SLAC–PUB–6459
May 1994
(N)

LATTICES FOR SYNCHROTRON RADIATION SOURCES*+

MAX CORNACCHIA
CORNACCHIA@SSRL01.SLAC.STANFORD.EDU
Stanford Linear Accelerator Center
Stanford Synchrotron Radiation Laboratory
Stanford, CA 94309, USA

1. Introduction

This chapter introduces the reader to the motion of electrons[1] in a storage ring, and to the connection between electron beam dynamics and the properties of synchrotron radiation.

The system of magnetic lenses that guides and focuses an electron beam is called the *lattice*. The choice of a lattice for a synchrotron radiation source is, arguably, the single most important decision in the history of a project. The lattice determines the emittance of the electron beam, the brightness of the photon beam, the beam lifetime, the quality of the experimental conditions, the number of insertion devices that can be accommodated in the straight sections, and the size and cost of the accelerator. The best choice of lattice is not a straightforward affair, involving complex performance and cost trade-offs, and a certain amount of intuition and subjectivity. The various types of lattices, and future directions in lattice design, will be covered, however briefly, in this review.

Synchrotron radiation is emitted from the bending sections of the electron trajectory and in the straight sections, where insertion devices might be installed (see Chapter 12). From the *source points* the radiation is channeled into a *beam line* for experimental use. The lattice and the electron beam energy define the trajectory and, together with the natural divergence of the radiation, the size and divergence of the source.

After a broad overview in Section 2, the magnetic forces acting on the electrons and the associated differential equations of motion are discussed in Section 3. The solutions of the equations are given without derivation; the method of solution is outlined, and references for deeper studies are given.

Section 4 shows how the dynamics of electron motion in magnetic lattices and the emission of radiation define the beam emittance. Examples of lattices for machines in operation or under construction are given in Section 5. Throughout this chapter, the electrons are assumed to be ultra-relativistic.

---

* Work supported by the Department of Energy contract DE–AC03–76SF00515. SSRL is funded by the Department of Energy, Office of Basic Energy.

+ Chapter 2 of Synchrotron Radiation Sources: a Primer, H. Winick, ed., World Sci. Pub. Co.

2. Overview of Electron Dynamics in a Storage Ring

2.1. The Storage Ring

A storage ring accumulates and stores electrons that have been pre-accelerated and transported from an Injection System (see Chapter 3). The electrons are injected and stored in packets called bunches, which are held together in the direction of motion by the bunching effect of the radio-frequency system (see Chapter 4). The latter also provides the energy lost by radiation and, if acceleration is needed, the energy gain required by the particles to keep in step with the magnetic field

The electrons circulate inside a doughnut-shaped chamber, in which a high vacuum is maintained, delimited by metallic walls (see Chapter 8). The chamber is surrounded by magnets alternating with empty, or drift, spaces. The magnets curve the electron trajectories (dipole field) and keep them close together in the plane perpendicular to the direction of motion (quadrupole field).

2.2. Collective and Individual Motion, Frame of Reference.

Figure 1 gives simplified top and cross sectional views of electron bunches, frozen in time, circulating in a vacuum chamber surrounded by magnets. The picture is much out of proportion. The circumference of a ring is on the order of 50–250 m for UV and soft x-radiation sources (beam energy: 0.5–3.0 GeV) and 800–1500 m for hard x-ray sources (6–8 GeV). The bunch length is on the order of centimeters or smaller. Tens to hundreds of bunches may circulate in a storage ring.

The motion[2] of the electrons is described in a reference system with an azimuthal axis tangent to the orbit, and the transverse horizontal (x) and vertical (y) coordinates, lying in the plane perpendicular to the orbit (indicated in Fig. 2). The azimuthal coordinate s is the independent variable, and is the distance along the orbit from a reference point s_o. Lattice designers and orbit scientists spend much of their time studying the functions $x(s)$, $x'(s) = dx/ds$, $y(s)$, and $y'(s)$.

The trajectory of an individual electron in a storage ring is qualitatively shown in Fig. 2. It consists of oscillations around an orbit that closes on itself after one revolution, appropriately called the *closed orbit*. The oscillations are called *betatron oscillations* and take place in both the horizontal and vertical planes. The orbit has horizontal and vertical components.[3] In an accelerator with neither vertical bends nor magnetic errors or misalignments, the orbit lies in the horizontal plane (x-s in Fig. 2), and the vertical closed orbit is zero everywhere. This ideal situation does not occur in practice, and there always is an orbit component in the vertical plane.

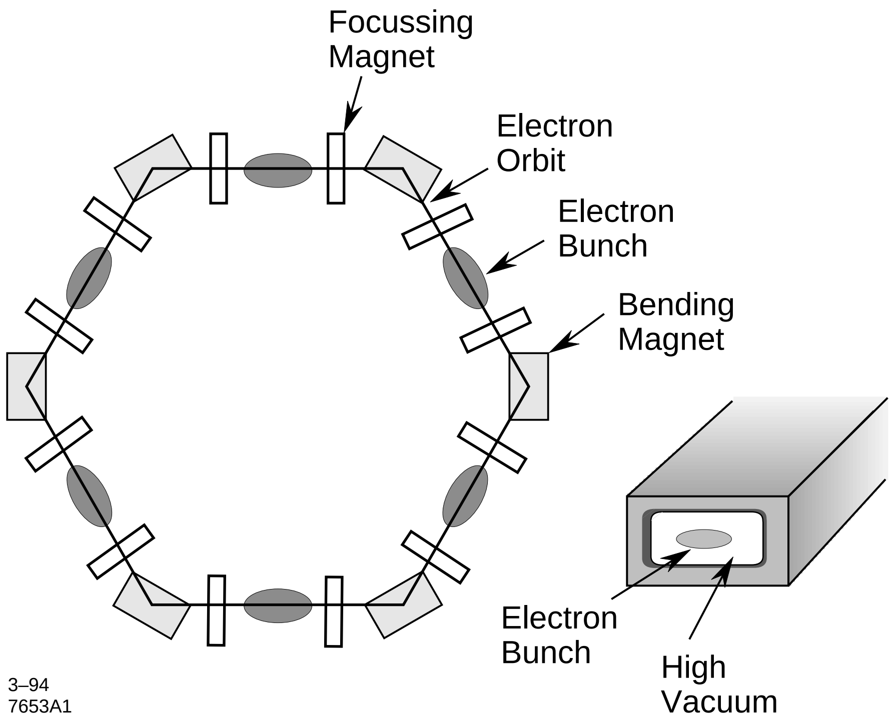

Fig.1 Simplified top and cross sectional views of electron bunches circulating in a storage ring.

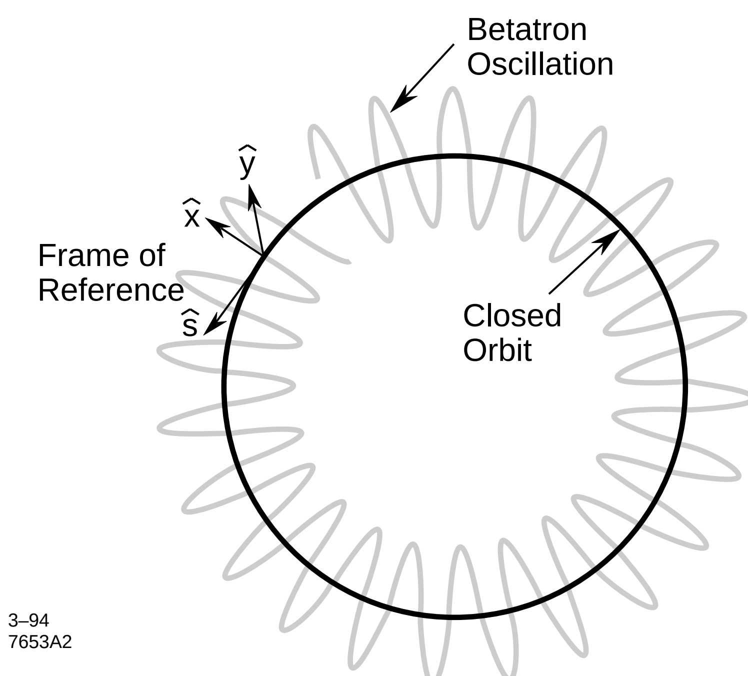

Fig.2 Descriptive view of the closed orbit and a betatron oscillation. The open section is meant to emphasize that the betatron oscillation is not closed.

