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Probing Anomalous Chromomagnetic Top Quark Couplings at the NLC *

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Abstract

The Next Linear Collider(NLC) will provide a excellent tool for probing the detailed nature of the top quark. By extending the recent analysis of Dokshitzer, Khoze and Sterling, we perform a preliminary examination of the influence of an anomalous chromomagnetic moment for the top, κ, on the spectrum of gluon radiation associated with $t\bar{t}$ production. In particular, we analyze the sensitivity of future data to non-zero values of κ and estimate the limits that can be placed on this parameter at the NLC with center of mass energies $\sqrt{s} = 500$ and 1000 GeV.

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Direct searches for the top quark at the Tevatron have led to a lower bound on the top mass of 131 GeV and, quite recently, to evidence that the top has been found with a mass near 175 GeV[1, 2]. If verified, this will be a remarkable success for the Standard Model(SM) since this value of the mass lies close to the center of the range predicted by precision electroweak data[3]. Due to its large mass, the top itself has been proposed as a probe for new physics beyond the SM. Detailed analyses of top quark couplings to gauge bosons through its direct production and subsequent decay at both hadron[4] and e^+e^- colliders[5] have been advocated in the literature for this very purpose. The present indirect constraints from low energy processes still allow for sizeable deviations from SM predictions[6]. In fact, the somewhat larger than expected cross section for $t\bar{t}$ production[7] obtained by the CDF Collaboration has already prompted several theoretical analyses[8] in which new dynamics involving the top quark have been discussed. Thus it is possible that the top may show us the first glimmer of new physics beyond the standard model.

The possibility of gluon emission during heavy quark production in e^+e^- collisions has been entertained for quite some time. In a recent paper, Dokshitzer, Khoze and Sterling(DKS)[9] have considered the spectrum of energetic gluon jets produced in association with $t\bar{t}$ at the Next Linear Collider(NLC). In the present paper we will extend their analysis and consider the possibility that the top possesses a non-zero anomalous chromomagnetic dipole moment, κ, in its coupling to gluons. (As in the DKS study, we will ignore the effects of top decay in this analysis.) Such a scenario has recently been shown to lead to significant modifications in the characteristics of the $t\bar{t}$ production at the Tevatron [10]. Here in this preliminary analysis we will show that the gluon energy distribution is a sensitive probe of κ, as a non-zero value of this parameter leads to an enhancement in the number of high energy gluons produced along with $t\bar{t}$ at the NLC. Since the emission rate for gluons from a chromomagnetic dipole term in the Lagrangian scales approximately like $\kappa\sqrt{s}/m_t$ in

the amplitude, large values of κ will lead to a breakdown in perturbation theory. We will also see that for a fixed top quark mass, increasing the NLC's center of mass energy will provide an additional lever arm in obtaining sensitivity to non-zero values of κ. In the *absence* of additional high energy gluon jets in comparison to ordinary QCD expectations we will demonstrate that reasonable limits on the value of κ are obtainable at the NLC. One might expect that substantially improved limits from fits to the gluon energy spectrum itself may be obtainable. We show that vast improvements in the constraints on κ from this approach are unlikely at the $\sqrt{s} = 500$ GeV NLC due to a conspiracy between SM and κ-dependent contributions to the cross section. For the $\sqrt{s} = 1$ TeV case, however, we find that fits to the gluon energy distribution yield greatly increased sensitivity to non-zero κ.

It is important to note that since the anomalous $t\bar{t}g$ coupling does not occur at the primary production vertex at the NLC and appears only in a higher order process, the limits we expect on the value of κ should be inferior to those obtained in the literature on anomalous *electroweak* top couplings[5]. While this expectation is realized, the limits we obtain, particularly for the $\sqrt{s} = 1$ TeV e^+e^- collider scenario, are reasonably strong.

It is also important to remember, of course, that an 'anomalous' chromomagnetic moment for the top (or any quark) is induced at the one-loop level in conventional QCD and is of order $\frac{\alpha_s}{\pi}$. In the context of the present paper, by 'anomalous' we mean a value over and above that given within the SM context, usually with a magnitude somewhat larger than that induced via conventional perturbative loop diagrams.

To begin our analysis and in order to set our conventions, we note that the piece of the Lagrangian which governs the $t\bar{t}g$ coupling with a non-zero value of κ is given by :

$$\mathcal{L} = g_s \bar{t} T_a \left(\gamma_\mu + i \frac{F_2(k^2)}{2m_t} \sigma_{\mu\nu} k^\nu \right) t G_a^\mu \,, \tag{1}$$

where g_s and T_a are the usual $SU(3)_c$ coupling and generators, m_t is the top quark mass, k is the out-going gluon momentum, and F_2 represents a general, k^2-dependent, form factor. (A potential $F_1(k^2)$-type form-factor has already been set to unity.) For on-shell gluons, we define $F_2(k^2 = 0) = \kappa$ following the standard notation. The rest of the notation below follows closely that of DKS. Let p_1, p_2, and k be the momenta of the t, $\bar{t}$, and g in the final state such that $q = p_1 + p_2 + k$ with $q^2 = s$. The kinematics of the $e^+e^- \to t\bar{t}g$ process then imply the usual defining relationships

$$\begin{aligned} z_i &= 2q \cdot p_i/s \,, \\ z &= 2q \cdot k/s \,, \\ 2k \cdot p_1/s &= 1 - z_2 \,, \\ 2k \cdot p_2/s &= 1 - z_1 \,, \\ 2p_1 \cdot p_2/s &= 1 - z - \frac{2m_t^2}{s} \,, \end{aligned} \tag{2}$$

where $z_1 + z_2 + z = 2$ is the statement of energy conservation. Following DKS, to leading order in α_s we can factorize the weighted, angular integrated, double differential cross section for the process of interest into the separate contributions due to the vector and axial-vector couplings of the top quark to the s-channel exchange gauge bosons as

$$\frac{d^2W}{dz_1 dz_2} = F_v \frac{d^2W_v}{dz_1 dz_2} + F_a \frac{d^2W_a}{dz_1 dz_2} \,, \tag{3}$$

where $F_{v,a}$ are the 'weighting' factors telling us the fraction of events arising from the vector and axial-vector couplings of the top to γ and Z. Note that we have scaled our result to the lowest order $t\bar{t}$ production cross section, *i.e.*, $W = \sigma/\sigma_0$, where $\sigma_0 = \sigma(e^+e^- \to t\bar{t})$. (Clearly, we must also have $F_v + F_a = 1$ in the above expression to conserve probability.) This factorization approach continues to remain valid even in the presence of a non-zero κ.

