SLAC–PUB–7123

The Frequency of Neutral Meson and Neutrino Oscillation *

Boris Kayser

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

and

† Division of Physics, National Science Foundation

4201 Wilson Boulevard, Arlington, Virginia 22230

Abstract

Interference between the different mass eigenstate components of a neutral K meson causes its decay probability to oscillate with time. Related oscillations occur in the decay chain $\phi \to KK \to f_1 f_2$ (where $f_{1,2}$ are decay channels), in neutral B decay, in the chain $\Upsilon(4s) \to BB \to f_1 f_2$, and in massive neutrino propagation. Since the mass eigenstates comprising a neutral K, a neutral B, or a neutrino have different masses, they have different speeds at any given momentum. Thus, classically, they become separated in space and time. This circumstance can tempt one to evaluate their contributions to the K or B decay, or to the neutrino interaction with a detector, at different spacetime points. However, these quantum-mechanically interfering contributions must always be evaluated at precisely the same point. Evaluating them at different points can lead to predicted oscillation frequencies double their true values.

Preliminary

*Work partially supported by the Department of Energy, contract DE–AC03–76SF00515.

†Permanent address.

The neutral K meson produced in a typical kaon experiment is a superposition of two mass eigenstates: the long-lived K_L, and the shorter-lived and very slightly lighter K_S. When the K meson decays, its K_L and K_S components contribute coherently. The interference between their coherent contributions causes the probability of decay into a given final state to oscillate with time. The frequency of this oscillation is $\Delta m_K \equiv m_L - m_S$, the difference between the K_L and K_S masses.

For a given momentum, the K_L and K_S components of a kaon travel at different speeds, due to their differing masses. Hence, *classically*, in the laboratory frame of reference they will arrive at the point x where the kaon decays at slightly different times, t^L and t^S. One might then be tempted to evaluate the K_L and K_S contributions to the kaon decay amplitude at these classical arrival times. But then one would be evaluating these two coherent contributions at *different* times. This would be an incorrect procedure. Following it can lead, as we shall see, to the erroneous conclusion that the frequency of oscillation in the decay probability is not Δm_K, but $2\Delta m_K$ [1].

The oscillation in the probability for decay of a single kaon has an analogue in the decay chain

$$\begin{array}{llll} \phi & \rightarrow & K \;+\; K & , \\ & & \;\; \llcorner\!\!\rightarrow f_1 \;\; \llcorner\!\!\rightarrow f_2 & \end{array}$$

where the kaons are neutral, and $f_{1,2}$ are final states of interest. There are further analogues in single neutral B decay, in the decay chain $\Upsilon(4s) \rightarrow BB \rightarrow f_1 f_2$ (where the B mesons are neutral), and in neutrino oscillation. In each of these situations, one has a propagating particle (or particles) which is a coherent superposition of several mass eigenstates with different masses. In every case, one must evaluate the coherent contributions of the different mass eigenstates to the amplitude for decay or detection of the propagating particle at precisely the same spacetime point. Calculating these contributions at different points and then adding them coherently is an erroneous procedure which in every instance can yield an oscillation frequency double the true value.

In this Letter, we will examine the oscillation in neutral meson decay, focusing on the physics related to the oscillation frequency. We will see that a recent claim that in $\phi \to KK \to f_1 f_2$ this frequency is $2\Delta m_K$, while in isolated single K decay it is Δm_K, cannot be correct, because the B-meson analogue of this claim is decisively contradicted by experiment. We will then turn to the analysis on which the claim is based, and discover that for $\phi \to KK \to f_1 f_2$, this analysis entails the evaluation at different spacetime points of the coherent K_L and K_S contributions to decay of a kaon. We will see explicitly that when the contributions of the different mass eigenstate components of a propagating particle to the decay or detection of this particle are not calculated at the same spacetime point, the predicted oscillation frequency can be twice its true value, not only in $\phi \to KK \to f_1 f_2$, but quite generally in both one- and two-neutral-K and neutral-B processes, and in neutrino oscillation.