A large number of electrons (10^{10} or more per bunch) oscillate around a closed orbit with all possible phases and amplitudes. The amplitudes are within a given range defined by the transverse size of the vacuum chamber or, as explained in Section 3, by the maximum stable amplitude.

Not all the particles in a bunch have the same energy. Due to the quantum emission of radiation, there is a distribution of energies. To each energy, there corresponds a closed orbit, around which off-energy particles execute betatron oscillations.

2.3. Lattice Definition

The lattice of a storage ring is defined to be the sequence of magnetic lenses designed to insure that electrons circulate for a period of several hours (corresponding to billions of revolutions) while maintaining the appropriately small dimensions of the beam. The former is requisite to guarantee the users long periods of uninterrupted emission of radiation, the latter to provide a small and hence bright source of light.

The magnetic properties of the lattice, together with the electron energy, determine the transverse size and divergence of the beam which, after convolution with the divergence of the radiation (see Chapter 14), define the photon beam size.

3. Equations of Motion and Solution

3.1. Basic Magnetic Elements In a Lattice

The basic lattice of a storage ring consists of a sequence of dipole (bending) and quadrupole (focusing or defocusing) magnets joined by field-free regions, or *drift spaces*. The sequence closes on itself to allow the electrons repeated revolutions around a determined reference orbit and within a confined region around this orbit.

The *bending magnets* are characterized by a magnetic field that is perpendicular to the direction of motion and is uniform in the region occupied by the beam (see Chapter 5). A bending magnet causes a charged particle to follow a circular trajectory along its length. Straight trajectories are joined by sections of circles. The bending magnets are positioned in such a way that there exists a trajectory that is a closed curve that satisfies given geometrical constraints. An electron on this trajectory repeats its motion every revolution. This trajectory is the *closed orbit*, already introduced in Section 2.2. In the absence of magnetic, alignment, and other imperfections, it is also the ideal (or design, or reference) orbit. The beam lines are positioned to receive the light emitted from, and are centered tangential to, the closed orbit in the bending magnets and insertion devices.

If the bending magnets were the only elements of the lattice, particles with spatial coordinates different from those of the ideal orbit would move progressively away from this orbit. Since a beam of electrons contains a distribution of particles having different positions and angles, as well as energies, eventually the whole beam would spread out and be lost. For this reason, focusing elements are required to keep together this collection of particles having different coordinates. These elements are *quadrupole* *magnets*, and they are characterized by a magnetic field whose components are linear functions of the x and y coordinates.[4]

The components of the magnetic field of the basic lattice components, dipoles, and quadrupoles, are:

$$
\begin{aligned}
&B_y = B_0, \\
&B_x = 0, && \text{for dipoles, and} \\
&B_y = Gx, \\
&B_x = Gy, && \text{for quadrupoles,}
\end{aligned} \tag{1}
$$

where B_o and G are constant. In a quadrupole, the field is zero at $x = y = 0$. This point defines the magnetic axis in the azimuthal direction. A quadrupole for which a particle away from the magnetic axis is deflected back onto(away from) it is called focusing(defocusing). It is a consequence of Maxwell's equations that a quadrupole field that is horizontally focusing is vertically defocusing, and vice versa. A sequence of focusing and defocusing quadrupoles, appropriately designed, can focus the beam in both planes. This is demonstrated in the theory of strong focusing,[5] on which all modern synchrotrons are based.

3.2. The Synchronous Orbit

For a given bending field, there is one value of the electron energy for which the particle follows the ideal orbit. We call this the *synchronous* energy and the particle the synchronous particle. The energy is given by the expression equating the centrifugal force to the Lorentz force:

$$E_0 = ecB_0\rho_0, \tag{2}$$

where E_o is the electron energy, e the electron charge, c the speed of light in a vacuum, B_o the bending field, and ρ_o the radius of curvature in the field of the dipole magnets.[6]

In a more commonly used, form, Eq. (2) can be written as

$$E_0[GeV] = 0.3B[T]\rho_0[m], \tag{2a}$$

where GeV, T, and m denote giga-electron-volt, Tesla, and meter, respectively.

3.3. Equations of the Synchronous Orbit and Their Solutions

An electron that, at a given initial azimuthal position s_o, has the same energy as the synchronous particle, but is displaced (in position or angle in the transverse coordinates x, x', y, y') with respect to the ideal orbit, executes *betatron oscillations* around this orbit. These oscillations occur in the horizontal and vertical planes and are defined by the following differential equations of motion:

$$
\begin{aligned}
&x'' + K_x(s)x = 0, \\
&y'' + K_y(s)y = 0.
\end{aligned} \tag{3}
$$

The focusing strengths $K_{x,y}(s)$ are proportional to the quadrupole fields (focusing or defocusing) and also include relatively small effects of the dipoles not discussed here.[7]

Because the magnets have constant fields along the direction of motion, these functions are dichotic ($K_{x,y}(s)$= constant = 0 in magnet-free regions, or $\neq 0$ in a magnetic field). Equation (3) was solved in the original, classical paper, in which the principles of strong focusing were described.[5]

$$x(s) = \sqrt{\varepsilon_x \beta_x(s)} \cos[\phi_x(s) + \phi_{0x}],$$
$$y(s) = \sqrt{\varepsilon_y \beta_y(s)} \cos[\phi_y(s) + \phi_{0y}]. \tag{4}$$

Here, *x* and *y* are the transverse displacements from the closed orbit defined earlier. The meaning of the constants ε_x and ε_y is discussed in Section 3.5, together with the functions β_x and β_y. The betatron phases ϕ_x and ϕ_y are functions of the distance s along the closed orbit, and ϕ_{ox} and ϕ_{oy} are the initial phases. The $\phi_{x,y}$ are given by

$$\phi_{x,y} = \int_0^s \frac{ds}{\beta_{x,y}(s)}. \tag{5}$$

The motion is a pseudo-harmonic oscillator, with instantaneous amplitudes proportional to the square root of the β-functions and instantaneous wavelength $\lambda_{x,y}(s) = 2\pi\,\beta_{x,y}(s)$.

3.4. The β–Function

The reader who has been exposed to accelerator terminology will have heard the term β–function used often. It was seen in Eq. 4 that these functions (horizontal and vertical) are related to the maximum amplitude of the oscillations at a given location s:

$$x,y_{max}(s)=\sqrt{\varepsilon_{x,y}\beta_{x,y}(s)}\,. \tag{6}$$

Similarly, the maximum angle of the oscillation at a location s is given by

$$x',y'_{max}(s)=\sqrt{\varepsilon_{x,y}/\beta_{x,y}(s)}\,. \tag{7}$$

The β-functions are periodic in s and follow the periodicity of the lattice. Together with the constants $\varepsilon_{x,y}$, they determine the maximum amplitude of the betatron oscillations. The units in use are meter-radian for $\varepsilon_{x,y}$ and meter/radian for the β-functions. Examples of β-functions are given in Fig. 10.

3.5. The Emittance

Figure 3 shows the locus of all possible positions and angles (x,x' or y,y') of a particle that is going around the accelerator, as it would be monitored by an observer placed at an azimuth s. All the points fall in an ellipse whose area, it can be shown, is equal to the constant $\varepsilon_{x,y}$ multiplied by π. The shape and orientation of the ellipse changes as a function of s. In an optical system without acceleration, emission of radiation, collective effects, or horizontal-vertical coupling, $\varepsilon_{x,y}$ remains constant as the particle revolves around the accelerator (a consequence of Liouville's theorem). In reality, this “constant “ is perturbed by radiation emission and acceleration, and this leads to the statistical concepts discussed in Section 4.

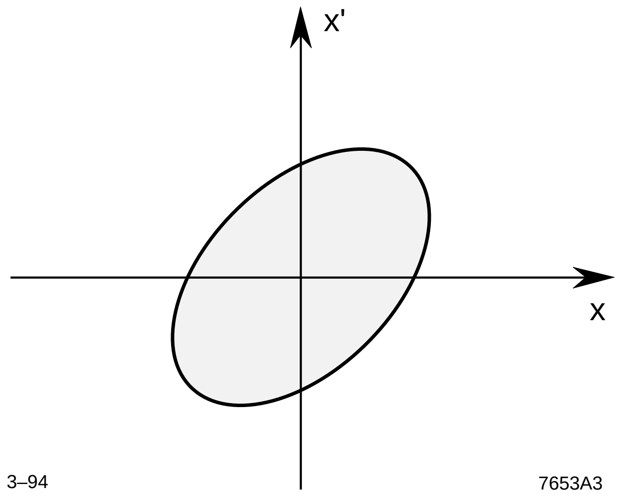

Fig.3 Phase Space ellipse.

