed in Ref. [30], obtaining a rate comparable to $\chi_1^\pm \chi_2^0$ results in "poor" kinematics, in that the leptons arising from $\tilde{l}_R^\pm \to l^\pm \chi_1^0$ are fairly soft since $m_{\tilde{l}_R} - m_{\chi_1^0}$ is reduced. Note that for $\not{R} < 1$ chargino production becomes even more important than $\tilde{l}_R^+ \tilde{l}_R^-$ production.

In the minimal model pair production of $\chi_1^\pm \chi_2^0$ and $\chi_1^+ \chi_1^-$ are the most important modes at a hadron collider. The cascade decays of $\chi_1^\pm$ and χ_2^0 are largely fixed by the form of the superpartner spectrum and couplings. If open, $\chi_1^\pm$ decays predominantly through its Higgsino components to the Higgsino components of χ_1^0 by $\chi_1^\pm \to \chi_1^0 W^\pm$. Likewise, χ_2^0 can also decay by $\chi_2^0 \to \chi_1^0 Z^0$. However, if open $\chi_2^0 \to h^0 \chi_1^0$ is suppressed by only a single Higgsino component in either χ_2^0 or χ_1^0, and represents the dominant decay mode for $m_{\chi_2^0} \gtrsim m_{h^0} + m_{\chi_1^0}$. The decay $\chi_2^0 \to \tilde{l}_R^\pm l^\mp$ is suppressed by the very small B-ino component of χ_2^0, and is only important if the other two-body modes given above are closed. If the two body decay modes for $\chi_1^\pm$ are closed, it decays through three-body final states predominantly through off-shell $W^{\pm *}$. Over much of the parameter space the minimal model therefore gives rise to the signatures $p\bar{p} \to W^\pm Z^0 + \not{E}_T$, $W^\pm h^0 + \not{E}_T$, and $W^+ W^- + \not{E}_T$. If decay to the Goldstino takes place well outside the detector, the minimal model yields the "standard" chargino signatures at a hadron collider [43].

If the intrinsic supersymmetry breaking scale is below a few 1000 TeV, the lightest standard model superpartner can decay to its partner plus the Goldstino within the detector [41, 42]. For the case of χ_1^0 as the lightest standard model superpartner, this degrades somewhat the missing energy, but leads to additional visible energy. The neutralino χ_1^0 decays by $\chi_1^0 \to \gamma + G$ and if kinematically accessible $\chi_1^0 \to (Z^0, h^0, H^0, A^0) + G$. In the minimal model $m_{A^0}, m_{H^0} > m_{\chi_1^0}$ so the only two body final states potentially open are $\chi_1^0 \to (\gamma, Z^0, h^0) + G$. However, as discussed in section 3.2.1, with MGM boundary conditions, electroweak symmetry breaking implies that χ_1^0 is mostly B-ino, and therefore decays predominantly to the gauge boson final states. The decay $\chi_1^0 \to h^0 + G$ takes place only through the small Higgsino components. In appendix C the decay rate to the h^0 final state is shown to be suppressed by $\mathcal{O}(m_{Z^0}^2 m_{\chi_1^0}^2 / \mu^4)$ compared with the gauge boson final states, and is therefore insignificant in the minimal model. Observation of the decay $\chi_1^0 \to h^0 + G$ would imply non-negligible Higgsino components in χ_1^0, and be a clear signal for deviations from the minimal

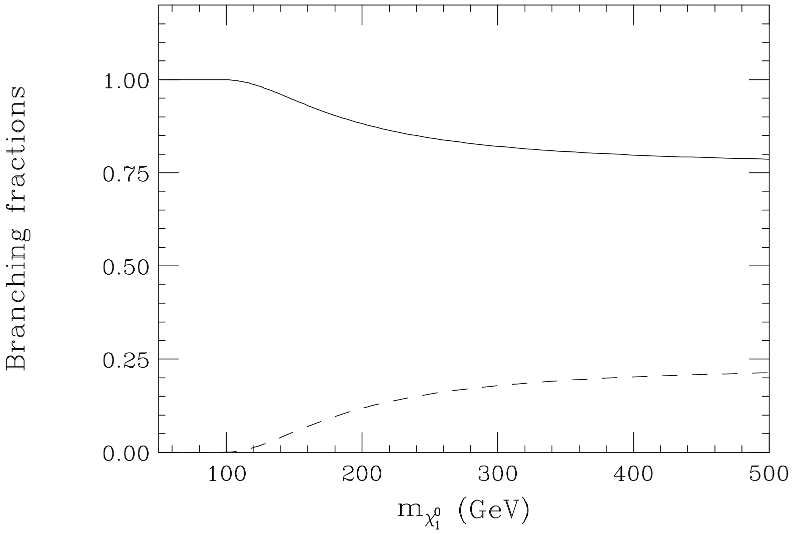

Figure 20: The branching ratios for $\chi_1^0 \to \gamma + G$ (solid line) and $\chi_1^0 \to Z^0 + G$ (dashed line) as a function of $m_{\chi_1^0}$ for $\Lambda = M$.

model in the Higgs sector. For example, as discussed in section 4.3, Δ_- large and negative leads to a mostly Higgsino χ_1^0, which decays predominantly by $\chi_1^0 \to h^0 + G$. The branching ratios in the minimal model for $\chi_1^0 \to \gamma + G$ and $\chi_1^0 \to Z^0 + G$ are shown in Fig. 20 as a function of $m_{\chi_1^0}$ for $\Lambda = M$.

