The Parton Distribution Functions in the Limit $x_{Bj} \to 1$

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Abstract

We review both the counting rule and the influence of the evolution in Q^2 for the large x_{Bj} behaviour of the valance quark distribution functions. Based on a factorization procedure we present a more general perturbative treatment to compute this behaviour. A complete analysis is performed in the scalar $\phi_{[6]}^3$-theory for the parton distribution function of the "meson", which shows that logarithmical corrections arise from the distribution amplitude and that the reference momentum square Q_0^2 is fixed by x_{Bj}.

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1   Introduction

Deep inelastic lepton-hadron scattering (DIS) experiments have shown that hadrons consist of point-like particles called partons. These partons were identified as quarks and gluons, which are the fundamental degrees of freedom of a relativistic $SU_c(3)$ gauge field theory, quantum chromodynamics (QCD). For instance, corresponding to this quark-parton model a nucleon is formed by a colour singlet state that consists of three valence quarks as well as sea quarks and gluons.

Since the bound state problem remains unsolved at present it is not possible to compute the DIS cross section completely from the first principle of QCD. However, with the help of the operator product expansion [1] it could be shown that the hadronic part factorises as a convolution of perturbative computable Wilson coefficients and nonperturbative parton distribution functions [2].[2] These distribution functions are defined as forward matrix elements of string light-cone (LC) operators [3, 4]. Moreover, from the renormalization group (RG) equation of these operators one obtains the Gribov-Lipatov-Altarelli-Parisi (GLAP) equation [5], which controls the evolution of the distribution functions in Q. However, it remains a theoretical non-predictable input, the distribution functions at a reference momentum $Q_0 \sim 1$ GeV.

At large x_{Bj} the theoretical situation is much better. Therefore, it is possible, with the help of the naive quark-parton model, to derive a counting rule [6] that predicts the power behaviour of the distribution functions in $(1 - x_{Bj})$ for both mesons and baryons [7]. These predictions will be reviewed in Section 2.

In Section 3 we define the distribution functions as expectation values of LC operators and discuss, with the help of their RG equation (or GLAP equation), the change of the large x_{Bj} behaviour by evolution in Q.

In Section 4 the distribution functions will be factorized as a convolution of distribution amplitudes and a perturbative computable hard amplitude, which is computed for the "meson" distribution function of the scalar $\phi_{[6]}^3$-theory in six dimensions. Apart from the leading power behaviour the convolution also provides logarithmical correction in $(1 - x_{Bj})$ that arises from the evolution of the distribution amplitude, and also the value of the reference momentum square Q_0 as a function of x_{Bj}.

2   Counting rule

It is well known that the hadronic part of the unpolarized charged lepton DIS is described by the hadronic tensor

$$W_{\mu\nu}(x_{Bj}, Q^2) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x_{Bj}, Q^2)$$
$$+ \left(P_\mu - \frac{q_\mu Pq}{q^2}\right)\left(P_\nu - \frac{q_\nu Pq}{q^2}\right) F_2(x_{Bj}, Q^2)/Pq, \tag{1}$$

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[2]At first this factorization was performed in terms of the moments (expectation values of local operators) from which the mentioned convolution can be obtained with the help of the Mellin transformation.

where P is the four-momentum of the hadron, and q denotes the four-momentum transfer. Here $F_1(x_{Bj}, Q^2)$ and $F_2(x_{Bj}, Q^2)$ are the structure functions which depend on the Bjorken variable $0 \leq x_{Bj} = -q^2/2qP \leq 1$ and the momentum transfer $Q^2 = -q^2$.

In the Bjorken region, i.e., large momentum transfer Q and fixed x_{Bj}, the two structure functions can be expressed according to the quark-parton model by the non-perturbative quark distribution functions $q_i(x_{Bj}, Q^2)$ and anti-quark distribution functions $\bar{q}_i(x_{Bj}, Q^2)$:

$$F_1(x_{Bj}, Q^2) = F_2(x_{Bj}, Q^2)/(2x_{Bj}) = \frac{1}{2} \sum_i e_i^2 \left(q_i(x_{Bj}, Q^2) + \bar{q}_i(x_{Bj}, Q^2) \right), \tag{2}$$

where $i = u, d, ...$ is the flavor index, and e_i is the corresponding electrical charge.

