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Testing Violation of CPT and Quantum Mechanics in the K_0–$\bar{K}_0$ System

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ABSTRACT

I report a recent study[1)]made in collaboration with M.E. Peskin, on the time dependence of a kaon beam propagating according to a generalization of quantum mechanics due to Ellis, Hagelin, Nanopoulos and Srednicki, in which *CP*- and *CPT*-violating signatures arise from the evolution of pure states to mixed states. Constraints on the magnitude of its parameters are established on the basis of existing experimental data. New facilities such as ϕ factories are shown to be particularly adequate to study this generalization from quantum mechanics and to disentangle its parameters from other *CPT* violating perturbations of the kaon system.

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1. Introduction

Developments in the quantum theory of gravity have led S. Hawking to propose a generalization of quantum mechanics which allows the evolution of pure states to mixed states.[3] This formulation was shown by D. Page to be in conflict with CPT conservation.[4] Ellis, Hagelin, Nanopoulos, and Srednicki (EHNS)[5] were first to observe that systems which exhibit quantum coherence over a macroscopic distance are most appropriate to probe the violation of quantum mechanics of the type proposed by S. Hawking. One of the simplest system exhibiting this property is a beam of neutral kaons. EHNS set up a generalized evolution equation for the K_0–$\bar{K}_0$ system in the space of density matrices which contains three new CPT violating parameters α,β and γ. These parameters have dimension of mass and could be as large as $m_K^2/m_{\rm Pl} \sim 10^{-19}$ GeV.

Recently, Ellis, Mavromatos, and Nanopoulos (EMN)[6,7] reconsidered this analysis. Exploiting experimental data on K_L and K_S, they presented an allowed region in the space of the parameters α, β, and γ. This region is compatible with the expected order of magnitude above and with the possibility that violation of quantum mechanics accounts for all CP violation observed in the K_0–$\bar{K}_0$ system. However, as these authors were aware, this analysis does not include data from the intermediate time region of the kaon beam. This time region has proven to be the main source of constraints on quantum mechanical CPT violating perturbations.[10]

The present talk is based on a recent work[1] done in collaboration with M.E. Peskin, which develops a general parameterization to incorporate CPT violation from both within and outside quantum mechanics and uses it to analyze past, present and future experiments on the K_0–$\bar{K}_0$ system. Studies of the time dependence of the kaon system of the early 1970's are combined with recent results from CPLEAR to constrain the EHNS parameters β and γ, limiting the contribution of violation of quantum mechanics to no more than 10% of the CP violation observed in the K_0–$\bar{K}_0$ system.

ϕ factories are new facilities[12] dedicated to the study of the properties of the kaon system; they are expected to give particularly incisive tests of CPT violation.[10] It is shown that these future facilities are especially suitable to test violation of quantum mechanics and to disentangle the EHNS parameters from other CPT violating perturbations.

2. general formalism

2.1 QUANTUM-MECHANICAL EVOLUTION

In this section we describe the quantum mechanical time evolution of a beam of kaon and generalize it to allow a pure state to evolve to a mixed state.

Let $\rho_K(\tau)$ be the density matrix of a kaon beam at proper time τ. Its value at the source is fixed by the production mechanism. Its values along the beam are then obtained from an effective hamiltonian $H = M - \frac{i}{2}\Gamma$, which incorporates the natural width of the system, and the laws of quantum mechanics

$$i\frac{d}{d\tau}\rho_K = H\,\rho_K - \rho_K\,H^\dagger. \tag{2.1}$$

The solution of (2.1) is generally expressed in terms of the properties of K_L and K_S as

$$\rho_K(\tau) \;=\; A_L\rho_L^{(\diamond)}e^{-\Gamma_L\tau} + A_S\rho_S^{(\diamond)}e^{-\Gamma_S\tau} + A_I\rho_I^{(\diamond)}e^{-\bar{\Gamma}\tau}e^{-i\Delta m\tau} + A_{\bar{I}}\rho_{\bar{I}}^{(\diamond)}e^{-\bar{\Gamma}\tau}e^{+i\Delta m\tau}. \tag{2.2}$$