In the minimal model, with χ_1^0 decaying within the detector, the signatures are the same as those given above, but with an additional pair of $\gamma\gamma$, γZ^0, or $Z^0 Z^0$. At an e^+e^- collider $e^+e^- \to \chi_1^0\chi_1^0 \to \gamma\gamma + \not{E}$ becomes the discovery mode [41, 42, 44]. At a hadron collider the reduction in $\not{E}_T$ from the secondary decay is more than compensated by the additional very distinctive visible energy. The presence of hard photons significantly reduces the background compared with standard supersymmetric signals [41, 42, 30, 45, 46]. In addition, decay of $\chi_1^0 \to \gamma + G$ over a macroscopic distance leads to displaced photon tracks, and of $\chi_1^0 \to Z^0 + G$ to displaced charged particle tracks. Measurement of the displaced vertex distribution gives a measure of the supersymmetry breaking scale.

In the minimal model, for large $\tan\beta$ the $\tilde{\tau}_1$ can become significantly lighter than $\tilde{e}_R$ and $\tilde{\mu}_R$. This enhances the $\tilde{\tau}_1^+\tilde{\tau}_1^-$ production cross section at a hadron collider. The ratio $\sigma(p\bar{p} \to \tilde{\tau}_1^+\tilde{\tau}_1^-)/\sigma(p\bar{p} \to \tilde{e}_R^+\tilde{e}_R^-)$ for $\sqrt{s} = 2$ TeV is shown in Fig. 21 as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115$ GeV and $\Lambda = M$. Measurement of this ratio gives a measure of the $\tilde{\tau}_1$ mass. Within the minimal model this allows an indirect probe of $\tan\beta$.

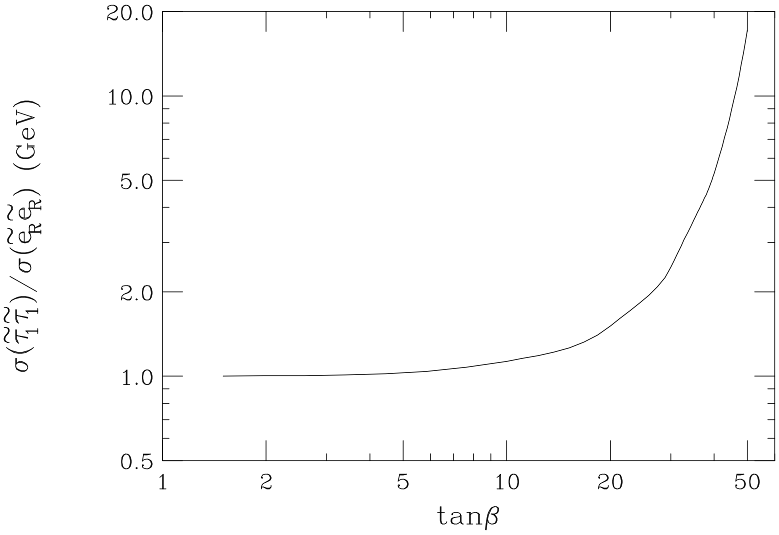

Figure 21: The ratio $\sigma(p\bar{p} \to \tilde{\tau}_1^+\tilde{\tau}_1^-)/\sigma(p\bar{p} \to \tilde{e}_R^+\tilde{e}_R^-)$ as a function of $\tan\beta$ for $m_{\tilde{B}}(M) = 115$ GeV and $\Lambda = M$. The center of mass energy is 2 TeV.

5.2.2 Heavy Charged Particle Signatures

In the minimal model the $\tilde{\tau}_1$ becomes lighter than χ_1^0 for $\tan\beta$ large enough, as discussed in section 3.2.1. The $\tilde{\tau}_1$ is then the lightest standard model superpartner. This is not a cosmological problem since the $\tilde{\tau}_1$ can decay to the Goldstino component of the gravitino, $\tilde{\tau}_1 \to \tau + G$, on a cosmologically short time scale. However, if the supersymmetry breaking scale is larger than a few 1000 TeV, at a collider this decay takes place well outside the detector. The signature for supersymmetry in this case is heavy charged particles passing through the detector, rather than missing energy. At an e^+e^- collider the most relevant mode is then $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$. At a hadron collider the signatures are very different since χ_1^0 decays by $\chi_1^0 \to \tilde{\tau}_1^\pm\tau^\mp$. Over much of the parameter space the dominant chargino production then gives rise to the signatures $p\bar{p} \to W^\pm Z^0\tau^+\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-$, $W^\pm h^0\tau^+\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-$, and $W^+W^-\tau^+\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-$. The additional cascade decays $\chi_2^0 \to \tilde{\tau}_1^\pm\tau^\mp$ and $\chi_1^\pm \to \tilde{\tau}_1^\pm\nu_\tau$ are also available through the $\tilde{\tau}_L$ component of $\tilde{\tau}_1$. Chargino production can therefore also give the signatures $p\bar{p} \to W^\pm\tau^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp$, $Z^0\tau^\pm\tilde{\tau}_1^\pm\tilde{\tau}_1^\mp + \not{E}_T$, $h^0\tau^\pm\tilde{\tau}_1^\pm\tilde{\tau}_1^\mp + \not{E}_T$, $\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\mp + \not{E}_T$, $W^\pm\tau^\pm\tilde{\tau}_1^\mp\tilde{\tau}_1^\pm + \not{E}_T$, and $\tilde{\tau}_1^+\tilde{\tau}_1^- + \not{E}_T$. Finally, direct pair production gives the signature $p\bar{p} \to \tilde{\tau}_1^+\tilde{\tau}_1^-$, while $\tilde{l}_R^+\tilde{l}_R^-$ production gives $p\bar{p} \to l^+l^-\tau^+\tau^-\tilde{\tau}_1^+\tilde{\tau}_1^-$ for $l = e, \mu$.