In order to proceed, only the quantities $\frac{d^2W_{v,a}}{dz_1 dz_2}$ and $F_{v,a}$ need to be computed. Since $F_{v,a}$ are insensitive to the existence of the anomalous chromomagnetic moment of the top quark, they are given solely by the kinematics and the electroweak couplings of the top. These factors can be read off directly from the general cross section expression for the production of heavy fermion pairs in e^+e^- collisions, *e.g.*, in[11]:

$$\begin{aligned} F_v &= \frac{\frac{1}{2}\beta(3-\beta^2)A_v}{\beta^3 A_a + \frac{1}{2}\beta(3-\beta^2)A_v}\,, \\ F_a &= \frac{\beta^3 A_a}{\beta^3 A_a + \frac{1}{2}\beta(3-\beta^2)A_v}\,, \end{aligned} \tag{4}$$

and $\beta = \sqrt{(1-4m_t^2/s)}$, with s being the square of the center of mass energy. $A_{v,a}$ are directly determined by the vector and axial vector couplings of the electron and top quark as well as the gauge boson propagator function:

$$\begin{aligned} A_v &= \sum_{ij} (v_i v_j + a_i a_j)_e (v_i v_j)_t P_{ij}\,, \\ A_a &= \sum_{ij} (v_i v_j + a_i a_j)_e (a_i a_j)_t P_{ij}\,, \\ P_{ij} &= s^2 \frac{[(s-M_i^2)(s-M_j^2) + (\Gamma M)_i(\Gamma M)_j]}{[(s-M_i^2)^2 + (\Gamma M)_i^2][(s-M_j^2)^2 + (\Gamma M)_j^2]}\,. \end{aligned} \tag{5}$$

The sum in the expression above is over the s-channel $\gamma(i,j=1)$ and $Z(i,j=2)$ gauge boson exchanges, including finite width effects, and for numerical purposes in our analysis the couplings are normalized to the running electromagnetic charge. In these numerical calculations, we will use the values of the various parameters as given in Ref.[3]. We assume that the vector and axial vector couplings of the top quark are given by their conventional SM values with no alterations being present in the $t\bar{t}\gamma, Z$ vertices.

Given all of the above, the evaluation of the square of the matrix element is quite straightforward. Defining the overall normalization coefficients

$$\begin{aligned} N_v &= \frac{2\alpha_s}{3\pi}(2m_t^2 s^2 x_1^2 x_2^2)^{-1}[\frac{1}{2}\beta(3-\beta^2)]^{-1}\,, \\ N_a &= \frac{2\alpha_s}{3\pi}(2m_t^2 s^2 x_1^2 x_2^2)^{-1}[\beta^3]^{-1}\,, \end{aligned} \tag{6}$$

where $x_i = 1 - z_i$, we obtain the following expressions for the above distributions:

$$\begin{aligned} \frac{d^2W_v}{dz_1 dz_2} &= N_v\Big[-8m_t^6(x_1+x_2)^2 - 4sm_t^4[x_1^2(1+2x_2)+x_2^2(1+2x_1)] + 2s^2m_t^2x_1x_2 \\ &\quad [(1-x_1)^2+(1-x_2)^2+\kappa(x_1-x_2)^2] + \kappa^2 s^3 x_1^2 x_2^2(1-x_1-x_2)\Big]\,, \\ \frac{d^2W_a}{dz_1 dz_2} &= N_a\Big[16m_t^6(x_1+x_2)^2 + 2sm_t^4[(\kappa^2+2\kappa+2)x_1x_2(x_1+x_2)^2 + 8x_1x_2 \\ &\quad (x_1+x_2) - 2(x_1^2+x_2^2+6x_1x_2)] + 2m_t^2 s^2 x_1x_2[(1-x_1)^2+(1-x_2)^2 + \\ &\quad \kappa(x_1^2+x_2^2-4) + \kappa^2 x_1x_2(x_1+x_2-3)] + \kappa^2 s^3 x_1^2 x_2^2(1-x_1)(1-x_2)\Big]\,, \end{aligned} \tag{7}$$

which, when combined with the other results above, gives the complete $t\bar{t}g$ double differential cross section, normalized to that for $t\bar{t}$ production, including the contributions from finite κ. For $\kappa = 0$, we reproduce the standard results in the literature[9]. These expressions have no co-linear singularities, due to the finite top mass, but are still, overall, infrared singular in the limit of zero gluon momenta. It is already clear from Eqs. 6 and 7 that as s/m_t^2 gets large the last terms in the expressions for $\frac{d^2W_{v,a}}{dz_1 dz_2}$ will become dominant over the conventional QCD result. Note that these terms are infrared finite due to the fact that the chromomagnetic coupling in the Lagrangian is proportional to the gluon momenta. Semiquantitative bounds on κ follow immediately upon integrating these two terms over phase space thus obtaining

the ratio of the $t\bar{t}g$ to $t\bar{t}$ cross sections in the large s/m_t^2 limit, *i.e.*,

$$\frac{\sigma_{t\bar{t}g}}{\sigma_{t\bar{t}}} \simeq \frac{\alpha_s}{\pi}\frac{\kappa^2 s}{18m_t^2}\frac{v^2+1.25a^2}{v^2+a^2}\,, \tag{8}$$

which leads for $m_t = 175$ GeV and $\sqrt{s} = 500$ GeV (assuming that the large s/m_t^2 limit is crudely valid in this case) to $\simeq 5.0(\kappa/3)^2$ as the coefficient of $\frac{\alpha_s}{\pi}$. Correcting for a color factor and a missing $\frac{1}{\pi}$, this limiting behavior is in agreement with that obtained by Grifols and Mendez[12] for an anomalous magnetic moment of the τ in the decay $Z \to \tau\bar{\tau}\gamma$. This crude estimate tells us that for perturbation theory to make sense for $t\bar{t}g$ production at the NLC, the value of κ must approximately satisfy $|\kappa| \leq 3$.

There are several ways to see the effects of a non-zero κ on the gluon jet energy spectrum. For the moment, let us follow DKS and calculate the average value of the scaled gluon energy, *i.e.*, z_{ave}, (where $z = 2E_{glu}/\sqrt{s}$) as a function of β^2 for pure vector or pure axial vector couplings; these are shown in Figs. 1a and 1b (assuming $\alpha_s = 0.1$ for purposes of direct comparison with DKS). Note that this quantity, z_{ave}, is infrared safe for all values of β. For $\kappa = 0$, the results of DKS are immediately recovered but for non-zero κ, significant deviations are observed which grow quite large with increasing β^2. It is, of course, just in this phase space regime where we expect the largest deviations from the conventional QCD results since large β^2 implies large s/m_t^2. Of course, even in standard QCD, the region close to $\beta = 1$ becomes non-perturbative (unless further cuts are applied) since it corresponds to the location in phase space where multiple soft co-linear gluon emission can occur. However, long before this non-perturbative range is reached we see that for non-zero values of κ there is an upward shift in the average value of z, for both vector and axial vector couplings. This increased divergence in the average value of z for large β^2 is only symptomatic of a more widespread phenomena, *i.e.*, the the presence of a non-zero κ not only increases the rate for the $t\bar{t}g$ final state but also hardens the gluon jet energy distribution. We further note that

the region of small β^2 is also nonperturbative as this corresponds to the threshold regime where resummation techniques need to be applied. The possible signatures for a non-zero κ from detailed threshold region studies is beyond the scope of the present paper.