Let us recall the standard results for neutral meson decay. The probability $\Gamma\,[K^0 \to f \text{ at } \tau]$ for an isolated neutral K to decay to a final state f at proper time τ, if at time $\tau = 0$ this K was a pure K^0, is given by [2]

$$\Gamma[K^0 \to f \text{ at } \tau] \propto e^{-\Gamma_S \tau} + |\eta_f|^2 e^{-\Gamma_L \tau} + 2|\eta_f| e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)\tau} \cos(\Delta m_K \tau - \phi_f) \; . \qquad (1)$$

Here, Γ_S and Γ_L are, respectively, the widths of the K_S and K_L, and

$$\eta_f \equiv |\eta_f| e^{i\phi_f} \equiv \frac{\langle f|T|K_L\rangle}{\langle f|T|K_S\rangle} \; . \qquad (2)$$

The oscillatory last term in Eq. (1) comes from the interference between the K_S and K_L contributions to the decay. Next, suppose we have a neutral kaon pair produced in a p wave via the process $\phi \to KK$. The joint probability $\Gamma[\text{One } K \to f_1 \text{ at } \tau_1;$ $\text{Other } K \to f_2 \text{ at } \tau_2]$ for one member of this pair to decay to the final state f_1 at proper time τ_1 after its birth in the ϕ decay, while the other decays to final state f_2 at time τ_2, is given by [3, 4, 5]

$$\begin{aligned} &\Gamma[\text{One } K \to f_1 \text{ at } \tau_1;\ \text{Other } K \to f_2 \text{ at } \tau_2] \\ &\propto \; \Big| e^{-i\lambda_L \tau_1} e^{-i\lambda_S \tau_2} A(K_L \to f_1)\, A(K_S \to f_2) \\ &- \; e^{-i\lambda_S \tau_1} e^{-i\lambda_L \tau_2} A(K_S \to f_1)\, A(K_L \to f_2) \Big|^2 \; . \end{aligned} \qquad (3)$$

Here, $\lambda_{L,S} \equiv m_{L,S} - \frac{i}{2}\Gamma_{L,S}$ are the complex masses of $K_{L,S}$, and $A(K_L \to f_1)$, etc., are decay amplitudes. The interference term in the probability (3) has the oscillatory time dependence

$$e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)(\tau_2+\tau_1)} \cos[\Delta m_K(\tau_2 - \tau_1) + \phi_0] \ , \tag{4}$$

where ϕ_0 is time-independent. Note that the frequency of this oscillation, Δm_K, is the same as that of the oscillatory term in the decay rate (1) for a single kaon in isolation.

The standard result for the probability $\Gamma[B^0 \to f \text{ at } \tau]$ for an isolated neutral nonstrange B to decay to a final state f at proper time τ, if at $\tau = 0$ this B was a pure B^0, is Eq. (1) with the K-meson quantities replaced by their B-meson counterparts [6]. The frequency of the oscillation is now $\Delta m_B \equiv m(B_H) - m(B_L)$, the difference between the masses of the heavier mass eigenstate (B_H) and the lighter mass eigenstate (B_L) of the $B^0 - \overline{B^0}$ system.

For $\overset{(\text{---})}{B^0}$ mesons, the counterpart to a K pair produced in a p wave via $\phi \to KK$ is a B pair produced in a p wave via $\Upsilon(4s) \to BB$ [7]. The standard result for the joint probability $\Gamma[\text{One } B \to f_1 \text{ at } \tau_1; \text{ Other } B \to f_2 \text{ at } \tau_2]$ for the decays of the members of this pair is Eq. (3) with K_L and K_S replaced by the B mass eigenstates, and λ_L and λ_S by their complex masses [8, 9, 6, 5, 10]. The interference term (4) in this joint probability now has frequency Δm_B. As in the K system, this frequency is the same as in the decay of a single B in isolation.

Stodolsky and I have presented an approach to the treatment of a decay chain such as $\phi \to KK \to f_1 f_2$ (or $\Upsilon(4s) \to BB \to f_1 f_2$) in which the amplitude for the entire chain is calculated directly [5, 10]. Unlike the more traditional treatment [3, 4, 8, 9] of such a chain, our method does not entail the introduction of a wave function, or state vector, for the KK system. Consequently, this method avoids the somewhat mysterious "collapse of the KK wave function," in which the decay of one K at a certain instant determines the $K^0 - \overline{K^0}$, or $K_S - K_L$, content of the other K at the same instant [3]. The amplitude approach also has the advantage of manifest Lorentz covariance. Applying this approach, one readily reproduces the standard expression (3) for the decay probability, without having to invoke the collapse of the wave function. Obtaining (3) by the amplitude methods makes it evident that the times in this decay rate are proper times in the K rest frames, not times in the ϕ

rest frame. In the "collapse" approach, this point is not so clear, and one must guess astutely.

In a Comment [11] on Ref. [5], and in other work [12] to which this Comment calls attention, Srivastava, Widom, and Sassaroli (SWS) argue that in $\phi \to KK \to f_1 f_2$, the frequency of the oscillation in the decay rate is not Δm_K, as in (4), but $2\Delta m_K$. Thus, these authors disagree with the early analyses of this process, and with the analysis in Ref. [5]. However, they agree that in the decay of an isolated single neutral kaon that is not part of a kaon pair produced via $\phi \to KK$, the oscillation frequency is indeed Δm_K, as in the standard result, Eq. (1). Hence, they predict that once the oscillation frequency in $\phi \to KK \to f_1 f_2$ is measured, it will prove to be twice the already-determined oscillation frequency in the decay of an isolated single kaon.