If the supersymmetry breaking scale is well below a few 1000 TeV the $\tilde{\tau}_1$ decays within the detector to a Goldstino by $\tilde{\tau}_1 \to \tau + G$. The signature of heavy charged particle production is then lost, but missing energy results since the Goldstinos escape the detector. In the signatures given above then all the $\tilde{\tau}_1^\pm$ are replaced by $\tau^\pm + \not{E}_T$.

The signature of heavy charged particles can also result with multiple generations in the messenger sector. As discussed in section 4.2, messenger sectors with larger matter representations result in gauginos which are heavier relative to the scalars than in the minimal model. For $N \geq 3$, and over much of the parameter space of the $N = 2$ model, a right handed slepton is the lightest standard model superpartner. Because of the larger Yukawa coupling, the $\tilde{\tau}_1$ is always lighter than $\tilde{e}_R$ and $\tilde{\mu}_R$. However, for small to moderate $\tan\beta$ $m_{\tilde{\mu}_R} - m_{\tilde{\tau}_1} < m_\tau + m_\mu$, and the decay $\tilde{\mu}_R^\pm \to \tilde{\tau}_1^+ \tau^- \mu^\pm$ through the B-ino component of off-shell χ_1^{0*} is kinematically blocked, and likewise for $\tilde{e}_R$ [46]. In addition, the second order electroweak decay $\tilde{\mu}_R^+ \to \tilde{\tau}_1^+ \nu_\tau \bar{\nu}_\mu$ is highly suppressed and not relevant for decay within the detector. In this case all three sleptons $\tilde{e}_R$, $\tilde{\mu}_R$, and $\tilde{\tau}_1$, are effectively stable on the scale of the detector for a supersymmetry breaking scale larger than a few 1000 TeV. At an e^+e^- collider the most relevant signature becomes $e^+e^- \to \tilde{l}_R^+ \tilde{l}_R^-$ with the sleptons leaving a greater than minimum ionizing track in the detector. At a hadron collider $\chi_1^\pm \chi_2^0$ and $\chi_1^+ \chi_1^-$ production gives the signatures $p\bar{p} \to W^\pm Z^0 l^+ l'^- \tilde{l}_R^- \tilde{l}'^+_R$, $W^\pm h^0 l^+ l'^- \tilde{l}_R^- \tilde{l}'^+_R$, and $W^+ W^- l^+ l'^- \tilde{l}_R^- \tilde{l}'^+_R$, while direct slepton pair production gives $p\bar{p} \to \tilde{l}_R^+ \tilde{l}_R^-$. If $\tan\beta$ is large then $m_{\tilde{\mu}_R} - m_{\tilde{\tau}_1} > m_\tau + m_\mu$, so that the decay $\tilde{\mu}_R^+ \to \tilde{\tau}_1^\pm \tau^\mp \mu^+$ can take place within the detector, and likewise for $\tilde{e}_R$. All the cascades then end with $\tilde{\tau}_1^\pm$. The additional $\tau^\pm l^+$, $\tau^\pm l^-$ which result from $\tilde{l}_R^\pm$ decay are very soft unless the splitting $m_{\tilde{l}_R} - m_{\tilde{\tau}_1}$ is sizeable.

If the supersymmetry breaking scale is below a few 1000 TeV, the sleptons can decay to the Goldstino by $\tilde{l}_R \to l + G$ within the detector. A missing energy signature then results from the escaping Goldstinos, and all the $\tilde{l}_R^\pm$ in the above signatures are replaced by $l^\pm +$ $\not{E}_T$. If the decay $\tilde{l}_R \to l + G$ takes place over a macroscopic distance the spectacular signature of a greater than minimizing ionizing track with a kink to a minimum ionizing track results [42, 46]. Again, measurement of the decay length distribution would give a measure of the supersymmetry breaking scale. All these interesting heavy charged particle signatures should not be overlooked in the search for supersymmetry at future colliders.

6 Conclusions

Gauge-mediated supersymmetry breaking has many consequences for the superpartner mass spectrum, and phenomenological signatures. In a large class of gauge-mediated models (including all the single spurion models given in this paper) the general features include:

- The natural absence of flavor changing neutral currents.