Perhaps a better probe of non-zero κ is the gluon jet energy spectrum itself. This we show in Fig. 2a assuming $m_t = 175$ GeV and an NLC center of mass energy of $\sqrt{s} = 500$ GeV. From the kinematics, the maximum gluon energy for these input parameters is $z_{max} = 1 - 4m_t^2/s = 0.51$ or 127.5 GeV. Note that in the Figure we have again assumed $\alpha_s = 0.1$. To get the 'best' result to compare with experiment (ignoring higher order corrections) we need to obtain the correct value of α_s at this energy and simply rescale all of the curves in Fig. 2a by an overall factor. One possible approach, which we will follow below, is to use the method of Brodsky, LePage, and Mackenzie(BLM)[13] in order to set the scale, using the value of $\alpha_s(M_z)$[3] as input, and to make use of the three-loop renormalization group equations. Without doing a complete calculation, it is possible to estimate the correct BLM scale(Q^*) at which to evaluate α_s by simple phase space considerations [14]. One finds $Q^* \simeq 0.435(\sqrt{s} - 2m_t)/3$, not too different than the Q^* for Z decay studies, so that we arrive at $\alpha_s(Q^*) = 0.121$ as our estimate (using the value of α_s at the Z scale extracted from R_h[3] as input). This implies that all the curves in Fig. 2 should be scaled upward by about 21%. For other preferred values of α_s, an appropriate rescaling should be performed.

Of course, the overall scale is not the interesting part of these figures as far as κ sensitivity is concerned since κ non-zero results in a distortion in the gluon spectrum, especially at larger z values. Generally one sees that the effect of κ is to flatten the spectrum so that there is an excess of gluon jets with high energies. For $\kappa = -1$, however, we see a sharper fall off in the spectrum than in standard QCD; this is a result of a destructive interference between the ordinary and κ-dependent amplitudes which takes place for the specific values of m_t and $\sqrt{s}$ we have used as input. This implies that for a range of negative κ, the SM result

and the one where κ is non-zero will be very difficult to distinguish. Since the deviation due to κ is at larger z values and the spectrum diverges as $z \to 0$, we can apply a cut on the minimum gluon energy, z_{cut}, which we use to define our event sample and integrate the spectrum for gluon energies above that value. This results in Fig. 2b, which should also be scaled upwards by 21% if the BLM approach is used. As expected, the curves for non-zero κ are generally higher than the standard QCD result. Taking the BLM value for α_s and a value of $z_{cut} = 0.2$, *i.e.*, only events with gluon jets having energies in excess of 50 GeV, we show in Fig. 2c the κ dependence of the resulting integrated cross section. Assuming an integrated luminosity of $30fb^{-1}$, this corresponds to a sample of $375(I/0.02)$ $t\bar{t}g$ events before further cuts are applied or 457 events in the standard QCD case with $\kappa = 0$. Of course to identify top pair production, we will demand at least one high-p_t lepton in the event($B = 0.44$) and, perhaps, an additional b-tag(with an assumed efficiency of $\epsilon = 0.8$) to remove backgrounds.

If the SM result is realized, we can use the estimates of the $t\bar{t}g$ event rate above to place bounds on the value of κ. (We might expect better limits would most likely be obtainable by a direct measurement of the gluon energy spectrum instead of a simple rate estimate; we will return to this possibility below.) Allowing for a 2% error on the determination of α_s in the NLC era, a 5% systematic error from higher-order QCD uncertainties, and the rates above to calculate statistical errors, the value of I would be determined to be $2.44 \pm 0.23\%$, which at 95% CL would restrict κ to lie in the range $-2.1 \leq \kappa \leq 0.6$. Varying the cuts leads to numerically similar results there being a relative trade off between increased(decreased) statistics and decreased(increased) sensitivity. The reason for the poor limit is clear; as we saw above a range of negative κ exists for which the resulting energy distribution is almost identical to the SM one. This is a result of a conspiracy between the various contributions to the differential cross section and the particular values of m_t and $\sqrt{s}$ we are examining.

This situation is not much alleviated by an actual fit to the gluon energy distribution

itself. For $z \geq 0.15$, we generate 'data' assuming $\kappa = 0$ by Monte Carlo taking a bin size of $\Delta z = 0.05$. The phase space region of interest is then covered by a total of seven z bins, the last one covering the range $0.45 \leq z \leq \beta^2 = 0.51$. We then fit the κ-dependent gluon spectrum to the data and perform a χ^2 analysis. The resulting χ^2 is a quartic function of κ and is shown explicitly in Fig. 2d where one sees that two essentially degenerate minima exists. This is due to the conspiracy discussed above and, as a result, only rather poor bounds on κ are obtainable. Explicitly, from this procedure we obtain the allowed range $-1.98 \leq \kappa \leq 0.44$ at 95% CL, which is only a slight improvement in the limit obtained above from simple counting. Similar limits are obtained if different bin sizes are chosen for the Monte Carlo study but we have not tried to optimize this choice in our analysis. In Fig. 2e, we compare the data generated by Monte Carlo with the energy spectra predicted for the κ values corresponding to the two approximately degenerate χ^2 minima, *i.e.*, $\kappa = -1.69$ and 0.12. As can be seen the fit is quite good in both cases.

What happens when we go to larger values of $\sqrt{s}$ where significantly greater sensitivities to κ are expected? In Figs. 3a-d, we examine the case of $t\bar{t}g$ production with non-zero κ for an NLC with $\sqrt{s} = 1$ TeV. (In these figures, the BLM value of $\alpha_s = 0.100$ has been assumed following the above procedure so that no overall rescaling is necessary in this case.) Fig. 3a clearly shows that at large values of z, the gluon jet energy distribution is even more enhanced for fixed values of κ than for the $\sqrt{s} = 500$ GeV case; this is exactly what we should have expected. Note the approximate symmetry of the curves under the interchange $\kappa \to -\kappa$; this occurs naturally in the large s/m_t^2 limit as seen above. Applying the z_{cut} approach as before yields Fig. 3b where the $\kappa \to -\kappa$ symmetry is even more obvious. The value of I is so large in the $|\kappa| = 3$ case for values of $z_{cut} < 0.4$ that we should most likely not trust our lowest order perturbative result for this range of parameters. Taking $z_{cut} = 0.4$, which corresponds to a gluon jet of energy 200 GeV, we show in Fig. 3c the

explicit κ dependence of our cross section ratio. If we assume an integrated luminosity of $200 fb^{-1}$ and make the same assumptions as in the 500 GeV case, the realization of the SM result can again be used to place significant constraints on κ. In this case we would obtain $I = 5.24 \pm 0.36\%$, which at 95% CL would restrict κ to lie in the range $-1.0 \leq \kappa \leq 0.25$, which is about a factor of two better than that obtained for the 500 GeV NLC. We see again that because small, negative values of κ and the SM case lead to very similar gluon energy distributions, we do not obtain a very narrow allowed range for κ. However, if we try the Monte Carlo approach as we did above and fit the entire spectrum for $z \geq 0.4$ in nine z bins (the last bin covering the range $0.8 \leq z \leq \beta^2 = 0.8775$) we obtain the χ^2 plot shown in Fig. 3d. Unlike the $\sqrt{s} = 500$ GeV case, the second local χ^2 minima is no longer degenerate so that we can now obtain a substantially improved bound on κ: $-0.12 \leq \kappa \leq 0.21$. Again, we have not made any attempt to optimize the bin size in this Monte Carlo study. In Fig. 3e, we compare the Monte Carlo generated data with the predicted gluon spectrum for the choice of $\kappa = 0.06$, corresponding to the χ^2 minimum. As can be seen, the fit is quite good.

