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Light-Ray Operators and their Application in QCD

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Abstract

The nonperturbative parton distribution and wave functions are directly related to matrix elements of light-ray (nonlocal) operators. These operators are generalizations of the standard local operators known from the operator product expansion. The renormalization group equation for these operators leads to evolution equations for more general distribution amplitudes which include the Altarelli-Parisi and the Brodsky-Lepage equations as special cases. It is possible to derive the Altarelli-Parisi kernel as a limiting case of the extended Brodsky-Lepage kernel. As new application of the operator product expansion the virtual Compton scattering near forward direction is considered.

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1 Light-Ray Operators in QCD

It is generally accepted that nonlocal composite operators are necessary to describe hadrons in QCD. For example a meson operator $M(x_1, x_2)$ could be represented by quark-antiquark fields connected by a path-ordered phase factor $U(x_2, x_1)$:

$$M(x_1, x_2) = \overline{\psi}(x_2)\Gamma U(x_2, x_1)\psi(x_1) \tag{1}$$

with

$$U(x_2, x_1) = Pexp(-ig\int_{x_1}^{x_2} dx^\mu A_\mu(x)); \tag{2}$$

where Γ abbreviates γ- and λ-matrices describing the spin and flavor structure of the meson. $A^\mu = A^\mu_a t^a$ denotes the gluon field, and the t^a are the generators of the colour group. For large internal momenta the main contribution comes from those operators for which the quark fields and the path-ordered phase factor lie on a light-like straight line $(x_1 - x_2)^2 = 0$. Such operators are known from the Operator Product Expansions [1]. They are highly singular and posses new properties. For simplicity let us write the corresponding Light-Cone Expansion for a toy model with scalar fields ϕ and scalar currents j:

$$Tj(x)j(0) = \sum_{n_1,n_2} C_{n_1,n_2} O_{n_1,n_2} + ... \quad , \tag{3}$$

where the $C_{n_1,n_2}(x^2)$ are singular coefficient functions, and O_{n_1,n_2} are the operators

$$O_{n_1,n_2} =: [(\tilde{x}\partial_x)^{n_1}\phi(x)][(\tilde{x}\partial_x)^{n_2}\phi(x)]|_{x=0} : \quad , \tag{4}$$

and $\tilde{x}$ is a light-like vector that approaches the vector x. This local light-cone expansion can be summed up to a nonlocal integral representation

$$Tj(x)j(0) = \int_0^1 d\kappa_1 d\kappa_2 F(x^2, \kappa_1, \kappa_2) O(\kappa_1, \kappa_2) + ..., \tag{5}$$

where

$$\begin{aligned} O(\kappa_1, \kappa_2) &= \ :\phi(\kappa_1\tilde{x})\phi(\kappa_2\tilde{x}): & (6)\\ &= \sum_{n_1,n_2} \frac{\kappa_1^{n_1}}{n_1!}\frac{\kappa_2^{n_1}}{n_2!} O_{n_1,n_2} & (7) \end{aligned}$$

and

$$C_{n_1,n_2}(x^2) = \frac{1}{n_1!n_2!}\int_0^1 d\kappa_1 d\kappa_2 F(x^2, \kappa_1, \kappa_2)\kappa_1^{n_1}\kappa_2^{n_1}. \tag{8}$$

Such a nonlocal light-ray operator product expansion has been independently introduced by [2] and [3] from different points of view. The operators $O(\kappa_1, \kappa_2)$ reflect the renormalization properties of all summed up local operators. The support of this operator is a light-like straight line which introduces new singularities and is responsible for the changed renormalization properties. The operator $O(\kappa_1, \kappa_2)$ depends on two parameters κ_1 and κ_2 so that in a multiplicative renormalization scheme the Z-factors and the anomalous dimensions depend on two initial and two final parameters, i.e.,

$$O(\kappa_1,\kappa_2)^{ren} = \int_0^1 d\kappa_1' d\kappa_2' Z(\kappa_1,\kappa_2;\kappa_1',\kappa_2') O(\kappa_1',\kappa_2')^{un}, \tag{9}$$