The coefficients $A_{L,S}$ and $A_{I,\bar{I}}$ describe the initial conditions of the beam and we defined

$$\bar{\Gamma} = \frac{\Gamma_L + \Gamma_S}{2}\,, \qquad \Delta\Gamma = \Gamma_S - \Gamma_L \qquad \text{and} \qquad \Delta m = m_L - m_S\,. \tag{2.3}$$

For later purpose, it is convenient to introduce the complex quantity

$$\begin{aligned} d &= \Delta m + \frac{i}{2}\Delta\Gamma = \big((3.522 \pm 0.016) + i(3.682 \pm 0.008)\big) \times 10^{-15}\text{GeV} \\ d &= |d|e^{i(\pi/2-\phi_{SW})} \qquad \phi_{SW} = (43.73 \pm 0.15)^\circ \end{aligned}\quad . \tag{2.4}$$

The density matrices $\rho_L^{(\diamond)}$, $\rho_S^{(\diamond)}$ and $\rho_I^{(\diamond)}$, $\rho_{\bar{I}}^{(\diamond)}$ are expressible in terms of the pure states $|K_L\rangle$ and $|K_S\rangle$ as follows

$$\begin{aligned} \rho_L^{(\diamond)} &= |K_L\rangle\langle K_L| \qquad & \rho_I^{(\diamond)} &= |K_S\rangle\langle K_L| \\ \rho_S^{(\diamond)} &= |K_S\rangle\langle K_S| \qquad & \rho_{\bar{I}}^{(\diamond)} &= |K_L\rangle\langle K_S|\,. \end{aligned} \tag{2.5}$$

The state $|K_{L(S)}\rangle$ is the sum of the CP *even(odd)* state $|K_{1(2)}\rangle$ and a small CP *odd(even)* component proportional to the state $|K_{2(1)}\rangle$

$$\begin{aligned} |K_S\rangle &= N_S\big(|K_1\rangle + \epsilon_S\,|K_2\rangle\big) \\ |K_L\rangle &= N_L\big(|K_2\rangle + \epsilon_L\,|K_1\rangle\big) \end{aligned} \tag{2.6}$$

where $\epsilon_S = \epsilon_M + \Delta$ and $\epsilon_L = \epsilon_M - \Delta$. The parameter* ϵ_M is odd under CP but even under CPT while the parameter Δ is odd under both CP and CPT. N_S, N_L are real, positive normalization factors.

Any observable of the kaon beam can be computed by tracing ρ_K with an appropriate operator $\mathcal{P}$, we write

$$\langle \mathcal{P}\rangle = \mathrm{tr}\big[\rho_K \mathcal{O}_{\mathcal{P}}\big]. \tag{2.7}$$

This expression for expectation values will remain true in the generalization of quantum mechanics described below. The most relevant observables are the semi leptonic decays $K \to \pi^{\pm}\ell^{\mp}\nu$ and the charged and neutral pion decay $K \to \pi^+\pi^-$, $\pi^0\pi^0$. The operators for the semileptonic decays are

$$\mathcal{O}_{\ell^\pm} = |a|^2\,|K_0\rangle\langle K_0| = \frac{|a\pm b|^2}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}, \tag{2.8}$$

and for the 2 pion decays, they are

$$\mathcal{O}_{+-} = |X_{+-}|^2\begin{pmatrix} 1 & Y_{+-} \\ Y_{+-}^* & |Y_{+-}|^2 \end{pmatrix}, \qquad \mathcal{O}_{00} = |X_{00}|^2\begin{pmatrix} 1 & Y_{00} \\ Y_{00}^* & |Y_{00}|^2 \end{pmatrix}, \tag{2.9}$$

$\star$ We use the notation of ref. 17.

where

$$X = \left\langle \pi\pi \,\middle|\, K_1 \right\rangle \;, \qquad Y = \frac{\langle \pi\pi \mid K_2 \rangle}{\langle \pi\pi \mid K_1 \rangle}. \tag{2.10}$$

More explicitly,[1]

$$\begin{aligned} Y_{+-} &= (\frac{\mathrm{Re} B_0}{A_0}) + \epsilon' \\ Y_{00} &= (\frac{\mathrm{Re} B_0}{A_0}) - 2\epsilon' \end{aligned} \tag{2.11}$$

