SLAC–PUB–6463
March 1994
M

HARD DIFFRACTION*

J. D. BJORKEN

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

ABSTRACT

An introduction to the rapidly developing subject of diffractive processes containing jets is given, with emphasis on an s-channel picture of the dynamics.

Invited Lectures presented at the
SLAC Summer Institute on Particle Physics:
Spin Structure in High Energy Processes
Stanford, California — July 26–August 6, 1993

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

1 Introduction; What is Diffraction?

This talk is intended as a re-introduction and generalization of simple and ancient ideas on diffraction for a generation of physicists trained mainly for the study of hard collision processes.

Diffraction is shadow physics; hence it is most important when opaque objects collide. Elastic scattering of hadrons at high energies is the most immediate example. But the subject is much more subtle. Inelastic diffraction exists as well, as anticipated long ago by Good and Walker,[1] as a consequence of the composite nature of hadrons. If, in a peripheral collision of hadrons, part of the wave-function of the projectile is attenuated more than the rest, then the internal wave function of the outgoing projectile no longer is the ground-state eigenfunction. Therefore, excitations will exist, even in "shadow" processes.

Because there are many kinds of diffractive processes, and because there is not a uniform terminology on what is meant by the word "diffractive," it is appropriate to start with definitions of what at least I mean by it:

> *A process is diffractive if and only if there is a large rapidity gap in the final-state phase space which is not exponentially suppressed.*

Some elaboration of this definition is clearly needed. The final-state phase space variables implied in the definition are the lego-variables:[2] (pseudo)-rapidity η, azimuthal angle ϕ, and transverse momentum p_t. "Large" will mean much greater than 2, at least 4 to 6 units of rapidity. "Non-exponentially suppressed" means that the probability of finding the gap in the final state is not a strong function of gap width, when the remaining contents of the lego plot are held fixed. Let us elaborate more on this last point:

In general, when a rapidity gap exists, the frame of reference can be chosen so that $\eta = 0$ is in the middle of the gap. Then all collision products are divided into either left-movers or right-movers; all of whose production angles are very small. There are no "wee" hadrons produced. So the above statement "non-exponentially suppressed" means that in the diffraction process

$$a + b \rightarrow A + B \,, \tag{1}$$

where a, A move to the right and b, B move to the left, the cross section

$$\frac{d\sigma_{ab}(\{A\},\{B\},s)}{d\Gamma_A d\Gamma_B} \tag{2}$$

does not fall as a power of s, with $s \propto e^{\Delta\eta}$. (In the above definition, $\{A\}$ and $\{B\}$ stand for sets of internal phase-space variables of systems A and B, and $d\Gamma_A, d\Gamma_B$ their differential phase-space volumes.

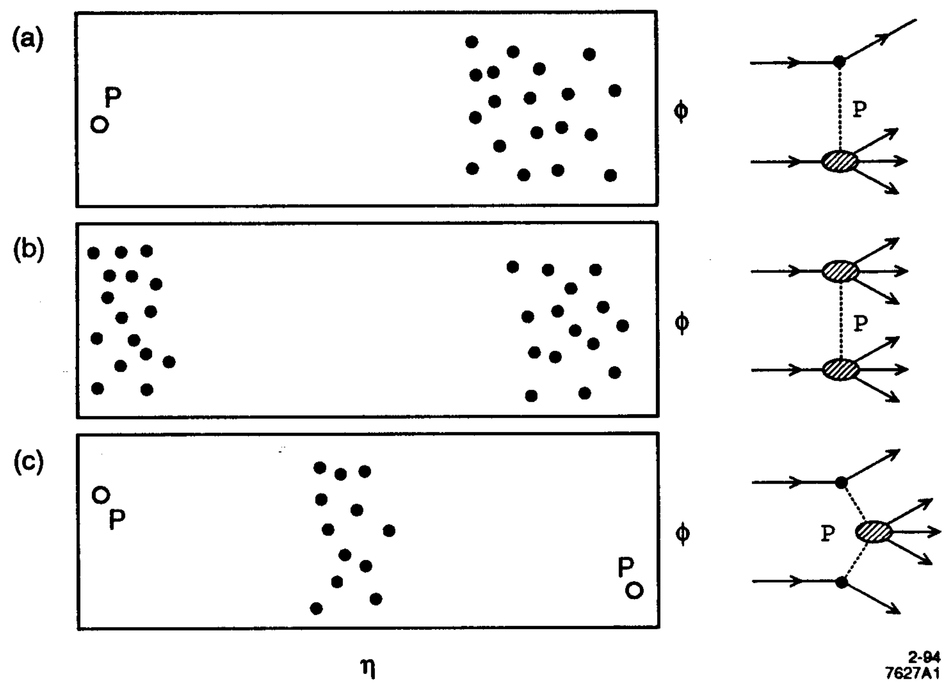

Figure 1. Examples of diffractive processes: (a) single diffractive dissociation, (b) double diffractive dissociation, (c) double Pomeron exchange.