$$\gamma(\kappa_1,\kappa_2;\kappa_1',\kappa_2') = \frac{1}{2}\int_0^1 d\kappa_1'' d\kappa_2'' \left(\mu\frac{d}{d\mu} Z(\kappa_1,\kappa_2;\kappa_1'',\kappa_2'')\right) Z^{-1}(\kappa_1'',\kappa_2'';\kappa_1',\kappa_2'), \tag{10}$$

where μ denotes the renormalization point. It is possible to derive the standard local anomalous dimensions from the nonlocal anomalous dimension.
In a very formal way we may write the renormalization group equation for the operator $O(\kappa_1,\kappa_2)^{ren}(\mu) = RTO(\kappa_1,\kappa_2) exp\{iS_I\}$ according to

$$\mu\frac{d}{d\mu} RTO(\underline{\kappa}) e^{iS_I} = \int d^2\underline{\kappa}' \left(\gamma(\underline{\kappa},\underline{\kappa}';g(\mu)) - 2\gamma_\psi(g(\mu))\delta^{(2)}(\underline{\kappa}-\underline{\kappa}')\right) RTO(\underline{\kappa}') e^{iS_I}, \tag{11}$$

where R denotes the renormalization procedure, T the time ordering, and S_I is the interacting part of the action. The new variables are $\kappa_\pm = (\kappa_2 \pm \kappa_1)/2$. For convenience we split the anomalous dimension of the operator into the anomalous dimension of the 1PI vertex function (with two external momenta) $\gamma(\underline{\kappa},\underline{\kappa}') = \gamma(\kappa_+,\kappa_-,\kappa_+',\kappa_-')$ and a part which is proportional to the anomalous dimension of the quark field γ_ψ. As short notations we use $d^2\underline{\kappa}' = d\kappa_+' d\kappa_-'$, and $\delta^{(2)}(\underline{\kappa}-\underline{\kappa}') = \delta(\kappa_+ - \kappa_+')\delta(\kappa_- - \kappa_-')$.

2 Generalized Distribution Functions and Evolution Equations

Before defining generalized distribution functions we repeat the definitions of the well-known meson wave function and the parton distribution function in QCD.

The meson wave function $\Phi^a(x,Q^2)$ which depends on the distribution parameter x ($0 \leq x \leq 1$) can be defined as the expectation value of a nonlocal (light-ray) operator lying on the light-cone [4, 5, 6]:

$$\Phi^a(x=\frac{1+t}{2},Q^2) = \int \frac{d\kappa_-(\tilde{n}P)}{2\pi(\tilde{n}P)} e^{i\kappa_-(\tilde{n}P)t} < 0|RTO^a(\kappa_-;\tilde{n}) e^{iS_I}|P> |_{\mu^2=Q^2}. \tag{2.1}$$

Here $|P>$ denotes the one-particle state of a scalar meson of momentum P, and

$$O^a(\kappa_-;\tilde{n}) = :\bar{\psi}(-\kappa_-\tilde{n})(\tilde{n}\gamma)\lambda^a U(-\kappa_-\tilde{n},\kappa_-\tilde{n})\psi(\kappa_-\tilde{n}): \tag{2.2}$$

is the light-ray operator with the same flavor content. As the renormalization point we choose the typical large momentum Q of the basic process to which the wave function contributes.

The introduced factor $1/(\tilde{n}P)$ compensates the $\tilde{n}$-dependence of the factor $\tilde{n}\gamma$ in (2.2). The vector $\tilde{n}$ with $\tilde{n}^2 = 0$ defines the light-ray pointing in the direction of the large momentum flow of the process. The path ordered phase factor is taken along the straight line with direction $\tilde{n}$, and λ^a is a generator of the flavor group corresponding to the considered meson.

Analogously we define the quark distribution function $q^a(z, Q^2)$ [7] with the distribution parameter z. For simplicity we consider the flavor nonsinglet distribution only defined by

$$q^a(z, Q^2) = \int \frac{d\kappa_-(\tilde{n}P)}{2\pi(\tilde{n}P)} \, e^{2i\kappa_-(\tilde{n}P)z} < P|RTO^a(\kappa_-; \tilde{n})e^{iS_I}|P > |_{\mu^2=Q^2}. \tag{2.3}$$

If we choose the index a so that the matrix λ^a is diagonal, then for a positive resp. a negative distribution parameter z this function represents a linear combination of the quark resp. antiquark distribution functions [7].