The quantities $\mathrm{Re}(B_0/A_0)$ and $\mathrm{Re}(b/a)$ parameterize CPT-violating decay amplitudes[10] while ϵ' is the standard CP violating parameter in the pion decay. To illustrate how to use these operators, we compute the charged pion decay rate at large time $\tau \gg 1/\Gamma_S$ in the evolution of the beam

$$\begin{aligned} \Gamma(K_L \to \pi^+\pi^-) &\propto \mathrm{tr}\, \rho_L \mathcal{O}_{+-} \\ &\propto |\epsilon_L|^2 + |Y_{+-}|^2 + 2\mathrm{Re}\, Y_{+-}\epsilon_L^* \;, \\ &\propto |\eta_{+-}^{(\diamond)}|^2 \end{aligned} \tag{2.12}$$

where $\eta_{+-}^{(\diamond)}$ is the complex number $\epsilon_L + Y_{+-} = \epsilon_L + \mathrm{Re}(B_0/A_0) + \epsilon'$.

2.2 Generalized time evolution

Violation of quantum mechanics is clearly a small perturbation of the formalism described above. EHNS proposed to account for the loss of coherence in the evolution of the beam by adding terms on the RHS of (2.1) which preserves the linearity of the time-evolution

$$i \frac{d}{d\tau} \rho_K = H \, \rho_K - \rho_K \, H^\dagger \, + \, \delta\not{h}\, \rho_K. \tag{2.13}$$

Requirements that such terms do not break conservation of probability and do not decrease the entropy of the system make $\delta\not{h}$ expressible in terms of six parameters. In order to lower this number to a more tractable one, EHNS further assume that these terms conserve strangeness, reducing $\delta\not{h}$ to three unknown parameters α, β and γ.

The solution of (2.13) has the general form given in (2.2) but with the eigenmodes ρ_L, ρ_S and ρ_I changed to,[†] in first order in small quantities

$$\rho_L = \rho_L^{(\diamond)} + \frac{\gamma}{\Delta\Gamma}\rho_S^{(\diamond)} + \frac{\beta}{d}\rho_I^{(\diamond)} + \frac{\beta}{d^\star}\rho_{\bar{I}}^{(\diamond)} \tag{2.14}$$

$$\rho_S = \rho_S^{(\diamond)} - \frac{\gamma}{\Delta\Gamma}\rho_L^{(\diamond)} - \frac{\beta}{d^\star}\rho_I^{(\diamond)} - \frac{\beta}{d}\rho_{\bar{I}}^{(\diamond)} \tag{2.15}$$

$$\rho_I = \rho_I^{(\diamond)} - \frac{\beta}{d^\star}\rho_S^{(\diamond)} + \frac{\beta}{d}\rho_L^{(\diamond)} - \frac{i\alpha}{2\Delta m}\rho_{\bar{I}}^{(\diamond)} \tag{2.16}$$

$$\rho_{\bar{I}} = \rho_{\bar{I}}^{(\diamond)} - \frac{\beta}{d}\rho_S^{(\diamond)} + \frac{\beta}{d^\star}\rho_L^{(\diamond)} + \frac{i\alpha}{2\Delta m}\rho_I^{(\diamond)} . \tag{2.17}$$

The corresponding eigenvalues are corrected by the shifts

$$\begin{aligned} \Gamma_L &\to \Gamma_L + \gamma , & \Gamma_S &\to \Gamma_S + \gamma , \\ \bar{\Gamma} &\to \bar{\Gamma} + \alpha , & \Delta m &\to \Delta m \cdot (1 - \tfrac{1}{2}(\beta/\Delta\Gamma)^2) . \end{aligned} \tag{2.18}$$

The shifts of Γ_L, Γ_S, and Δm can be absorbed by redefinition of these parameters. The shift of Δm is of relative size 10^{-6} and so is negligible in any event. If we redefine $\bar{\Gamma}$ to be the average of the new Γ_S and Γ_L, then the interference terms ρ_I and $\rho_{\bar{I}}$ fall off at the rate $\bar{\Gamma} + (\alpha - \gamma)$. This correction is not relevant to current experiments unless α is as large as $10^{-2}\bar{\Gamma}$; in that case α would be 10 times larger than the familiar CP-violating parameters.