In this paper we have analyzed the influence of an anomalous chromomagnetic dipole moment for the top quark, κ, on the cross sections and associated gluon jet energy distributions for $t\bar{t}g$ events produced at both 500 and 1000 GeV e^+e^- linear colliders assuming a top quark mass of 175 GeV. Making a cut on the gluon jet energy of 50(200) GeV and demanding at least one b-tag as well as one high-p_t lepton tag as a top signal, an integrated luminosity of $30(200) fb^{-1}$ leads to a bound on κ of $-2.1 \leq \kappa \leq 0.6(-1.0 \leq \kappa \leq 0.25)$ at the NLC with $\sqrt{s} = 500(1000)$ GeV, assuming no excess $t\bar{t}g$ events are observed. One might have expected that a complete fit of the gluon energy spectrum to the κ-dependent distribution would generally lead to substantial improvements in these limits. We found, however, that this was not the case at a $\sqrt{s} = 500$ GeV NLC due to a conspiracy between the SM and κ-dependent terms in the cross section and that only slight improvements were

obtainable: $-1.98 \leq \kappa \leq 0.44$. For the case of a $\sqrt{s} = 1$ TeV machine, this conspiracy no longer took place and the power of fitting to the gluon energy spectrum was realized yielding a vastly improved bound of $-0.12 \leq \kappa \leq 0.21$.

The results of this analysis are only preliminary. In a more complete Monte Carlo study the effects of the top decay, possible gluon emission from the final state bottom quarks, and detector resolution and efficiencies need to be included. However, by demanding a very high energy additional gluon jet with the rest of the event reconstructing to $t\bar{t}$, the results of such an analysis should closely mimic those we have obtained above. We have seen that it is quite likely that the NLC will be able to place reasonably strong constraints on the existence of a top quark anomalous chromomagnetic moment. Of course, the more exciting possibility of observing a hardening of the gluon jet energy spectrum would be a spectacular signature for an anomalous magnetic moment for the top or other new physics beyond the standard model.

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Figure Captions

Figure 1. Average value of the scaled gluon jet energy, z, as a function of β^2 for both purely vector(a) or axial vector(b) heavy quark couplings. The upper(lower) dotted, dashed, and dot-dashed curves correspond to κ values of 3(-3), 2(-2), and 1(-1) respectively while the solid curve is conventional QCD with $\kappa = 0$. $\alpha_s = 0.10$ has been assumed.

Figure 2. (a)Gluon jet energy spectrum assuming $\alpha_s = 0.10$ for $m_t = 175$ GeV at a $\sqrt{s} = 500$ GeV NLC. (b)Integrated gluon energy spectrum for the same input parameters as in (a) as a function of the minimum gluon energy, z_{cut}. In both (a) and (b), the labelling of the various curves is as in Fig. 1. (c) Same as (b) but as a function of κ assuming $z_{cut} = 0.2$ and $\alpha_s = 0.121$ as suggested by the BLM approach. (d) χ^2 fit to the gluon energy spectrum for the value of κ as described in the text. $\alpha_s = 0.121$ is again assumed. (e) Best fit gluon κ-dependent spectra through the points generated by Monte Carlo. The dashed(dash-dotted) curve corresponds to to $\kappa = -1.69(0.12)$.

Figure 3. Same as Fig. 2, but for an NLC with $\sqrt{s} = 1$ TeV. In (c) the value of $z_{cut} = 0.4$ is assumed along with $\alpha_s = 0.100$. (e) Best fit gluon spectrum through the points generated by the Monte Carlo analysis corresponding to $\kappa = 0.06$.