Some familiar examples of diffractive processes are shown in Fig. 1. The description of these processes, especially the two-body processes, can be made either with emphasis on the s-channel or the t-channel. An s-channel description tends to be more like classical optics, and typically uses optical-model concepts.[3] We will emphasize this view later. The t-channel descriptions typically utilize the theory of complex angular momentum, Regge-pole theory,[4] developed in the 1960s. While Regge theory nowadays is somewhat unfashionable, there is no reason, either theoretical or experimental, for its neglect. Indeed, since its foundations are built on the general properties of renormalizable local field theory, the emergence of QCD as the appropriate description of strong interactions argues even more strongly for the relevance of Regge-pole concepts to the phenomenology.

If one tries to express the energy dependence of a candidate rapidity-gap process as a power of s

$$\frac{d\sigma_{ab}(A,B,s)}{d\Gamma_A d\Gamma_B} \sim F_a(A)\left(\frac{s}{s_0}\right)^{\alpha} F_b(B) \tag{3}$$

it is natural (just as in deep-inelastic scattering) to take moments of the amplitudes with respect to the s variable—*i.e.* introduce Mellin transforms and study the properties of the scattering amplitude in the transform space. If one describes the process in terms of exchange in the t-channel of a particle of spin J, the amplitude would depend upon s like $(s/t)^J$. To see this, write

$$A_J(A,B) \approx g(A)\,\frac{P_J(\cos\theta_t)}{t - M^2(s)}\,g(B)\ . \tag{4}$$

If one works out the kinematics to find $\cos\theta_t$ one finds, for massless particles

$$\cos\theta_t = 1 + \frac{2s}{t} \tag{5}$$

with inessential complications for more general cases. Thus as $s \to \infty$,

$$P_J(\cos\theta_t) \to (\text{const.})\cdot\left(\frac{s}{t}\right)^J\ . \tag{6}$$

Therefore one can expect the asymptotic s dependence of scattering amplitudes will be related to the angular momentum of the objects exchanged in the t-channel.

When these objects are composite, the theoretical consequences are very beautiful. One considers together the set of orbital excitations with the same number of nodes in their radial wave functions. A good example in real life is the set of resonances which comprise the ρ-meson and its radial excitations:

$$^3S_1(770),\quad ^3P_2(1320),\quad ^3D_3(1690)\,,\quad \ldots \tag{7}$$

Since the radial wave equation for this set of resonances can in principle be solved for *non-integer* angular momentum (only the strength of the centrifugal barrier term $\ell(\ell+1)/R^2$ is affected), the mass of this set of resonances may be considered as a *continuous* (indeed analytic) function of J (or vice versa). The function $J = J(M^2)$ is called the Regge trajectory of this set of resonances:

With some nice mathematics (Watson-Sommerfeld transform), it then follows that the exchanges of this *set* of resonances has a dependence on s, at given t, which is just given by the Regge trajectory $J(M^2)$ extended from timelike M^2 to spacelike t. An example is given in Fig. 2. The most important feature is that high spin exchanges no longer lead to severe cross-section growth at high energy.