The functions in (2.1) and (2.3) are used for different physical processes and have different interpretations. Nevertheless it seems to be natural to introduce the more general distribution amplitude

$$\begin{aligned} q^a(t, \tau, \mu^2) &= \\ &\int \frac{d\kappa_-(\tilde{n}P_+)}{2\pi(\tilde{n}P_+)} \, e^{i\kappa_-(\tilde{n}P_+)t} < P_2|RTO^a(\kappa_-; \tilde{n})e^{iS_I}|P_1 > |_{\tilde{n}P_-=\tau\tilde{n}P_+}, \end{aligned} \tag{2.4}$$

with $P_\pm = P_2 \pm P_1$. This function depends on the distribution parameter t and the quotient $\tau = \tilde{n}P_-/\tilde{n}P_+$ of the projection of momenta onto a light-like direction $\tilde{n}$, the renormalization point μ, and the scalar products of the external moments P_iP_j. For physical states $|P_1>$ and $|P_2>$ the additional variable τ is restricted by

$$|\tau| = \left|\frac{\tilde{n}P_-}{\tilde{n}P_+}\right| = \left|\frac{P_-^0 - P_-^\parallel}{P_+^0 - P_+^\parallel}\right| \leq 1, \quad P^\parallel = \frac{\vec{\tilde{n}}\vec{P}}{|\vec{\tilde{n}}|}. \tag{2.5}$$

With the help of the α-representation it is possible to investigate the support properties of the function $q^a(t, \tau, \mu^2)$ with respect to the variable t. It turns out that $q^a(t, \tau, Q^2) = 0$ for $|t| > 1$. This distribution amplitude is of relevance for e.g., the description of nonforward processes near the forward case, where the parton picture with two distribution functions cannot be applied. On the other hand, it is possible to apply such a function in the limit $P_2 \to 0$ too. Of course, then this state is not the vacuum state, but for the mathematical aspects of the connection of evolution equations for forward and nonforward processes this is very useful. In the limit of forward scattering we reach the "forward distribution amplitude". This amplitude is real for $P_1 = P_2$, and for this reason it simultaneously plays the role of the parton distribution function in deep inelastic scattering (For the virtual Compton scattering amplitude approximated by a light-cone expansion the formation of its absorptive part is nontrivial for the hard scattering part only.). So we can perform two limits:

$$\begin{aligned} \Phi^a(x = (1+t)/2, Q^2) &= \lim_{\tau \to -1} q^a(t, \tau, Q^2) \qquad \text{meson wave function,} \\ q^a(z, Q^2) &= \lim_{\tau \to 0} q^a(z, \tau, Q^2) \qquad \text{quark distribution function.} \end{aligned} \tag{2.6}$$

Next we need evolution equations valid for the distribution amplitude (2.4). As input we can use the renormalization group equations (11) for the light-ray operators (2.2). As special cases we should find the Brodsky-Lepage (BL) and the Altarelli-Parisi (AP) equations.

For the derivation of the evolution equation we differentiate the general distribution amplitude $q^a(t,\tau,\mu^2)$ with respect to the renormalization parameter. Thereby we take into account its representation in terms of matrix elements of light-ray operator (2.4). The differentiation of this operator can be performed with the help of its renormalization group equation (11). A straightforward calculation starts from

$$\mu\frac{d}{d\mu}q^a(t,\tau,\mu^2) = \tag{2.7}$$
$$\int\frac{d\kappa_-(\tilde{n}P_+)}{2\pi(\tilde{n}P_+)}\, e^{i\kappa_-(\tilde{n}P_+)t}\mu\frac{d}{d\mu} < P_2|RTO^a(\kappa_+=0,\kappa_-;\tilde{n})e^{iS_I}|P_1>|_{\tilde{n}P_-=\tau(\tilde{n}P_+)}.$$