The major effect of violation of quantum mechanics is embodied in the eigenmodes ρ_L, ρ_S, ρ_I. These density matrices are no longer pure density matrices in contrast to their quantum mechanical counterparts. This loss of purity alters the decay properties of the beam. For example, the properties of the beam at large time, $\tau \gg 1/\Gamma_S$, are dominated by the properties of ρ_L. The second term on the RHS of (2.14) is proportional to $\rho_S^{(\diamond)}$ and is even under CP conjugation. As a

† We use a diamond superscript to label quantum mechanical quantities.

result, we expect an enhancement in the decay into 2 pion at late time in the evolution of the beam. More specifically, in leading order in small quantities, we find for the decay rate at large time

$$\begin{aligned}\Gamma(K_L \to \pi\pi) &\propto \operatorname{tr} \rho_L \mathcal{O}_{\pi\pi} \\ &\propto \operatorname{tr} \rho_L^{(\diamond)} \mathcal{O}_{\pi\pi} + \frac{\gamma}{\Delta\Gamma} \operatorname{tr} \rho_S^{(\diamond)} \mathcal{O}_{\pi\pi} + 2\mathrm{Re}\left(\frac{\beta}{d} \operatorname{tr} \rho_I^{(\diamond)} \mathcal{O}_{\pi\pi}\right), \\ &\propto |\eta_{+-}^{(\diamond)}|^2 + \frac{\gamma}{\Delta\Gamma} + \text{higher order} = R_L\end{aligned} \tag{2.19}$$

There is, indeed, an enhancement of the 2 pion decay rate proportional to $\frac{\gamma}{\Delta\Gamma}$. Similarly, in the intermediate time region ($\tau \sim 1/\bar{\Gamma}$), the dominant contribution to the 2 pion decay comes from ρ_I and its hermitian conjugate $\rho_{\bar{I}}$. From its expression (2.16), ρ_I has a piece proportional to $\rho_S^{(\diamond)}$ which shifts the 2 pion decay rate by an amount $\propto \beta/|d| \cos\phi_{SW}$ and $\beta/|d| \sin\phi_{SW}$ in the intermediate time region.

From the simple arguments above, we expect the phenomenology of the kaon beam to be affected in leading order in the parameters $\frac{\beta}{d}$ and $\frac{\gamma}{\Delta\Gamma}$. In particular, we expect corrections of order $\frac{\beta}{d}$ in the intermediate time region. We will exploit this fact in the next section to establish some experimental bounds on violation of quantum mechanics.

3. Experimental constraints on α and β

This section is a summary of an analysis made in ref. 1 which establishes constraints on the EHNS parameters using present experimental data. In this analysis we combine very accurate measurements on the time dependent kaon system made in the early 1970's by the CERN-Heidelberg collaboration[13] with new data from the CPLEAR experiment.[16] Provided that the EHNS parameters do not accidentally cancel against the effects of the quantum mechanics CPT violating parameters in direct decay Y and b/a introduced in (2.8)-(2.11), we are able to give stringent bounds on β and γ which limit their effects to be at most 10% of the observed CP violation in the K_0–$\bar{K}_0$ system. To simplify the following discussion, we temporarily neglect quantum mechanics CPT violation in direct decays: $Y = b/a = 0$. We will reintroduce them at the end of this section.

For this analysis, we need two observables. the time dependent 2 pion decay rate

$$\begin{aligned}\frac{\Gamma(K(\tau)\to\pi\pi)}{\Gamma(K(0)\to\pi\pi)}=&\frac{\mathrm{tr}\,\rho_K(\tau)\mathcal{O}_{+-}}{\mathrm{tr}\,\rho_K(0)\mathcal{O}_{+-}}\\=&e^{-\Gamma_S\tau}+2|\eta_{+-}|^2\cos(\Delta m\tau-\phi_{+-})+R_Le^{-\Gamma_L\tau}\end{aligned}\tag{3.1}$$

and the semileptonic decay rate at large time $\tau \gg 1/\Gamma_S$

$$\begin{aligned}\delta_L=&\frac{\Gamma(K_L\to\pi^-\ell^+\nu)-\Gamma(K_L\to\pi^+\ell^-\nu)}{\Gamma(K_L\to\pi^-\ell^+\nu)+\Gamma(K_L\to\pi^+\ell^-\nu)}\\=&\frac{\mathrm{tr}\,\rho_K(\tau)(\mathcal{O}_{\ell^+}-\mathcal{O}_{\ell^-})}{\mathrm{tr}\,\rho_K(\tau)(\mathcal{O}_{\ell^+}+\mathcal{O}_{\ell^-})}\end{aligned}\quad.\tag{3.2}$$

