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CORRECT CHROMATICITIES OF CIRCULAR ACCELERATORS WITHOUT SEXTUPOLES†

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Abstract

A new method of correcting chromaticities of circular accelerators is introduced. Instead of using two families of sextupoles, as the standard way to correct chromaticities, two pairs of TM_{210} mode RF cavities are used. The betatron phase advances (both horizontal and vertical) between the two cavities are set to be a multiple of π, and a proper momentum compaction is required. With this method, sextupole nonlinear terms are eliminated. There are octupole terms left by this method. However, they are explicit and should be easy to compensate. An example lattice demonstrates the method. The power required for the RF cavities is estimated.

1 INTRODUCTION

The chromaticity of circular accelerators represents the betatron oscillation frequency change due to the particle relative momentum deviation. It results from the quadrupole strength varing according to different particle momentum. To cure the chromaticity, the standard method is to introduce sextupoles in a dispersive region, that give additional focussing to particles with different momentum.

The side effect of using sextupoles to correct chromaticity is the introduction of nonlinearities in the accelerator. The development of modern storage rings, such as light sources, damping rings and heavy quark factories, diamonds even smaller emittances. This requires stronger focussing, which results in higher chromaticities. To correct the chromaticities, stronger sextupoles are required. Therefore, the stronger nonlinearity is introduced to the storage ring.

This paper introduces another method to correct chromaticities. Instead of using sextupoles, quadrupole (TM_{210} mode) RF cavities are employed. With this method, no sextupole terms are introduced. There are some octupole and coupling terms left, but they are easily calculated and should be easy to compensate.

2 THE FIELD AND EFFECT OF TM_{210} MODE CAVITY

The electro-magnetic field can be solved for analytically for a cylindrical cavity with radius R and length d. For the TM_{210} mode, we have

$$\mathbf{E}_S = -\frac{U_0\alpha_2}{2d}(x^2-y^2) \tag{1}$$

$$\mathbf{B}_X = -\frac{U_0\alpha_2}{c\ d}\ y\ s \tag{2}$$

$$\mathbf{B}_y = -\frac{U_0\alpha_2}{c\ d}\ x\ s \tag{3}$$

in which, U_0 is the transverse kicking field, α_2=5.136/R, and c is the speed of light.

Here, a linear approximation is applied, assuming the bunch length is much shorter than the wave length of the cavity, and particles are near axis.

Thus, the changes to the variables are:

$$\Delta x' = -\frac{eU_0\alpha_2}{E_S}\ x\ s = -2K\ x\ s \tag{4}$$

$$\Delta y' = \frac{eU_0\alpha_2}{E_S}\ y\ s = 2K\ x\ s \tag{5}$$

$$\Delta\delta = -\frac{eU_0\alpha_2}{2E_S}(x^2-y^2) = -K(x^2-y^2) \tag{6}$$

where $K=\frac{eU_0\alpha_2}{2E_S}$, E_S is the beam energy. From this, we can find the second order transfer matrix (using TRANSPORT[1] convention) elements:

$$T_{215} = T_{251} = -K \tag{7}$$

$$T_{435} = T_{453} = K \tag{8}$$

$$T_{611} = -K \tag{9}$$

$$T_{633} = K \tag{10}$$

Therefore, the TM_{210} cavity can be included in beam line as a symplectic second order matrix.

3 THE CHROMATICITY CORRECTION SCHEME

To correct chromaticity, a quadrupole field with a strength proportional to the particle momentum is required. As shown in previous section (eq. 4, 5), the TM_{210} cavity provides the quadrupole field with a strength proportional to the particle longitudinal position. Therefore, a special lattice arrangement is designed to achieve the proper effect.

We consider two TM_{210} cavities located at each end of a section of lattice with horizontal and vertical phase advance odd multiples of π, and a reasonable value of momentum compaction factor. The two cavities have identical field strength but 180° out of phase, *i.e.*, the first cavity has a strength *K*, as defined in previous section, while the other cavity has a strength of *-K*. The transform through the lattice can be found as follows:

After the first cavity:

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$$
\begin{aligned}
x_1 &= x_0 \\
x_1' &= x_0' - 2Kx_0 s_0 \\
y_1 &= y_0 \\
y_1' &= y_0' + 2Ky_0 s_0 \\
s_1 &= s_0 \\
\delta_1 &= \delta_0 + K(x_0^2 - y_0^2)
\end{aligned}
$$

(11)