Now, the B-meson analogue of the SWS argument for kaons would predict that in $\Upsilon(4s) \to BB \to f_1 f_2$, the oscillation frequency in the decay rate is twice as large as in the decay of an isolated single neutral B. However, this prediction is contradicted by experiment. To see this, consider first the decay of an isolated B, or, equivalently, a B which is part of a complicated many-body state produced in a Z^0 decay or a high-energy $p\overline{p}$ collision. Suppose that through tagging this B is known to be a pure B^0 at proper time $\tau = 0$. Owing to $B^0 - \overline{B^0}$ mixing, at subsequent times the B will be a B^0, $\overline{B^0}$ superposition. Suppose that at time $\tau > 0$, the B decays into a final state f_B which can come only from its B^0 component, or into the CP-conjugate state $\overline{f_B}$, which, of course, can come only from its $\overline{B^0}$ component. The probabilities $\Gamma[B^0 \to \overset{(\overline{})}{f_B} \text{ at } \tau]$ for these two decays are given by [6]

$$\Gamma[B^0 \to \overset{(\overline{})}{f_B} \text{ at } \tau] \propto e^{-\Gamma\tau}[1 \overset{+}{(-)} \cos(\omega_1 \tau)] \ . \tag{5}$$

Here, ω_1 is the oscillation frequency, and all authors, including SWS, agree that $\omega_1 = \Delta m_B$. The quantity Γ is the decay width that B_H and B_L (unlike K_L and K_S) have in common. The results (5), with $\omega_1 = \Delta m_B$, may be obtained from the K-meson formula (1) by substituting Γ for Γ_S and Γ_L, and Δm_B for Δm_K, and then using the easily demonstrated fact that the B-meson analogue of the parameter η_f of Eq. (2) is +1 for the final state f_B, and −1 for $\overline{f_B}$. Analysis of the decay of neutral B

mesons produced in Z^0 decays at LEP, and in $p\overline{p}$ collisions at the Tevatron, in terms of Eqs. (5) has yielded the value [13]

$$\omega_1 = (0.458 \pm 0.020) ps^{-1} \ . \tag{6}$$

This analysis has included direct observation of the oscillation with time. The proper time τ of a B decay is determined by measuring the distance x which the B has traveled and the B momentum p. Then

$$\tau = x \, \frac{m_B}{p} \ , \tag{7}$$

where m_B is the average neutral B mass, and does not distinguish between B_H and B_L.

Consider, next, the chains $\Upsilon(4s) \to BB \to f_B \stackrel{(\overline{\quad})}{g_B}$, where the final states f_B and g_B can come only from a B^0, and $\overline{g_B}$ only from a $\overline{B^0}$. For these chains, all agree that

$$\begin{aligned} &\Gamma[\text{One } B \to f_B \text{ at } \tau_1; \text{ Other } B \to \stackrel{(\overline{\quad})}{g_B} \text{ at } \tau_2] \\ &\quad \propto e^{-\Gamma(\tau_2+\tau_1)} \left[1 \ \overline{(+)} \ \cos(\omega_2(\tau_2 - \tau_1)) \right] \ , \end{aligned} \tag{8}$$

except for a disagreement concerning the value of the oscillation frequency ω_2. All early treatments [9, 6] and the more recent amplitude approach [5, 10] predict that $\omega_2 = \Delta m_B$. Indeed, Eqs. (8) with $\omega_2 = \Delta m_B$ follow trivially from the B-meson analogue of the standard K-meson formula (3) and the usual expressions for B_H and B_L in terms of B^0 and $\overline{B^0}$. However, by following the SWS approach, as described in Ref. [12], we find that they would predict that $\omega_2 = 2\Delta m_B$. This is a B-meson analogue of the oscillation frequency doubling they claim to be present in $\phi \to KK$. Apart from the value of ω_2, their approach yields precisely the same decay probabilities, Eqs. (8), as found by others. (In the SWS treatment, these probabilities are initially expressed in terms of the B pathlengths x and momenta p, but they coincide with Eqs. (8), with $\omega_2 = 2\Delta m_B$, once we use Eq. (7) to express them in terms of the experimentally measured proper times.)