- A large hierarchy among scalars with different gauge charges, $m_{\tilde{q}_R}/m_{\tilde{l}_R} \lesssim 6.3$, and $m_{\tilde{l}_L}/m_{\tilde{l}_R} \lesssim 2.1$, with the inequalities saturated for a messenger scale of order the supersymmetry breaking scale.

- Mass splittings between scalars with different gauge quantum numbers are related by various sum rules.
- "Gaugino unification" mass relations.
- Precise degeneracy among the first two generation scalars, and sum rules for the third generation that test the flavor symmetry of masses at the messenger scale.
- Radiative electroweak symmetry breaking induced by heavy stops, even for a low messenger scale.
- Small A-terms.
- The lightest standard model superpartner is either χ_1^0 or $\tilde{l}_R^{\pm}$.
- The possibility of the lightest standard model superpartner decaying within the detector to its partner plus the Goldstino.

The mass relations and sum rules hold in a very large class of gauge-mediated models and represent fairly generic features. The possibility that the lightest standard model superpartners is a charged slepton leads to the dramatic signature of heavy charged particles leaving a greater than minimum ionizing track in the detector. This signature should not be overlooked in searches for supersymmetry at future colliders. The possibility that the lightest standard model superpartner decays within the detector, either $\chi_1^0 \to (\gamma, Z^0, h^0) + G$ or $\tilde{l}_R \to l + G$, leads to very distinctive signatures, and provides the possibility of indirectly measuring the supersymmetry breaking scale.

The minimal model of gauge-mediated supersymmetry breaking is highly constrained, and gives the additional general features:

- Gauginos are lighter than the associated scalars, $m_3 < m_{\tilde{q}}$, $m_2 < m_{\tilde{l}_L}$, and $m_1 < m_{\tilde{l}_R}$.
- The Higgsinos are heavier than the electroweak gauginos, $3m_1 \lesssim |\mu| \lesssim 6m_1$.
- Absence of a light stop.
- The mass of the lightest Higgs boson receives large radiative corrections from the heavy stops, $80\,\text{GeV} \lesssim m_{h^0} \lesssim 140\,\text{GeV}$.

- Unless $\tan\beta$ is very large, the lightest standard model superpartner is the mostly B-ino χ_1^0, which decays predominantly by $\chi_1^0 \to \gamma + G$.
- At a hadron collider the largest supersymmetric production cross section is for $\chi_1^{\pm}\chi_2^0$ and $\chi_1^+\chi_1^-$.
- Discernible deviation in $Br(b \to s\gamma)$ from the standard model with data from future B-factories.

If superpartners are detected at a high energy collider, one of the most important tasks will be to match the low energy spectrum with a more fundamental theory. Patterns and relations among the superpartner masses can in general give information about the messenger sector responsible for transmitting supersymmetry breaking. As discussed in this paper, gauge-mediated supersymmetry breaking leads to many distinctive patterns in the superpartner spectrum. Any spectroscopy can of course be trivially mocked by postulates of non-universal boundary conditions at any messenger scale. However, gauge-mediation in its minimal form represents a *simple* anzatz which is highly predictive. In addition, if decay of the lightest standard model superpartner takes place within the detector, implying a low supersymmetry breaking scale, the usual gauge interactions are likely to play some role in the messenger sector.

The overall scale for the superpartner masses is of course a free parameter. However, the Higgs sector mass parameters set the scale for electroweak symmetry breaking. Since all the superpartner masses are related to a single overall scale with gauge-mediated supersymmetry breaking, it is reasonable that the states transforming under $SU(2)_L$ have mass of order the electroweak scale. From the low energy point of view, masses much larger than this scale would appear to imply that electroweak symmetry breaking is tuned, and that the electroweak scale is unnaturally small. Quantitative measures of tuning are of course subjective. However, when the overall scale is large compared to m_{Z^0}, tuning among the Higgs sector parameters arises in the minimization condition (16) as a near cancelation between $(\tan^2\beta - 1)|\mu|^2$ and $m_{H_u}^2 - \tan^2\beta m_{H_d}^2$, resulting in $m_{Z^0}^2 \ll |\mu|^2$. In this regime the near cancelation enforces constraints among some of the Higgs sector parameters in order to obtain proper electroweak symmetry breaking. As the overall superpartner scale is increased these tuned constraints are reflected by ratios in the physical spectrum which become independent of the electroweak scale. This tuning is visually apparent in Fig. 10 as the linear dependence of m_{A^0} on $m_{\chi_1^0}$ at large overall scales. The "natural" regime in which the Higgs sector parameters are all the same order as the electroweak and superpartner scale can be seen in Fig. 10 as the non-linear dependence of m_{A^0} on $m_{\chi_1^0}$. In Fig. 5 this "natural" non-linear regime with light superpartners is in the far lower left corner, and hardly discernible in the linearly scaled plot.

Although no more subjective than any measure of tuning, this bodes well for the prospects indirectly detecting the effects of superpartners and Higgs bosons in precision measurements, and for directly producing superpartners at future colliders.