As mentioned, at large x_{Bj} it is possible to compute the parton distribution functions. But one deals with two large scales, $Q \to \infty$ and $1/(1 - x_{Bj}) \to \infty$, so one has to clarify in which way the two limits will be performed. With the help of the hadronic final state mass $M_x^2 = (P + q)^2 = M_H^2 + (1 - x_{Bj})Q^2/x_{Bj} \geq M_H^2$, where M_H is the mass of the hadron, one finds the following classification:

1. $(1 - x_{Bj})Q^2 \to 0$, $\qquad M_x^2 = M_H^2$,

2. $(1 - x_{Bj})Q^2 \to$ finite, $\quad M_x^2 = M_H^2 +$ finite,

3. $(1 - x_{Bj})Q^2 \to \infty$, $\qquad M_x^2 \to \infty$.

In this paper we consider only the third case, for which the factorization in Q via the operator product expansion remains true so that, as usual, higher order twist terms can be neglected.

Now let us review the counting rule for the large x_{Bj} behaviour of the structure function. Corresponding to the naive quark-parton model we have the struck quark with momentum p, and the spectator quarks carry the momentum $P - p$:

$$p = \left(x_{Bj}P_+, \frac{M^2 + \vec{k}_\perp^2}{x_{Bj}P_+}, \vec{k}_\perp \right), \qquad P - p = \left((1 - x_{Bj})P_+, \frac{\mathcal{M}^2 + \vec{k}_\perp^2}{(1 - x_{Bj})P_+}, -\vec{k}_\perp \right). \tag{3}$$

Here we have introduced light-cone coordinates, i.e., $p = (p_+ = p_0 + p_3, p_- = p_0 - p_3, \vec{p}_\perp)$, so that $x_{Bj} = p_+/P_+$. The essential assumption for deriving the counting rule is the requirement that the spectator quarks have a non-vanished finite mass $\mathcal{M}$. Then from the kinematical identity for the hadron mass $M_H^2 = ((P - p) + p)^2$ it follows that for $x_{Bj} \to 1$ the mass of the struck quark

$$M^2 = p^2 = \frac{x_{Bj}(1 - x_{Bj})M^2 - x_{Bj}\mathcal{M}^2 - \vec{k}_\perp^2}{1 - x_{Bj}} \sim -\frac{\mathcal{M}^2 + \vec{k}_\perp^2}{1 - x_{Bj}} \tag{4}$$

is far off shell. Note that this behaviour also remains in the infinite momentum frame.

The power behaviour of the structure functions at $x_{Bj} \to 1$ follows directly from the far off shell behaviour of the wave function. The iteration of the Bethe-Salpeter equation provides that this behaviour is determined by the minimal number of gluon exchanges required to stop the spectator (see Fig. 1 for a nucleon). Thus, without taking into account the spin of the partons one gets the following counting rule [7]:

$$F_2(x_{Bj}) \sim q(x_{Bj}) \sim (1 - x_{Bj})^{2n_s - 1} \quad \text{for } x_{Bj} \to 1, \tag{5}$$

where n_s is the number of spectators.

Corresponding to the counting rule (5) a nucleon has a power behaviour of 3, which is consistent with the experimental results. A meson should have a power behaviour of 1. But it was shown in Ref. [8] that spin effects provide an additional suppression so that the power is 2. It is interesting to note that experimentally one finds for the pion the power behaviour is closer to 1 than to 2 [9]; so the details of this prediction are not quite clear (see also Ref. [7]).