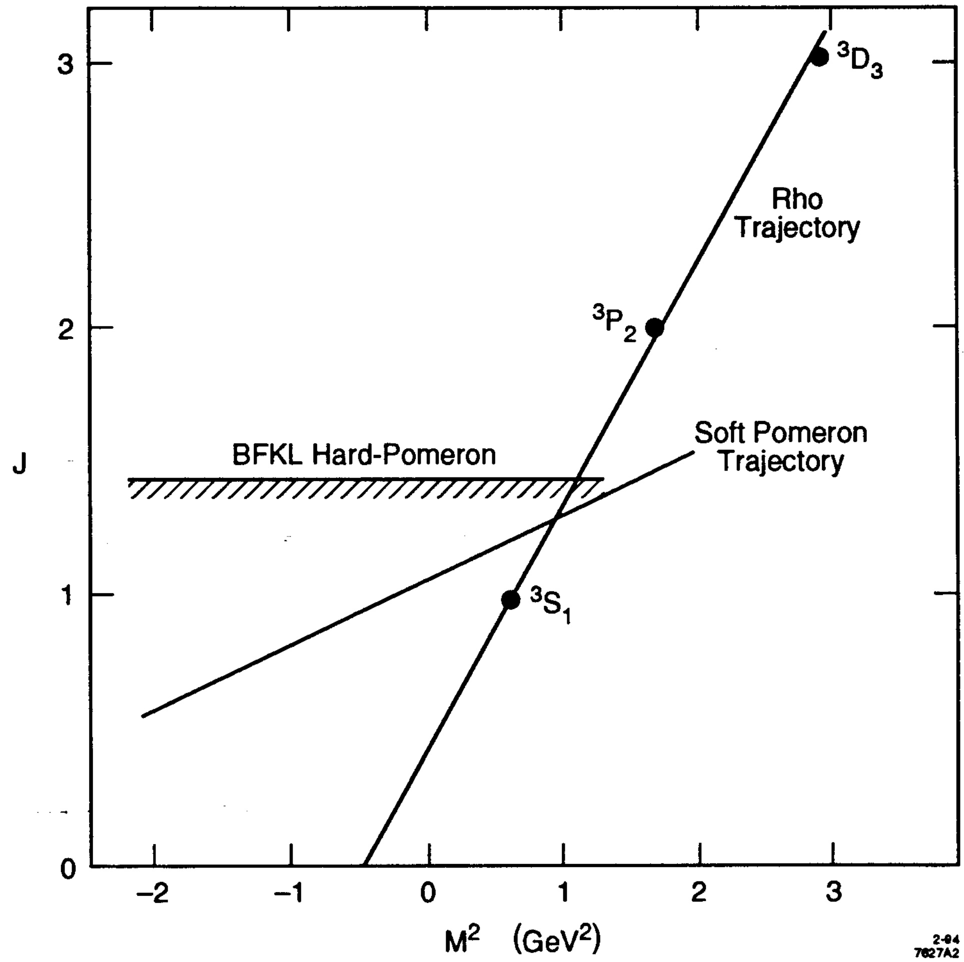

Figure 2. Chew-Frautschi plot of J versus M^2 for a meson trajectory, for the Donnachie-Landshoff "soft Pomeron," and for the BFKL perturbative or hard Pomeron, discussed in Section 2.

This picture works best when the exchanged object is a *bona fide* two-body bound state, in which case one gets a pure power law. (The jargon is a "pole in the j-plane," *i.e.* a pole singularity in the Mellin (or better Legendre) transform of the original amplitude.)

There is an abundant body of experimental data which decisively exhibits the validity of these ideas when applied to exchange of mesons which are "non-singlet," *i.e.* which contain non-vacuum quantum numbers such as charge or flavor.[5]

For high energy diffractive processes, for which the exchanged object is "singlet," *i.e.* carries vacuum quantum numbers, the situation is less clear. Theoretically one does not expect a pure power of s asymptotically, but instead some kind of logarithmic growth of the total cross section. (The jargon for this is "a cut in the j-plane.") However Donnachie and Landshoff[6] have had great phenomenological success assuming the exchanged object, the soft Pomeron (named in honor(?) of I. Ya. Pomeranchuk, who contributed much to the early history of this subject), has a pure Regge trajectory $J(t)$ which intercepts $t = 0$ at $J = 1.08$, with a slope in t (or M^2) four times less than that of the ρ trajectory. They also find simplicity upon assuming that this soft Pomeron couples to constituent quarks in a way quite similar to the photon coupling to quarks.

2 Hard Diffraction

Hard diffraction is a subject which is now only coming into existence. It has an obvious definition:

Hard diffraction is the set of strong-interaction diffractive processes which contain jets in the final-state phase space.

Ingelman[7] suggests a further distinction:

1. *Diffractive hard scattering*[8] *has jets only on one side of the rapidity gap.*
2. *Hard diffractive scattering has jets on both sides of the rapidity gap.*

When the distinction is not to be made, I try to remember to use the two words "hard diffraction," not three.