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A Soft Masses for a General Messenger Sector

The messenger sector of the minimal model may be generalized to fields Q and $\overline{Q}$ forming any vector representation of the standard model gauge group. With a coupling to a single background spurion, $W = \lambda S Q \overline{Q}$, the general expression for gaugino masses at the messenger scale is

$$m_{\lambda_i} = \sum_m C_{2,i} \Lambda \, \frac{\alpha_i}{4\pi} \tag{36}$$

where $\mathrm{Tr}(T^a T^b) = C_2 \delta^{ab}$, and $\sum_m$ is over all messenger representations. For a fundamental of $SU(N_c)$ $C_2 = 1/2$, for a two index antisymmetric tensor $C_2 = N_c/2 - 1$, and for an adjoint $C_2 = N_c$. The general expression for scalar masses at the messenger scale is

$$m^2 = 2\Lambda^2 \sum_{m,i} C_{2,i} C_{3,i} \left(\frac{\alpha_i}{4\pi} \right)^2 \tag{37}$$

where $\sum_a (T^a T^a) = C_3 \mathbf{1}$ is over the visible sector field. For a fundamental of $SU(N_c)$ $C_3 = (N_c^2 - 1)/(2N_c)$, for a two index antisymmetric tensor $C_3 = (N_c+1)(N_c-2)/N_c$, and for an adjoint $C_3 = N_c$.

The successful prediction of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge coupling unification [4] is unaffected at lowest order by the messenger sector if all the $C_{2,i}$ $i=1,2,3$ are identical. In this case the effect of the messengers on the renormalization group evolution of gauge couplings can be absorbed in a shift of the gauge coupling at the unification scale. Messengers which can be embedded in GUT representations are a subset of all such possible messenger sectors. All messenger sectors of this type with a single spurion also yield the "gaugino unification" mass relations at lowest order, $m_3 : m_2 : m_1 = \alpha_3 : \alpha_2 : \alpha_1$. In addition, with a single spurion the ratios of gaugino to scalar masses at the messenger scale depend on C_2. For $\alpha_3 \gg \alpha_2 > \alpha_1$ the approximate relations $m^2 \simeq 2 C_2 C_{3,i} m_{\lambda_i}^2$ for $i = 1,2,3$, result at the messenger scale. With a single spurion $\not{R} = \sqrt{C_2}$.

B A Non-minimal Messenger Sector

The minimal model of gauge-mediated supersymmetry breaking represents a very constrained and predictive theory for the soft supersymmetry breaking parameters. However, as discussed in section (2.2), non-minimal messenger sectors can modify the relations among the soft terms. Among the possible deviations away from the minimal model are *i*) messenger masses which arise from a sector not associated with supersymmetry breaking, thereby introducing dependence on the messenger Yukawa couplings, *ii*) an approximate $U(1)_R$ symmetry, leading to gauginos much lighter than the scalars, *iii*) messenger sectors which can be embedded in a GUT theory, but do not lead to the "standard" GUT relations among gaugino masses. Here we present a single model which illustrates all of these features. The model is a generalization of the one given in Ref. [30] with two generations of $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$ and two fields, X and S, with a superpotential coupling

$$W = X(\lambda_2 \ell_1 \bar{\ell}_1 + \lambda_3 q_1 \bar{q}_1 + \xi^2) + \lambda' S(\ell_1 \bar{\ell}_2 + q_1 \bar{q}_2 + \ 1 \leftrightarrow 2\ ) \tag{38}$$

The field S is taken to transform as a singlet under $SU(5)$, while X and the spurion ξ^2 transform as the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet component of $\mathbf{24}$'s of $SU(5)$. The $(\mathbf{8},\mathbf{1})+(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})+(\mathbf{1},\mathbf{3})$ components of X can gain a mass at the GUT scale. With these representations $SU(5)$ invariance implies that X couples to the messenger fields proportional to $U(1)_Y$, so that $3\lambda_3 = -2\lambda_2$ at the GUT scale. The messenger Yukawa couplings are therefore $SU(5)$ invariant, but not $SU(5)$ singlets. For $\xi \neq 0$ supersymmetry is broken by the O'Raifeartaigh mechanism. For $\lambda' S > \xi$, the ground state is at $q_i = \bar{q}_i = \ell_i = \bar{\ell}_i = 0$, with X and S undetermined at tree level. For $S \neq 0$ there is a $U(1)_R$ symmetry which is broken only for $X \neq 0$.

The model exhibits the features mentioned above with $(\lambda_i X), \xi \ll \lambda' S$. In this limit the messengers obtain a mass mainly from the S expectation value, and receive soft supersymmetry breaking masses only from the auxiliary component of X. This misalignment of the scalar and fermion mass matrices introduces dependence on the relative magnitude of the Yukawa couplings λ_2 and λ_3. In this limit the X superfield may be treated as an insertion in the graphs which give rise to visible sector soft terms. The resulting gaugino masses are in the ratios

$$m_3 : m_2 : m_1 = \lambda_3^2 \alpha_3 : \lambda_2^2 \alpha_2 : \frac{1}{5}(2\lambda_3^2 + 3\lambda_2^2)\alpha_1 \tag{39}$$