We obtain the evolution equation

$$Q^2\frac{d}{dQ^2}q^a(t,\tau,Q^2) = \int_{-1}^{1}\frac{dt'}{|2\tau|}\left(\gamma\left(\frac{t}{\tau},\frac{t'}{\tau}\right) - 2\gamma_\psi\delta\left(\frac{t}{\tau}-\frac{t'}{\tau}\right)\right)q^a(t',\tau,Q^2) \tag{2.8}$$

with the evolution kernel

$$\gamma(t,t') = \int dw_-\ \gamma(w_+ = t'w_- - t, w_-), \tag{2.9}$$

$$w_+ = \frac{\kappa'_+ - \kappa_+}{\kappa_-},\quad w_- = \frac{\kappa'_-}{\kappa_-}, \tag{2.10}$$

where $\gamma(w_+,w_-)$ is directly related to the original anomalous dimension $\gamma(\underline{\kappa},\underline{\kappa}')$. Using translation and scale invariance we get

$$\gamma(\kappa_+,\kappa_-;\kappa'_+,\kappa'_-) = \frac{1}{\kappa_-^2}\gamma(0,1,\frac{\kappa'_+ - \kappa_+}{\kappa_-},\frac{\kappa'_-}{\kappa_-}) = \frac{1}{\kappa_-^2}\gamma(w_+,w_-). \tag{2.11}$$

This property is essential for reducing the two-variable renormalization group equation (11) to a one-variable evolution equation (2.8). From the renormalization group invariance of $\int dt\ q^a(t,\tau,\mu^2)$ it follows that

$$\int dt\ \gamma(t,t') = 2\gamma_\psi, \tag{2.12}$$

so that the standard "+"-definition for the generalized function

$$[\gamma(t,t')]_+ = \gamma(t,t') - \delta(t-t')\int dt''\ \gamma(t'',t') = \gamma(t,t') - 2\gamma_\psi\delta(t-t') \tag{2.13}$$

arises in a very natural way. We see that instead of the original anomalous dimension there appears an evolution kernel that contains the original anomalous dimension as an essential input. It seems to be natural to denote the general evolution kernel as an extended BL-kernel. The reason for this is the following: restricting $\gamma(t,t')$ to the parameter region $|t|,|t'|\leq 1$, it coincides with the evolution kernel for the hadron wave function, but eqs. (2.9) and (2.11) provide us with a meaningful definition outside this region.

In this way, we obtain evolution equations for all forward and nonforward matrix elements. To our knowledge, in the literature up to now only two examples have been investigated, the case of forward scattering and the case of the meson wave function. Here, both cases are contained as limits $P_2 \to P_1$ or $P_2 \to 0$. We write these limits down in a short form:

- Evolution equation for the quark distribution function (forward scattering) with the AP-kernel $P(z/z')$

$$\begin{aligned} q^a(z,Q^2) &= \lim_{\tau\to 0} q^a(z,\tau,Q^2), \\ Q^2\frac{d}{dQ^2}q^a(z,Q^2) &= \int_{-1}^{1}\frac{dz'}{|z'|}\,P\Big(\frac{z}{z'};\alpha_s(Q^2)\Big)\,q^a(z',Q^2) \qquad (2.14)\\ |z'|^{-1}P\Big(\frac{z}{z'}\Big) &= \lim_{\tau\to 0}\frac{1}{|2\tau|}\left[\gamma\Big(\frac{z}{\tau},\frac{z'}{\tau}\Big)\right]_+ \qquad (2.15)\end{aligned}$$

- Evolution equation for meson wave functions with the BL-kernel $V_{BL}(x,y)$

$$\begin{aligned} \Phi^a(x=(1+t)/2,Q^2) &= \lim_{\tau\to -1} q^a(t,\tau,Q^2), \\ Q^2\frac{d}{dQ^2}\Phi^a(x,Q^2) &= \int_0^1 dy\; V_{BL}(x,y;\alpha_s(Q^2))\Phi^a(y,Q^2). \qquad (2.16)\\ V_{BL}(x,y) &= [\gamma(2x-1,2y-1)]_+|_{0\le x,y\le 1}, \qquad (2.17)\end{aligned}$$

  where $\alpha_s(Q^2)$ denotes the running coupling constant. Here, the variables $x=(1+t)/2$ and $y=(1+t')/2$ are restricted to $0\le x,y\le 1$. Note, however, that in fact the quantum numbers must change discontinuously, so that the limit is only formal.

Note that both kernels (2.15) and (2.17) are calculated separately in QCD. From the above considerations it is obvious, however, that they have a common origin, namely the anomalous dimension $\gamma(w_+,w_-)$.