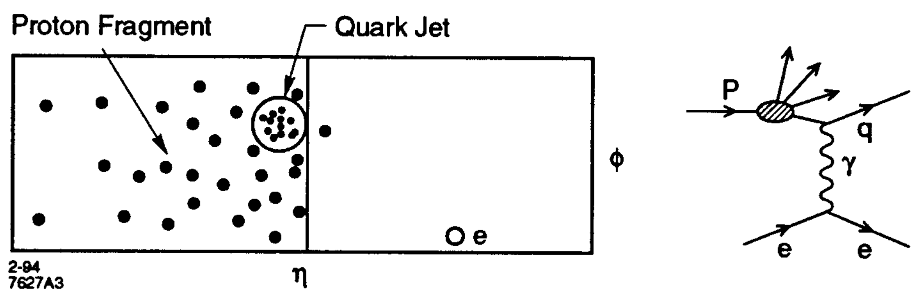

Figure 3. Deep inelastic scattering at HERA: structure of the lego plot.

Before giving some examples of candidate hard-diffraction processes, there are some preliminary kinematic considerations. These have to do with where in the phase-space one most expects the jets to occur. This is best introduced by considering electron-proton deep-inelastic scattering in collider mode, as now occurring at HERA. A lego plot of a typical deep-inelastic final state is shown in Fig. 3. Even for the generic final states which, in γ^*-proton collinear frames of reference, do not contain jets, there will be in the laboratory frame a jet along the direction of the struck quark. If one draws the conventional circle-of-radius 0.7 around the jet core to isolate the jet contents, and if one then constructs a tangent to the circle as shown, then almost all reaction products lie on the side of the tangent common to the nucleon beam fragments;[9] there is a rapidity gap between the quark jet and the electron.

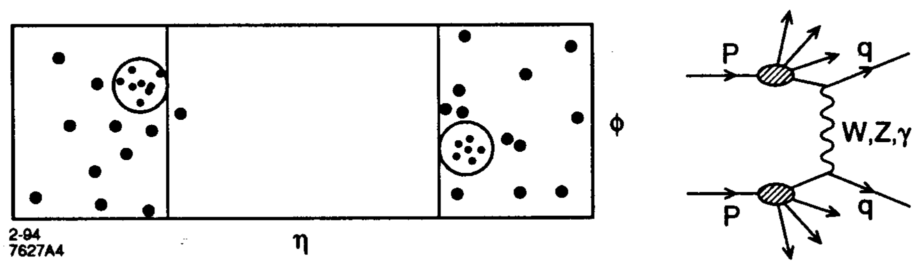

Figure 4. pp scattering via γ, W exchange.

To generalize to hadron-hadron scattering, we first look at hard scattering via photon (or W, Z) exchange. From the above discussion we expect the final state, naively, to look like Fig. 4. The jets are separated by a rapidity gap, as before. But probably this electroweak exchange is smothered by two-gluon exchange, where the two gluons are a net color-singlet, thus simulating a photon.

This two-gluon exchange is the prototypical example of hard diffraction scattering. We shall return to it later, but before doing so must give warning that the factorized form of the scattering process is very naive. We have neglected all the spectator-constituents in the projectiles; if they interact during the collision the rapidity gap will usually be filled in. Thus any parton-level estimate of the hard-diffraction cross-section must be multiplied by the "survival probability of the rapidity gap" $\langle |S|^2 \rangle$.

Various estimates of the quantity $\langle |S|^2 \rangle$ exist.[10] But they all employ essentially the same method, which is to assume no correlations in the impact plane. With this assumption one simply weights the hard cross-section luminosity at a given impact parameter with the transmission probability of the two projectiles at that impact parameter. The latter quantity is measured in elastic scattering. However the assumption of no correlation in the impact-plane is very suspect, and the generic estimate $\langle |S|^2 \rangle \approx 0.1$ for TeVatron energies may be too high.[11]

3 Experiments

The pioneering experiment on hard diffraction stemmed from the seminal proposal of Ingelman and Schlein[12] to measure the "structure function of the Pomeron." They assumed that the Pomeron exchange can be regarded in a way similar to ordinary hadron exchange. If so, it makes sense to measure its structure function via a hard collision process. Indeed such a process can be considered to *define* its structure function.

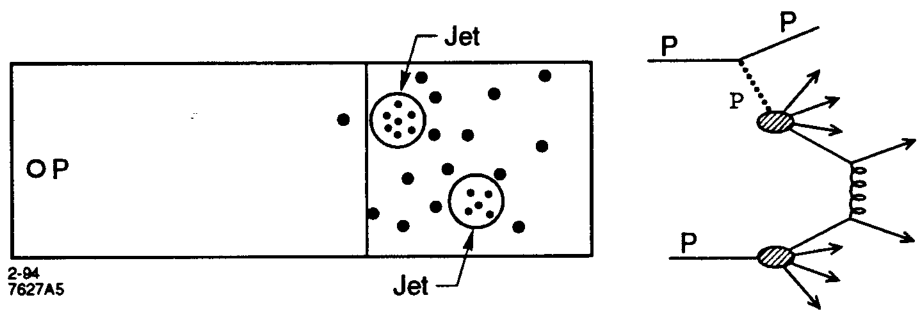

Figure 5. The Ingelman-Schlein process for determining the Pomeron structure function.