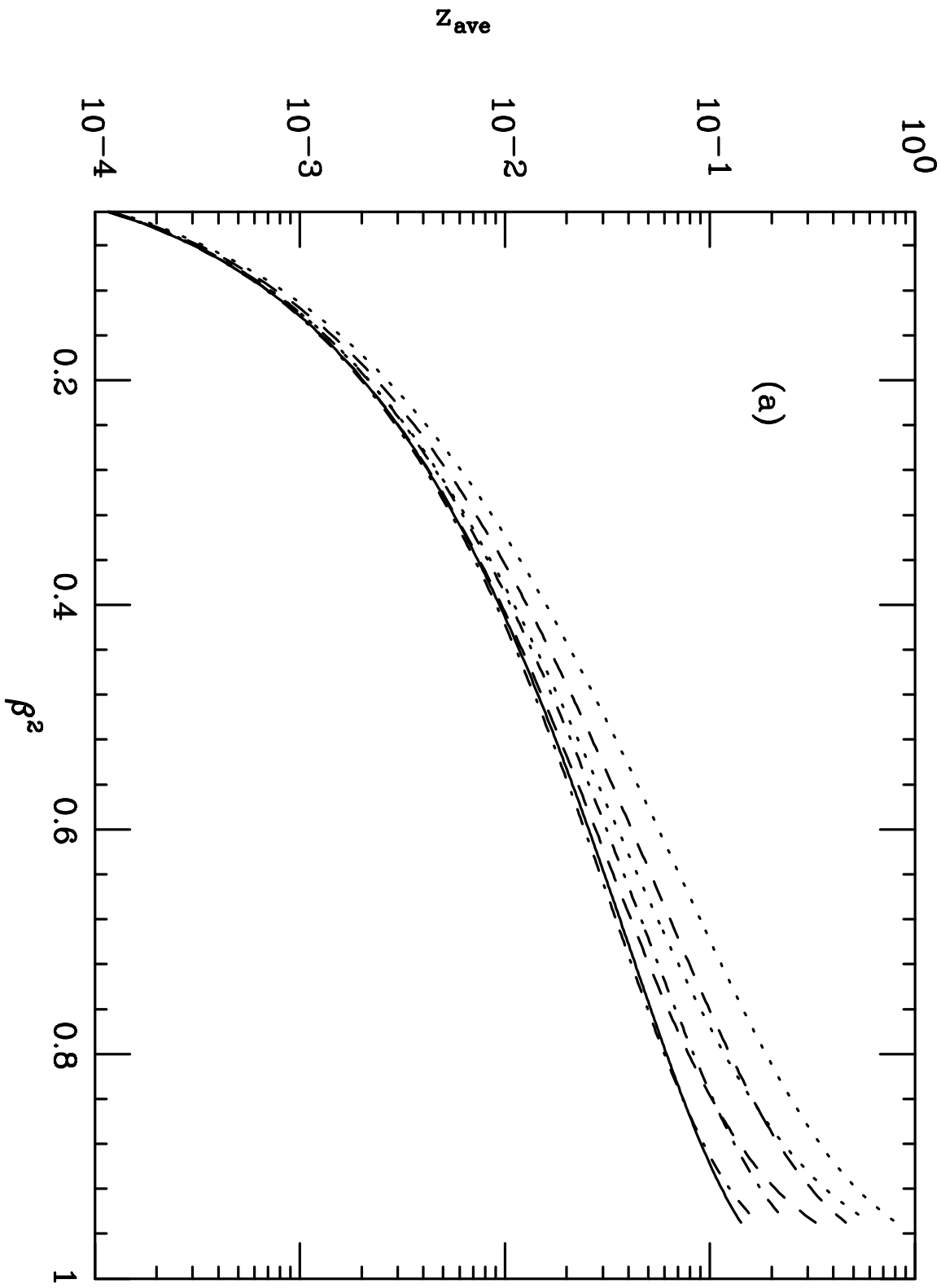

Figure 1a

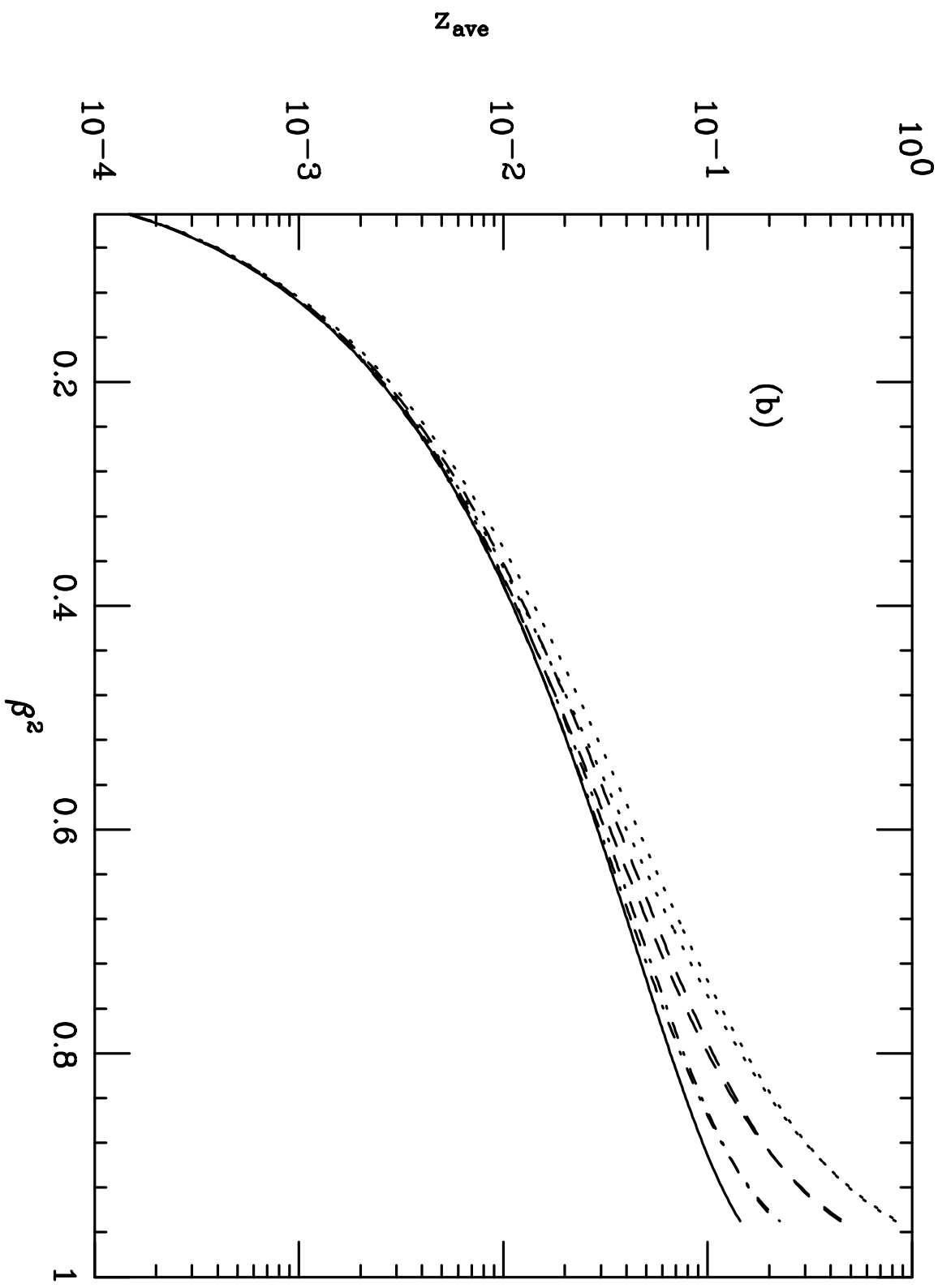

Figure 1b

