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Charge form factors of pseudoscalar mesons

Felix Schlumpf

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

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Abstract

The elastic charge form factors of the pion, kaon, D- and B-mesons are calculated within a relativistic constituent quark model formulated on the light-cone. Our parameter free predictions agree well with the available data. The results are approximately independent of the assumed form of the light-cone wave function, in particular they do not depend on the high momentum tail of the wave function for energies accessible to present experiments.

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I. INTRODUCTION

There has recently been renewed interest in the charge form factors of pseudoscalar mesons because CEBAF is planning experiments to measure independently the pion (E-93-021) and kaon (E-93-018) charge form factors in the range of $Q^2 < 3$ GeV^2. On the other hand, theoretical predictions of the kaon form factor given in Refs. [1–3] differ significantly. It is therefore reasonable to analyze the kaon form factor, and in general the pseudoscalar form factors, in a different model.

Since an *ab-initio* QCD calculation for hadronic wave functions is currently not feasible, it is important to have QCD-inspired models which realistically describe hadron static properties. Such a model has been described in Refs. [4,5] and many observables have been calculated within this model [6–8] and found in good agreement with experiment.

In this paper we extend the analysis of Ref. [5] to pseudoscalar mesons in which the constituent quarks have unequal masses. The main uncertainty in this model is due to the uncertainty in the undetermined momentum wave function. We find that the charge form factors are approximately insensitive for a large class of wave functions. This is a generalization of the finding in Ref. [9] which studies the connection between the radius and the decay constant of the pion.

In Ref. [7] the charge form factors for the pion and kaon are given in the parameterization $F(Q^2) = F(0)/(1 - Q^2/\Lambda_1^2 - Q^4/\Lambda_2^4)$. Unfortunately, this specific representation is only valid for $Q^2 < 0.25$ GeV^2 for the pion and $Q^2 < 0.75$ GeV^2 for the kaon. This paper presents the full calculation.

In the next section we describe the model and give the formula for the charge form factor. In Sec. III the numerical results are presented and compared with available experimental data and other theoretical calculations. Finally, we conclude this paper in Sec. IV with a summary.

II. RELATIVISTIC CONSTITUENT QUARK MODEL

The light-cone constituent quark model given in Ref. [5] provides a framework for representing the general structure of the two-quark wave functions for mesons. The wave function is constructed as the product of a momentum wave function, which is spherically symmetric and invariant under permutations, and a spin-isospin wave function, which is uniquely determined by symmetry requirements. A Wigner (Melosh [10]) rotation is applied to the spinors, so that the wave function of the meson is an eigenfunction of J^2 and J_z [11]. To represent the range of uncertainty in the possible form of the momentum wave function we choose two simple functions of p^2; a gaussian or harmonic oscillator (HO) wave function and a power-law wave function:

$$\psi_{\rm HO}(p^2) = N_{\rm HO} \exp(-p^2/2\beta^2), \tag{2.1}$$

$$\psi_{\rm Power}(p^2) = N_{\rm Power}(1 + p^2/\beta^2)^{-s}, \tag{2.2}$$

where β sets the scale of the nucleon size, and p is the three-vector on the light-cone defined below. This may seem quite arbitrary, but as we will show below, the form factors are essentially independent of the shape of the wave function for $s \geq 2$.

Since the four-momentum $Q = p' - p$ is spacelike it is always possible to orient the axes in such a manner that $Q^+ = 0$. The charge form factor $F(Q^2)$ of the pseudoscalar meson is then given by the matrix element ($Q^2 = -q^2$)

$$F(Q^2) = \langle p + q|J^+(0)|p\rangle. \tag{2.3}$$

The general formula for the pseudoscalar meson is then

$$F_{\bar{q}Q}(Q^2) = e_{\bar{q}} I(m_{\bar{q}}, m_Q, \beta_{\bar{q}Q}, Q^2) + e_Q I(m_Q, m_{\bar{q}}, \beta_{\bar{q}Q}, Q^2), \tag{2.4}$$

with [12]

$$I(m_1, m_2, \beta_{\bar{q}Q}, Q^2) = \int d^3p \left(\frac{E_1' E_2' M_0}{E_1 E_2 M_0'}\right)^{1/2} \psi^\dagger(p')\psi(p) \times \\ \frac{p_\perp \cdot p_\perp' + (\xi m_2 + (1-\xi) m_1)^2}{\xi(1-\xi)\sqrt{M_0^2 - (m_1 - m_2)^2}\sqrt{M_0'^2 - (m_1 - m_2)^2}}.$$