The relevant measurable quantities are R_L, δ_L and η_{+-}. R_L and δ_L reflect the properties of the beam which evolved at large time; the complex number $\eta_{+-} = |\eta_{+-}|\exp(i\phi_{+-})$ is a property of the intermediate time region.

In quantum mechanics, they are related according to

$$\begin{aligned}R_L&=|\eta_{+-}|^2\\\frac{\delta_L}{2}&=\mathrm{Re}\,\eta_{+-}\end{aligned}\quad.\tag{3.3}$$

After allowance has been made for quantum mechanics violation, they relate according to

$$\begin{aligned}R_L&\simeq|\eta_{+-}|^2+\frac{\gamma}{\Delta\Gamma}+4\frac{\beta}{|d|}|\eta_{+-}|\\\frac{\delta_L}{2}&=\mathrm{Re}\,(\eta_{+-}-2\frac{\beta}{d})\end{aligned}\quad.\tag{3.4}$$

The corrections of order β, γ are the ones we described in the previous section but including some second order corrections. The geometry of these corrections is given in Fig. 1a . The current experimental situation is discussed in ref. 1 and shown on Fig. 1b.

According to equations (3.4), the parameter β is proportional to the distance of the ellipse to the vertical band while the distance of the ellipse to the arc provides a measurement of γ. This comparison leads to the bounds

$$\begin{aligned}\beta =&(0.12 \pm 0.44) \times 10^{-18} \text{ GeV} \\ \gamma =&(-1.1 \pm 3.6) \times 10^{-21} \text{ GeV}\end{aligned} \quad . \tag{3.5}$$

To obtain these bounds, we set to zero the CPT quantum mechanics perturbations of the decay amplitude b/a and B_0/A_0 introduced in (2.8)-(2.11). If we restore these parameters in the above analysis, we find instead

$$\begin{aligned}\beta + \frac{|d|}{\sin\phi_{SW}}\mathrm{Re}(\frac{b}{a} - \frac{B_0}{A_0}) = (0.12 \pm 0.44) \times 10^{-18} \text{ GeV} \\ \gamma - \frac{|d|}{\sin\phi_{SW}}\mathrm{Re}(\frac{b}{a} - \frac{B_0}{A_0}) = (-1.1 \pm 3.6) \times 10^{-21} \text{ GeV}\end{aligned} \quad . \tag{3.6}$$

Thus, our previous constraints on β and γ now appear as constraints on combinations of CPT-violating parameters. Unless, unnatural cancellations occur among these CPT violating parameters, they can be independently constrain, in which case, neither of them contributes more than 10% of the total CP violation observed in the K_0–$\bar{K}_0$ system.

With we advent of a new generation of facilities such as ϕ factories, which can perform incisive tests on CPT violation,[12] come new ways of probing quantum mechanics. This is the object of the next section.

4. Tests of quantum mechanics at a ϕ-factory

At a ϕ factory, a spin-1 meson decays to an antisymmetric state of two kaons which propagates with opposite momentum. If the kaons are neutral, we can describe the resulting wavefunction, in the basis of CP eigenstates $|K_1\rangle$, $|K_2\rangle$, as

$$\phi \to \frac{1}{\sqrt{2}}\Big(|K_1, p> \otimes |K_2, -p> - |K_2, p> \otimes |K_1, -p>\Big) \ . \tag{4.1}$$

The two-kaon density matrix resulting from this decay is a 4×4 matrix P. The quantum mechanical time evolution of P is contained in Eq. (2.1) while, in the context of generalization of quantum mechanics, it is contained in Eq. (2.13) .