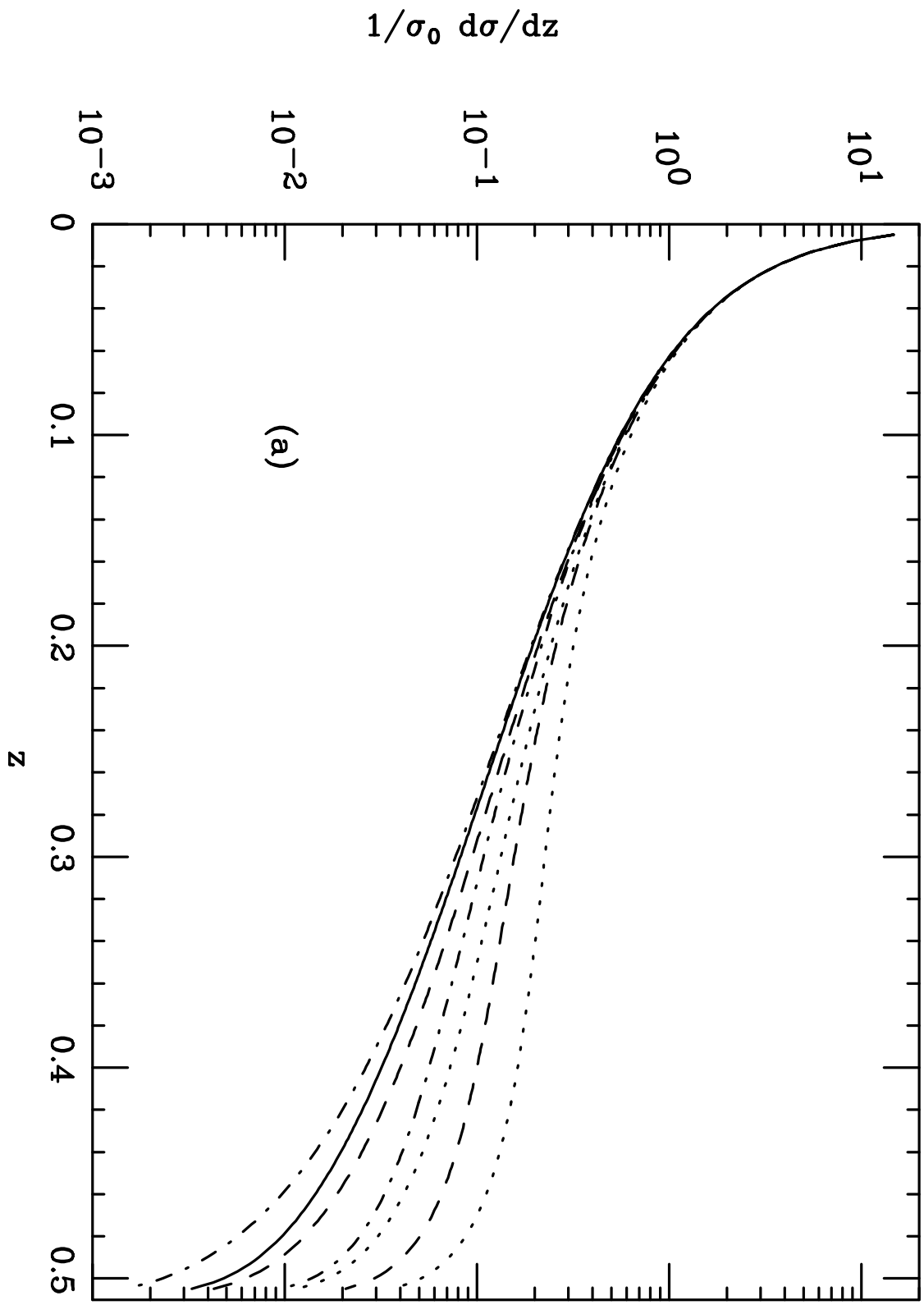

Figure 2a

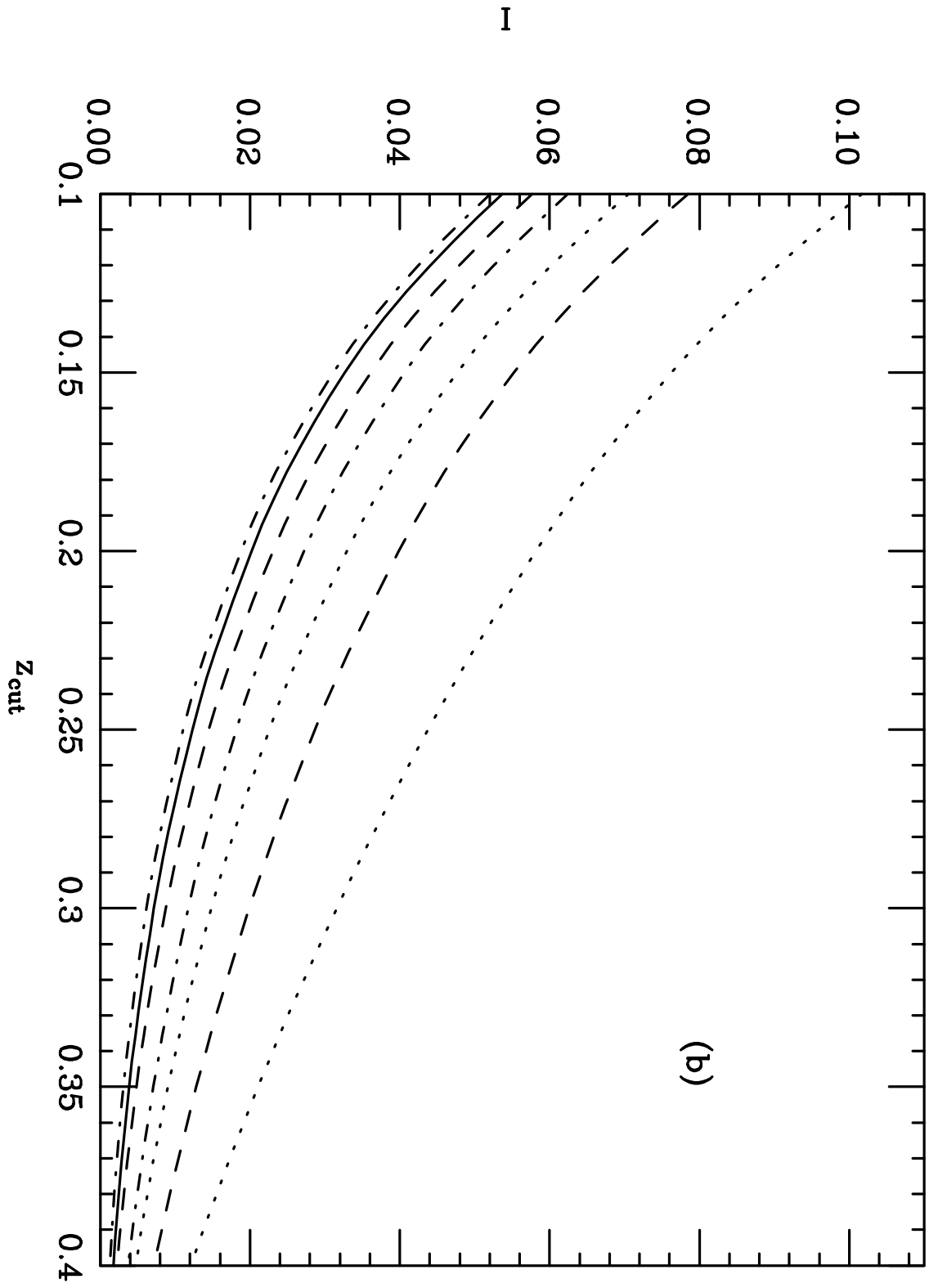

Figure 2b