The relative three-momentum $(p_\perp, p_z)$ in this formula is defined as

$$\xi = p_1^+/P^+, p_\perp = p_{1\perp} - \xi P_\perp, p_z = \xi E_2 - (1-\xi)E_1, \tag{2.5}$$

where P and p_1 are the four-momentum of the meson and the first constituent quark, respectively. The energy E_i is given by $E_i^2 = p_\perp^2 + p_z^2 + m_i^2$, the invariant mass M_0 is given by $M_0 = E_1 + E_2$, and $p'_\perp = p_\perp - (1-\xi)q_\perp$.

The wave function is normalized as $\int d^3p|\psi|^2 = 1$ so that $F(0)$ gives the charge of the particle, because $I(m_1, m_2, \beta_{\bar{q}Q}, 0) = 1$ and $F(0) = e_{\bar{q}} + e_Q$. The radius of the particle is defined by $r^2 = -6dF(Q^2)/dQ^2|_{Q^2=0}$. The expression for the decay constant is given in Eq. (3.3) of Ref. [7].

III. NUMERICAL RESULTS

The parameters of the model have to be determined by comparison with experimental data. They are the constituent masses m_q of the quarks and the confinement scale $1/\beta$ of the bound meson. The parameters used in this paper are given in Table I. For the $u-, d-, s-$quark sector the parameters from Ref. [7] are chosen. These four parameters have been obtained by fitting the decay constants f_π, f_ρ, f_K and the decay rate for $K^{*+} \to K^+\gamma$. With these parameters all the processes described in Ref. [7] can be described in addition to the charge form factors discussed in this paper. The $c-$quark mass is taken from Ref. [8] where the decays $D \to Ke\nu$ and $D \to K^*e\nu$ are analyzed. There is an uncertainty in β, which we determined by fitting the decay constant f_D as given by recent lattice calculations [13]. The $b-$quark mass is taken from recent nonrelativistic lattice calculations [14]. Again we fit β to obtain the decay constant f_B obtained in Ref. [13]. Table I also shows the βs for the power-law wave function in Eq. 2.2 with $s = 2$, which are determined to yield the same decay constants as for the harmonic oscillator wave function. The weak decay constants for the D and B mesons for different values of the parameter β are given in Table II. There is a strong dependence on β so that our model does not restrict the decay constants of the D and B mesons very well.

The weak decay constants and the charge radii are given in Table III and compared with data. All the radii agree with experiment were available. We also achieve the result $f_B \approx f_D$ given in Ref. [15]. This differs from heavy quark effective theory which predicts $f_D/f_B = \sqrt{M_B/M_D} = 1.68$. It has already been noted in Ref. [16] that the Wigner rotation of the spin lowers this ratio.

Figures 1 – 6 show the charge form factors of the pion, kaon, D- and B-mesons, respectively, together with experimental data where available. The results for the pion are compared in Fig. 1 with data for low Q^2 [17] and in Fig. 2 with data for high Q^2 [18]. The solid line represents the HO wave function in Eq. 2.1 which fits the data best for both low and high momentum transfer. The other lines give the result for the power-law wave function in Eq. 2.2 with $s = 1$ (dotted line), $s = 2$ (dashed line), and $s = 3$ (dot-dashed line). The power-law wave function for $s \geq 2$ can describe the data as well. The result is approximately independent of the wave function for the experimental accessible Q^2-region [19]. This wave function independence is even better fulfilled for the heavy quark sector as seen in Fig. 5 and 6. This is astonishing since for baryons it was found in Ref. [20] that the high momentum tail of the wave function is important for $Q^2 > 2$ GeV^2. On the other hand, Ref. [21] describes that static properties are exactly independent of the wave function even for the light quark sector. The result of Fig. 1 is expected from Ref. [9] which confirms the approximate relation $r_\pi = \text{const}/f_\pi$ for different wave functions.

Figure 2 shows that the high momentum tail of the wave function does not matter for energies accessible to present experiments. Important is the parameter β which determines the overall shape of the wave function. This has independently been found by Jean [22]. Indeed all the different curves in Fig. 4 of Ref. [23] can be obtained with a HO wave function with fixed quark mass and different βs.