Let us first describe the quantum mechanical time dependence of P. We find

$$\begin{aligned} P^{(\diamond)}(\tau_1, \tau_2) = \frac{1 + 2\mathrm{Re}(\epsilon_S \epsilon_L)}{2} \Big(&\rho_S^{(\diamond)} \otimes \rho_L^{(\diamond)} e^{-\Gamma_S \tau_1} e^{-\Gamma_L \tau_2} + \rho_L^{(\diamond)} \otimes \rho_S^{(\diamond)} e^{-\Gamma_L \tau_1} e^{-\Gamma_S \tau_2} \\ &- \rho_I^{(\diamond)} \otimes \rho_{\bar{I}}^{(\diamond)} e^{-i\Delta m(\tau_1 - \tau_2)} e^{-\bar{\Gamma}(\tau_1 + \tau_2)} - \rho_{\bar{I}}^{(\diamond)} \otimes \rho_I^{(\diamond)} e^{+i\Delta m(\tau_1 - \tau_2)} e^{-\bar{\Gamma}(\tau_1 + \tau_2)}\Big) \ . \end{aligned} \tag{4.2}$$

As in the one kaon system, any observable is obtained by tracing the density matrix with a suitable hermitian operator. The basic observables computed from P are double differential decay rates, the probabilities that the kaon with momentum p decays into the final state f_1 at proper time τ_1 while the kaon with momentum $(-p)$ decays to the final state f_2 at proper time τ_2. We denote this quantity as $\mathcal{P}(f_1, \tau_1; f_2, \tau_2)$. If we denote the expression (4.2) schematically as $P = \sum A_{ij}\, \rho_i \otimes \rho_j$ where i, j run over $S, L, I, \bar{I}$, and write the corresponding eigenvalues as λ_i, then the double decay rate is given by

$$\mathcal{P}(f_1, \tau_1; f_2, \tau_2) = \sum_{i,j} A_{ij}\ \mathrm{tr}[\rho_i \mathcal{O}_{f_1}]\, \mathrm{tr}[\rho_j \mathcal{O}_{f_2}] e^{-\lambda_i \tau_1 - \lambda_j \tau_2} \ . \tag{4.3}$$

A situation of particular importance is the decay into two identical final states

$f_1 = f_2$, the double decay rate takes the simple form[11)]

$$\mathcal{P}^{(\diamondsuit)}(f,\tau_1;f,\tau_2) = C \times \left[e^{-\Gamma_S\tau_1-\Gamma_L\tau_2} + e^{-\Gamma_L\tau_1-\Gamma_S\tau_2} - 2\cos(\Delta m(\tau_1-\tau_2))e^{-\bar{\Gamma}(\tau_1+\tau_2)}\right] . \tag{4.4}$$

This quantity has no dependence on the CP and CPT parameters and depends on the two times in a manner completely fixed by quantum mechanics irrespective of the properties of the decay amplitudes. In particular, at $\tau_1 = \tau_2$, the double distribution *vanishes*, as a consequence of the antisymmetry of the initial state wavefunction which is preserved in the evolution of the beam.

The previous steps can be performed taking into account violation of quantum mechanics, and expressions corresponding to (4.2) and (4.4) are provided in ref. 1. In addition to the natural replacement $\rho_K^{(\diamondsuit)} \to \rho_K$, new structures appear in the time dependence of the system . We will content ourselves to illustrate these structures in the case both kaons decay to $\pi^-\ell^+\nu$ or to $\pi^+\ell^-\bar{\nu}$. These are examples of decays to identical final states whose quantum mechanical time dependence has the general structure depicted in (4.4). We find, to first order in small parameters

$$\begin{aligned}
\mathcal{P}(\ell^\pm,\tau_1;\ell^\pm,\tau_2) &= \frac{|a|^4}{8} \\
\times\Big\{&(1 \pm 4\mathrm{Re}\,\epsilon_M)\left[e^{-\Gamma_S\tau_1-\Gamma_L\tau_2} + e^{-\Gamma_L\tau_1-\Gamma_S\tau_2} - 2\cos\Delta m(\tau_1-\tau_2)e^{-(\bar{\Gamma}+\alpha-\gamma)(\tau_1+\tau_2)}\right] \\
&\pm 4\frac{\beta}{|d|}\sin(\Delta m\tau_1 - \phi_{SW})e^{-(\bar{\Gamma}+\alpha-\gamma)\tau_1}e^{-\Gamma_S\tau_2} + (1 \leftrightarrow 2) \\
&\pm 4\frac{\beta}{|d|}\sin(\Delta m\tau_1 + \phi_{SW})e^{-(\bar{\Gamma}+\alpha-\gamma)\tau_1}e^{-\Gamma_L\tau_2} + (1 \leftrightarrow 2) \\
&+ 2\frac{\alpha}{\Delta m}\sin\Delta m(\tau_1+\tau_2)\,e^{-(\bar{\Gamma}+\alpha-\gamma)(\tau_1+\tau_2)} + 2\frac{\gamma}{\Delta\Gamma}\left[e^{-\Gamma_L(\tau_1+\tau_2)} - e^{-\Gamma_S(\tau_1+\tau_2)}\right]\Big\}
\end{aligned} \tag{4.5}$$