The ARGUS and CLEO collaborations have studied the time integrals over the decay rates (8). These groups report the ratio

$$r = \frac{N(f_B \, g_B) + N(\overline{f_B} \, \overline{g_B})}{N(f_B \, \overline{g_B}) + N(\overline{f_B} \, g_B)} \ , \tag{9}$$

where

$$N(f_B\, g_B) = \int_0^\infty d\tau_1 \int_0^\infty d\tau_2\, \Gamma[\text{One } B \to f_B \text{ at } \tau_1;\ \text{Other } B \to g_B \text{ at } \tau_2] \tag{10}$$

is the total number of events in which one B decays to f_B and the other to g_B, and similarly for the other quantities. The state f_B is chosen to be $D^{*-}\ell^+\nu$, and g_B to be the inclusive state $X\ell^+\nu$. In employing the measured r to learn about ω_2, one makes use of the fact that the omitted proportionality constants in the two relations (8) are equal. This equality holds so long as CP violation in the decay amplitudes $A\left(\overset{(-)}{B^0} \to \overset{(-)}{g_B}\right)$ may be neglected. Since in the Standard Model semileptonic B^0 decay is heavily dominated by a single tree-level diagram, which is unlikely to have significant competition from beyond the Standard Model, this should be an excellent approximation. Similarly, so should the CP relations $N(\overline{f_B}\,\overline{g_B}) = N(f_B\, g_B)$ and $N(\overline{f_B}\, g_B) = N(f_B\, \overline{g_B})$. Thus, integrating Eqs. (8), we conclude that

$$r = \frac{\xi^2}{2+\xi^2}\ , \tag{11}$$

where $\xi \equiv \omega_2/\Gamma$. From the combined ARGUS-CLEO result [14],

$$r = 0.179 \pm 0.039\ , \tag{12}$$

we then learn that [14]

$$\xi = 0.66 \pm 0.09\ . \tag{13}$$

Now, the common $B_H - B_L$ lifetime, $1/\Gamma$, has been found to be [15]

$$\frac{1}{\Gamma} = 1.57 \pm 0.05\, ps\ . \tag{14}$$

Combining this lifetime with the value of ξ, Eq. (13), we find that

$$\omega_2 = (0.42 \pm 0.06)\, ps^{-1}\ . \tag{15}$$

This frequency is in excellent agreement with the frequency ω_1, Eq. (6), found in single B decay. Had ω_2 been twice ω_1, as expected by SWS, the measured ω_1, Eq. (6), would have predicted that $\omega_2 = (0.92 \pm 0.04) ps^{-1}$, in very strong disagreement with the measured ω_2 value, $(0.42 \pm 0.06) ps^{-1}$. Thus, strictly on the basis of experimental data, we can conclude that the oscillation frequency in $\Upsilon(4s) \to BB \to f_1 f_2$ is the same as in single B decay, rather than twice as great. Since the physics of $\phi \to KK \to f_1 f_2$ is virtually identical to that of $\Upsilon(4s) \to BB \to f_1 f_2$, we can also conclude that the oscillation frequency in $\phi \to KK \to f_1 f_2$ is the same as in single K decay.

From the standpoint of the "collapse" approach, it would have been very surprising if ω_2 had differed from ω_1. In this approach, the decay of one B in a B pair produced via $\Upsilon(4s) \to BB$ fixes the state of the remaining B at the same instant. Subsequently, this remaining B oscillates between B^0 and $\overline{B^0}$ states. Since this B is now alone, it would be surprising if the frequency of this oscillation were different than in the case of a single B which is alone from the very beginning [16].

Let us now compare the SWS treatment with the amplitude approach of Refs. [5] and [10], and identify the features which lead SWS to expect that the oscillation frequency in $\phi \to KK \to f_1 f_2$ is $2\Delta m_K$, twice that in single K decay, while from the amplitude approach we expect both frequencies to be Δm_K. In treating $\phi \to KK$, SWS introduce a state vector for the KK system, and use field-theoretic propagators to describe the propagation of the kaons. In the amplitude approach, the introduction of a KK wave function or state vector is carefully avoided, and field theory is not used. Complete decay chains such as $\phi \to KK \to f_1 f_2$, including the kaon decay amplitudes, which are omitted by SWS, are treated. Nevertheless, it appears from Ref. [12] that SWS would agree with the authors of the amplitude approach that the oscillation frequency in $\phi \to KK \to f_1 f_2$ is Δm_K, and not $2\Delta m_K$, if the former only treated proper times as do the latter.