Likewise the scalar masses at the messenger scale are in the approximate ratios

$$m_{\tilde{q}}^2 : m_{\tilde{l}_L}^2 : m_{\tilde{l}_R}^2 \simeq \frac{4}{3}\lambda_3^2 \alpha_3^2 : \frac{3}{4}\lambda_2^2 \alpha_2^2 : \frac{3}{25}(2\lambda_3^2 + 3\lambda_2^2)\alpha_1^2 \tag{40}$$

In addition to the sensitivity to the relative magnitude of the Yukawa couplings, there is an approximate $U(1)_R$ symmetry in this limit. Since the gaugino masses require an insertion of both the scalar and auxiliary components of X, while the scalars require only auxiliary components, the gauginos are lighter than the scalars by $\mathcal{O}(\lambda_i X/\lambda' S)$. Finally, even though the model may be embedded in a GUT theory, the gauginos do not satisfy the "standard" gaugino unification relation. This results from the misalignment of the messenger scalar and fermion mass matrices, and the fact that the couplings transform under $SU(5)$.

This model may also be generalized to a messenger sector with two generations of $\mathbf{10}+\overline{\mathbf{10}}$ of $SU(5)$, with superpotential couplings

$$W = X(\lambda_{3,2}Q_1\bar{Q}_1 + \lambda_3 u_1\bar{u}_1 + \lambda_1 e_1\bar{e}_1 + \xi^2) + \lambda' S(Q_1\bar{Q}_2 + u_1\bar{u}_2 + e_1\bar{e}_2 + \ 1 \leftrightarrow 2\ ) \tag{41}$$

With these representations $SU(5)$ invariance again implies that X couples proportional to $U(1)_Y$, so that $2\lambda_1 = 12\lambda_{3,2} = -3\lambda_3$ at the GUT scale. In the limit $(\lambda_i X), \xi \ll \lambda' S$, the gaugino masses are in the ratio

$$m_3 : m_2 : m_1 = (2\lambda_{3,2}^2 + \lambda_3^2)\alpha_3 : 3\lambda_{3,2}^2\alpha_2 : \frac{1}{5}(\lambda_{3,2}^2 + 8\lambda_3^2 + 6\lambda_1^2)\alpha_1 \tag{42}$$

This leads to the interesting hierarchy $m_2 \ll m_1 < m_3$, which does not even approximately satisfy "gaugino unification." The large B-ino mass arises from the large messenger positron Yukawa coupling. The scalar masses are in the approximate ratios

$$m_{\tilde{q}}^2 : m_{\tilde{l}_L}^2 : m_{\tilde{l}_R}^2 \simeq \frac{4}{3}(2\lambda_{3,2}^2 + \lambda_3)\alpha_3^2 : \frac{9}{4}\lambda_2^2\alpha_2^2 : \frac{3}{25}(\lambda_{3,2}^2 + 8\lambda_3^2 + 6\lambda_1^2)\alpha_1 \tag{43}$$

Again, in this model the gauginos are lighter than the scalars by $\mathcal{O}(\lambda_i X/\lambda' S)$.

C Decay to the Goldstino

The spontaneous breaking of global supersymmetry leads to the existence of a massless Goldstino fermion, the Goldstino. The lowest order couplings for emission or absorption of a single on-shell Goldstino are fixed by the supersymmetric Goldberger-Treiman low energy theorem to be proportional to the divergence of the supercurrent [47]

$$\mathcal{L} = -\frac{1}{F}\partial_\mu G^\alpha j^\mu{}_\alpha \ + \ h.c. \tag{44}$$

where

$$j^\mu{}_\alpha = \sigma^\nu_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}\psi_\beta D_\nu\phi + \frac{1}{2\sqrt{2}}\sigma^\nu_{\alpha\dot{\alpha}}\bar{\sigma}^{\rho\dot{\alpha}\beta}\sigma^\mu_{\beta\dot{\beta}}\lambda^{a\dot{\beta}^*}F^a_{\nu\rho} \tag{45}$$

is the supercurrent, and the components of the chiral superfields and vector superfield strengths are given by

$$\Phi = \phi + \sqrt{2}\theta^\alpha\psi_\alpha + \theta^2 F \tag{46}$$

$$W^a_\alpha = \lambda^a_\alpha + \left(\delta_\alpha{}^\beta D^a - \frac{i}{2}\sigma^\mu_{\alpha\dot\alpha}\bar\sigma^{\nu\dot\alpha\beta}F^a_{\mu\nu}\right)\theta_\beta + i\theta^2\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\lambda^{a\dot\alpha *} \tag{47}$$

The physical Goldstino and supersymmetry breaking scales are given by

$$G^\alpha = \frac{1}{F}\sum_{\psi_i}\psi^\alpha_i\langle F_i\rangle + \frac{1}{\sqrt{2}}\sum_{\lambda^a_i}\lambda^{a\alpha}_i\langle D^a_i\rangle \tag{48}$$

$$F \equiv \sum_i\langle F_i\rangle + \frac{1}{\sqrt{2}}\sum_{i,a}\langle D^a_i\rangle \tag{49}$$

where the sums are over all chiral and vector multiplets, with auxiliary components F_i and D^a_i respectively, in both the supersymmetry breaking and visible sectors.

