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BEAM-BEAM DISRUPTION AND THE CASE FOR A PLASMA LENS IN e^-e^- COLLISIONS

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After a short review of the expected beam-beam disruption effects in e$^-$e$^-$ collisions we show estimates that a plasma lens will improve the luminosity for a SLC- and NLC-energy e$^-$e$^-$ collider.

1. Introduction

In an e^-e^- collider, the mutual disruption of the beams will significantly decrease the luminosity, different from the case of e^+e^- colliders, where the mutual attraction of the oppositely-charged beams leads to a self-focusing effect. In this paper we estimate the beam disruption at SLC and NLC energies using ABEL[1] simulations. We then give estimates of how a plasma lens might improve the luminosity in e^-e^- collisions.

2. Beam-Beam Disruption in e^-e^- Collisions

In the disruption process, particle trajectories are bent by the collective field provided by the oncoming beam. We first consider the case of e^+e^- collisions. A single electron of energy E moving in opposite direction of a single positron at an impact parameter b will receive a momentum kick of[2]

$$\Delta p_\perp = -\frac{2e^2}{bc} \tag{1}$$

and will be deflected by an angle

$$\theta = \frac{\Delta p_\perp}{p} = -\frac{2r_e}{\gamma b}, \tag{2}$$

where $r_e = e^2/m_e c^2$ is the classical radius of the electron, and γ is $E/m_e c^2$. A 'focal length' f at which the electron trajectory intersects that of the positron, is then

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given by

$$f = -b/\theta = \gamma b^2/2r_e. \tag{3}$$

Similarly, a single particle being deflected by a Gaussian beam of transverse r.m.s. sizes σ_x, σ_y, longitudinal r.m.s. size σ_z, and containing N electrons, can be shown to have a focal length

$$f_{x,y} = \frac{\gamma\sigma_{x.y}(\sigma_x + \sigma_y)}{2r_e N}. \tag{4}$$

This motivates the definition of dimensionless and Lorentz-invariant *disruption parameters* D_x, D_y,[3] which are given by

$$D_{x,y} = \frac{\sigma_z}{f_{x,y}} = \frac{2r_e N\sigma_z}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}. \tag{5}$$

If D is much less than one, the disruption is very weak, and the focal point behind the beam; for $D \gg 1$, the particles oscillate in the focusing potential of the oncoming beam. For a 500 GeV Next Linear Collider (NLC), for example, with 'flat beams,' a proposed parameter set[4] has $D_x = 0.08$ and $D_y = 8.2$; so there is considerable focusing in y, but very little in x.

A 'luminosity enhancement factor,' H_D, can be defined by the ratio of the effective luminosity in the presence of disruption, $\mathcal{L}$, to that in the absence of disruption, $\mathcal{L}_0$,

$$H_D = \frac{\mathcal{L}}{\mathcal{L}_0}. \tag{6}$$

For e^+e^- collisions with the NLC parameters just given, H_D turns out to be 1.4.

Similar considerations apply to e^-e^- collisions, but now, of course, the deflection of particles in the field of the oncoming beam is unbounded instead of oscillatory and confining.

To estimate the loss in luminosity for the flat NLC beams we consider the movement in the transverse coordinate y of a test particle starting out at $y_0 = \sigma_y$. Since the y-component of the electric field of the oncoming beam is near constant over the relevant length scale, the equation of motion of the test particle can be approximated as

$$\frac{d^2y}{dt^2} - \frac{D_y\sigma_y}{\sqrt{3}\sigma_z^2} = 0. \tag{7}$$

The solution is trivial:

$$y(t) = \frac{1}{2}\frac{D_y\sigma_y}{\sqrt{3}\sigma_z^2}t^2 + y_1 t + y_0. \tag{8}$$

The collision time is one half of the total bunch length of $l = 2\sqrt{3}\sigma_z$. Thus, for $t = \sqrt{3}\sigma_z, y_0 = \sigma_y, y_1 = 0$,

$$y(t) = \left(\frac{\sqrt{3}}{2}D_y + 1\right)\sigma_y =: y_{final}. \tag{9}$$

For $D_y = 8.2$ then, $y_{final} = 8.1\sigma_y \approx D_y\sigma_y$. Time-averaging y_{final} will reduce the average y by a factor of $\frac{1}{2}$, so that the effective beam size is enlarged by a factor

$$\langle y\rangle/\sigma_y = \left(\frac{\sqrt{3}}{2}D_y + 1\right) \Big/ 2 \approx 4 \tag{10}$$

and the effective luminosity reduced accordingly. This, however, is only a heuristic estimate. For a more reliable value one needs to resort to Monte Carlo simulations.

ABEL[1] simulations have been performed to calculate the luminosity enhancement factors for e^-e^- collisions, both as function of the disruption parameter for round beams, and for the flat beams as function of an offset in x or y between the flat beams, and are shown in Figures 1 and 2 below.