The treatment of proper times in the amplitude approach is most easily explained by discussing the decay of a single kaon, born as a pure K^0, into a final state f. For the amplitude for this process, A, the amplitude approach yields the readily understood result

$$A = \sum_{N=L,S} A(K^0 \text{ is } K_N)\, e^{-i\lambda_N \tau^N} A(K_N \to f) \ . \tag{16}$$

Here, the pure K^0 is a superposition of the mass eigenstates K_L and K_S, and $A(K^0$ is $K_N)$ is the amplitude for it to be, in particular, the mass eigenstate K_N. The factor $\exp(-i\lambda_N \tau^N)$ is the amplitude for this K_N to propagate for the proper time interval τ^N that elapses between its birth and decay [17]. Finally, $A(K_N \to f)$ is the amplitude for the K_N to decay into the state f. In principle, the proper time that elapses between the kaon birth and decay may depend on whether the kaon is a K_L or a K_S, so we denote it by τ^N, with $N = L$ or S. As explained in Ref. [5], the meaning of this proper time is somewhat subtle. We picture the kaon as being described by a wave packet, with some central momentum p [18]. Suppose that the kaon is born at the spacetime point (0,0), and decays at the point (t, x). Now, the K_L and K_S have different masses, so, for a given momentum p, it is not possible *classically* for both the K_L and K_S components of the kaon, born at the point (0,0), to arrive at the location x at the same time t. However, it *is* possible for the K_L and K_S components of the kaon wave packet to overlap at the point (t, x), with the center of the K_L component displaced relative to that of the K_S component. These overlapping components of the wave packet are two amplitudes, corresponding to the two terms in Eq. (16), for the kaon to be at the spacetime point (t, x). It is the interference between these amplitudes which leads to the oscillation in the decay probability. In calculating this interference, we must, of course, evaluate all phase factors at the common point (t, x) where the interference occurs. Thus, in the factor $\exp(-i\lambda_N \tau^N)$, the proper time τ^N is the kaon-rest-frame time which corresponds to the decay point (t, x). Hence, from the Lorentz transformation, we have for the mass eigenstate K_N with momentum p and corresponding energy $E_N(p) = (p^2 + m_N^2)^{1/2}$,

$$\tau^N = \frac{1}{m_N}\,(E_N(p)t - px)\ . \tag{17}$$

To first order in Δm_K, this relation is

$$\tau^{L(S)} = \tau^0 \ \overset{+}{(-)}\ \frac{\Delta m_K}{2m_K}\left[\frac{m_K}{E_K(p)}\,t - \tau^0\right]\ , \tag{18}$$

where $m_K \equiv (m_L + m_S)/2$ is the average neutral kaon mass, $E_K(p) \equiv (p^2 + m_K^2)^{1/2}$ is the corresponding energy at momentum p, and $\tau^0 \equiv [E_K(p)t - px]/m_K$ is the value of τ^L or τ^S for vanishing Δm_K. Now, τ^0 and t are, of course, related by time dilation: $\tau^0 = (m_K/E_K(p))t$. Thus, from Eq. (18) we see that to first order in Δm_K, $\tau^L = \tau^S = \tau^0$, and we shall refer to this proper time simply as τ. To first order in Δm_K, the relative phase of the factors $\exp(-i\lambda_N \tau^N)$ in the two interfering terms in

Eq. (16) is just

$$(m_L - m_S)\tau = (\Delta m_K)\tau \ . \tag{19}$$

Hence, the oscillation with τ induced by the interference has frequency Δm_K. Using the well-known fact that $A(K^0 \text{ is } K_L) = A(K^0 \text{ is } K_S)$, the amplitude of Eq. (16) immediately yields the decay rate of Eq. (1).

In practice, the observer-frame time of the decay, t, is not measured. The location of the decay, x, and the momentum of the kaon, p, are measured, and t is then inferred from x and p using the relation $t = (E_K(p)/p)x$. The proper time $\tau(\equiv \tau^0)$ of the decay is then found from t using the time-dilation relation $\tau = (m_K/E_K(p))t$. That is,

$$\tau = x \, \frac{m_K}{p} \ . \tag{20}$$

Equation (7) already gave the B-experiment analogue of this relation.

We have just seen that in K decay (and similarly in B decay), the variation of the proper time of a given decay point (t, x) from one contributing mass eigenstate to another is completely negligible. However, in the propagation of a neutrino, the proper time of a given neutrino interaction point (t, x) varies very importantly from one mass eigenstate to another [5]. Thus, whether the proper times associated with different mass eigenstates are equal or different depends on which problem one is treating.