When the ellipse represents the motion of that particle in a bunch with the highest value of $\varepsilon_{x,y}$ is the *emittance* of the beam. Its importance is immediately recognized: multiplying by the value of the β-function at a given location and taking the square root (Eq. 6) gives the value of the maximum amplitude of the oscillations in the beam, its size.[8]

In an electron storage ring the distribution of betatron oscillation amplitudes is Gaussian, and it is normal practice to define the emittance of the beam as the values of the constants $\varepsilon_{x,y}$ that are related to the standard deviation of the distribution of amplitudes and angular divergences. The relationships are given by the expressions (derived from Eqs. (6) and (7)),

$$\sigma_x = \sqrt{\varepsilon_x \beta_x}\ , \sigma_y = \sqrt{\varepsilon_y \beta_y}\ , \qquad (8a)$$

$$\sigma'_x = \sqrt{\varepsilon_x / \beta_x}\, , \sigma'_y = \sqrt{\varepsilon_y / \beta_y}\ , \qquad (8b)$$

where $\sigma_{x,y}$ and $\sigma'_{x\,y}$ are the distribution standard deviations of position and angle.

Table I shows the horizontal emittance of a few representative synchrotron radiation sources. One should note how the electron beam emittance of the various generations of storage rings has evolved towards smaller and smaller values, producing photon beams with smaller and smaller beam sizes and divergences, i.e., brighter photon beams.

3.6. Tunes and Resonances

3.6.1. Definition of Tunes

The numbers of horizontal and vertical betatron oscillations per ring revolution are called the tunes and are denoted by the symbols ν_x and ν_y. From Eq. (5), the tunes are given by

Table I.		
Facility	Energy (GeV)	Emittance ($\times 10^{-9}$ meter-radian)
SRS	2.0	108
BESSY I	0.8	38
NSLS VUV	0.7	138
NSLS x-ray	2.5	102
SPEAR	3.0	135
Photon Factory	2.5	130
ALS	1.5	4
APS	7.0	8
ESRF	6.0	7
SPRING-8	8.0	7

$$\nu_{x,y} = \frac{1}{2\pi}\int_0^C \frac{ds}{\beta_{x,y}(s)}. \tag{9}$$

The integral is extended to the entire lattice length C. The tunes play an important role in the stability of the motion. Their values vary, depending on the optics, but their integer part is on the order of 5–15 units in UV and soft x-ray light sources. In larger, hard x-ray machines, like for instance ESRF, they have the values 36.2 (horizontal) and 11.2 (vertical). Sometimes the symbol $Q_{x,y}$ is used for the tunes.

3.6.2. Survey and Magnetic Imperfections, Linear and Nonlinear Resonances

The analysis of the motion shows[9] that there are certain values of the tunes that potentially threaten the stability of the motion. Considering for a moment the transverse plane only (i.e. neglecting energy oscillations) they are those that satisfy the relationship

$$m\nu_x \pm n\nu_y = p, \tag{10}$$

where m, n, and p are integer values. Equation (10) expresses the phenomenon that, if there are magnetic perturbations in the accelerator (unavoidable), the perturbing effect (colloquially called the kick in accelerator jargon) can add up at each revolution, causing the amplitude of the oscillations to grow. For this to happen, the numerical relationship of Eq. (10) must be satisfied, otherwise the perturbations tend to cancel each other over a sufficiently large number of turns. Since m and n can take any integer values, it appears very difficult to find a pair of tunes values that escape Eq. (10). Fortunately, it happens that the perturbing effect becomes weaker and weaker as the *order of the resonance*, defined as the sum of |m| and |n|, becomes larger. In general, in electron accelerators, one does not worry about resonances for which |m| + |n| > 5. This is because radiation damping tends to neutralize the resonant amplitude growth when it and the damping rates are of the same order (see Section 4.3).

It is not necessary for Eq. (10) to be perfectly satisfied for a magnetic perturbation to be felt. There is a region around a resonance line, defined by Eq. (10), where the trajectory can be perturbed. Fortunately, this band (called *stop-band width*) becomes narrower the higher the order of the resonance.

3.6.2.1. Linear Resonances, Orbit and Focusing Perturbations. The resonances for which |n| + |m| ≤ 2 are driven by *linear imperfections* in the lattice. The resonances $\nu_{x,y}$ = integer are particularly disruptive. They are driven by magnetic imperfections of the dipole type, by survey imperfections in the transverse locations of the quadrupoles, and by rotational errors in the placement of the dipoles. These resonances cause closed orbit distortions and are responsible for the movement of the photon beam that is so disruptive to the experimentalists. The orbit distortions act like $1/\left(\nu_{x,y}^2 - p^2\right)$ (where p is any integer) and, although the tunes are normally set at a respectable distance from an integer value, orbit distortions can be, and are, driven at any tune values. For this reason, *dipole correctors* are used to correct the orbit distortions and are a necessary part of any accelerator (Chapter 13 is devoted to the important aspects of orbit correction). To reduce the amplitude of the orbit distortions, tight tolerances are set for the random relative variation of the bending field (on the order of a few times 10^{-4} rms), for the transverse positioning of the quadrupoles (typically 0.10–0.15 mm rms) and for the rotation angle of the bending magnets (0.5–1.0 mrad rms),

Magnetic imperfections of the quadrupole type drive second order resonances. They perturb the β-functions, couple the horizontal and vertical motion (see Section 3.6.2.3), and, if strong enough, may lead to an unstable lattice. The tolerances on the variation of the field gradient (G in Eq. (1)) from quadrupole to quadrupole are specified to limit this effect, and are typically on the order of 10^{-3}.

3.6.2.2. Non-linear Resonances. Those resonances for which |n| + |m| > 2 are driven by nonlinear fields, for example the two third integer resonances:

$$\begin{aligned} 3\nu_x &= p, \\ 2\nu_y \pm \nu_x &= p. \end{aligned} \tag{11}$$

They are driven by sextupoles fields that have the form

$$\begin{aligned} B_y &= S\left(x^2 - y^2\right), \\ B_x &= S2xy, \end{aligned} \tag{12}$$

where S is the sextupole strength, normally expressed in T/m^2. Sextupole magnets are part of any storage ring lattice because, as we shall see in Section 3.9.1, they are needed to correct the chromatic aberrations of the quadrupoles.

Higher order resonances are driven by magnetic fields with higher order non-linearities. For instance, octupole fields (those that have cubic dependence on the displacement, $B_y(x,0) = \text{constant}\ x^3$) drive fourth order resonances, for which |n| + |m| = 4. Decapole fields (quartic dependence on displacement) drive fifth order resonances, and so on. Some resonances require a slight rotation of the magnetic axis in order to be driven, and this often sets the survey tolerances.

At the construction stage the magnet builder requires a set of tolerances from the accelerator physicists for the purity of the magnetic field.[10] This is typically on the order of a few times 10^{-4}. The subject of beam-stability in the presence of non-linear fields is discussed further in Section 3.9.2.

Two more points need to be mentioned concerning Eq. (10). It can be shown that only the + sign (*sum resonances*) on the left hand side of the equation leads to indefinite growth in both the horizontal and vertical directions. The – sign resonances (*difference resonances*) lead to a transfer of oscillation amplitudes from the horizontal into the vertical, and vice versa, but the motion is bounded. The behavior is much like that of a coupled pendulum, with the maximum amplitudes beating between the two directions of transverse motion. Sum resonances are in general much more dangerous.

For a resonance condition to be established, the perturbation (dipoles, quadrupoles, non-linear fields) must have a p-th (integer of Eq. (10)) Fourier component, analyzed as a function of the azimuth, that is non-zero. This is the harmonic that drives the resonance. Linear and non-linear resonances are corrected by canceling out, with appropriate magnets, the more dangerous harmonics of the field errors.