These simulations indicate that the luminosity enhancement factor H_D is only about 0.5 for e^-e^- collisions, compared to 1.4 for e^+e^- collisions. Thus the luminosity of an e^-e^- collider is $0.5/1.4 \approx 1/3$ of that of an e^+e^- collider with the same beam parameters.

We note that the luminosity enhancement for flat beams is dominated by the strong disruption (and therefore beam deformation) in y, while the very mild x-disruption ($D_x \approx D_y/100$) barely changes the beam size in the x-dimension. On the other hand, the electromagnetic field of a flat beam is dominated by σ_x, as represented by the 'beamstrahlung parameter' Υ:

$$\Upsilon = \frac{5}{6}\frac{Nr_e\lambda_e\gamma}{\sigma_z(\sigma_x+\sigma_y)} \approx \frac{Nr_e\lambda_e\gamma}{\sigma_z\sigma_x} \tag{11}$$

which, as we argued, barely changes due to disruption. (Here λ_e is the electron-Compton wavelength.) For e^+e^- collisions at NLC, the nominal $\Upsilon_{e^+e^-}$ is ≈ 0.09; the effective $\bar{\Upsilon}_{e^+e^-}$ with disruption effects is about the same. Although the field strengths in the e^+e^- and e^-e^- cases should be similar as discussed above, the mean fields that the particles experience on the average are expected to be somewhat different. This is due to the fact that in the e^+e^- case, the beams are pinched, the particles are moving inwards and therefore experience the weaker part of the beam field near the beam center; while in the e^-e^- case, the particles are deflected outwards and tend to experience the maximum field of the beam for a longer time. This suggests that the effective beamstrahlung parameter $\bar{\Upsilon}_{e^-e^-}$ for e^-e^- collisions should be no more than twice that of e^+e^- collisions, that is $\bar{\Upsilon}_{e^-e^-} \lesssim 0.18$. Hence also the various beamstrahlung-related effects should be no more than twice as strong in the e^-e^- as in the e^+e^- mode. In particular, we expect the number of beamstrahlung photons emitted per electron to be about two, and the spin depolarization ΔP to be $\lesssim 4\%$.

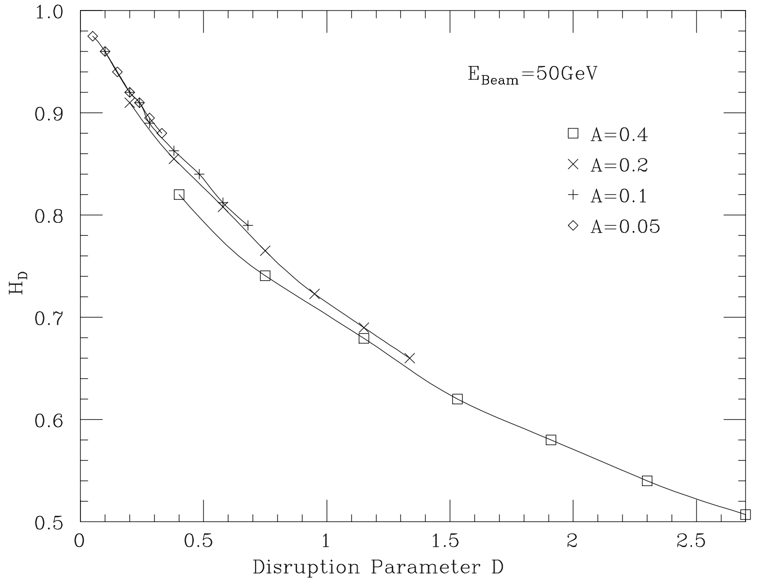

Fig. 1. Luminosity enhancment factor H_D as function of disruption parameter D, for e^-e^- collisions of round beams at $E_{Beam} = 50$ GeV. The parameter $A_{x,y} = \sigma_z/\beta^*_{x,y}$, where $\beta^*_{x,y}$ are the β-functions at the interaction point, is a measure of the inherent divergence of the beam.