Then, going through the arc:

$$
\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \\ s_2 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -L \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \\ s_1 \\ \delta_1 \end{pmatrix}
$$

$$
= \begin{pmatrix} -x_0 \\ -x_0' + 2Kx_0 s_0 \\ -y_0 \\ -y_0' - 2Ky_0 s_0 \\ s_0 - KL(x_0^2 - y_0^2) \\ \delta_0 + K(x_0^2 - y_0^2) \end{pmatrix}
$$

(12)

where L is the product of momentum compaction factor and the orbit length. Finally, apply the second cavity:

$$
\begin{aligned}
x_3 &= -x_0 \\
x_3' &= -x_0' - 2KLx_0\delta_0 - 2K^2L(x_0^3 - x_0 y_0^2) \\
y_3 &= -y_0 \\
y_3' &= -y_0' + 2KLy_0\delta_0 + 2K^2L(x_0^2 y_0 - y_0^3) \\
s_3 &= s_0 - L\delta_0 - KL(x_0^2 - y_0^2) \\
\delta_3 &= \delta_0
\end{aligned}
$$

(13)

Notice that at the end of the lattice, we obtained the terms we wanted—$-2KLx_0\delta_0$ and $2KLy_0\delta_0$. They are quadrupole terms with a strength proportional to momentum.

The quadrupole terms result in a tune change by

$$
\begin{aligned}
\Delta Q_x &= -\frac{\beta_x}{4\pi} 2KL\delta_0 \\
\Delta Q_y &= \frac{\beta_y}{4\pi} 2KL\delta_0
\end{aligned}
$$

(14)

Therefore, we have the following corrections to chromaticities:

$$
\begin{aligned}
Q_x' &= \frac{dQ_x}{d\delta} = -\frac{\beta_x KL}{2\pi} \\
Q_y' &= \frac{dQ_y}{d\delta} = \frac{\beta_y KL}{2\pi}
\end{aligned}
$$

(15)

To properly correct chromaticities in both directions, two pairs of the cavities are necessary. One pair is located at large β_x location and the other pair is located at large β_y. This is similar to the use of two families of sextupoles to correct both chromaticities.

The design makes chromatic correction possible by taking advantage of momentum compaction in the arc. The quadrupole mode cavity provides a quadrupole field with a strength varying with the longitudinal *position*, while we need a quadrupole field with strength varying with the *momentum*. Therefore, the lattice is designed to cancel the quadrupole effects of the two cavities. However, due to the momentum compaction effect of the bending magnets in the arc, particle's longitudinal positions have been changed when they move from the first cavity to the second. This change results in partial cancellation of the kicks, and what is left is just we want for the chromaticity correction because the change is proportional to the particle momentum.

4 AN EXAMPLE

To demonstrate this chromaticity correction method, a storage ring with the TM_{210} cavities is designed. The lattice is essentially the same as the synchrotron light source HiSOR at Hiroshima[2]. The reason for choosing this lattice is that its parameters are just right for implementing this concept. HiSOR is a 1.5 GeV electron storage ring. The natural chromaticities are -11.01 and -7.75 respectively for horizontal and vertical directions. The momentum compaction factor is 0.0118.

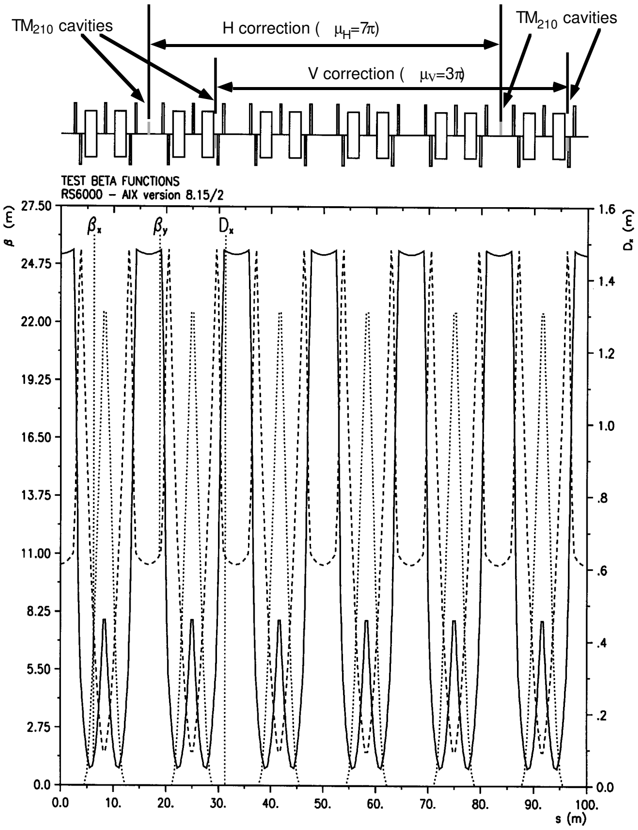

Figure 1. The lattice layout and machine functions.