3 Improving the counting rule result by evolution

The predicted behaviour of the structure function at large x_{Bj} will be modified by the evolution of the distribution functions and by perturbative corrections of the Wilson coefficients. Although at $x_{Bj} \rightarrow 1$ higher order corrections are important [10], in this paper we discuss only the modification by the evolution at leading order. Interesting results from perturbative all-order investigations can be found in the literature for both the evolution of the distribution functions [11] and corrections of the Wilson coefficients [12].

First let us sketch why the distribution functions are defined as expectation values of LC operators and why the GLAP equation arises from the RG equation of these operators. The dispersion relation

$$
\begin{aligned}
W_{\mu\nu}(x_{Bj}, Q^2) &= \frac{1}{4\pi} \mathrm{Im} T_{\mu\nu}(x_{Bj}, Q^2), \\
T_{\mu\nu}(x_{Bj}, Q^2) &= i \int d^4x < P|\mathrm{T}\{j_\mu(x)j_\nu(0)\}|P > e^{iqx},
\end{aligned}
\tag{6}
$$

provides that the hadronic tensor can be expressed by the imaginary part of the forward compton scattering amplitude $T_{\mu\nu}(x_{Bj}, Q^2)$. Furthermore, the leading Q^2 behaviour of $W_{\mu\nu}(x_{Bj}, Q^2)$ is determined by the light-cone singularities of the time ordered product $\mathrm{T}\{j_\mu(x)j_\nu(0)\}$. Thus, the non-local light-cone expansion of this operator product [13] provides a definition of the quark distribution functions in terms of gauge invariant string operators that are a resummation of local leading twist operators (see, e.g., [14]). In the axial gauge $\tilde{x}A = 0$ these operators are simplified to be bi-local:

$$
q_i(x, Q^2) = \int \frac{d\kappa}{2\pi} e^{2i\kappa(\tilde{x}P)x} < P|\bar{\psi}_i(-\kappa\tilde{x})\tilde{x}\gamma\psi_i(\kappa\tilde{x})|P >_{|\mu^2=Q^2}, \ 0 \le x \le 1,
\tag{7}
$$

$$
\bar{q}_i(x, Q^2) = -\int \frac{d\kappa}{2\pi} e^{-2i\kappa(\tilde{x}P)x} < P|\bar{\psi}_i(-\kappa\tilde{x})\tilde{x}\gamma\psi_i(\kappa\tilde{x})|P >_{|\mu^2=Q^2}, \ 0 \le x \le 1.
\tag{8}
$$

Here the more arbitrary light-like vector $\tilde{x}$ arises from the projection of x onto the light-cone and can be chosen as $\tilde{x} = (\tilde{x}_+ = 0, \tilde{x}_- = 2, \vec{0}_\perp)$ so that $\tilde{x}P = P_+$.

In the following we consider only the evolution in the flavor non-singlet sector. For technical simplification we define formally the flavor non-singlet operators

$$
O^a(x, \mu^2) = \int \frac{d\kappa}{2\pi} e^{2i\kappa(\tilde{x}P)x} \bar{\psi}_i(-\kappa\tilde{x})\tilde{x}\gamma\lambda_{ij}^a\psi_i(\kappa\tilde{x}),
\tag{9}
$$

4

where λ_{ij}^a is a generator of the flavor group. If we keep in mind the forward case then the renormalization group equation can be formally written as

$$\mu \frac{d2}{d\mu^2} O^a(x, \mu^2) = \int_x^1 \frac{dy}{y} P(x/y; \alpha_s(\mu^2)) O^a(y, \mu^2), \tag{10}$$

where the integral kernel is known as perturbative expansion in α_s,

$$P(x; \alpha_s) = C_F \frac{\alpha_s}{2\pi} \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right) + O(\alpha_s^2), \quad \text{where } C_F = 4/3. \tag{11}$$