The first term in the brackets has a form quite close to the canonical form (4.4) predicted by quantum mechanics, while the remaining terms give systematic corrections to this result. The three following lines contain totally new dependence. These new terms signal the breakdown of the antisymmetry of the final state wave function, that is, the breakdown of angular momentum conservation. This is expected in the framework of density matrix evolution equations, as was explained in ref. 8. However, in the ϕ factory experiments, one does not need to wait for the problems of energy-momentum conservation to built up to a macroscopic violation; one can instead track these violations directly in the frequency dependence of corrections.

The above peculiar dependence on τ_1 and τ_2 is a unique signature of violation of quantum mechanics and provides an unambiguous method to isolate the EHNS parameters from the CPT violating perturbations of the decay rates from within quantum mechanics introduced in (2.8) - (2.11).

One can, for instance, interpolate the double decay rates into identical final states $\mathcal{P}(f,\tau_1;f,\tau_2)$ on the line of equal time $\tau_1 = \tau_2$. This quantity vanishes identically according to the principles of quantum mechanics and thus is of order α, β and γ. As an illustration, the semileptonic double decay rate at equal time yields

$$\begin{aligned}
\mathcal{P}(\ell^{\pm},\tau;\ell^{\pm},\tau)/\mathcal{P}(\ell^{\pm},\tau;\ell^{\mp},\tau) = & \\
&\frac{1}{2}\big[1-e^{-2(\alpha-\gamma)\tau}\big(1-\frac{\alpha}{\Delta m}\sin 2\Delta m\tau\big)\big] \\
&+\frac{1}{2}\frac{\gamma}{\Delta\Gamma}\big[e^{+\Delta\Gamma\tau}-e^{-\Delta\Gamma\tau}\big] \\
&\pm 4\frac{\beta}{|d|}\big[\sin(\Delta m\tau-\phi_{SW})e^{-\Delta\Gamma\tau/2}+\sin(\Delta m\tau+\phi_{SW})e^{+\Delta\Gamma\tau/2}\big]
\end{aligned} \tag{4.6}$$

The three coefficients α, β, and γ are selected by terms which are monotonic in τ, oscillatory with frequency Δm,and oscillatory with frequency $2\Delta m$.

There seems to be no difficulty in constraining CPT violation from outside quantum mechanics in a ϕ factory independently of other CPT violating perturbation of quantum mechanics . However, the reverse is not true. Any observable at a ϕ factory is expected to receive α, β and γ corrections. These corrections are, however, easily computed and can be systematically taken into account using the knowledge gained on these parameters using methods of the type described above.

As an illustration, we present the corrections to the formula predicting the value of $3\mathrm{Re}\epsilon'/\epsilon$ using the integrated time distribution[11] at fixed $\Delta\tau = \tau_1 - \tau_2$ of the asymmetric decay into charged and neutral pions

$$\bar{\mathcal{P}}(\Delta\tau) = \int_{|\Delta\tau|}^{\infty} d(\tau_1 + \tau_2)\, \mathcal{P}(\pi^+\pi^-; \pi^0\pi^0; \Delta\tau) \ . \tag{4.7}$$

This time interval distribution is very useful for obtaining the standard *CP* violation parameters of the neutral kaon system. In particular, one predicts[18]

$$\lim_{|\Delta\tau|\Gamma_S \gg 1} \frac{\bar{\mathcal{P}}^{(\diamond)}(\Delta\tau > 0) - \bar{\mathcal{P}}^{(\diamond)}(\Delta\tau < 0)}{\bar{\mathcal{P}}^{(\diamond)}(\Delta\tau > 0) + \bar{\mathcal{P}}^{(\diamond)}(\Delta\tau < 0)} = 3\mathrm{Re}\, \frac{\epsilon'}{\epsilon}. \tag{4.8}$$