The results for the kaon are compared in Fig. 3 with data for low Q^2 [24] and shown in Fig. 4 for high Q^2. Again, the solid and the dashed lines give the result for the HO and power-law wave function for $s = 2$, respectively. This should represent the range of uncertainty in the possible form of the momentum wave function. Our prediction is compared with the calculation in Ref. [3] (dotted line) and in Ref. [2] (dot-dashed line). While the former curve nearly coincides with our power-law wave function result, the latter calculation gives a much smaller charge form factor. The CEBAF experiment (E-93-018) may be able to distinguish between these different predictions.

Our formalism is suited for both light and heavy quarks, so that we also present the charge form factors for the D- and B-mesons in Fig. 5 and 6. Note the different scale in the momentum transfer. The wave function dependence for the D- and B-mesons is very small even for very high momentum transfer.

A main emphasis of this paper is to choose the same parameters as Ref. [7], so that all the precise results obtained in that paper are preserved. For instance, the explicit calculation of the weak $K \to \pi$ transition form factor gives $F(0) = 0.965$ and $r^2 = 0.318$ fm^2 for the HO wave function [7]. For K_{e3} decay, one may neglect the electron mass in the calculation of the rate, which then depends only on the form factor $F(Q^2)$. The corresponding value V_{us} of the KM quark-mixing matrix is $V_{us} = 0.2199 \pm 0.0017$ [7]. The error for V_{us} originates in the uncertainty of the strange quark mass in the constituent quark model.

IV. CONCLUSION

We extend the simple relativistic quark model described in Ref. [5–7] to investigate the charge form factors of the pseudoscalar mesons. Most of the parameters are fixed by these previous studies of the model. We notice an approximate independence of the momentum wave function for a large Q^2-range. The charge form factors do not depend on the high momentum tail of the wave function, but rather on the confinement scale $1/\beta$.

The validity of this model has also been tested in other systems such as mesons having nonzero angular momentum [7] and baryons [20]. In conclusion, this model provides a remarkably good description of the electroweak properties of hadrons, although it is both conceptually and computationally simple.

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TABLES

TABLE I. Table of the parameters of the relativistic constituent quark model.

q	m_q (GeV)	$\beta_{u\bar{q}}^{\rm HO}$ (GeV)	$\beta_{u\bar{q}}^{\rm Power}$ (GeV)
u, d	0.25	0.3194	0.335
s	0.37	0.395	0.41
c	1.85	0.49	0.50
b	4.7	0.55	0.54

TABLE II. The weak decay constants of the D and B mesons for different values of the parameter β for the HO wave function.

β (GeV)	0.4	0.5	0.6	0.7
f_D (MeV)	118	149	178	205
f_B (MeV)	89	117	147	177

TABLE III. Charge radii and weak decay constants for the pseudoscalar mesons. The parameters have been chosen in such a way that both HO and power-law wave function give the same weak decay constant. The charge radii are given for the HO wave function.

Observable	Expt.	Calculation
f_π (MeV)	92.4 ± 0.2 [26]	92.4
f_K (MeV)	113.4 ± 1.1 [26]	113.4
f_D (MeV)	< 219 [27]	146
f_B (MeV)	–	132
r^2_π (fm^2)	0.432 ± 0.016 [17]	0.449
	0.44 ± 0.03 [28]	
$r^2_{K^+}$ (fm^2)	0.34 ± 0.05 [24]	0.327
$r^2_{K^0}$ (fm^2)	-0.054 ± 0.101 [24]	0.000
r^2_D (fm^2)	–	0.024
r^2_B (fm^2)	–	0.005

FIGURES

FIG. 1. The square of the pion charge form factor for low values of Q^2 compared with data [17]. The solid line represents the HO wave function, the other lines give the result for the power-law wave function with $s = 1$ (dotted), $s = 2$ (dashed), and $s = 3$ (dot-dashed).