HiSOR consists of six superperiods. In each superperiod, the horizontal phase advance is 1.75π and the vertical phase advance is 0.75π. By choosing four superperiods as the arc, the horizontal and vertical phase advances are 7π and 3π, respectively. This satisfies the -I transfer matrices condition.

Two pairs of TM_{210} cavities are inserted in the lattice, both spaced by four superperiods. One pair is put at the beginning of one superperiod, where high β_x helps horizontal chromaticity correction. The other pair is located next to a vertical focussing quadrupole to provide correction for the vertical chromaticity. Because the storage ring has only six superperiods and the arc needs four superperiods, the two pairs of cavities are interleaved. Figure 1 shows the lattice and machine functions.

Lattice optics calculation is done with MAD[3]. The original sextupoles are turned off. The TM_{210} cavities are entered as arbitrary matrix elements, giving the second order matrix elements according to eq. (7)-(10). Two variables, K_H, the strength of the first pair cavities, and K_V, the strength of the second pair cavities, are adjusted to make both chromaticities zero. The solution is K_H=4.383, K_V=-4.238.

5 DISCUSSION

In the previous section, calculation has demonstrated the new method of chromaticity correction. Instead of using 24 sextupoles in the original design, four RF cavities have effectively the same function at second order.

The new method not only physically removes all the sextupole magnets, but also the sextupole components and the associated higher order terms from the sextupoles. However, this method is not nonlinearity-free. As one can see in eq. (13), there are extra terms, $x_3' \sim -2K^2L(x_0^3 - x_0 y_0^2)$ and $y_3' \sim 2K^2L(x_0^2 y_0 - y_0^3)$, created by this scheme. They are octupole terms. How strong they are in comparison with the crossing terms of sextupoles and what the consequences are in terms of beam dynamics is not transparent, because they strongly depend on lattice designs. In any case, the new method gives clear, single magnet terms, that are easy to model, and can be compensated by putting an actual octupole with an inverted field strength right next to the second cavity. Meanwhile, because of the removal of the sextupoles, the problems associated with misalignment will be dramatically reduced too.

Another extra term in eq. (13) is $s_3 \sim -KL(x_0^2 - y_0^2)$. It introduces synchro-betatron coupling, primarily exciting $2Q_{x/y} \pm Q_s$ family resonances. It also changes radiation damping properties. If the storage ring does not operate near those resonances, this term may not have significant effects on the machine performance.

One important issue is the RF power required for the cavities. It is obvious that there won't be much power going into the beam, and the amount depends on the beam sizes. For a round beam profile at the cavity, for example, the power into the beam is zero. The most power needed is to build up the field, which could be high, depending on the lattice. In this case, a superconducting cavity is ideal. In principle, such a superconducting cavity can be built. However, to the author's knowledge, no experimental work has been done for such cavities yet.

There has been some design work for room temperature TM_{210} cavities[4]. Using such a conventional cavity, the major concern is the power dissipated on the cavity walls when a high field is established inside. It could demand huge amount of power from the RF source.

Govil *et al.* [4] defined R/Q in terms of the energy ΔW transfered to a particle off-axis a specified distance r_0, and the power dissipated in the walls, P_{loss}, as: $R/Q = (\Delta W)^2 / P_{loss}$. ΔW is related to the constant K of interest for this work according to

$$\Delta W = K r_0^2 E_{beam}.$$

Combining these two relations one can determine the steady state power required for CW operation, P_{loss},.

Ref. 4 considered a number of designs and calculated R/Q for r_0=2mm. A typical design has R/Q~1kΩ at 2856MHz, and a beam passing aperture of 1cm diameter. This corresponds to 690kW in the

HiSOR example. Thsi power requirement can be reduced by using multicell cavities.

Another approach is to design the lattice from the start with this technique in mind. A larger momentum compaction factor, a longer path length, a higher β-function at the cavities will help to reduce the power. For instance, the power estimated for SPEAR is much less. Suppose we take half of the SPEAR ring to place a pair of the cavities to correct the half of the chromaticity. We have E_{beam}=3 GeV, β=20 m, α=0.04, and orbit path length is 100 m. To correct a chromaticity of -5, we need K=0.4. Therefore, even using single cell, the dissipated power is 23 kW. This is much easier to achieve and more economical.

An optimized cavity design which provides a higher shunt impedence can also reduce the power requirement. Certainly, a better answer may be the use of superconducting cavity.

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