Only terms which break supersymmetry in the low energy effective theory contribute to $\partial_\mu j^\mu{}_\alpha$, and therefore contribute to Goldstino couplings. The derivative form of the coupling (44) may be obtained by applying a space-time dependent supersymmetry transformation to cancel the non-derivative Goldstino couplings arising from the effective operators which couple the visible and supersymmetry breaking sectors. Either basis can in principle be used to compute Goldstino couplings. The derivative basis is more often convenient since supersymmetry breaking in the low energy theory then appears only on on-shell external states through equations of motion. This basis is especially useful for the lightest neutralino in the standard model, χ^0_1, which is a mixture of states that receive mass from both supersymmetric and supersymmetry breaking terms in the low energy theory.

In local supersymmetry the Goldstino becomes the longitudinal component of the gravitino, giving a gravitino mass of

$$m_G = \frac{F}{\sqrt{3}M_p} \tag{50}$$

where $M_p = m_p/\sqrt{8\pi}$ is the reduced Planck mass. For a Messenger scale well below the Planck scale, the gravitino is much lighter than the electroweak scale, and is naturally the lightest supersymmetric particle (LSP). The lightest standard model superpartner is the the next to lightest supersymmetric particle (NLSP). In this case the gravitino mass and couplings of the spin $\frac{3}{2}$ transverse components are completely irrelevant for accelerator experiments. The global description in terms of the the spin $\frac{1}{2}$ Goldstino component is therefore sufficient.

The Goldstino acts on supermultiplets like the supercharge, and therefore transforms a superpartner into its partner. Since the Goldstino couplings are suppressed compared with electro-weak and strong interactions, decay to the Goldstino is only relevant for the lightest standard model superpartner (NLSP). Assuming R-parity conservation, the NLSP is quasi-stable and can only decay through coupling to the Goldstino. For $\sqrt{F}$ below a few 1000 TeV, such a decay can take place within a detector. With gauge-mediated supersymmetry breaking the natural candidate for the NLSP is either a slepton or electro-weak neutralino. For a slepton NLSP the decay rate for $\tilde{l} \to l + G$ is

$$\Gamma(\tilde{l} \to l + G) = \frac{m_{\tilde{l}}^5}{16\pi F^2} \tag{51}$$

It is also possible that the NLSP is the lightest electro-weak neutralino, χ_1^0. It can decay through the gaugino components by $\chi_1^0 \to \gamma + G$ and $\chi_1^0 \to Z^0 + G$ if kinematically accessible, and through the Higgsino components by $\chi_1^0 \to Z^0 + G$, $\chi_1^0 \to h^0 + G$, $\chi_1^0 \to H^0 + G$, and $\chi_1^0 \to A^0 + G$ if kinematically accessible. The decay rates for gauge boson final states are

$$\Gamma(\chi_1^0 \to \gamma + G) = |\cos\theta_W N_{1\tilde{B}} + \sin\theta_W N_{1\tilde{W}}|^2 \; \frac{m_{\chi_1^0}^5}{16\pi F^2} \tag{52}$$

$$\begin{aligned} \Gamma(\chi_1^0 \to Z^0 + G) = {} & \left(|\sin\theta_W N_{1\tilde{B}} - \cos\theta_W N_{1\tilde{W}}|^2 + \frac{1}{2} |\cos\beta N_{1d} - \sin\beta N_{1u}|^2 \right) \\ & \times \; \frac{m_{\chi_1^0}^5}{16\pi F^2} \left(1 - \frac{m_{Z^0}^2}{m_{\chi_1^0}^2} \right)^4 \end{aligned} \tag{53}$$

The amplitudes for the decay $\chi_1^0 \to Z^0 + G$ from gaugino and Higgsino components do not interfere since the gaugino admixtures couple only to transverse Z^0 components, while the Higgsino admixtures couple only to longitudinal Z^0 components. Even though χ_1^0 is a fermion, the decay to a gauge boson is isotropic in the rest frame. This follows since χ_1^0 is Majorana, and can therefore decay to both Goldstino helicities. The rate to the two Goldstino helicities sums to an isotropic distribution.

The decay rates for Higgs boson final states are

$$\Gamma(\chi_1^0 \to h^0 + G) = |\sin\alpha N_{1d} - \cos\alpha N_{1u}|^2 \; \frac{m_{\chi_1^0}^5}{32\pi F^2} \left(1 - \frac{m_{h^0}^2}{m_{\chi_1^0}^2} \right)^4 \tag{54}$$

$$\Gamma(\chi_1^0 \to H^0 + G) = |\cos\alpha N_{1d} + \sin\alpha N_{1u}|^2 \; \frac{m_{\chi_1^0}^5}{32\pi F^2} \left(1 - \frac{m_{H^0}^2}{m_{\chi_1^0}^2} \right)^4 \tag{55}$$

$$\Gamma(\chi_1^0 \to A^0 + G) = |\sin\beta N_{1d} + \cos\beta N_{1u}|^2 \; \frac{m_{\chi_1^0}^5}{32\pi F^2} \left(1 - \frac{m_{A^0}^2}{m_{\chi_1^0}^2} \right)^4 \tag{56}$$

where the physical Higgs bosons, and Goldstone boson are related to H_u^0 and H_d^0 by [48]

$$\begin{aligned} H_d^0 &= \frac{1}{\sqrt{2}} \left(v_d + H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta - iG^0 \cos\beta \right) \\ H_u^0 &= \frac{1}{\sqrt{2}} \left(v_u + H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta + iG^0 \sin\beta \right) \end{aligned} \tag{57}$$

These expressions for the decay rates agree with those obtained in Ref. [45]. Because of the Goldstino derivative coupling to the gauge field strengths and derivative of the scalar components, the decay to massive final states suffers a β^4 suppression near threshold, where $\beta = \sqrt{1-4(m_f^2/m_i^2)/(1+m_f^2/m_i^2)^2}$ is the massive final state velocity in the decaying rest frame.