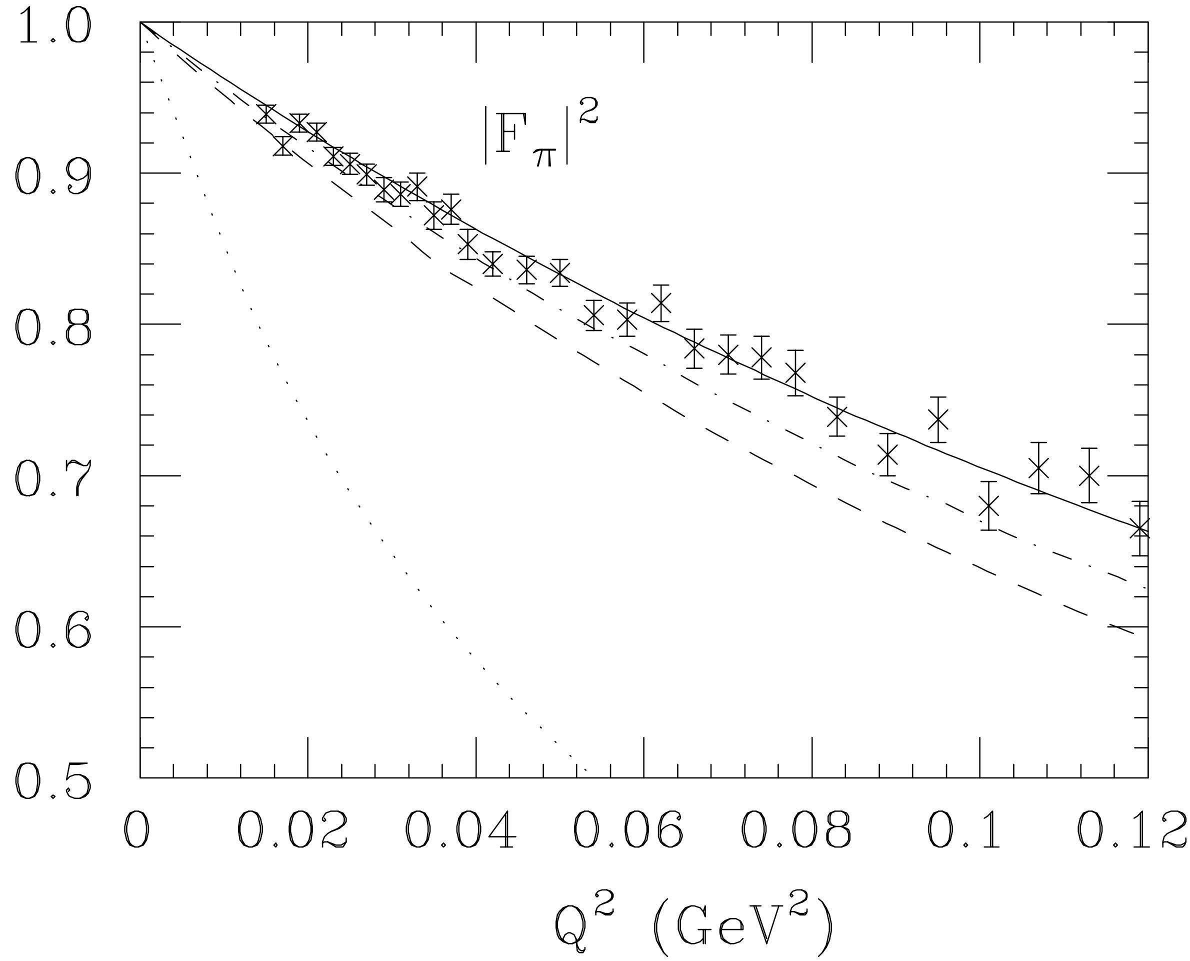
FIG. 2. The charge form factor for the pion compared with data taken from Ref. [18]. The same line code as in Fig. 1 is used.

FIG. 3. The square of the kaon charge form factor for low values of Q^2 compared with data [24]. Solid line, HO wave function; dashed line, power-law wave function with $s = 2$; and the dot-dashed line is taken from Ref. [2].

FIG. 4. The charge form factor for the K^+. The same line code as in Fig. 3 is used. In addition, the dotted line is taken from Ref. [3].

FIG. 5. The charge form factor for the $D^\pm, B^\pm$. Solid line, HO wave function; dashed line, power-law wave function with $s = 2$.

FIG. 6. The charge form factor for the D^+, B^+. The same line code as in Fig. 5 is used.