Figure 4 shows the working diagram of the ALS storage ring.[11] This is a plot of resonance lines defined by Eq. (10), with axes given by ν_x and ν_y. The *working point* is the point having the tune values as coordinates. The accelerator physicist chooses this working point to be at a suitable distance from resonance lines. It is worthwhile to mention the order of magnitude of the tolerable departure of the tunes from the design (or, in an existing machine, experimentally found, optimum) values. This tolerance varies greatly from storage ring to storage ring, but it could be as tight as 0.001 in tune. Remember that tune values are in the tens of units. Thus, this tolerance is rather tight and is reflected in the high stability required from the power supplies.

It is shown in Chapter 4 that, due to the restoring force of the radio-frequency field, particles oscillate in energy, describing *synchrotron oscillations*. The number of oscillations per revolution is denoted by the symbol ν_s (synchrotron wave number), and is on the order of 0.01 (100 turns per oscillation period). If the three-dimensional motion is considered (two transverse and one energy variable), then more resonances appear that involve the energy oscillations. The extended numerological condition for resonance is

$$m\nu_x \pm n\nu_y \pm k\nu_s = p. \tag{13}$$

When Eq. (13) applies, the resonance is called a synchrotron-betatron resonance. It may be driven, for instance, when the value of the dispersion at the location of the radio-frequency accelerating cavities is non-zero.

3.6.2.3. Horizontal-Vertical Coupling. It is important to note that in Eq. (3) horizontal and vertical motions were not coupled, i.e., the horizontal differential equation of motion did not depend on the vertical coordinates, and vice versa. This is only true in an ideal lattice in which the horizontal and vertical components of the magnetic field are perfectly aligned and in absence of field imperfections (see Eq. (1)). In practice, a small amount of coupling is always present.

Particularly important is the coupling due to a *rotated quadrupole*, i.e., a quadrupole that, because of survey tolerances, is slightly (on the order of one mrad or less) rotated around its magnetic axis. This imperfection excites the coupling resonances $\nu_x \pm \nu_y = p$. The sum resonance must be avoided. Although normally not "fatal," special attention is required also for the difference resonance $\nu_x - \nu_y = p$. This resonance couples horizontal and vertical motion. Since the vertical beam emittance is only a few percent of

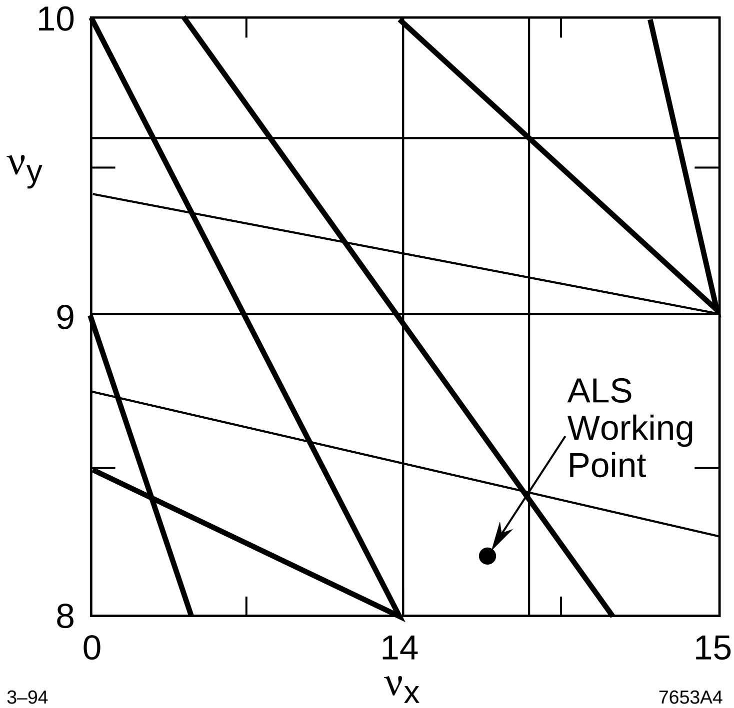

Fig. 4 Tune diagram of the ALS, showing resonances of the type $m\ \nu_x \pm n\ \nu_y = 12p$, up to order 6. (The ALS has a twelve-fold periodicity, and p is any integer.)

the horizontal one, this resonance may appreciably increase the vertical beam size and thus reduce the brightness of the photon beam.

To combat the linear coupling effect, most storage rings are provided with rotated quadrupoles (i.e., quadrupoles that are rotated by 45^o around the magnetic axis) placed at strategic positions to cancel the effect of the rotation errors of the lattice quadrupoles.

3.7. Effects of Insertion Devices on the Particle Motion

Synchrotron light facilities are making ever increasing use of wigglers and undulators (see Chapter 14), to the extent that these devices are becoming significant parts of the beam optical system. Theoretical studies,[12] confirmed by experimental observations, have shed light on the perturbations to the trajectory caused by the magnetic field of such devices. The analytical expressions for the field are given in Chapter 14, namely a dipole field (but rich in higher harmonic content) of alternating polarities that imposes an oscillatory trajectory to the electrons (Fig. 5a). In the general form[13] the expressions for the field in a planar undulator are[14]

$$B_y = B_0 \cosh k_x x \cosh k_y y \cos ks,$$

$$B_x = \frac{k_x}{k_y} B_0 \sinh k_x x \sinh k_y y \cos ks, \qquad (14)$$

$$B_z = -\frac{k_x}{k_y} B_0 \cosh k_x x \sinh k_y y \sin ks,$$

where $k_x^2 + k_y^2 = k^2 = (2\pi/l)^2$, and l is the length of the magnetic period.

If the poles are flat, and in the approximation that they are infinitely large, $k_x = 0$. Shaping the poles to provide horizontal focusing gives $k_x^2 > 0$.

The amplitudes of the orbit oscillations are, typically, on the order of a few microns in undulators and hundreds of microns in wigglers. Figures 5a and b give an impression of the trajectory.

The first requirement of the field is that the trajectory not be perturbed outside the length of the insertion device (Fig. 5b). This is so in a perfectly designed and built magnet, but inevitable field errors cause a perturbation to the orbit that, if uncorrected, propagates around the ring. Correcting dipole magnets and beam position monitors are normally added at the beginning and end of the insertion device to cancel any orbit distortion. More sophisticated corrections may also be included. These corrections are particularly important in modern light sources, in which the undulator field is often changed during the experiment, and it is important that one insertion device does not perturb the orbit for other users.

Even a perfectly built insertion device, however, has focusing terms that must be accounted for in the design of the optics and contains significant non-linear components, as implicit in Eqs. 14.

The focusing effect of insertion devices results in linear[15] tune shifts. In parallel pole devices (no horizontal focusing) the linear tune shift only occurs in the vertical plane. This tune variation can be significant, particularly in wigglers, and must be corrected with quadrupole magnets, preferably locally (i.e., in the same straight section that houses the insertion device). For a given insertion device the tune shift scales inversely with the square of the energy. For this reason it is higher in UV and soft x-ray sources (1.5–2.0 GeV) than in hard x-ray facilities (6–8 GeV). On the other hand, larger, higher energy facilities tend to accomodate more undulators. The tune shifts can be on the order of 0.01–0.03 in the lower energy storage rings and a factor of 20 or so smaller in hard x-ray sources.

The non-linear terms of Eq. 14 drive mainly third and fourth order resonances (see Section 3.6.2.2) and cause tune dependence on the betatron amplitude; the optics designer must make sure that they have a negligible effect on the stability of the beam.

Wigglers have stronger fields than undulators, but often also longer periods (see Chapter 14). In wigglers the tune shifts are higher than in undulators, but the non-linear effects are weaker, since the deflecting field of undulators has fast azimuthal variations (short poles) that tend to enhance the non-linear components.