Forming forward matrix elements and setting the renormalization point $\mu = Q$ one gets from the RG equation (10) the GLAP equation. Take into account Eq. (11) one finds for $x \to 1$

$$Q^2 \frac{d}{dQ^2} q^{NS}(x, Q^2) \tag{12}$$

$$= C_F \frac{\alpha_s(Q^2)}{2\pi} \left(\left(2\ln(1-x) - \frac{3}{2} \right) q^{NS}(x, Q^2) + 2 \int_x^1 dy \frac{q^{NS}(y, Q^2) - q^{NS}(x, Q^2)}{y - x} \right).$$

This equation is to be implemented by a initial condition that has, corresponding to Eq. (5), the form $q^{NS}(x, Q_0^2) \propto (1-x)^\nu$, where the value of ν is approximately determined by the counting rule result. Using the ansatz $q^{NS}(x, Q^2) = A(Q, Q_0)(1-x)^{\nu + \xi(Q, Q_0)}$, where $\xi(Q_0, Q_0) = 0$, it is straight forward to derive the solution [15]

$$q^{NS}(x, Q^2) \propto \frac{\Gamma(\nu+1)}{\Gamma(\nu+1+\xi(Q, Q_0))} e^{(3/4 - \gamma_E)\xi(Q, Q_0)} (1-x)^{\nu + \xi(Q, Q_0)}, \tag{13}$$

where

$$\xi(Q, Q_0) = 2C_F \int_{Q_0^2}^{Q^2} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} = \frac{4C_F}{\beta_0} \ln\left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \right), \quad \beta_0 = 11 - (2/3)n_f, \tag{14}$$

is a function increasing with Q. Thus, with rising Q the $(1-x)$ power behaviour increases and the prefactor decreases.

4 Factorization of the distribution function

In this section we present a more rigorous treatment that allows the perturbative computation of the large x_{Bj} behaviour of the parton distribution functions and also provides a smooth inclusive exclusive connection. As we saw in Section 2, at $x_{Bj} \to 1$, the parton model suggests that the momentum of the struck quark is far off shell. Thus, using the light-cone quantization and the representation of the wave function by a superposition of fock states, a factorization can be performed based on the iteration of the equation of motion [3],[3]

$$q(x, Q^2) \sim \int_0^1 du \int_0^1 dv \phi^*(u, \lambda^2) T_H(x, u, v, \lambda^2, Q^2) \phi(v, \lambda^2), \tag{15}$$

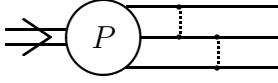[3]A similar treatment is used for the factorization of exclusive large momentum transfer processes.

Fig. 1: This graph yields to the leading $(1-x)^3$ power behaviour of the nucleon structure function for $x \to 1$.
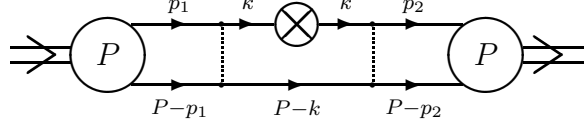

Fig. 2: Parton distribution function of the meson at large x. The crossed circle symbolizes the LC operator insertion.

where the factorization scale is given by $\lambda^2 \sim m^2/(1-x)$. Here the hard amplitude $T_H(x, u, v, \lambda^2, Q^2)$ includes the LC operator $O^a(x, \mu = Q)$ and provides, in principal (see below), the power behaviour in $(1-x)$.

For simplicity, in the following we restrict ourselves to the parton distribution functions of the meson. Since the large momentum arises from the kinematics of the partons inside the meson, the leading order approximation of $T_H(x, u, v, \lambda^2, Q^2)$ is determined by the two gluon exchanges (see Fig. 2). Note that T_H is not dependent on the reference momentum Q_0 and that the Q-dependence will be induced by the renormalization of the LC operator. Furthermore, the meson distribution amplitude $\phi(u, \lambda^2)$ satisfies an evolution equation which controls the logarithmical corrections in λ^2 [3]:

$$\phi(u, \lambda^2) = \sum_{k=0}^{\infty} a_k (1-u) u C_k^{3/2}(2u-1) \left(\frac{\ln(\lambda^2/\Lambda^2)}{\ln(\lambda_0^2/\Lambda^2)} \right)^{\gamma_k^0/\beta_0}, \qquad (16)$$

where $\gamma_k^0 = C_F \left(3 + \frac{2}{(k+1)(k+2)} - 4 \sum_{i=1}^{k+1} \frac{1}{i} \right)$ and $C_k^{3/2}$ are Gegenbauer polynomials, the a_k are nonperturbative coefficients, and $\lambda_0^2 = m^2/(1-x_0)$ is an appropriate reference momentum square.

Now we study the large x behaviour of the parton distribution functions for the scalar $\phi_{[6]}^3$-theory. First we compute the hard amplitude T_H by including the tree approximation of the LC operator vertex, which is given by $\delta(x - k_+/P_+)$. Using the far off shell behaviour of the struck partons [see Eq. (4)] one finds corresponding to Fig. 2 (substitute the gluon lines by scalar lines) the following Feynman integral representation,

$$T_H(x, u, v) \sim \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \frac{1-x_1-x_2-x_3}{(D-B^2)^2} \delta(x - x_1 u - x_2 v - x_3),$$

$$D - B^2 \sim -(1 - x_1 - x_2 - x_3) \left(x_1 \frac{m^2}{1-u} + x_2 \frac{m^2}{1-v} - M_H^2 \right) - [1 - (x_1 + x_2) x_3] m^2, \quad (17)$$

where the mass of the spectator partons was set equal to the free particle mass m. The restriction of the integration region by the δ-function comes from the operator vertex and provides three contributions,

$$T_H(x, u, v) = \theta(u, v < x) T_H^1 + \left\{ \theta(u < x < v) T_H^2 + \{u \leftrightarrow v\} \right\} + \theta(x < u, v) T_H^3,$$

$$T_H^1(x, u, v) = (1-x) \left(\frac{1-x}{1-u} \right) \left(\frac{1-x}{1-v} \right) f_1(x, u, v),$$

$$T_H^2(x, u, v) = (1-x) \left(\frac{1-x}{1-u} f_2(x, u, v) + \frac{v-x}{v-u} \tilde{f}_2(x, u, v) \right),$$

$$T_H^3(x, u, v) = (1-x) f_3(x, u, v), \qquad (18)$$

6

where f_i, $i = 1, 2, 3$, and $\tilde{f}_2$ are smooth functions, which can be more or less replaced by constants.

The convolution (15) provides the $x \to 1$ behaviour; but because of the complicated structure of $T_H(x, u, v)$ in Eq. (18) the result depends on the end point behaviour of the distribution function $\phi(u)$. In the physical case, where the distribution amplitude $\phi(u)$ vanishes at the end points, the leading term comes from the integration region $u, v \leq x \sim 1$, which provides[4]

$$q(x) = \int_0^x du \int_0^x dv \phi^*(u, \lambda^2) T_H^1(x, u, v) \phi(v, \lambda^2) \sim (1-x)^3 \left| \int_0^1 du \frac{\phi(u)}{1-u} \right|^2 \quad \text{for } x \to 1. \quad (19)$$

It is also interesting to investigate the unphysical case, in which the distribution amplitude is constant or divergent at the end points, i.e., $\phi(u) \sim (1-u)^{-\epsilon}$ for $u \to 1$, $\epsilon \geq 0$. Then one gets for $1/2 > \epsilon$ [5] the behaviour

$$q(x) \sim (1-x)^3 |\phi(x)|^2 \sim (1-x)^{(3-2\epsilon)}. \quad (20)$$

Obviously, the obtained power behaviour is different from that given in Eq. (19).