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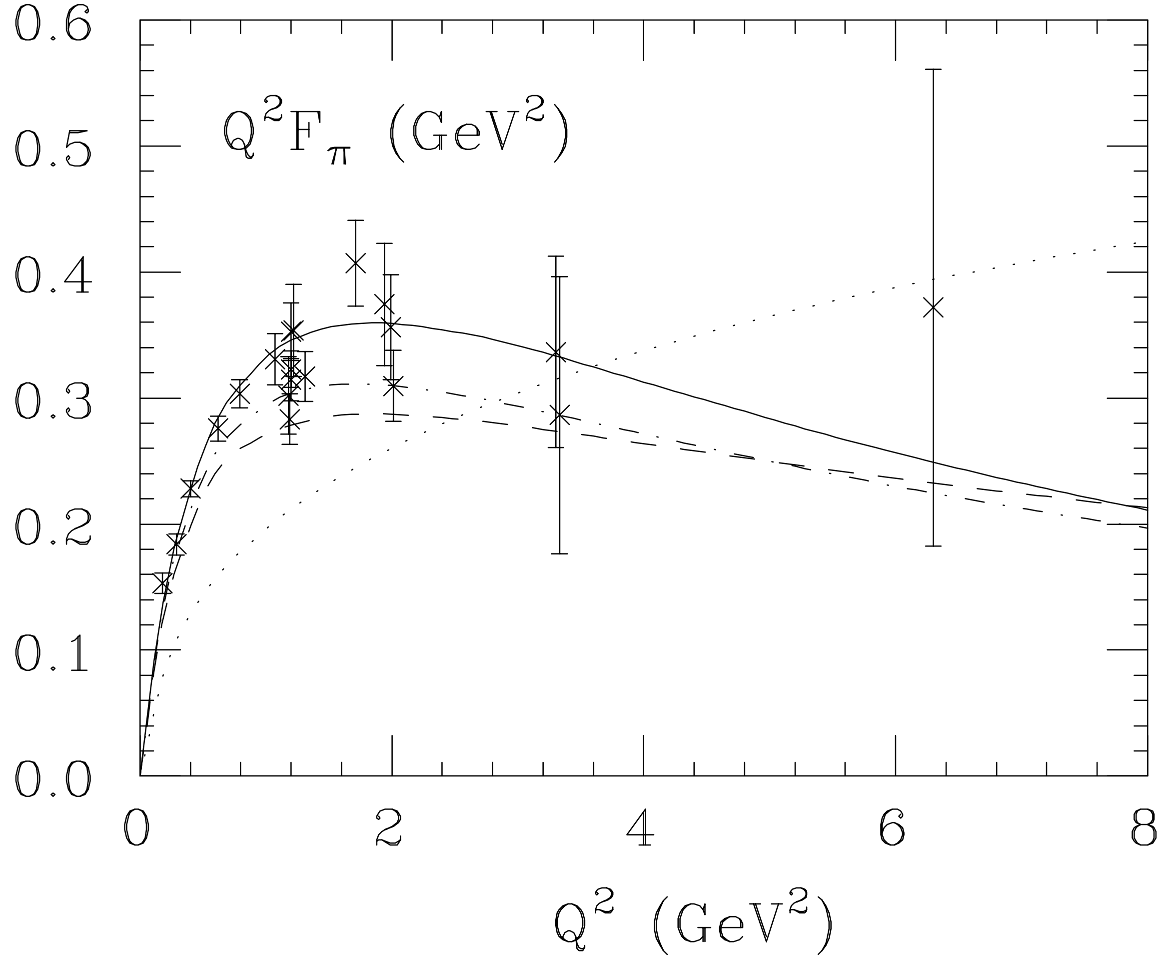