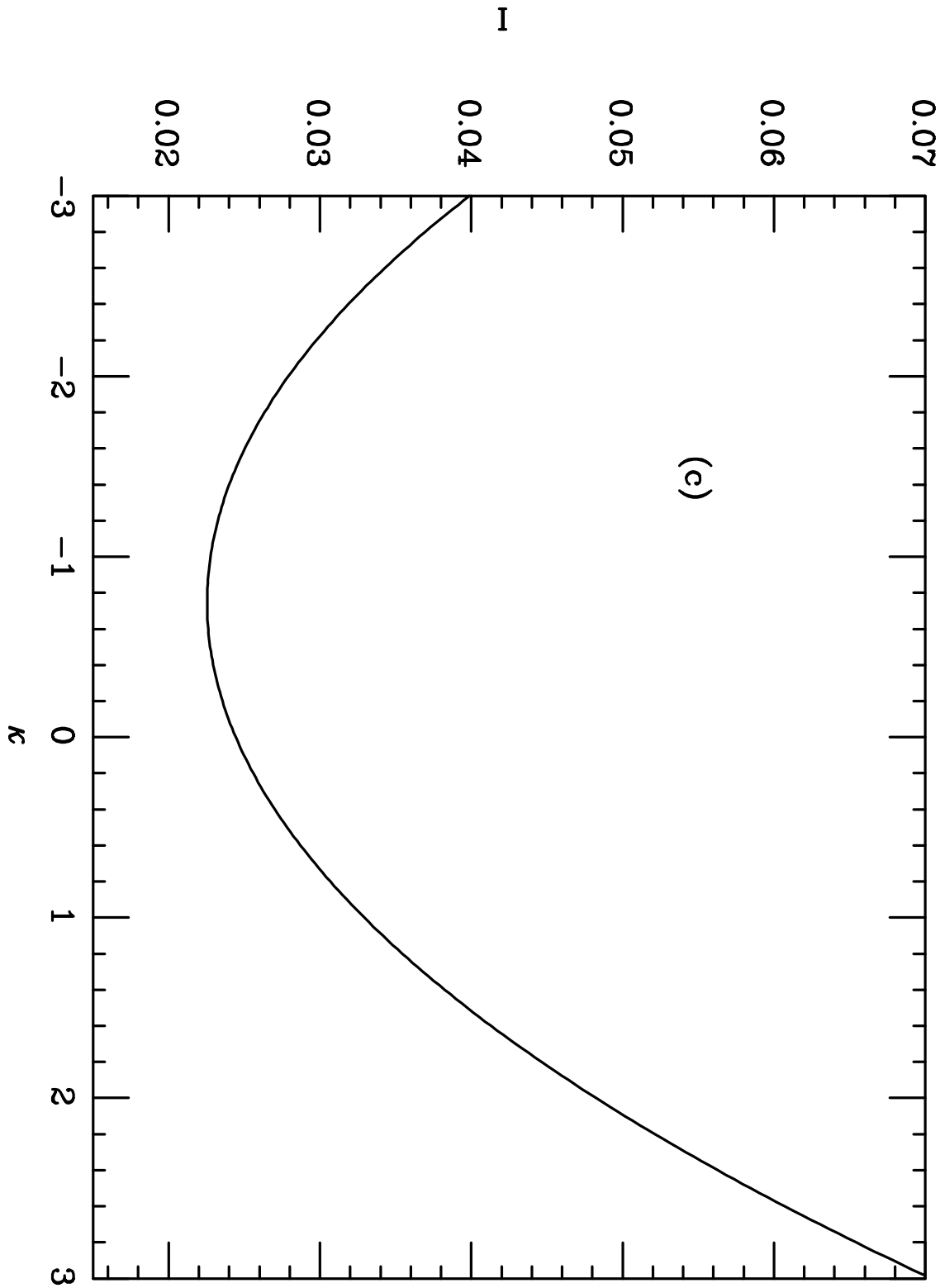

Figure 2c

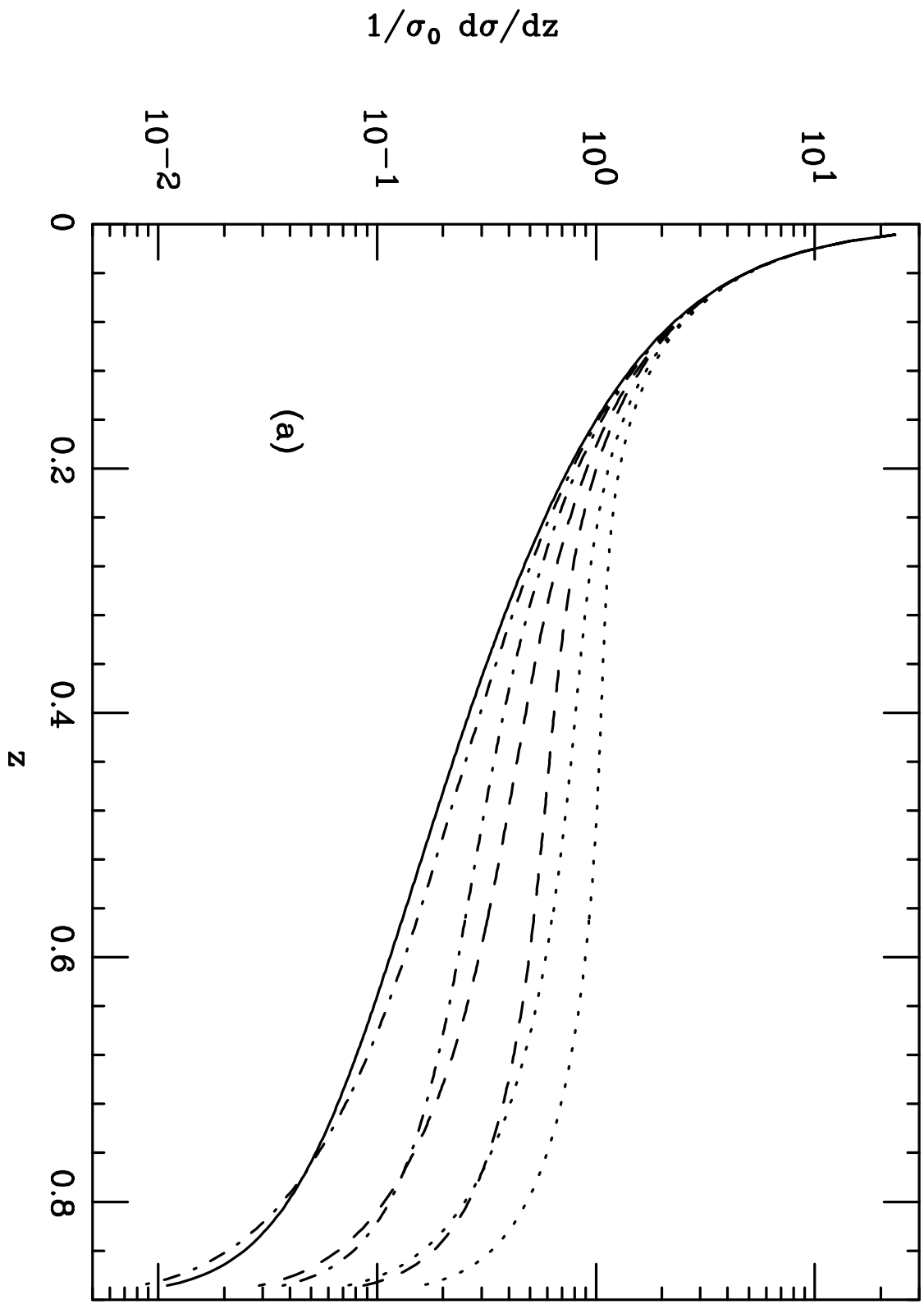

Figure 3a