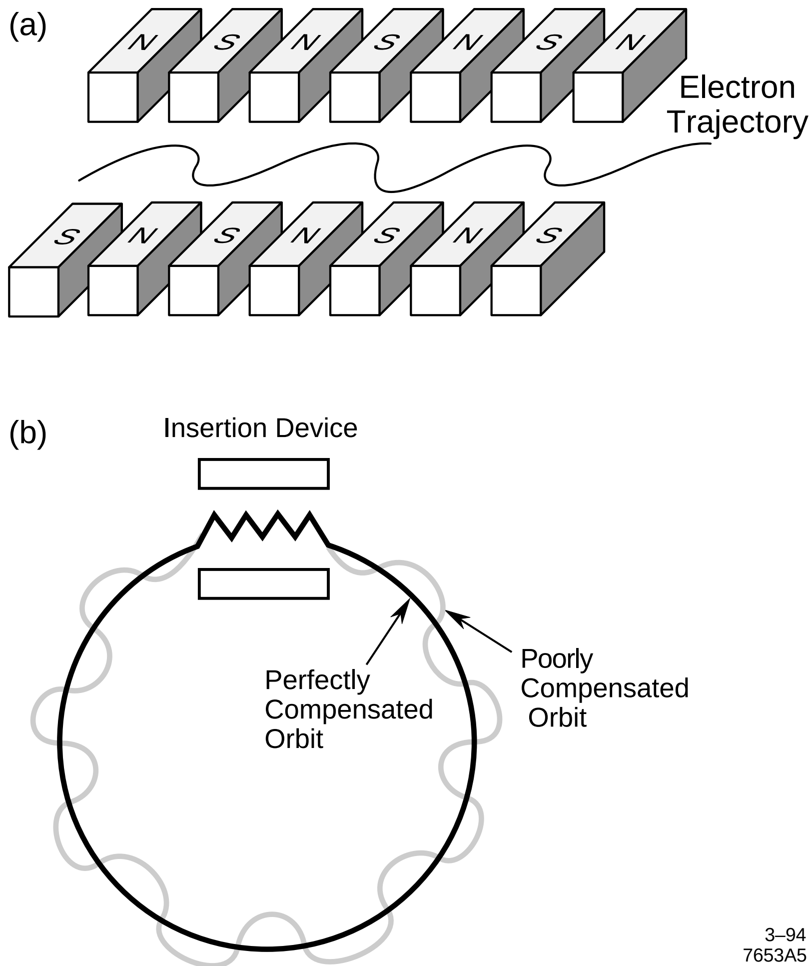

Fig. 5. Beam trajectory in an insertion device. The amplitudes of the oscillations are highly exaggerated.

3.8. Off-Energy Particle Motion, Dispersion, Beam Size, and Momentum Compaction

In this section, non-synchronous orbits, namely those of electrons having energy different from the one defined by Eq. 2, are discussed. The off-energy dynamics is not just of theoretical interest. It is important because, due to the quantum emission of radiation and the action of the radio-frequency system, the particle energy fluctuates around an average value. It is this energy fluctuation that, as we shall see, largely determines the electron beam emittance.[16]

Four important functions describe the motion of off-energy particles. Two are the *dispersion*, normally denoted by the symbol η, and its derivative η' with respect to the independent variable s. The others are the horizontal and vertical *chromaticities*.

3.8.1. The Dispersion.

If the momentum of a particle changes, the bending radius in the dipoles changes according to Eq. 2, and the closed orbit also changes. A particle whose energy differs from the reference value follows a different orbit. The differential equations of motion (Eq. 3) now becomes:

$$
\begin{aligned}
&x'' + K_x(s)x = \frac{1}{\rho_0(s)}\frac{\Delta E}{E_0}, \\
&y'' + K_y(s)y = 0.
\end{aligned}
\tag{15}
$$

They differ from Eq. (3) by the presence of a driving term in the x-axis and by a small (but important, see Section 3.9) change in the focusing terms K_x and K_y. The latter reflects the fact that a change in energy (denoted as the relative change $\Delta E/E_0$ with respect to the synchronous energy E_0) changes the focusing strength of the quadrupoles. The term $(1/\rho_0(s))(\Delta E/E_0)$ represents the perturbation introduced by the fact that the energy of the particle does not match the strength of the bending field, $\rho_o(s)$ being the bending radius in the dipoles of the synchronous particle with energy E_o. The vertical plane does not have such perturbation, unless vertical bends are present in the lattice. The function $1/\rho_0(s)$ follows the periodicity of the bending magnets. One of the solutions of Eq. 15 is periodic with the lattice periodicity, i.e., satisfies the conditions x(0) = x(C), x'(0) = x'(C), where C is the length of the orbit after one revolution and can be expressed in terms of the dispersion $\eta(s)$ and its derivative $\eta'(s)$ defined as:

$$
\begin{aligned}
&x_c(s) = \eta(s)\frac{\Delta E}{E_0}, \\
&x_c'(s) = \eta'(s)\frac{\Delta E}{E_0}.
\end{aligned}
\tag{16}
$$

The dispersion is expressed in units of meters. Its derivative is dimensionless.

The solutions of Eq. 15 are those of the non-homogeneous and the homogenous forms, the latter given by Eq. 4. In a general form that includes energy deviation and betatron oscillation, the horizontal motion of an electron can be described as the sum of a term which is a periodic function of *s* and of an oscillatory term:

$$
x(s) = \eta(s)\frac{\Delta E}{E_0} + \sqrt{\varepsilon_x\beta_x(s)}\cos[\phi_x(s) + \phi_{0x}]. \tag{17}
$$

The slope, dx/ds, is given by

$$
x'(s) = \eta'(s)\frac{\Delta E}{E_0} - \alpha(s)\sqrt{\frac{\varepsilon_x}{\beta_x(s)}}\cos[\phi_x(s) + \phi_{0x}] - \sqrt{\frac{\varepsilon_x}{\beta_x(s)}}\sin[\phi_x(s) + \phi_{0x}], \tag{18}
$$

with

$$
\alpha(s) = -\frac{1}{2}\frac{d\beta}{ds}.
$$

3.8.2. The Beam Size and Divergence.

Having introduced a function for the motion of off-energy particles, we are in a position to generalize the beam size and divergence expressed by Eqs. (6) and (7). Those equations ignored the contribution of the spread in energy that is always present in a beam. Like the distribution of betatron amplitudes, the distribution of the energy spread is Gaussian. If $\langle \Delta E \rangle$ is the root-mean-square of the energy deviation, and, as is normal practice, the emittance ε_x also defines the rms of betatron amplitudes, then, since these quantities are uncorrelated, they contribute quadratically to the overall beam size:

$$\begin{aligned} \sigma_x &= \sqrt{\varepsilon_x \beta_x + \eta^2 \left(\frac{\langle \Delta E \rangle}{E_0} \right)^2}, \\ \sigma'_x &= \sqrt{\frac{\varepsilon_x}{\beta_x} + \eta'^2 \left(\frac{\langle \Delta E \rangle}{E_0} \right)^2}. \end{aligned} \tag{19}$$

Typically, the relative energy spread is on the order of 10^{-3}, and the dispersion is measured in meters. One meter dispersion gives a contribution of 1 mm to the beam size. For comparison, an emittance of 5×10^{-9} meter-radians at a location at which β_x is, say, 10 m, gives a beam size of 0.22 mm. This is one of the reasons why insertion devices are normally located in “dispersion-free regions” where $\eta = \eta' = 0$, or is at least very small.

3.8.3. The Momentum Compaction Factor

Let us now introduce a quantity that is of fundamental importance for the longitudinal motion. This parameter is the *momentum compaction*. It is a measure of how the time taken by the particle to complete one turn in the accelerator varies with energy. In high-energy electron accelerators the velocity of the particle is nearly constant with energy, and the revolution time is determined by the longer (or shorter) path a higher (lower) energy particle has to travel. Only the curved sections contribute to a lengthening of the orbit with energy, and higher energy particles have a larger bending radius. The momentum compaction factor is defined as

$$\alpha_c = \frac{\Delta T / T_0}{\Delta E / E_0}, \tag{20}$$

where ΔE is the energy difference from the synchronous energy E_o, and T_o is the revolution period of the synchronous particle. The momentum compaction is determined by the properties of the lattice. In fact, it is the average of the dispersion in the bending section divided by the average machine radius.[17] The stronger the focusing, the lower this value α_c is. An approximation often used is $\alpha_c \approx 1/\nu_x^2$. The small value of the momentum compaction function in synchrotron radiation sources has important implications for the longitudinal motion (see Chapters 4 and 12).