With minimal boundary conditions, electro-weak symmetry breaking implies that both $|\mu|$ and m_{A^0} are somewhat larger than $m_{\chi_1^0}$ and m_{Z^0}. It is therefore interesting to consider the decay rates in this limit. For $\mu^2 - m_1^2 \gg m_{Z^0}^2$, χ_1^0 is mostly B-ino. In this limit the neutralino mass matrix may be diagonalized perturbatively to find the small Higgsino admixtures in χ_1^0. To $\mathcal{O}(m_{Z^0}\mu/(\mu^2-m_1^2))$ the χ_1^0 eigenvectors become

$$\begin{aligned} N_{1\tilde{B}} &\simeq 1 \\ N_{1\tilde{W}} &\simeq 0 \\ N_{1d} &\simeq \sin\theta_W \sin\beta \; \frac{m_{Z^0}(\mu + m_1\cot\beta)}{|\mu|^2 - m_1^2} \\ N_{1u} &\simeq -\sin\theta_W \cos\beta \; \frac{m_{Z^0}(\mu + m_1\tan\beta)}{|\mu|^2 - m_1^2} \end{aligned} \tag{58}$$

The decay rates to gauge boson final states are then dominated by the B-ino component

$$\Gamma(\chi_1^0 \to \gamma + G) \simeq \frac{\cos^2\theta_W m_{\chi_1^0}^5}{16\pi F^2} \tag{59}$$

$$\Gamma(\chi_1^0 \to Z^0 + G) \simeq \frac{\sin^2\theta_W m_{\chi_1^0}^5}{16\pi F^2}\left(1 - \frac{m_{Z^0}^2}{m_{\chi_1^0}^2}\right)^4 \tag{60}$$

For decay to Higgs bosons, with $m_{A^0} \gg m_{\chi_1^0}$, only $\chi_1^0 \to h^0 + G$ is open. For $m_{A^0}^2 \gg m_{Z^0}^2$ the Higgs boson decoupling limit is reached in which the h^0 couplings become standard model like. In this limit the $h^0 - H^0$ mixing angle is related to $\tan\beta$ by $\sin\alpha \simeq -\cos\beta$, and $\cos\alpha \simeq \sin\beta$ [48]. The h^0 components of H_u^0 and H_d^0 then align with the expectation values, as can be seen from (57). In the B-ino limit, using the approximate χ_1^0 eigenvalues given above, the $\chi_1^0 h^0 G$ coupling is then proportional to

$$\sin\alpha N_{1d} - \cos\alpha N_{1u} \simeq \frac{m_{Z^0}\sin\theta_W}{|\mu|^2 - m_1^2}\left(\frac{\mu m_{Z^0}^2 \sin 4\beta}{2m_{A^0}^2} - m_1\cos 2\beta\right) \tag{61}$$

where $\cos(\alpha-\beta) \simeq \sin 4\beta m_{Z^0}^2/(2m_{A^0}^2)$ [48]. For $m_{Z^0}^2/m_{A^0}^2 \ll m_1/\mu$ the decay rate in the large $|\mu|$ and m_{A^0} limits becomes

$$\Gamma(\chi_1^0 \to h^0 + G) \simeq \frac{\sin^2\theta_W \cos^2 2\beta m_{\chi_1^0}^5}{32\pi F^2}\left(\frac{m_{Z^0}m_{\chi_1^0}}{|\mu|^2 - m_{\chi_1^0}^2}\right)^2\left(1 - \frac{m_{h^0}^2}{m_{\chi_1^0}^2}\right)^4 \tag{62}$$

This is down by $\mathcal{O}(m_{Z^0}^2 m_{\chi_1^0}^2/\mu^4)$ compared with the gauge boson final states.

The branching ratio for $\chi_1^0 \to h^0 + G$ is therefore always quite small in the minimal model. For small $m_{\chi_1^0}$ the rate suffers the β^4 threshold suppression, and for large $m_{\chi_1^0}$ is suppressed by the rapid decoupling of h^0. For $m_{\chi_1^0}^2 \gg m_{Z^0}^2$ the branching ratios to gauge bosons therefore dominate the two body decays and approach

$$\mathrm{Br}(\chi_1^0 \to \gamma + G) \simeq \cos^2\theta_W \left(1 + 4\sin^2\theta_W m_{Z^0}^2/m_{\chi_1^0}^2\right)$$

$$\mathrm{Br}(\chi_1^0 \to Z^0 + G) \simeq \sin^2\theta_W \left(1 - 4\cos^2\theta_W m_{Z^0}^2/m_{\chi_1^0}^2\right) \tag{63}$$

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