3 The Extended BL-Kernel, Relations between the BL- and AP-Kernels

We have obtained a new evolution kernel (extended BL-kernel) for distribution amplitudes containing the (restricted) BL-kernel and the AP-kernel as special cases. The standard perturbative calculations of the BL-kernel define it as an evolution kernel for a restricted range of the variables. It turns out that this kernel already contains the essential information and that an explicit continuation procedure can be prescribed.

The first question is: What is the region in the (t,t')-plane where the kernel $\gamma(t,t')$ is defined? After some algebra we obtain the following representation in the (t,t')-plane [8]:

$$\begin{aligned}\gamma(t,t') = &[\theta(t-t')\theta(1-t)-\theta(t'-t)\theta(t-1)]f(t,t') \qquad (3.1)\\ &+[\theta(t'-t)\theta(1+t)-\theta(t-t')\theta(-t-1)]f(-t,-t')\\ &+[\theta(-t-t')\theta(1+t)-\theta(t'+t)\theta(-t-1)]g(-t,t')\\ &+[\theta(t+t')\theta(1-t)-\theta(-t'-t)\theta(t-1)]g(t,-t'),\end{aligned}$$

where the functions $f(t,t')$ and $g(t,t')$ are given by

$$\begin{aligned} f(t,t') &= \int_0^{\frac{1-t}{1-t'}} dw_-\;\gamma(w_+=t-w_-t',w_-),\\ g(-t,t') &= \int_0^{\frac{1+t}{1-t'}} dw_-\;\gamma(w_+=-t-w_-t',-w_-). \qquad (3.2)\end{aligned}$$

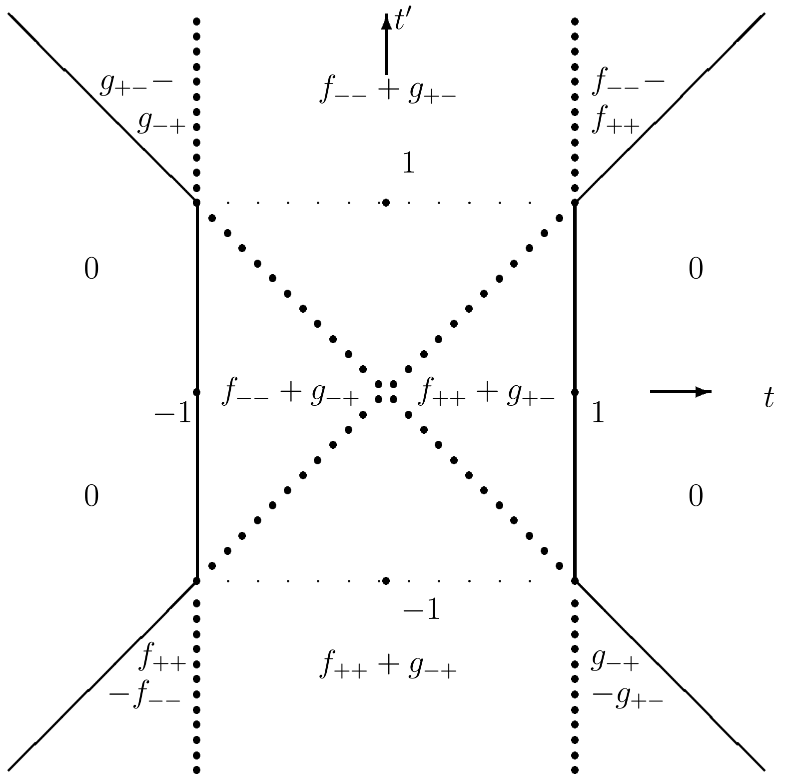

Figure 1: Support of $\gamma(t,t')$, where $f_{\pm\pm} = f(\pm t, \pm t')$, and $g_{\pm\mp} = g(\pm t, \mp t')$ are defined by eq. (3.1).

Note, that for $|t|, |t'| > 1$, because of support restrictions, the lower boundary in the integral representations (3.2) is not attained. However, in these regions only the differences $f(t,t') - f(-t,-t')$ and $g(-t,t') - g(t,-t')$ appear, so that such undetermined contributions eliminate each other. It is clear, therefore, that the kernel is defined in the complete (t,t')-plane as it can be seen in Fig. 1.