The process is shown in Fig. 5 along with the event pattern in the lego plot. The experiment was done[13] at the CERN $SP\overline{P}S$; the recoil proton was tagged with a "Roman pot" detector inserted very near the beam far downstream of the collision point. The diffracted system, which was required to contain dijets, was detected in the UA2 calorimeters. The experiment yielded a remarkable result. While the *a priori* estimates of the structure function were typically soft, $\sim (1-x)^5$, or mesonic[14] (actually $q\overline{q}$) $\sim (1-x)^1$, the experimenters saw evidence for a "super-hard" Pomeron, with 30% of the structure function being concentrated near $x = 1$, like $\delta(1-x)$ or possibly $(1-x)^{-1}$. A possible interpretation of this result will be sketched in the next section.

A similar example of hard diffraction is emerging from HERA, where deep-inelastic final states are seen which contain a leading dijet in the photon fragmentation region and nothing else visible, *i.e.* a rapidity gap toward the proton direction.[15] If interpreted as the process

$$\text{Pomeron} + \text{photon} \rightarrow \text{jet} + \text{jet} \tag{8}$$

the interpretation will again probably require a "superhard" gluon component within the Pomeron.

Finally, the DØ collaboration at the Fermilab TeVatron has searched for rapidity gaps between jet pairs, as described in the previous section. The data[16] is shown in Fig. 6. While the results appear encouraging that a non-exponentially suppressed rapidity gap has been observed, the DØ collaboration quotes only an upper limit. Thus far, only calorimetric information has been utilized, and their difficulties in cleanly defining nothing preclude a stronger statement. Eventually tracking information should be available to provide a more incisive conclusion.

4 The s-channel viewpoint

As we have seen, the Regge-pole approach has created a heritage of looking at diffractive processes from a t-channel viewpoint. In addition, the work of many theorists, especially Lipatov[17] and his associates[18] on the asymptotic behavior at short distances expected from perturbative QCD, also relies on a t-channel point of view. Roughly speaking, the summation of gluon-ladder exchanges leads to a strong growth with energy of parton-parton scattering cross-sections at fixed t as $s \rightarrow \infty$. Typically the behavior of the total cross-section (related to the absorptive part of the forward scattering amplitude) is