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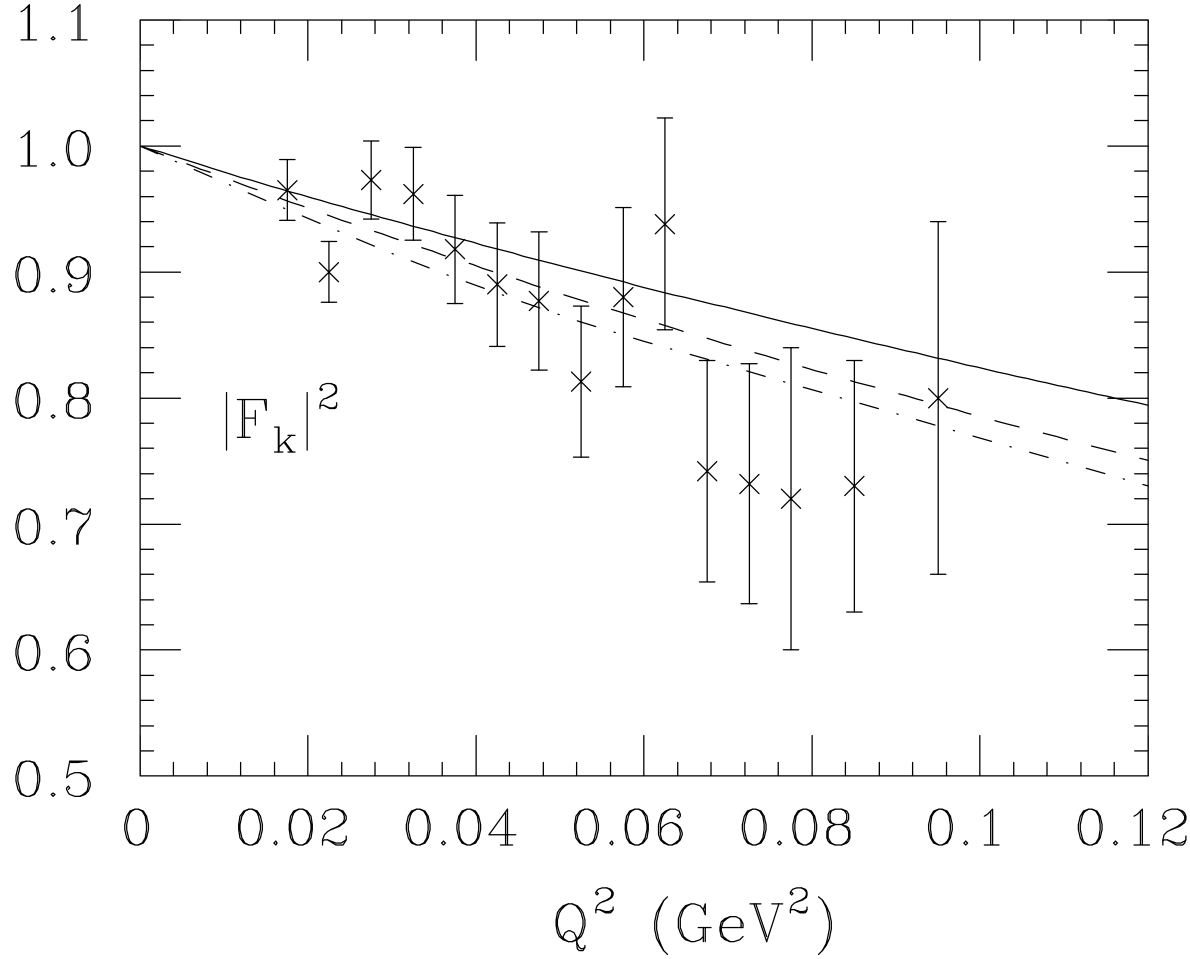

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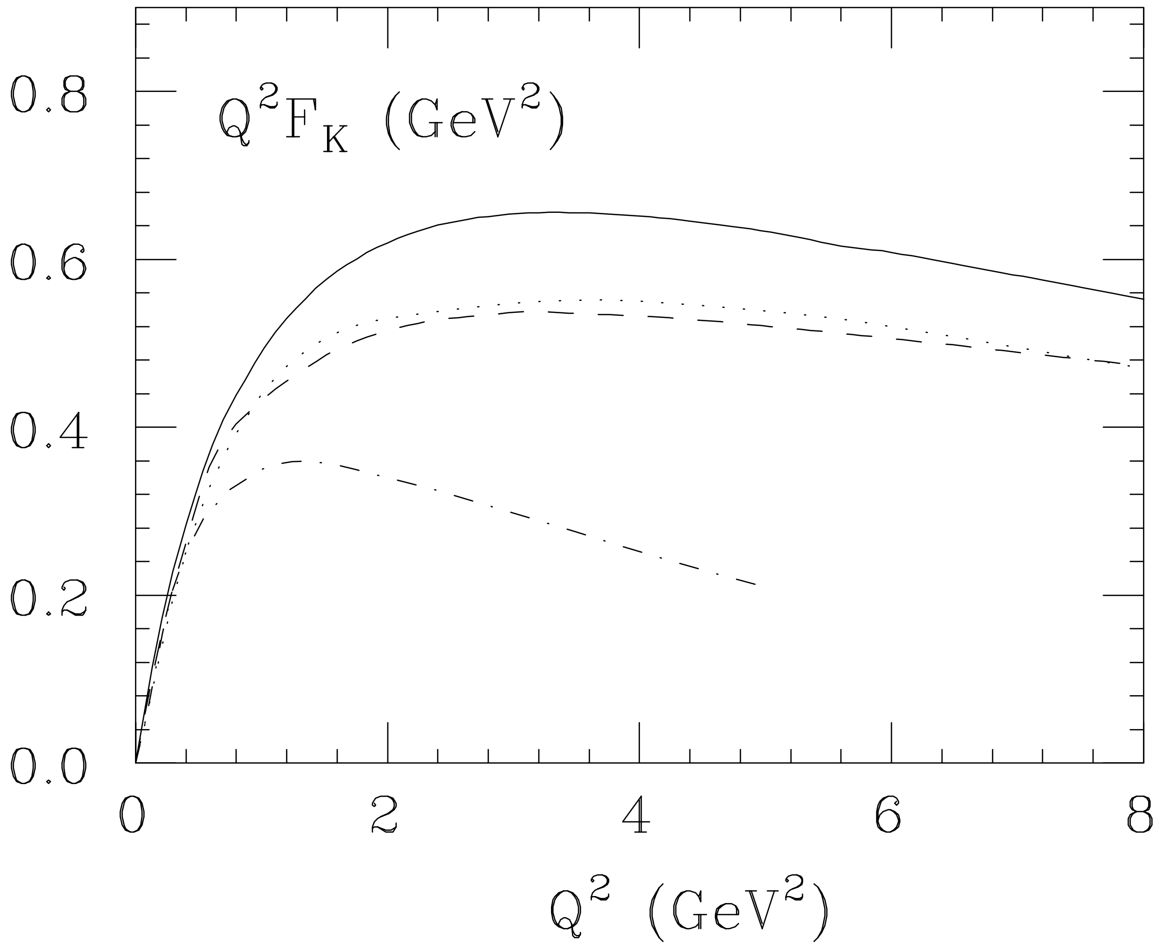