In one-loop approximation the extended BL-kernel reads

$$\begin{aligned} V_{BL}^{ext}(x,y) &= \gamma_0(t = 2x-1, t' = 2y-1) = \frac{\alpha_s}{2\pi} c_F [V_0(x,y)]_+ \\ V_0(x,y) &= \theta\left(1 - \frac{x}{y}\right) \theta\left(\frac{x}{y}\right) \mathrm{sign}(y) \frac{x}{y} \left(1 - \frac{1}{x-y}\right) + \left\{ \begin{matrix} x \to \overline{x} \\ y \to \overline{y} \end{matrix} \right\}. \end{aligned} \tag{3.3}$$

(c_F is the known group theoretic constant). This result can be obtained by an extension procedure or by direct calculation of $\gamma_0(t,t')$.

As an interesting example we shall determine the AP-kernel from the known extended BL-kernel. According to eq. (2.15) it is just this new region ($|t|, |t'| > 1$) which is essential for the determination of the AP-kernel. We first discuss the θ-structure and then consider the limiting process for the coefficient functions contained in eq. (3.3). The limit $\tau \to 0$, with $x = z/\tau$ and $y = 1/\tau$, for the θ-functions are

$$\left.\begin{array}{c}\theta(\frac{x}{y})\,\theta(1-\frac{x}{y})\\ \theta(\frac{1-x}{1-y})\,\theta(1-\frac{1-x}{1-y})\end{array}\right\} \quad \rightarrow \quad \theta(z)\,\theta(1-z), \tag{3.4}$$

$$\left.\begin{array}{c}\theta(\frac{1-x}{y})\,\theta(1-\frac{1-x}{y})\\ \theta(\frac{x}{1-y})\,\theta(1-\frac{x}{1-y})\end{array}\right\} \quad \rightarrow \quad \theta(-z)\,\theta(1+z). \tag{3.5}$$

Related θ-structures (structures that turn into each other under $x \leftrightarrow \overline{x} = 1-x$ and $y \leftrightarrow \overline{y} = 1-y$) have the same limit. Putting it all together we obtain an expression for the AP-kernel which follows from the extended BL-kernel (2.15), (3.3), and (3.4):

$$P(z) = \frac{\alpha_s}{2\pi} c_F [P_0(z)]_+ \tag{3.6}$$

and

$$\begin{aligned} P_0(z) &= \lim_{\tau \to 0} |\tau|^{-1} V_0 \left(\frac{z}{\tau}, \frac{1}{\tau} \right) \\ &= \theta(z)\,\theta(1-z) \frac{1+z^2}{1-z}. \end{aligned} \tag{3.7}$$

The same procedure can be performed for the two-loop contributions to the BL-kernel. The calculations are much more complicated. The result obtained for the AP-kernel [8] is equivalent to that of [9]. It also includes in a natural way the second order contributions stemming from the internal anti-quark lines. This constitutes a consistency check of the calculation of the diagonal local anomalous dimensions. The nondiagonal elements of the anomalous dimension matrix have been controlled by exploiting conformal symmetry breaking [10]. Note, that our regularization prescription for the extended BL-kernel, after performing the limiting process, turns automatically into the well accepted regularization prescription for the AP-kernel.

4 Evolution Equations and Structure Functions for Two-Photon Processes

Here we consider the virtual nonforward Compton scattering amplitude of a spinless particle (near forward direction) at a large off-shell behavior of the incoming photon. The considered region is a generalized Bjorken region, where (as in the case of deep inelastic scattering) this process is dominated by contributions from the light-cone. The kinematics of the process are the following:

$$\gamma^*(q_1) + \mathrm{H}(p_1) = \gamma^*(q_2) + \mathrm{H}(p_2), \tag{4.1}$$

$$\begin{aligned} P &= p_1 + p_2 = (2E_p, \vec{0}), \qquad q = (1/2)(q_1 + q_2), \\ \Delta &= (p_2 - p_1) = (q_1 - q_2) = (0, -2\vec{p}), \quad E_p = \sqrt{(m^2 + \vec{p}^2)}. \end{aligned} \tag{4.2}$$