For the decay chain $\phi \to KK \to f_1 f_2$, the amplitude approach yields the time-dependent probability of Eq. (3), in which the first term on the right-hand side is the amplitude for a K_L to decay into the final state f_1 at proper time τ_1 while a K_S decays into the state f_2 at proper time τ_2, and the second term is the amplitude for the process in which the roles of K_L and K_S are interchanged. The time dependence in Eq. (3) arises from the factors $\exp(-i\lambda_N \tau_j)$, with $N = L$ or S and $j = 1$ or 2. Just as in the case of single kaon decay, to first order in Δm_K the proper time τ_j in each of these factors does not depend on whether the factor is for propagation of a K_L or K_S. Accordingly, we have written this proper time simply as τ_j, not τ_j^N. Suppose that, in the ϕ rest frame, the kaons created in $\phi \to KK$ are observed to travel distances x_1 and x_2 before decaying. Then, if p is the ϕ-frame momentum of either kaon, the proper times τ_j of the two decays are given by

$$\tau_j = x_j \, \frac{m_K}{p} ; \; j = 1, 2 \; , \tag{21}$$

just as in Eq. (20) for a single K decay. To first order in Δm_K, the phase of the factor $\exp(-i\lambda_N \tau_j)$ is just $-m_N \tau_j$. Thus, to this order, the time-dependent part of the relative phase of the two interfering terms in Eq. (3) is

$$(m_L - m_S)(\tau_1 - \tau_2) = (\Delta m_K)(\tau_1 - \tau_2) \; . \tag{22}$$

Hence, the oscillation with proper time induced by the interference has frequency Δm_K—identical to the frequency in single kaon decay.

In the SWS approach, the kaon pair created in $\phi \to KK$ is described by a state vector $|\psi\rangle$ of the form [12]

$$|\psi\rangle \sim e^{-i\lambda_L \tau_1^L} e^{-i\lambda_S \tau_2^S} \, |K_L K_S\rangle - e^{-i\lambda_S \tau_1^S} e^{-i\lambda_L \tau_2^L} \, |K_S K_L\rangle \; . \tag{23}$$

To explain the notation, let us suppose, as before, that in the ϕ rest frame the two kaons are observed to travel distances x_1 and x_2 before decaying. In the first term in $|\psi\rangle$, $|K_L K_S\rangle$ is a state in which it is a K_L which has traveled to x_1 and a K_S to x_2, and in the second term the roles of K_L and K_S are interchanged. The proper times τ_j^N ($N = L$ or S and $j = 1$ or 2) of the decays are *not* the proper times τ_j of Eq. (21). Rather, SWS state [11, 12] that if the kaon which travels the distance x_j is the mass eigenstate K_N, the proper time τ_j^N of its decay, to be used in Eq. (23), is given by

$$\tau_j^N = x_j \, \frac{m_N}{p} \; . \tag{24}$$

Here, p is, as before, the ϕ-frame momentum of either member of the kaon pair.

Comparing Eqs. (23) and (3), we conclude that if the SWS expression for $|\psi\rangle$ had involved the same proper times τ_j as does the amplitude on the right-hand side of Eq. (3), SWS would have found the same oscillation frequency in $\phi \to KK \to f_1 f_2$ as we did using the amplitude method; namely, Δm_K. However, if one uses the SWS proper times of Eq. (24), then the time-dependent part of the relative phase of the two terms in the SWS $|\psi\rangle$ is

$$\left(m_L^2 - m_S^2 \right) \; \frac{x_1 - x_2}{p} = 2(\Delta m_K) \, m_K \, \frac{x_1 - x_2}{p} \; . \tag{25}$$

In terms of the proper times τ_j of Eq. (21) which will be inferred from measurements, this relative phase is just

$$2(\Delta m_K)(\tau_1 - \tau_2) \ . \tag{26}$$

Thus, SWS predict that in the oscillation as a function of experimentally determined proper times, the frequency is $2\Delta m_K$. We see that it is their treatment of proper time that has led to the spurious factor of two in this prediction.

What is wrong with the SWS proper time of Eq. (24)? To answer this puzzle, let us consider the reaction $\phi \rightarrow KK$ in the ϕ rest frame, which will be the detector frame at the ϕ factory DAΦNE. SWS assume that the pathlength x_j of a kaon produced in $\phi \rightarrow KK$ is determined by measurement, and is independent of whether the kaon is a K_L or a K_S. Similarly, they assume that the momentum p carried by the kaon is fixed by the kinematics of $\phi \rightarrow KK$, and is also independent of whether the kaon is a K_L or a K_S. They then take the proper time of the kaon decay to be the quantity τ_j^N of Eq. (24), which does depend on whether the kaon is a K_L or a K_S. Now, the Lorentz transformation implies that the proper time of the decay, τ_j^N, the ϕ-frame time of this decay, t_j^N, and the measured ϕ-frame location of the decay, x_j, are related by