$$\sigma_{PQCD}(s) \sim \frac{s^{\omega_P}}{\sqrt{\log s}} \tag{9}$$

or in deep-inelastic scattering

$$F_2(x) \sim \left(\ell n \frac{1}{x}\right)^{-1/2} x^{-\omega_P} \tag{10}$$

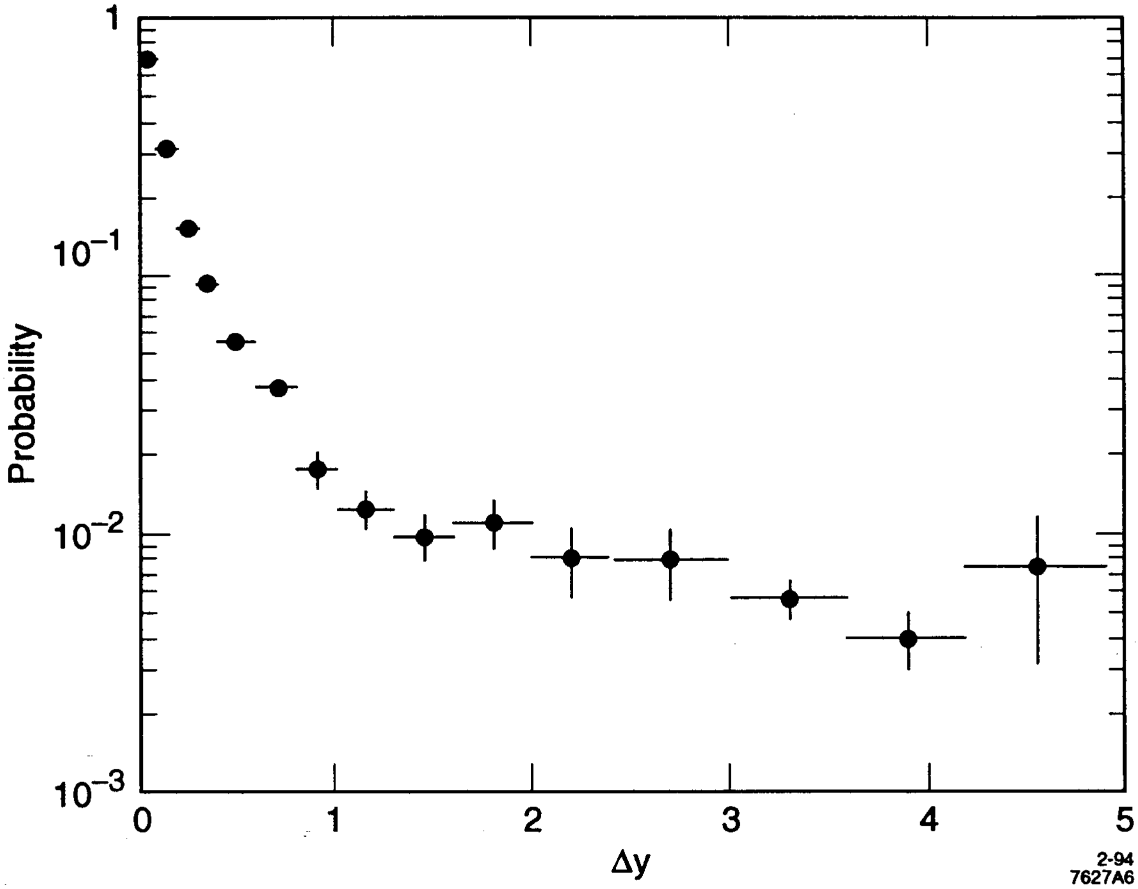

Figure 6. Probability of finding a rapidity gap between jet pairs versus gap width; data[16] from the DØ collaboration at the Fermilab collider.

with the intercept of this "BFKL Pomeron" given by

$$\omega_p \approx \frac{12\alpha_s}{\pi} \ell n\, 2 \approx 0.4 \ . \tag{11}$$

This behavior is consistent with what is being observed at HERA.

But even for this situation, an s-channel point of view appears to be very useful. In what follows, we do not enter into the ramifications of the BFKL Pomeron, but only look at its starting point, two-gluon exchange.

Let us return to our example of small-angle parton-parton scattering via single gluon exchange. At this level there will be no rapidity gap because color has been exchanged. Exchange of a second gluon, however, can provide the color neutralization.

To estimate all this, it is easiest[19] to work in impact-parameter space, not transverse-momentum space. The original one-gluon "Coulomb" amplitude is,

$$T(q) \approx \frac{\alpha_s}{q^2} \; . \tag{12}$$

The Fourier transform of $1/q^2$ is a logarithm, so one has

$$T(b) \sim \alpha_s \log bR \tag{13}$$

with R some screening radius, evaluation of which goes beyond the realm of perturbation theory.

The virtue of impact space is that at high energy the impact parameter is conserved during the collision process—it is essentially the classical limit of angular momentum. Thus the absorptive part of the two-gluon exchange amplitude is easily calculated via unitarity:

$$\mathrm{Im}\, T(b) = |T(b)|^2 \sim \alpha_s^2 \log^2 bR \; . \tag{14}$$

This contains a log and dominates the real amplitude. The ratio to the lowest order amplitude is also $\alpha_s \log bR$. So, since $\alpha_s \sim \log^{-1}$, we have

$$\frac{\sigma(2g\,\text{color singlet})}{\sigma(1g\,\text{color octet})} \sim \alpha_s^2 \log^2 bR = (\text{constant}) \cdot [1 + \mathcal{O}(\alpha_s)] \; . \tag{15}$$

The constant just consists of color factors, π's, etc. and is about 0.1. Since the ratio does not depend upon impact parameter, the same result will survive a Fourier-transform back to transverse-momentum space. This implies[20] that the fraction of two-jet events with a rapidity gap between them will be given in this simple approximation by

$$\frac{\sigma(2\,\text{jet, gap})}{\sigma(2\,\text{jet, no gap})} \approx 0.1 \left\langle |S|^2 \right\rangle \lesssim 0.01 \tag{16}$$

independent of p_t and of width of the gap. Indeed this result is even independent of whether the objects which scatter are quarks or gluons.