The last notations are the values of the momenta in the Breit frame. The generalized Bjorken region is given by

$$\nu = Pq = 2E_p q_0 \to \infty, \qquad Q^2 = -q^2 \to \infty, \tag{4.3}$$

with the scaling variables

$$\xi \;=\; \frac{-q^2}{Pq}, \qquad \eta = \frac{\Delta q}{Pq} = \frac{q_1^2 - q_2^2}{2\nu} = \frac{|\vec{p}|}{E_p}\cos\phi. \tag{4.4}$$

To understand this process better we introduce the angle between the vectors $\vec{p}$ and $\vec{q}$ in the Breit frame by $\cos\phi = \vec{p}\vec{q}/(|\vec{p}||\vec{q}|)$. In terms of these variables we get

$$q_1^2 = -\left(\xi - \frac{|\vec{p}|}{E_p}\cos\phi\right)\nu, \qquad q_2^2 = -\left(\xi + \frac{|\vec{p}|}{E_p}\cos\phi\right)\nu. \tag{4.5}$$

In contrast to deep inelastic scattering the variable ξ is not restricted to $0 \le \xi \le 1$. For example $q_2^2 = 0$ demands $\xi = -(|\vec{p}|/E_p)\cos\phi$. It may be shown that in this region the helicity amplitudes $T(\lambda', \lambda) = \varepsilon_2^{\mu}(\lambda')T_{\mu\nu} \times \varepsilon_1^{\nu}(\lambda)$ are given by $(1/2)\varepsilon_2(\lambda')\varepsilon_1(\lambda)T_{\mu}^{\mu}$ for the transverse helicities and vanish otherwise. Therefore, only the trace of the scattering amplitude

$$T_{\mu\nu}(P, \Delta, q) = i \int d^4x\, e^{iqx} \; < P_2|T\left(J_\mu\left(\frac{x}{2}\right) J_\nu\left(\frac{-x}{2}\right)\right)|P_1 > \tag{4.6}$$

has to be considered; ($J_\mu(x) = (1/2) : \overline{\psi}(x)\gamma_\mu(\lambda^3 - \lambda^8/\sqrt{3})\psi(x)$: is the electromagnetic current of the hadrons (for flavour $SU(3)$).

In our special case the light-ray operator product expansion contains in leading order only the following quark operator

$$O^a(\tilde{x}, \kappa_1, \kappa_2) =: \overline{\psi}(\kappa_1\tilde{x})(\tilde{x}\gamma)U(\kappa_1\tilde{x}, \kappa_2\tilde{x})\psi(\kappa_2\tilde{x}) : . \tag{4.7}$$

The singular coefficient functions F_a are determined perturbatively; with the Born approximation of the coefficient functions we obtain finally

$$T_{\mu}^{\mu}(P, \Delta, q) \approx 2\int_{-1}^{1} dt \left(\frac{1}{\xi + t} - \frac{1}{\xi - t}\right) e_a q_C^a(t, \eta; \mu^2 = Q^2), \tag{4.8}$$

where $e_a = (2/9)\delta_{a0} + (1/6)\delta_{a3} + (1/6\sqrt{3})\delta_{a8}$, and the distribution amplitudes

$$q_C^a(t, \tau; \mu^2) = \int \frac{d(\kappa_-\tilde{x}P)}{2\pi\tilde{x}P}\, e^{i\kappa_-\tilde{x}P\,t} < P_2|O^a(\tilde{x}, \kappa_-, \kappa_+ = 0)_{\mu^2}|P_1 >_{|\tilde{x}\Delta = \tau\tilde{x}P} \tag{4.9}$$

contain the long range behaviour of the process (which is related to the hadron states $|P_i>$). This amplitude generalizes the parton distribution function of deep inelastic scattering. It has the following properties:

- the support of $q_C^a(t, \tau, Q^2)$ is restricted by $|t| \le 1$ and $|\tau| \le 1$,
- $q^a(t, Q^2) = q_C^a(t, \tau = 0, Q^2)$ is essentially the parton distribution function used in deep inelastic scattering,
- $q_C^a(t, \tau = 0, Q^2)$ is real.

This generalized distribution function satisfies our new evolution equation Eq. (2.8). This conference contribution collects results of the article [11].

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