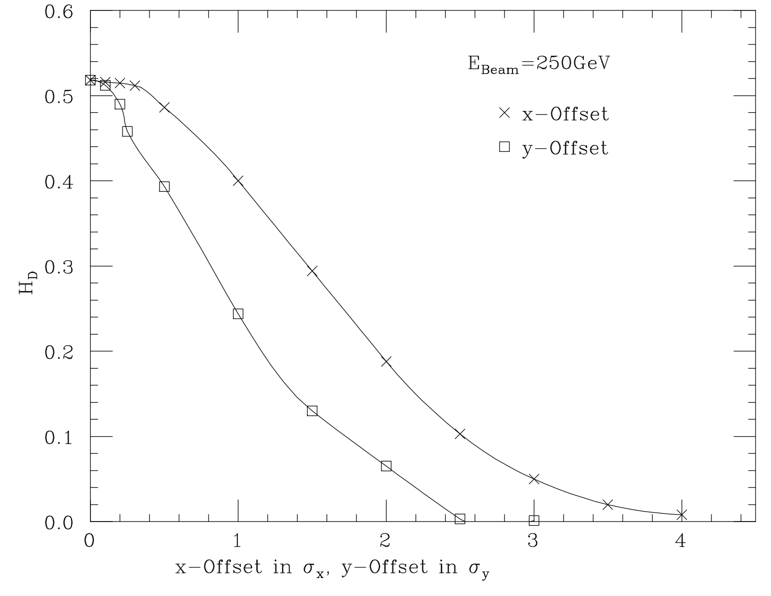

Fig. 2. Luminosity enhancement factor H_D as function of beam offset x/σ_x or y/σ_y.

3. Plasma Lens

The use of a plasma lens as a final-focusing device to increase the luminosity in e^+e^- linear colliders was proposed some time ago.[5] Conventional quadrupole magnets for final on-line focusing in high-energy accelerators have limited focusing strengths (a few hundred MG/cm), while plasma lenses are able to produce focusing strengths a few orders of magnitude higher, depending on the plasma density. A plasma-lens final-focus system might be even more important to recover the luminosity lost by disruption effects in a proposed e^-e^- collider. The self-focusing effect of a plasma on relativistic beams has been verified experimentally at Argonne National Laboratory,[6] at Tokyo University,[7] and more recently at UCLA.[8]

The issue of luminosity enhancement by a self-ionized plasma at the Stanford Linear Collider (SLC) has been studied numerically by particle-in-cell simulations; It was found that an enhancement factor of $\lesssim 5$ is possible at the SLC center-of-mass energy ($\sim$92 GeV).[9]

For the flat beams of NLC, with r.m.s. beam sizes $\sigma_x = 300$nm, $\sigma_y = 3$nm, $\sigma_z = 100\mu$m, it will be difficult to focus further in y; however, in the x-direction there is some possibility of improvement.

As an example, we consider a plasma lens of thickness $d = 1$mm, with a density $n_P = 5 \cdot 10^{18}\text{cm}^{-3}$, arranged so that the incoming beam is focused on the end of this lens. Such a lens would have a focusing strength $K = 2\pi r_e n_P/\gamma \approx 18.1$ cm^{-2}.

At the entrance to the lens, the NLC beta functions are changed from their value at the focus in absence of a lens, $\beta^*_{x0} = 10$mm, $\beta^*_{y0} = 100\mu$m, to

$$\beta_{x0} = \beta^*_{x0}\left(1 + \left(\frac{d}{\beta^*_{x0}}\right)^2\right) = 1.01\text{cm} \tag{12}$$

$$\beta_{y0} = \beta^*_{y0}\left(1 + \left(\frac{d}{\beta^*_{y0}}\right)^2\right) = 1.01\text{cm}. \tag{13}$$

The differential equation for the β-function in the plasma lens as a function of the distance s along the beam line, $\beta''' + 4K\beta' + K'\beta = 0$, can be solved with the initial conditions (matching β, β' at $s = -d$, the entrance to the plasma lens), yielding

$$\beta = \frac{\beta}{2} + \frac{1}{2K\beta^*_0} + \left(\frac{\beta}{2} - \frac{1}{2K\beta^*_0}\right)\cos[\nu(s+d)] - \frac{2d}{\nu\beta^*_0}\cos[\nu(s+d)] \tag{14}$$

where $\nu = 4K$. The maximal reduction in β^* can be found to be[9]

$$\frac{\beta^*}{\beta^*_0} = \frac{1}{1 + K\beta^*_0(\beta_0 - \beta_1)} \tag{15}$$

where β_0, β_1 are the β-function at the entrance and exit of the plasma lens, respectively. With the parameters given, one finds $\beta^*_x/\beta^*_{x0} = 0.23 = 1/4.3$, for an improvement in luminosity by a factor of ≈ 2.1. The corresponding $\beta^*_y/\beta^*_{y0} = 0.85 = 1/1.18$, is relatively small, as expected.

For a larger increase in luminosity, a plasma lens of higher density n_P and greater length d might be required. Note that the plasma lens parameters chosen above, and therefore, the luminosity enhancement, serve only as an illustration and are not necessarily optimized.

Here we consider only the use of a plasma lens as a device to focus one beam just before their interaction point; focusing both beams would, of course, double the effect. One could also imagine having a suitably shaped plasma at the interaction point itself.

4. Conclusions

We conclude that the disruption effects reduce the luminosity of e^-e^- colliders by about a factor of three compared to an e^+e^- collider with the same beam parameters; we estimate that a modest luminosity enhancement can be obtained by a plasma lens.

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