So the main message is that from both the theoretical and experimental point of view, it is easiest to estimate (and measure!) the ratio of the process with rapidity gap present to the *same* process with no rapidity gap. We conjecture that in fact this procedure generalizes. Suppose the final state consists of left-moving jets $\{A\}$ and right-moving jets $\{B\}$. Then the conjecture is that

$$\frac{d\sigma_{AB}}{d\Gamma_A d\Gamma_B}(\text{gap}) = (\text{constant}) \cdot \langle |S|^2 \rangle \frac{d\sigma_{AB}}{d\Gamma_A d\Gamma_B}(\text{no gap}) . \tag{17}$$

The constant should again be roughly 0.1 but may be somewhat different, because inelastic, not elastic, unitarity will be utilized. The strategy for the proof (not yet completed) is again to

1. Use the large N_c approximation to simplify the color-counting.
2. Fourier-transform to impact space.
3. Use (inelastic) unitarity to estimate the absorptive part of the color-singlet two-gluon exchange.
4. Argue that the result is valid over a large enough range of impact parameters to allow the Fourier transform to momentum space to be done without disrupting the result.

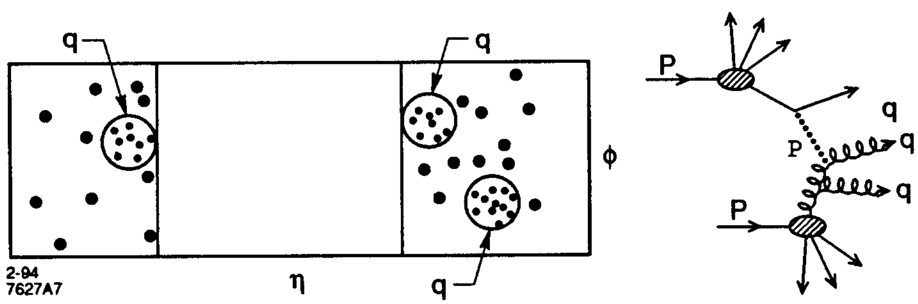

Figure 7. A generalized Ingelman-Schlein process.

Now let us interpret this s-channel result from the t-channel point of view. We suppose that there exist no snags in the proof, and consider the generalized Ingelman-Schlein process shown in Fig. 7. It is a little simpler to analyze than the real experiment because all essential parts of the process can be considered at the parton level. We see that, since the ratio of the Pomeron-exchange cross-section to the single-gluon-exchange cross-section does not depend on kinematic parameters, *the Pomeron acts as if it were a single gluon.*[21] Thus we reproduce the s-channel results (to logarithmic accuracy) by assuming the structure function of the Pomeron is

$$F_g(x) = (\text{constant}) \cdot \langle |S|^2 \rangle \; \delta(1-x) \tag{18}$$

or perhaps

$$F_g(x) \approx (\text{constant}) \cdot \langle |S|^2 \rangle \; (1-x)^{-1} \tag{19}$$

which to logarithmic accuracy is the same thing. Note that this parton distribution is unusual not only for its singular x-dependence but also for its normalization, which contains the factor $\langle |S|^2 \rangle$, something especially dirty and nonperturbative. From this point of view, one cannot hold out much hope that in general the t-channel "hard Pomeron" is a simple object. In a most interesting and important paper,[22] Collins, Frankfurt and Strikman argue that the feature of a singular x-distribution in the Pomeron structure function is not unreasonable; their arguments for this are not dissimilar from what we present here. But they go on to argue that the familiar property of factorization will not hold in general, so that there will not be a unique definition of the parton distribution inside a Pomeron, good for all hard processes. They in particular discuss an interesting example of deep-inelastic electroproduction of dijets, with p_t^2 large compared to the Pomeron t. Under these circumstances, the cross-section without gap is suppressed by a factor t/p_t^2, owing to the smallness in space ($\sim p_t^{-1}$) of the $q\bar{q}$ color dipole created by the photon relative to the resolution scale $t^{-1/2}$. This suppression mechanism acts on the second gluon exchanged as well, leading to a factor t/p_t^2 suppression in the *ratio* of the process with rapidity gap to that without the gap. An important lesson is that the rule we have conjectured, Eq. (16), will have exceptions when more than one distance (or p_t) scale is operative in the dynamics.

5 Summary Remarks

On the level for which we have considered the "hard Pomeron," namely, simple two-gluon exchange, the physics is extremely simple when viewed in the s channel. To logarithmic accuracy, one of the two gluons carries most of the momentum transfer. The other adds a "Coulomb phase' to the amplitude. In QED the Coulomb-phase contribution is at high energy innocuous (at low energies it makes

Keplerian orbits!). In QCD it is not innocuous because of color; color-singlet exchange occurs first in the second order. Nevertheless, it allows the Pomeron ≈ gluon picture to be credible at this level.