3.9. Chromaticity Correction and Dynamic Aperture

The focusing (or defocusing) action of the quadrupoles is inversely proportional to the particle energy. In analogy with optical lenses, this effect is called a *chromatic effect*. It leads to a dependence of the tunes on energy. This dependence is measured by the horizontal and vertical chromaticities, ξ_x and ξ_y:

$$\Delta\nu_x = \xi_x \frac{\Delta E}{E_0}, \quad \Delta\nu_y = \xi_y \frac{\Delta E}{E_0}, \tag{21}$$

where the $\Delta\nu_{x,y}$ are the shifts in tunes from those of the synchronous particle and are caused by a change in energy $\Delta E/E_0$.

Because the focusing action decreases with energy, the uncorrected chromaticities are negative numbers. Corresponding to a spread in energy within a beam of particles, Equation (21) implies that a spread in tunes follows, and this may have adverse effects if it results in crossing resonance lines (Section 3.6.2). Sextupole magnets, non-linear elements already introduced in Eq. (12), are used to correct the chromaticities. Most accelerators operate with zero or slightly positive chromaticities. The next section gives a simple treatment of how sextupoles are used to control the chromaticities.

3.9 1. How Sextupoles Correct the Chromaticities

Consider the field of a sextupole magnet (Eq. (12)):

$$B_y = S(x^2 - y^2),$$
$$B_x = 2Sxy.$$

If a particle is off-energy, its horizontal displacement x consists of two terms: a betatron oscillation x_β and an orbit shift x_E (Eq. (17)). The vertical displacement is a pure betatron oscillation y_β. The field seen by the particle can be decomposed into the components of the displacements

$$\begin{aligned} B_y &= Sx_\beta^2 + \mathbf{2Sx_E x_\beta} + Sx_E^2 - Sy_\beta^2, \\ B_x &= 2Sx_\beta y_\beta + \mathbf{2Sx_E y_\beta}. \end{aligned} \tag{22}$$

The terms in bold in Eq. (22) have the form of a quadrupole field, a field that is linear in the betatron displacements x_β and y_β. The "strength" of the quadrupole is 2 S x_E, and is proportional to the particle energy via its closed orbit displacement x_E. This fact is utilized to offset the (linear) energy dependence of the focusing strength of the quadrupoles. Figure 6 shows the quadratic dependence of the horizontally deflecting field.

Since the horizontal and vertical machine chromaticities are both negative, but the equivalent quadrupole of Eq. (22) has opposite focusing and defocusing effects in the horizontal and vertical axes, two families of sextupoles are required, both placed in dispersive regions. The horizontally correcting sextupoles are located in regions in which the horizontal β-function is high and the vertical β-function is low. The converse is true for the vertical chromaticity correcting sextupoles. Since, from Eq. 16, $x_E = \eta(s)\,\frac{\Delta E}{E_o}$, it is

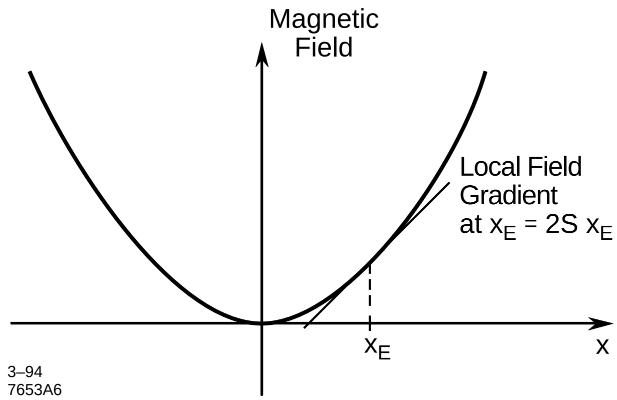

Fig.6 Sextupole field and local field gradient for an orbit displaced by X_E.

convenient, in order to reduce the sextupoles strength, to place the sextupoles at locations where the dispersion is high.

Equation (22) indicates that, besides the “useful” terms (in bold characters) that correct the chromaticities, unwanted, non-linear terms crop-up that perturb the motion. Some storage rings include more than two families of sextupoles, the additional families being used to neutralize some of the resonances create by the unwanted terms of Eq. 22.

3.9.2. The Dynamic Aperture Problem

Sextupoles are non-linear elements, and, while they correct for the linear part of the chromatic aberrations, they can also disrupt the motion and cause particle loss. Low emittance lattices are characterized by strong sextupoles, and the problem of the *dynamic aperture* is one of the most important design issues.

The dynamic aperture is defined to be the maximum betatron oscillation that can be sustained in the accelerator for a sufficient number of turns. In electron storage rings, the time scale is on the order of the damping time (see Section 4.2).

The amplitude may be limited by the transverse size of the vacuum chamber (physical aperture), or by the perturbing effect of the non-linear fields (dynamic aperture). The problem of determining the maximum stable amplitude of the oscillations in the presence of non-linear perturbations is not amenable to an exact mathematical solution. The *dynamic aperture limit* is estimated by computer simulation of the motion of the particles in the presence of the non-linear field of the sextupoles and other perturbing non-linearities, like those caused by magnetic imperfections. It is desirable to design a lattice and chromaticity correction sextupoles such that the maximum amplitude of the betatron oscillations is determined by the physical aperture of the chamber, and not by the non-linear perturbations.

The dynamic aperture is often plotted in a graph that depicts the maximum amplitudes of the vertical betatron oscillations that are stable as a function of the maximum stable horizontal amplitudes. Figure 7 shows the dynamic aperture of the Advanced Light Source (ALS), as computed for the Conceptual Design Report.[18]

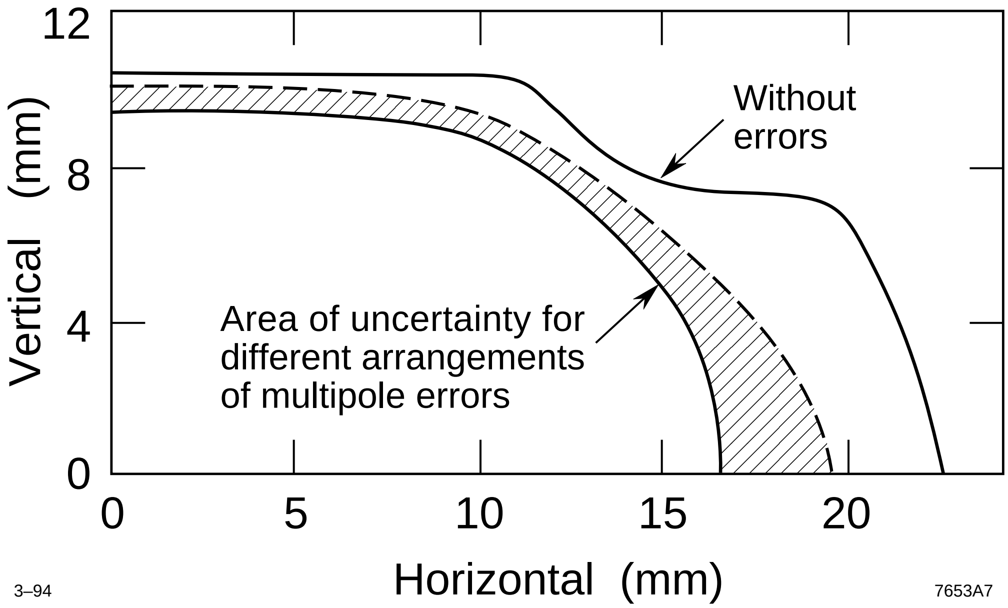

Fig.7 Dynamic aperture in presence of multipole errors in the ALS.

The dynamic aperture is sensitive to the degree of symmetry of an accelerator, high periodicity usually being associated with a larger dynamic aperture. Unfortunately, even in a machine designed with high periodicity, the regular lattice pattern is broken by magnetic imperfections, orbit errors, and the presence of different types of insertion devices in the straight sections. Figure 7 shows the dynamic aperture of the ALS lattice in which the only non-linear elements are the chromaticity sextupoles ("without errors" curve). The maximum stable amplitudes are reduced when magnet misalignments and field imperfections are included in the computation.

4. Emission of Radiation and the Equilibrium Emittance

4.1. Emission of Radiation, Damping Times and Equilibrium Emittance

A charged particle on a curved path emits radiation. In a storage ring, the orbit is curved in bending magnets and insertion devices. The radiation is emitted in the direction tangential to the direction of the motion and is concentrated in a narrow cone with an apex angle of about a milliradian (see Chapters 1 and 14).