We find instead*

$$\begin{aligned}\lim_{|\Delta\tau|\Gamma_S \gg 1} \frac{\bar{\mathcal{P}}(\Delta\tau > 0) - \bar{\mathcal{P}}(\Delta\tau < 0)}{\bar{\mathcal{P}}(\Delta\tau > 0) + \bar{\mathcal{P}}(\Delta\tau < 0)} \simeq & 3\mathrm{Re}\, \epsilon'/\epsilon \left[1 - \frac{\gamma}{|d||\eta_{+-}|^2} + 2\frac{\beta}{|d||\eta_{+-}|}\right] \\ & -3\, \mathrm{Im}\, \epsilon'/\epsilon \left[2\frac{\beta}{|d||\eta_{+-}|}\right]\end{aligned} \tag{4.9}$$

It is only under the assumption $\mathrm{Im}\epsilon'/\epsilon \ll \mathrm{Re}\,\epsilon'/\epsilon \times \left(|d||\eta_{+-}|/\beta\right)$ that (4.9) is a measurement of $3\mathrm{Re}\,\epsilon'/\epsilon$.

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* A more exact formula, as well as a more complete discussion, is given in ref. 1.

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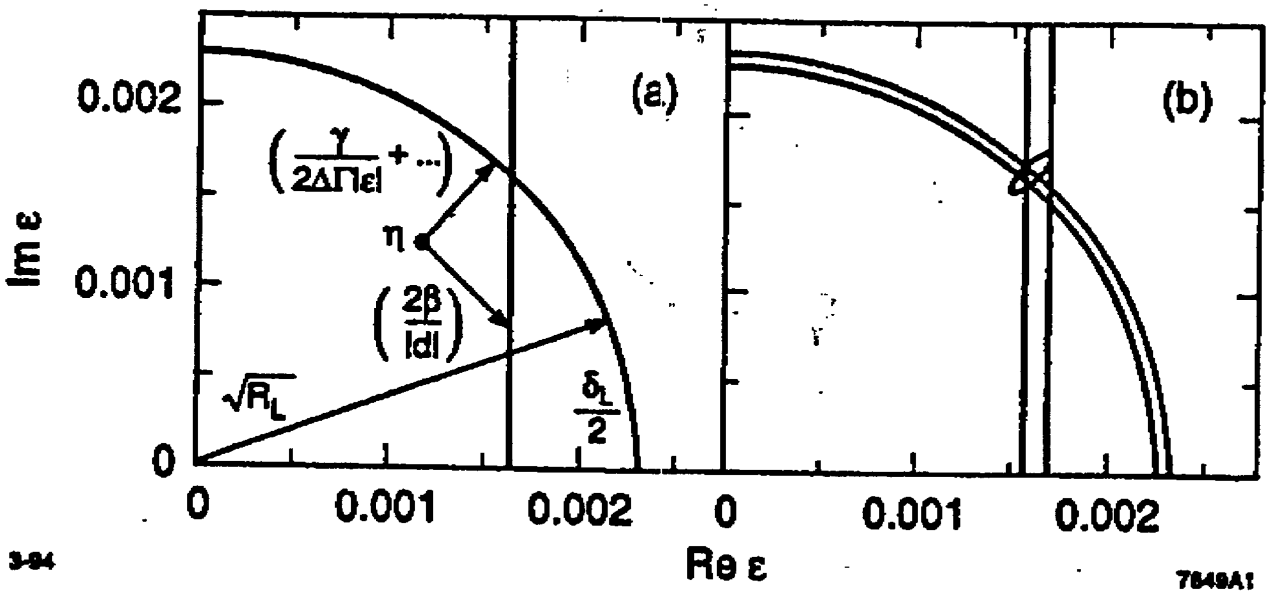
FIGURE CAPTION

-Fig.1- Theoretical predictions (a) and experimental data (b).



-Fig.1-
Theoretical predictions (a) and experimental data (b).