SLAC-PUB-95-6837

Coherent Synchrotron Radiation and Stability of a Short Bunch in a Compact Storage Ring*

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Abstract

It should be possible to observe coherent synchrotron radiation at millimeter wavelengths in a compact electron storage ring, provided that the bunch can be made sufficiently short. On the other hand, for a short bunch the radiation reaction is so strong that it could cause a longitudinal instability if the current exceeded some threshold. This might cause bunch lengthening, and cut off or reduce the coherent radiation. Using wake fields from simple models of the vacuum chamber, we estimate the threshold current for a proposed upgrade of the Brookhaven small x-ray light source, SXLS - Phase I.

I. Introduction

Coherent synchrotron radiation, normally suppressed by shielding due to the metallic vacuum chamber, can occur at frequencies ω greater than a certain threshold, roughly $\omega_c(R/h)^{1/2}$, where ω_c is the wave guide cutoff of the chamber, R is the bending radius of particle orbits, and h is the transverse size of the chamber [1]. Heretofore, only linac experiments with bending magnets [2] have provided suitable conditions to overcome the shielding and produce coherent radiation; namely, small bunch length σ_L, small R, and large h. Murphy and Krinsky [3] have proposed an experiment offering similar conditions in a compact storage ring, SXLS - Phase I at Brookhaven. This machine, now out of service, would be upgraded with a new r.f. system (1.5 MeV, 2855 MHz, harmonic number 81). In a parameter set for 150 MeV beam energy, the proposed bunch length is 0.32 mm; for 200 MeV operation the bunch length is 0.49 mm. On the basis of impedance and stability estimates not including coherent radiation (curvature wake field), Murphy and Krinsky anticipated an operating current of about $2\cdot 10^7$ particles per bunch. In earlier operation with the existing 50 kV, 211 MHz r.f system, longer bunches with currents up to $8.8\cdot 10^{10}$ (0.5 amp) were stored.

Here we try to estimate the threshold for a longitudinal instability, accounting for the wake field due to curvature, but neglecting other contributions to the wake field.

II. Wake Field for a Model of the Vacuum Chamber

Suppression of coherent radiation by shielding was recognized as early as the 1940's (Schwinger, Schiff), and has been studied theoretically in various simple models; see [1] for references. The models all involve simple geometries (smooth torus, pillbox, parallel plates), and rely on solutions of Maxwell's equations in the frequency domain in terms of Bessel functions. Examples of wake potentials from Fourier transforms of such solutions have been given by one of the authors [4].

The ring SXLS is built in race track form, with two large dipoles providing the bends. The bending radius is 0.6037 m. The vacuum chamber through the bends is rectangular in cross section (with an antechamber for pumping), the main chamber being 3.8 cm high and 8 cm in width. In the straight sections the chamber is round, with a diameter of 8.57 cm. To compute the curvature wake field, we assume a smooth, circular, toroidal chamber with rectangular cross-section, width 8 cm and height 3.8 cm. The beam is at the center of the cross-section, following a circular trajectory of radius $R = 0.6037$ m. The model includes resistive walls, with conductivity appropriate to the stainless steel chamber; (actually, resistivity has little effect on our conclusions, but it makes it easier to compute the fields near resonances). We hope that this model gives a good picture of the wake field due to coherent radiation in the bends. It is not clear that other sources of wake fields (r.f. cavity, transitions in chamber size, kicker, etc.) can be simply added to the curvature wake field. Some light might be thrown on this question by more elaborate models that could be treated by mode matching, for instance a smooth torus perturbed by one cavity.

The longitudinal coupling impedance for this toroidal model has been derived in [1]. It is given for a beam with zero extent in the radial direction, but with nonzero extent in the z-direction (we work in cylindrical coordinates, with the z-axis perpendicular to the plane of the orbit). The transverse distribution in the z-direction has little effect on the results. The fields are given as Fourier series in z and θ, with r-dependent coefficients expressed in terms of Bessel functions. Certain eigenmodes of the whole chamber are both resonant and synchronous with the beam, and show up as poles in the impedance (off the real axis when walls are resistive). There is a minimum frequency for a mode to be both resonant and synchronous, and that is the threshold frequency for coherent radiation. At lower frequencies the curvature impedance is purely reactive, and generally negligible.

Figure 1 shows the real part of the impedance of the toroidal chamber, multiplied by the Fourier transform of a Gaussian charge distribution with $\sigma_L = 0.3$mm. The density of resonance peaks is much higher than in the examples of [1], which were for bigger rings with smaller vacuum chambers. This has to do with more modes in the z-series being important; 13 modes were needed in the present case,

*Work supported by DOE contract DE-AC03-76SF00515

Presented at the 16th IEEE Particle Accelerator Conference (PAC95) and International Conference on High Energy Accelerators, Dallas, Texas, May 1-5, 1995.

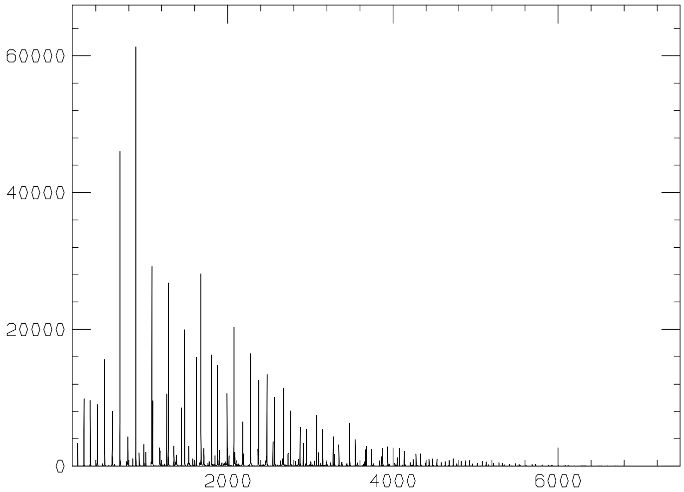

Figure. 1. Real part of longitudinal impedance in ohms, multiplied by Fourier transform of a Gaussian charge distribution, $\sigma_L = 0.3$ mm. The abscissa is the longitudinal mode number $n = \omega/\omega_0$, where ω_0 is the angular revolution frequency.

but only one in the previous examples. Infinitely many modes are required to retrieve the free-space synchrotron radiation.