To take into account the evolution in Q we determine $O(x, \mu^2/k^2, \alpha_s(\mu))$ for $x \to 1$ from the RGE and include it in the computation of T_H. For $x \to 1$ the integral kernel is completely determined by the anomalous dimension $\gamma_\phi(\alpha_s)$ of the field ϕ, $P(x; \alpha_s) = (\alpha_s/(2\pi))\gamma_\phi^0 \delta(1-x) + O(\text{const.}, \alpha_s^2)$. Thus, the RGE is simplified to

$$\mu^2 \frac{d}{d\mu^2} O(x, \mu^2/k^2, \alpha_s(\mu)) \Big|_{x \to 1} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_\phi^0 O(x, \mu^2/k^2, \alpha_s(\mu)). \quad (21)$$

The solution of this first order (partial) differential equation can be found in a text book:

$$O(x, \mu^2/k^2, \alpha_s(\mu)) = O(x, 1, \alpha_s(k^2)) \left(\frac{\ln(\mu^2/\Lambda^2)}{\ln(k^2/\Lambda^2)} \right)^{2\gamma_\phi^0/\beta_0},$$

$$= \left(\delta\left(x - \frac{k_+}{P_+}\right) + O(\alpha_s(k^2)) \right) \left(\frac{\ln(\mu^2/\Lambda^2)}{\ln(k^2/\Lambda^2)} \right)^{2\gamma_\phi^0/\beta_0}. \quad (22)$$

Here $O(x, 1, \alpha_s(k^2))$ represents the initial condition, which, assuming $\alpha_s(k^2)$ is small enough, can be expanded perturbatively.

Including the solution (22) in the computation of T_H and setting the renormalization point $\mu = Q$ provides for a physical distribution amplitude:

$$q(x, Q^2) \underset{x \to 1}{\sim} \left(\frac{\ln(Q^2/\Lambda^2)}{\ln(m^2/((1-x)\Lambda^2))} \right)^{2\gamma_\phi^0/\beta_0} (1-x)^3 \left| \int_0^1 du \frac{\phi(u, m^2/(1-x))}{1-u} \right|^2. \quad (23)$$

The comparison with the solution of the GLAP equation shows that the reference momentum square $Q_0^2 = m^2/(1-x)$.

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[4]Because of the space-time dimension six the obtained power behaviour is 3, and it agrees with the counting rule result.

[5]This restriction is necessary to avoid divergencies caused by the convolution.

5 Summary

Using both the definition of the parton distribution functions as forward matrix elements of light-cone operators and the parton model describing the hadronic bound state we have factorized (for $Q^2 > m^2/(1 - x_{Bj})$ and large $m^2/(1 - x_{Bj})$) this distribution functions as a convolution of a hard amplitude and a distribution amplitude. Here the factorization scale $\lambda \sim m^2/(1 - x_{Bj})$ originates from the kinematics of the parton inside the hadron as well as the assumption that the spectator mass does not vanished. Furthermore, we have computed the hard amplitude in the scalar $\phi^3_{[6]}$ toy theory for the distribution function of the "meson" and found that the resulting $(1 - x_{Bj})$ behaviour is governed by the end point behaviour of the distribution amplitude. For a physical distribution amplitude that vanishes at the end points, the obtained $(1 - x_{Bj})$ power behaviour agrees with the counting rule result. In the other case one finds a disagreement.

Moreover, to obtain the evolution in Q we have included the renormalized LC operator in the computation of the hard amplitude. Since the initial condition of the renormalization group equation can be computed perturbatively (this is not possible for the initial condition of the GLAP equation) we have shown that the reference momentum square Q_0^2 is given by the x_{Bj} dependent factorization scale, i.e., by $m^2/(1 - x_{Bj})$.

Note that the use of this treatment for QCD is straight forward. It will be interesting to investigate the question of the $(1 - x_{Bj})$ power behaviour on the dependence of the distribution amplitude and the effects of higher twist terms. It is to be expected that the value $Q_0^2 \sim m^2/(1 - x_{Bj})$ will be the same in QCD, so that the power behaviour of the valence quark distribution functions will be changed [see Eq. (13)].

6 Acknowledgments

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