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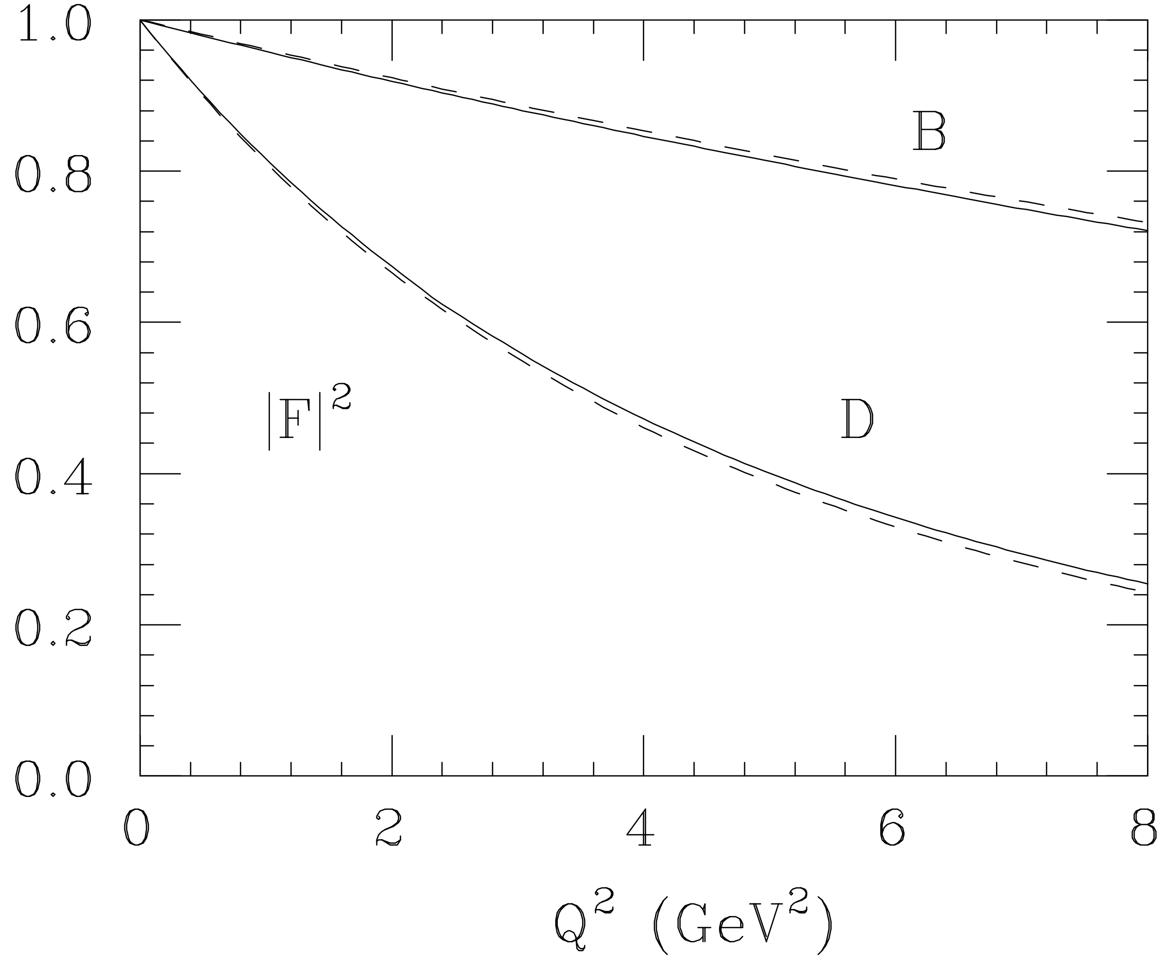