The emittance (horizontal or vertical) is not defined by the characteristics of the beam upon injection into the storage ring. Instead, after characteristic times, called the *damping times* (horizontal and vertical), the beam size and angular spread are determined by the lattice and the emission of radiation. In other words, the beam loses all memory of its previous dynamical history. The equilibrium emittance is the result of a balance between an anti-damping mechanism that tends to increase the beam size and a damping process that tends to reduce it.

If there is a transient perturbation (power supply, residual gas effects, fast magnetic kicks, etc.), the value of the emittance settles back to equilibrium after a damping time.

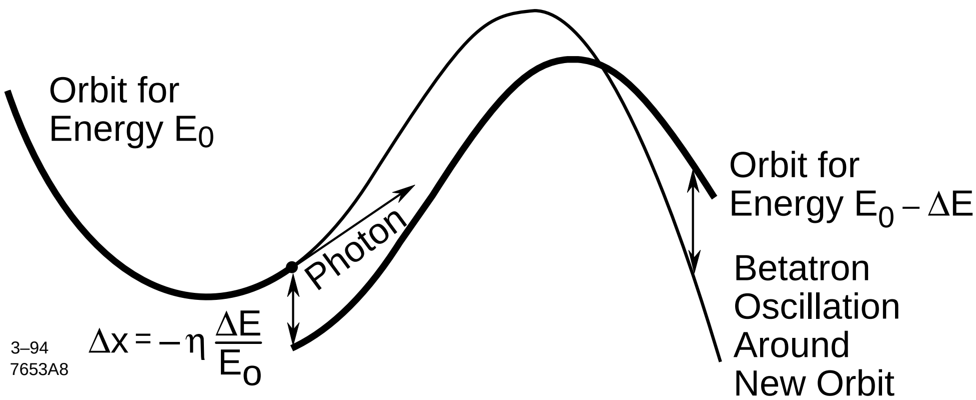

Fig.8 Sketch of a trajectory following the emission of a photon of energy ΔE.

4.2. Antidamping of Betatron Oscillations

Consider an electron that follows a horizontal betatron oscillation around the closed orbit defined by its energy (Eq. (17)). Suppose that, at a given azimuthal position *s*, the electron emits a photon. The photon can be emitted anywhere within a cone of aperture $1/\gamma$ centered around the tangent of motion (where γ is the ratio of the particle energy to its rest energy). Since the effect of this angular aperture is small compared to the average perturbation[19] we are justified in modeling the photon as if it were emitted tangential to the motion.

Figure 8 shows the trajectory of an electron emitting a quantum of energy ΔE at a point at which its betatron amplitude and angle are x and x' with respect to the closed orbit relevant to its energy E_0.

Since the emission occurs in the direction of the particle momentum, there is no change in displacement or slope following the emission of radiation. The particle energy, however, changes by an amount –ΔE. The closed orbit associated with the new energy, $E_o - \Delta E$, is now different, as shown in Fig. 8. After the emission of radiation, the particle trajectory is different from the one it would have followed if, hypothetically, no radiation emission had taken place. The motion is still described by the sum of periodic (closed orbit) and oscillatory terms (Eqs. 17 and 18), but the coordinates of the closed orbit are now those relevant to the new energy. Since the position and angle of the particle have not changed following the emission of radiation, and since Eqs. 17 and 18 are still valid, the amplitude and angle of the betatron oscillation must change.

A rigorous description of the statistical nature of the phenomenon[17,19] reveals that the quantum nature of the radiation emission leads to an increase of the betatron oscillation amplitudes of the ensemble of particles.

It is intuitive, and it can be rigorously shown, that *the greater the amplitude of the oscillations are greater, the greater the value of the dispersion and its derivative at the location where the radiation is emitted.*

Fortunately, this is not the whole story, as another phenomenon takes place that tends, instead, to decrease the amplitude of the betatron oscillations. This is discussed in the next section.

4.3. Damping of Betatron Oscillation

It is shown in Chapter 4 that the radio-frequency accelerating system restores the energy that the particles lose due to the emission of radiation. We have also mentioned that the radiation is predominantly emitted in a direction which is tangent to the direction of motion. For a particle executing betatron oscillations (horizontal or vertical), the radiation is emitted at an angle with respect to the closed orbit (Fig. 8). A particle loses part of its transverse momentum (p_x or p_y) if, in the course of the betatron oscillations, there are components of the momentum in the x or y direction. The electric field of the radio-frequency accelerating system restores the momentum in the direction of the ideal orbit, and thus tends to “align” the electrons in the direction of this orbit.

The overall effect is a transfer of momentum from the horizontal and vertical oscillations into the azimuthal direction, causing a reduction of the horizontal and vertical betatron amplitudes.

4.4. The Horizontal Emittance

The horizontal emittance of the beam in a storage ring is determined by the equilibrium between the two actions described in Sections 4.2 and 4.3, namely the *antidamping effect* of the quantum emission and the *damping* effect of the restoring radio-frequency field. Apart from collective effects and other perturbing factors, the emittance is completely determined by the energy, bending field, and lattice functions.

Once the beam is injected into a storage ring, it takes some time for the emittance to settle to the equilibrium value. This time depends on the rate of emission of radiation and is usually on the order of a few milliseconds. It is referred to as the *betatron damping time*. This time plays an important role in the injection process (see Chapter 3).

4.5. The Vertical Emittance

In a machine for which the bending occurs only in the horizontal plane, the vertical motion is privileged, since it experiences the damping effect of the radio-frequency field, but not the antidamping effect of the quantum emission of radiation. Without vertical bending there is no vertical dispersion. Theoretically, the vertical emittance should be almost zero in a storage ring with no vertical bends. In practice, the vertical emittance is small but finite, due to several factors, for instance vertical bends are present due to quadrupole misalignment and associated correction magnets, horizontal-vertical betatron coupling caused by quadrupoles that are rotated, due to the limitations of survey accuracy, around their magnetic axis.

Most electron storage rings are characterized by a very small vertical/horizontal emittance ratio, on the order of 0.01–0.03. The synchrotron light spot looks like an ellipse with a much larger horizontal axis.

Sometimes it becomes necessary to intentionally increase the coupling, and the vertical beam size, in order to reduce the charge density when intra-beam scattering (see Chapter 12) is important.

4.6. Design Criteria For Synchrotron Light Sources

Let us now qualitatively discuss how the horizontal emittance depends on the lattice characteristics. For a given energy, the energy loss per turn to synchrotron radiation is inversely proportional to the bending radius (Chapter 1). This energy loss *increases* the amplitude of the horizontal betatron oscillations via the anti-damping mechanism discussed in Section 4.2. We have also seen that this increase is greater, the greater the dispersion and its derivative at the location where the radiation is emitted.

The rate of energy loss is proportional to the fourth power of the particle energy (Chapter 1). It can be shown that this leads to a dependence of the emittance on the *square of the energy*, for a given lattice.

Based on these factors, the main criteria that dictate the design of low emittance synchrotron radiation lattices are:

1) A *large bending radius* to reduce the rate of energy loss by radiation. A large bending radius leads to larger rings. High energy storage rings for hard x-ray machines like the APS and the ESRF must have larger circumferences to achieve emittances comparable to low energy machines for UV and soft x-ray production.
2) A *small value of the dispersion* function and its derivative where the radiation is emitted (bending magnets and insertion devices). This requires large bending radii and/or *strong focusing* lattices to keep the value of the dispersion small. The new, 3-rd generation, sources, with their design emphasis on small emittance, are, in fact, characterized by strong focusing lattices.

There is a limit on how strongly focusing a lattice can be made. This is the subject of the next section.

4.7. Strong Focusing Lattices

We saw in the previous section that low emittance lattices require that the dispersion function and its derivative be kept small. The dispersion is created by the (bending magnets) excitation of a driving term on the right hand side of Eq. (15). This driving term is due to the fact that the energy does not match the design value defined by Eq. (2). According to Eq. (3), this perturbation propagates like a harmonic oscillator until it senses another perturbation by a bending magnet, and so on.

One way to keep the dispersion low is to have a large bending radius, with many short bending magnets. This approach leads to rather large accelerators. It has been proposed as a method to reduce the emittance limit of third generation light sources.[20]

The recent wave of third generation light sources (ALS, APS, ESRF, SRRC, ELETTRA, etc.) has chosen to reduce the amplitude of the dispersion function by increasing the focusing action of the lattice, given by the term $K_x(s)$ in Eq. (2). This is achieved by increasing the strength of the quadrupoles and by spacing them close together.