$$m_N \, \tau_j^N = E_N(p) \, t_j^N - p x_j \ , \tag{27}$$

where $E_N(p) \equiv (p^2 + m_N^2)^{1/2}$. For the τ_j^N of Eq. (24), this relation implies that

$$t_j^N = \frac{E_N(p)}{p} \, x_j \ . \tag{28}$$

We recognize that this t_j^N is just the classical arrival time of the mass eigenstate K_N at the decay point x_j. In particular, for SWS, the detector-frame time t_j^N of the decay depends on whether the decaying kaon is a K_L or a K_S. As a result, when the amplitude for $\phi \rightarrow KK \rightarrow f_1 f_2$ is calculated from the SWS state vector $|\psi\rangle$ of Eq. (23), the contributions to this amplitude from the two terms in $|\psi\rangle$ are evaluated for different detector-frame times. The contribution from the first term (which corresponds to a K_L decaying at x_1 and a K_S at x_2) is evaluated for the pair of spacetime decay points (t_1^L, x_1), (t_2^S, x_2). But the contribution from the second term (in which the roles of K_L and K_S are reversed) is evaluated for the points (t_1^S, x_1), (t_2^L, x_2). The two contributions are then added coherently.

As noted at the beginning of this Letter, such coherent adding of an amplitude for decays at one pair of spacetime points to that for decays at a different pair of points is an incorrect procedure. This is true even if, as here, the difference between the pairs is too small to be resolved experimentally. Note, for example, that one cannot reliably calculate the intensity of a two-component electromagnetic wave by evaluating the amplitudes for the two components at slightly different times and then adding the results coherently. If, as in kaon decay, the two components have very rapid time dependence, such a procedure would yield completely incorrect results. Similarly, quantum mechanical amplitudes to be added coherently must correspond to the same final states and *precisely* the same spacetime points.

SWS describe an isolated single kaon by a state vector [12] with a K_L and a K_S term, each of which has the same time dependence as the corresponding term in the decay amplitude of Eq. (16). Thus, we see from Eq. (24) that, were SWS to interpret proper times in single kaon decay as they do in $\phi \to KK \to f_1 f_2$, the relative phase of the K_L and K_S contributions to their single kaon decay amplitude would be

$$\left(m_L^2 - m_S^2\right) \frac{x}{p} = 2(\Delta m_K)\,\tau \ . \tag{29}$$

Here, x and p are, respectively, the measured pathlength and momentum of the kaon, and τ is the measured proper time of its decay, inferred from x and p using Eq. (20). We see that SWS would then predict that the oscillation frequency in single kaon decay is $2\Delta m_K$, just as they do for $\phi \to KK \to f_1 f_2$. The reason that they actually predict a frequency of Δm_K for single kaon decay is that, for this case, they assume the K_L and K_S components of the kaon to have, not a common momentum, but a common *speed* [12]. Thus, these components cover the measured kaon pathlength x in the same time, and, since their time dilation factors are equal, in the same proper time as well. The relative phase of the K_L and K_S pieces of the state vector is then just $(\Delta m_K)\tau$, so that the oscillation frequency is Δm_K. While this agrees with the standard result, we cannot understand the basis for taking the K_L and K_S to have equal speeds. In the kaon wave packet, these components have equal *momenta*, and the true explanation for the frequency Δm_K is as given earlier.

A spurious factor of two has appeared, not only in the SWS analysis of $\phi \to KK \to$

$f_1 f_2$, but also in other discussions of neutral meson or neutrino oscillation. In every case, this spurious factor can be traced to the mistake of taking the spacetime point of meson decay or neutrino detection to be different for different mass eigenstates, rather than being defined by the experiment and common to all the mass eigenstates.

To see this, let us first look briefly at neutrino oscillation. A neutrino of definite flavor (that is, a ν_e, a ν_μ, or a ν_τ) is a superposition of mass eigenstates ν_m. For a given momentum p, the eigenstate ν_m has an energy $E_m(p) = (p^2 + M_m^2)^{1/2}$. Assuming that the neutrino masses are small, $M_m \ll p$, $E_m(p) \cong p + M_m^2/2p$. In the time t that elapses between the birth of the neutrino and its detection, the ν_m component of the neutrino state vector acquires a phase factor $\exp[-iE_m(p)t] \cong \exp[-ipt(1 + M_m^2/2p^2)]$. In practice, t is not measured. Rather, it is the pathlength x traversed by the neutrino before its detection which is measured. Since the neutrino is highly relativistic, one may then infer that $t \cong x$. Thus, the relative phase of the ν_m and $\nu_{m'}$ components of the neutrino state vector is $(M_m^2 - M_{m'}^2)x/2p$. From this relative phase, the usual expressions for neutrino oscillation follow.