Figure 5

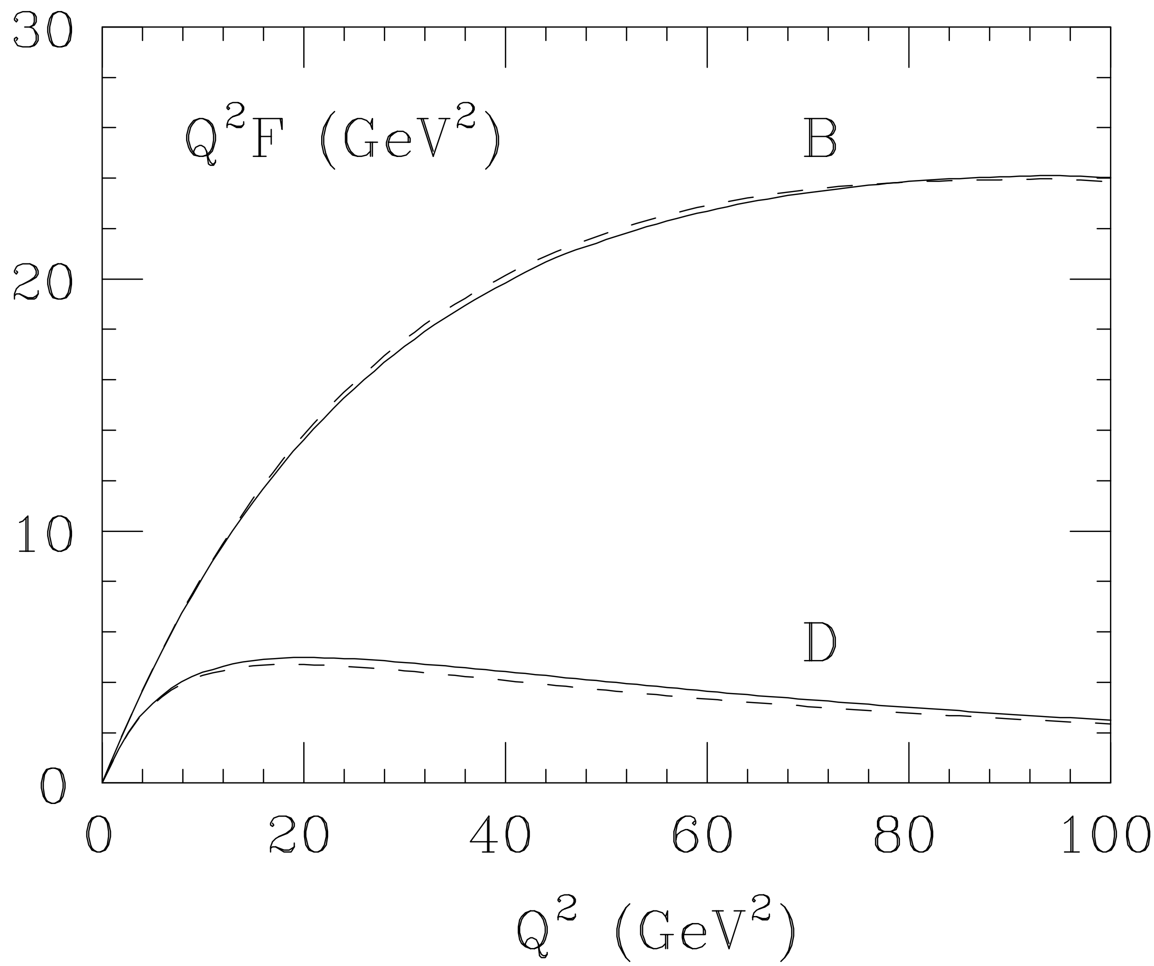

Figure 6