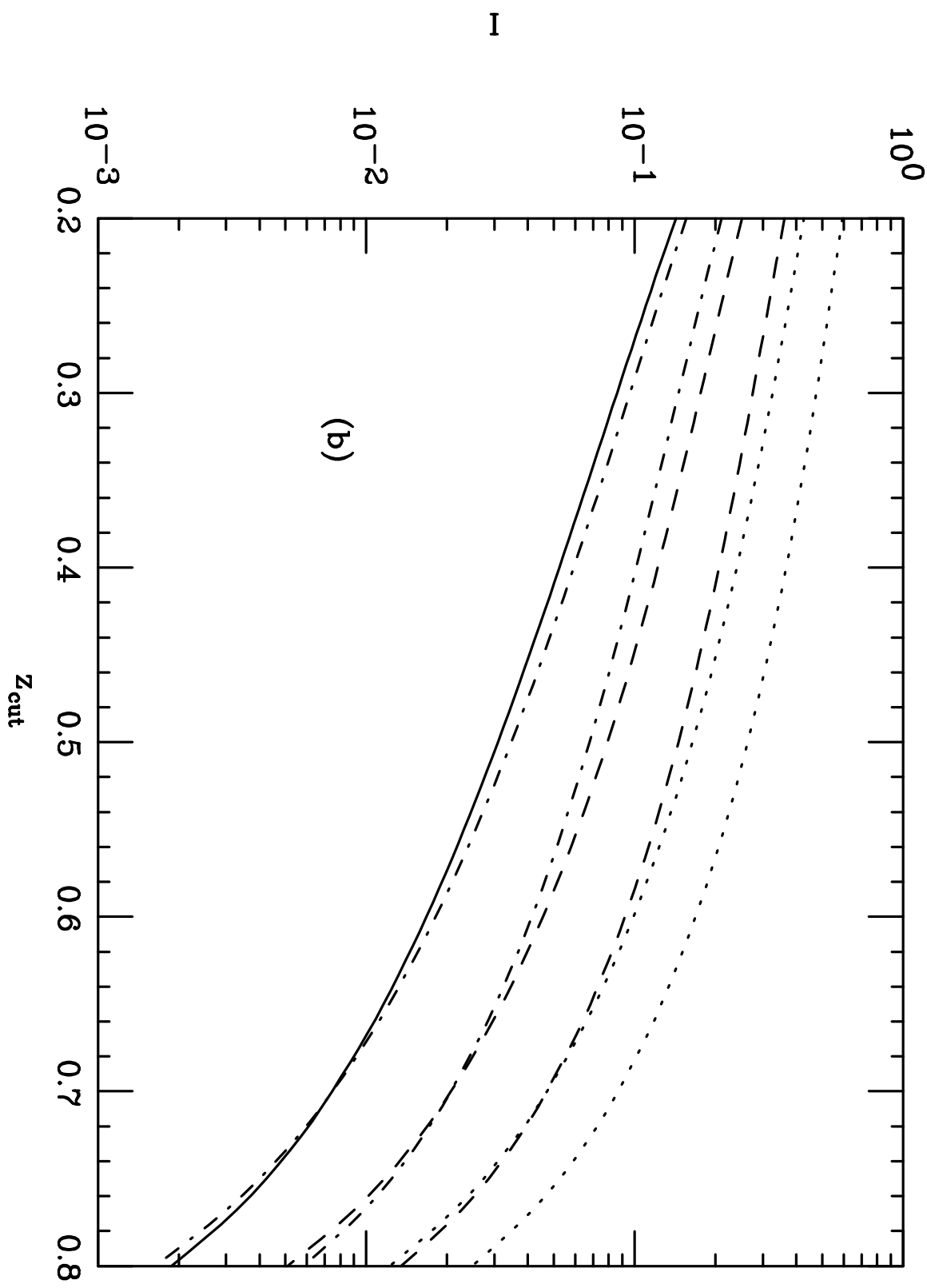

Figure 3b

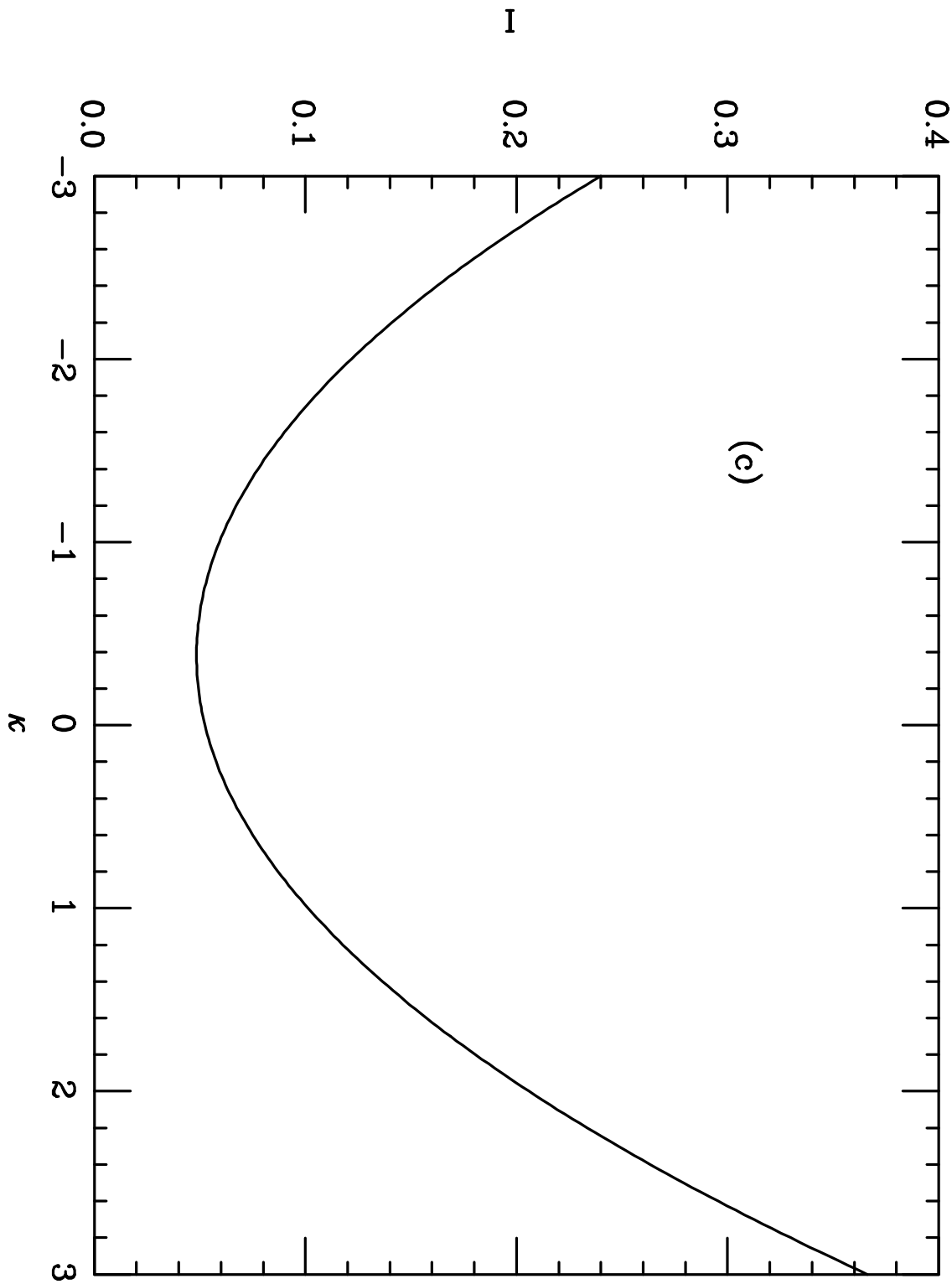

Figure 3c