Suppose, now, that we do not consider the spacetime point of detection, (t, x), to be a fixed point common to all the mass eigenstate components of the neutrino state vector. Suppose that, instead, we make the mistake of evaluating the different mass eigenstate components at their differing classical times of arrival at the measured detection location x. That is, we now evaluate the ν_m component of the state vector at time $t^m = x(E_m(p)/p)$. This component then has the phase factor

$$\exp[-iE_m(p)t^m] = \exp[-iE_m^2(p)x/p] = \exp[-ipx(1 + M_m^2/p^2)] \ .$$

The relative phase of the ν_m and $\nu_{m'}$ components is now $(M_m^2 - M_{m'}^2)x/p$, twice as big as before. Correspondingly, the oscillation frequency is twice its true value. Note that the source of the spurious factor of two is the incorrect assumption that the times of a given detection can be taken to be different for different components of a single neutrino state.

We have already seen that a spurious factor of two will arise in single kaon decay if one takes the proper time of the decay to be given by Eq. (24), so that in the experimental observer's frame, the K_L and K_S components of the kaon decay at different times. We also observe from the discussion by Lipkin [1] that the spurious factor of two in the frequency arises precisely when the different mass eigenstate components of the meson are taken to decay at different times, and is absent if these components are taken to decay at the same spacetime point.

In summary, the oscillation frequency in $\phi \to KK \to f_1 f_2$, and that in single K decay, are both Δm_K. Similarly, the frequencies in $\Upsilon(4s) \to BB \to f_1 f_2$ and single B decay are both Δm_B. That the latter two frequencies are equal is an experimental fact. That they are equal to Δm_B, and their K-meson analogues to Δm_K, follows from quantum mechanics.

In treating neutral meson or neutrino oscillation, it is important to take the different mass eigenstate components of the oscillating particle to decay or be detected at precisely the same spacetime point. Otherwise, a spurious factor of two in the oscillation frequency can result.

Acknowledgments

The analysis reported here has grown out of a very enjoyable ongoing collaboration with Leo Stodolsky. I would like to thank Professor Stodolsky for his many insights, and in particular for his very useful comments on the topic of the present paper. I am grateful to the Stanford Linear Accelerator Center for its hospitality during part of this work. I wish to thank Helen Quinn for a number of particularly helpful discussions of neutral meson oscillation, Richard Blankenbecler for several useful conversations, and Sharon Jensen for her excellent typing of the manuscript.

References

[1] A thoughtful discussion of single neutral meson decay, which notes the possibility of obtaining an erroneous oscillation frequency if correct procedures are not followed, has been given in H. Lipkin, *Phys. Lett.* **B348** (1995) 604.

[2] See, for example, K. Kleinknecht, *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1989) p. 41.

[3] H. Lipkin, *Phys. Rev.* **176** (1968) 1715.

[4] I. Dunietz, J. Hauser, and J. Rosner, *Phys. Rev.* **D35** (1987) 2166.

[5] B. Kayser and L. Stodolsky, *Phys. Lett.* **B359** (1995) 343.

[6] See, for example, I. Bigi, V. Khoze, N. Uraltsev, and A. Sanda, *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1989) p. 175.

[7] A wave packet treatment of this system has been given in R. Blankenbecler, unpublished.

[8] A. Carter and A. Sanda, *Phys. Rev. Lett.* **45** (1980) 952; *Phys. Rev.* **D23** (1981) 1567.

[9] I. Bigi and A. Sanda, *Nucl. Phys.* **B193** (1981) 85.

[10] B. Kayser, to appear in the *Proceedings of the Moriond Workshop on Electroweak Interactions and Unified Theories*, Les Arcs, France, March, 1995.

[11] Y. Srivastava, A. Widom, and E. Sassaroli, Northeastern University preprint NUB–3130 (hep-ph/9511294).

[12] Y. Srivastava, A. Widom, and E. Sassaroli, *Z. Phys.* **C66** (1995) 601. We thank the authors of this paper for bringing it to our attention.

[13] S. L. Wu, University of Wisconsin preprint WISC–EX–96–343 (hep-ex/9602003), and private communication.

[14] H. Schröder, *B Decays* (Revised Second Edition), ed. S. Stone (World Scientific, Singapore, 1994) p. 449.

[15] J. Kroll, Fermilab preprint FERMILAB–CONF–96–032 (hep-ex/9602005).

[16] We thank H. Quinn for making essentially this argument.

[17] L. Stodolsky, *Gen. Rel. & Grav.* **11** (1979) 391.

[18] For an early wave-packet treatment of neutral kaons, see R. Sachs, *Ann. Phys.* **22** (1963) 239.