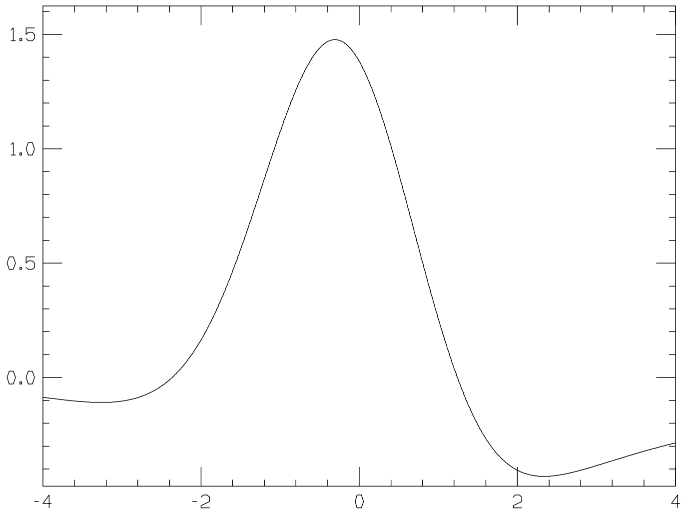

Figure. 2. Wake voltage for one turn, in kilovolts per picocoulomb, versus distance s from bunch center in units of σ_L. Here s is positive in front of the bunch, and a positive wake voltage corresponds to energy lost by the test particle.

Figure 2 shows the corresponding wake voltage per turn, as a function of the distance s between a test particle and the center of the bunch. The bunch length is 0.3mm. The distance s is expressed in units of σ_L, and is positive in front of the bunch. A positive value for the wake voltage means that energy is lost by the test particle. This wake voltage is quite substantial, being larger per unit length than that of the SLAC linac.

The peaks of the impedance shift in position quite noticeably under very small changes in the trajectory radius R, since the dispersion curve for the structure runs almost parallel to the synchronism line [1]. A particular mode goes out of synchronism when R changes by a very small amount, but the mode spectrum is very dense, so that another mode, synchronous at a slightly different frequency, can step in to take its place. The result is that the wake voltage does not vary appreciably as R sweeps over values corresponding to a typical bunch width. Indeed, if we average the wake voltage over many values of R, extending over a typical bunch size, we get something very close to the wake voltage for a single R. The corresponding averaged impedance has an ever more dense distribution of peaks as the number of R values in the average is increased.

It is also interesting to observe that the wake voltage for a beam circulating in the midplane between two infinite parallel plates (perfectly conducting) is very nearly the same as that for the resistive torus, at least within a few σ_L of the beam center. The impedance is totally different in appearance, however. Being an open structure, the parallel-plate system does not have eigenmodes and poles of the impedance. Radiation to infinity takes the place of radiation into eigenmodes.

We have also evaluated the transverse forces. The radial wake force is a substantial fraction of the longitudinal force, but the beam of 150 MeV is so "stiff" in the longitudinal direction that the radial force will not give much transverse displacement. A circular orbit in equilibrium, with a centripetal force due to the radial self field acting against the bending field, has a radius differing by less than a micron from that in the presence of the bending field alone.

III. Estimate of Current Threshold for Instability

We have applied a computer code written by K. Oide [5] to estimate the threshold for a longitudinal instability. The code solves a Vlasov equation, linearized about the equilibrium distribution as determined from the Haïssinski equation. Although one might prefer to do a direct multi-particle simulation, that is difficult in the present instance because of the long damping time of SXLS. Oide's routine to solve the Haïssinski equation assumes that there is no field in front of the bunch, which is not the case in the toroidal chamber. Modes of the whole chamber ring for a long time (for an infinite time if the walls are perfectly conducting), so there is always some field in front of the bunch. This precursor field is relatively weak, however, so we merely cut it off to get a suitable wake potential for Oide's code. A separate solution of the Haïssinski equation by our own iterative code, which does not require that the precursor field vanish, showed that the precursor does not have a big effect on the potential well distortion. Indeed, the entire potential well distortion is pretty small up to our estimated threshold current. The bunch stays nearly Gaussian, and moves a little toward the peak of the r.f., to compensate the energy loss of the essentially resistive wake field.

The result of running the code is that an instability sets in (i.e., the coherent frequency acquires a positive imaginary part) at a current of about $3\cdot 10^7$ particles per bunch. This is the result without account of momentum spread, as described by the Fokker-Planck term. According to Oide's rough estimate of the frequency shift due to the Fokker-Planck term, the momentum spread may raise the threshold to about $4 \cdot 10^7$, but this is a very uncertain matter. In fact, we are not even entirely confident of the result without the Fokker-Planck term, since we were not able to observe unambiguous convergence as the number of angle modes in the distribution function was increased, and the mesh in the action variable was refined. Of course, a positive imaginary part from the linearized Vlasov equation does not necessarily imply a permanent and fatal instability. Nonlinear stabilization after initial growth is always a possibility.

IV. Acknowledgment

We wish to thank Katsunobu Oide for providing his Vlasov code, and for helping us to use it. We also wish to thank Jim Murphy for many enjoyable discussions.

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