When BFKL iterations are added to make the two-gluon exchange into a ladder, the situation is still most easily described in the s channel and the importance of the "Coulomb-phase" point of view survives a variety of complications.[23]

But the most dramatic consequence of the BFKL iterations is the strong s-dependence which is generated, an extra $s^{0.4}$ behavior of parton-parton interactions at fixed large t. This means eventually that even partons become black.[24] When the energies are reached such that black partons collide and are opaque relative to each other, there may well occur really new strong-interaction phenomena—phenomena too novel for theorists to anticipate.[25] At that point the purely *experimental* study of strong interactions will become extremely interesting.

I think that in the long run the study of hard diffraction may well take a place of importance relative to soft diffraction analogous to the importance of deep-inelastic processes relative to elastic electron-proton scattering, or other such exclusive processes. It is worth a great deal of theoretical and experimental attention.

References

[1] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).

[2] We shall not distinguish between true rapidity y and pseudo-rapidity η.

[3] For a review of this approach, see A. Bialas and W. Czyz, Acta Phys. Pol. **B12**, 459 (1981).

[4] P. Collins and E. Squires, *An Introduction to Regge Theory and High Energy Physics*, Cambridge, University Press, Cambridge (1977).

[5] For a nice review of the data and phenomenology, see A. Irving and R. Worden, Phys. Rept. **34**, 117 (1977).

[6] A. Donnachie and P. Landshoff, Phys. Lett. **B296**, 227 (1992).

[7] G. Ingelman, unpublished.

[8] See E. Berger, J. Collins, D. Soper, and G. Sterman, Nucl. Phys. **B286**, 704 (1987) for an extensive discussion of these processes.

[9] See J. Bjorken, Phys. Rev. **D45**, 4077 (1992) appendix, for the argument. Monte Carlo exercises (G. Greenbaum, unpublished) confirm this analytic argument.

[10] J. Bjorken, Phys. Rev. **D47**, 101 (1992); E. Gotsman, E. Levin, and U. Maor, Phys. Lett. **B309**, 199 (1993).

[11] There is also concern that somehow soft perturbative QCD gluons might get radiated into the gap. This should not happen; see D. Zeppenfeld, MAD–PH–806, November 1993.

[12] G. Ingelman and P. Schlein, Phys. Lett. **152B**, 256 (1985).

[13] A. Brandt *et al.*, UA8 Collaboration, Phys. Lett. **B297**, 417 (1992).

[14] A. Donnachie and P. Landshoff, Nucl. Phys. **B303**, 634 (1988).

[15] M. Derrick *et al.* (ZEUS collaboration), Phys. Lett. **B316**, 207 (1993).

[16] S. Abachi *et al.* (DØ collaboration), FERMILAB–PUB–94–005–E, January 1994 (to be published).

[17] L. Lipatov, Sov. Phys. KETP **63**, 904 (1986).

[18] Ya. Balitsky and L. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978); E. Kuraev, L. Lipatov, and V. Fadin, Sov. Phys. JETP **45**, 199 (1977).

[19] This goes all the way back to the original works on light-cone quantization: J. Kogut and D. Soper, Phys. Rev. **D1**, 2901 (1970). In the context of QCD, it has been developed by N. Nikolaev and B. Zakharov, Zeit. Phys. **C49**, 607 (1991); **C53**, 331 (1992).

[20] J. Bjorken, Ref. 10.

[21] L. Frankfurt and M. Strikman, Phys. Rev. Lett. **63**, 1914 (1989), **64**, 815 (1990); L. Frankfurt, presented to meeting of the FAD working group, Dallas, Texas (unpublished).

[22] J. Collins, L. Frankfurt, and M. Strikman, Phys. Lett. **B307**, 161 (1993).

[23] N. Nikolaev and B. Zakharov, *op. cit.,*; also A. Mueller, Columbia University preprint CU–TP–609 (August 1993); L. Lipatov, University of Padua preprint DFPD/93/TH/70 (October 1993).

[24] The opacity of a constituent quark has been considered by V. Anisovich, E. Levin, M. Ryskin, Yad. Fiz. **29**, 1311 (1979); see especially V. Shekhter, *ibid.*, **33**, 817 (1981).

[25] See, for example, J. Bjorken, Acta. Physica Polonica **B23**, 637 (1992).