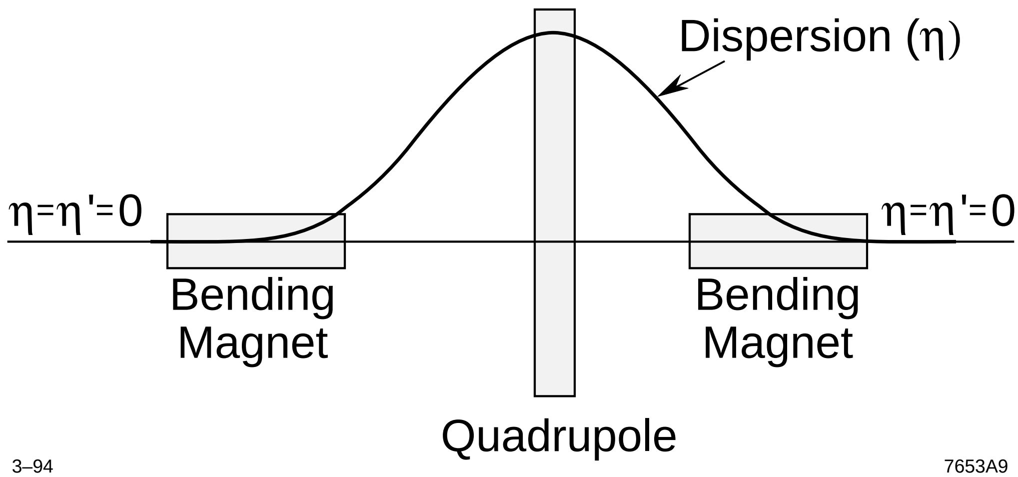

Fig.9 The dispersion in simplified, Chasman-Green type, cell.

5. Characteristics of Lattices For Synchrotron Radiation Sources

5.1. General Considerations

The basic building block of a lattice is the *cell*. This is a sequence of magnetic elements characterized by certain requirements and by the fact that the lattice functions (β-function, dispersion, their derivatives, etc.) have the same values at the beginnings and ends of the cells. If this is the case, then one can build a lattice by aligning the cells one after another always repeating the lattice functions with one exception: the more unit cells from which one builds a lattice, the smaller the bending radius of the dipoles must be, since the total bending angle in the ring (2 π) is unchanged. This leads to a strong, *cubic dependence* of the emittance on the number of cells. Remembering that the emittance acts like the square of the energy yields the following important expression for the horizontal emittance:

$$\varepsilon_x = k \frac{E^2}{N_c^3}, \tag{23}$$

where E and N_c are the electron energy and the number of identical cells, respectively. The constant *k* depends on the type of cell.

The first task of a lattice designer is, then, to build a cell. When this task is accomplished, the designer has to decide on the *periodicity* of the accelerator, i.e., how many cells to build into the lattice.

5.2. The Cell

In designing a cell, the lattice designer must keep several considerations in mind. If the goal of the accelerator is to achieve as small an emittance as possible, the designer will need to keep the dispersion function as small as possible in the bending magnets. Fig. 9 depicts a simplified cell, two bending magnets with a quadrupole in between.

If the quadrupole strength is chosen judiciously, a dispersion function that has coordinates $\eta=\eta'=0$ at the beginning of the cell ends up with the same coordinates at the end of the cell. The bending magnets create the dispersion (due to the fact that particles of different energy are subject to different deflections). To prevent it from growing too large, the dipoles must be kept short and the quadrupole placed as close as possible to the magnets. This in turn requires strong quadrupole strengths, increases the chromatic aberrations, requires stronger sextupoles, and creates potential dynamic aperture problems.

The other requirement to be kept in mind is the need to inject the beam into the storage ring. This is discussed in Chapter 3. We mention here that the injection process requires special elements (fast kicker magnets and a septum magnet). The cell must include sufficient free space to accommodate these elements along with special optics requirements to be considered.

The presence of straight sections (free of magnetic elements) to accommodate insertion devices is an essential feature of the new generation of light sources. The length of the sections depends on the length of the insertion devices and on possible limitations on the circumference of the ring. In most third generation storage rings the length varies between 3 and 7 meters. One of the reasons for the upper limit is the need to prevent the β-function from growing too large. Symmetry considerations show that the β-function has a minimum in the middle of a straight section (waist). Away from the waist, where $\beta=\beta^*$, the function grows as

$$\beta^* = \beta^* + \frac{s^2}{\beta^*}. \tag{24}$$

Why does one keep the β–function from growing too large? There are two reasons: 1) beam size (Eq. (6)), with the associated hardware requirements of larger aperture, more costly magnets, etc., and 2) the fact that chromatic aberrations and sensitivity to survey errors become more important as the β-functions in the quadrupoles increase. In future light sources, these limitations could be overcome if focusing (horizontal and vertical) could be introduced in the magnetic fields of insertion devices. This would open up the possibility of installing very long undulators.

It has been common practice in 3-rd generation light sources to design the optics of the cells such that the dispersion and its derivative are zero in the straight sections. This approach minimizes the beam size and divergence, as shown in Eq. (19). A non-zero dispersion in the field of wiggler magnets also leads to an emittance increase, according to the mechanism described in Section 4.2. In both cases the result is a reduction of photon beam brightness.

The process of determining the magnet strengths and positions to satisfy design conditions for the lattice functions is called *matching*. Often, to match the dispersion and its derivative to achieve zero values in the straight sections requires ingenuity and cost trade-offs in the number of quadrupoles and their strengths. Recently, the need for a rigorous zero value of the dispersion in the region of the insertion devices has come under scrutiny, and has been challenged as a too high a price to pay for a relatively small improvement in beam quality.[21]

5.3. Types of Cells

In this section the types of cells most commonly used for synchrotron light sources are reviewed[22] and critiqued, together with indications for future directions.

5.3.1. The FODO Cell

The simplest of the cell layouts is the so called FODO structure. Its name stands for Focusing-Drift-Defocusing-Drift. Fig. 10a shows the version built at the Daresbury SRS2 facility. The drift spaces may contain bending magnets or, in some cases, be left empty for dispersion matching purposes. Straight sections for insertion devices can be accommodated by special insertion optics matched to a sequence of FODO cells.

This type of cell is inherited from high energy accelerators, where it is commonly used. The Photon Factory (Tsukuba) adopted FODO cells for the regular part of the lattice, as did the Daresbury Light Source. FODO optics are also the choice of the Duke University FEL Storage Ring and of damping rings for linear colliders, which share many design criteria with synchrotron light sources.

FODO lattices are easily tuned and well understood. More recent third generation low emittance sources, however, have shied away from this concept. One of the reasons, in the author's view, is the engineering compactness of the design required to achieve a small emittance and the difficulty in providing sufficient space for radiation ports at bending magnets.

5.3.2. The Double-Bend Achromat (DBA)

Rina Chasman and Ken Green of Brookhaven National Laboratory proposed for the National Synchrotron Light Source (NSLS) a cell specifically designed for synchrotron radiation sources.[23] In various forms, it has become very popular. The NSLS x-ray ring version is shown in Fig. 10b. It consists of two bending dipoles separated by a single quadrupole. In other designs, a doublet may replace the single quadrupole. The straight section contains quadrupoles to match the β-functions. The dispersion is zero in the straight sections, increases as it goes through the dipoles, and is focused by the middle quadrupole(s). The Chasman-Green lattice is compact and economizes the number of magnets. The dispersion in the bending sections can be kept small (as demanded by a low emittance optics) by using short dipoles and keeping the space between dipoles to a minimum. This ingenious layout has served the scientific community well. Examples of light sources employing the Chasman-Green lattice are the NSLS (Brookhaven) VUV and x-ray rings, MAX at Lundt, NIJI III at SUMITOMO Industries (Japan), HISOR (Hiroshima), LSU CAMD (Louisiana University), the proposed LNLS facility in Campinas, and others.

Its drawbacks, for very small emittance applications, are represented, in the author's view, by the limited length of the region where the dispersion is non-zero and by the small decoupling[24] between the horizontal and vertical β–functions. This may lead to strong sextupole requirements, and the dynamic aperture may be a problem for very low emittance rings. It is also rather limited in tunability.[25]

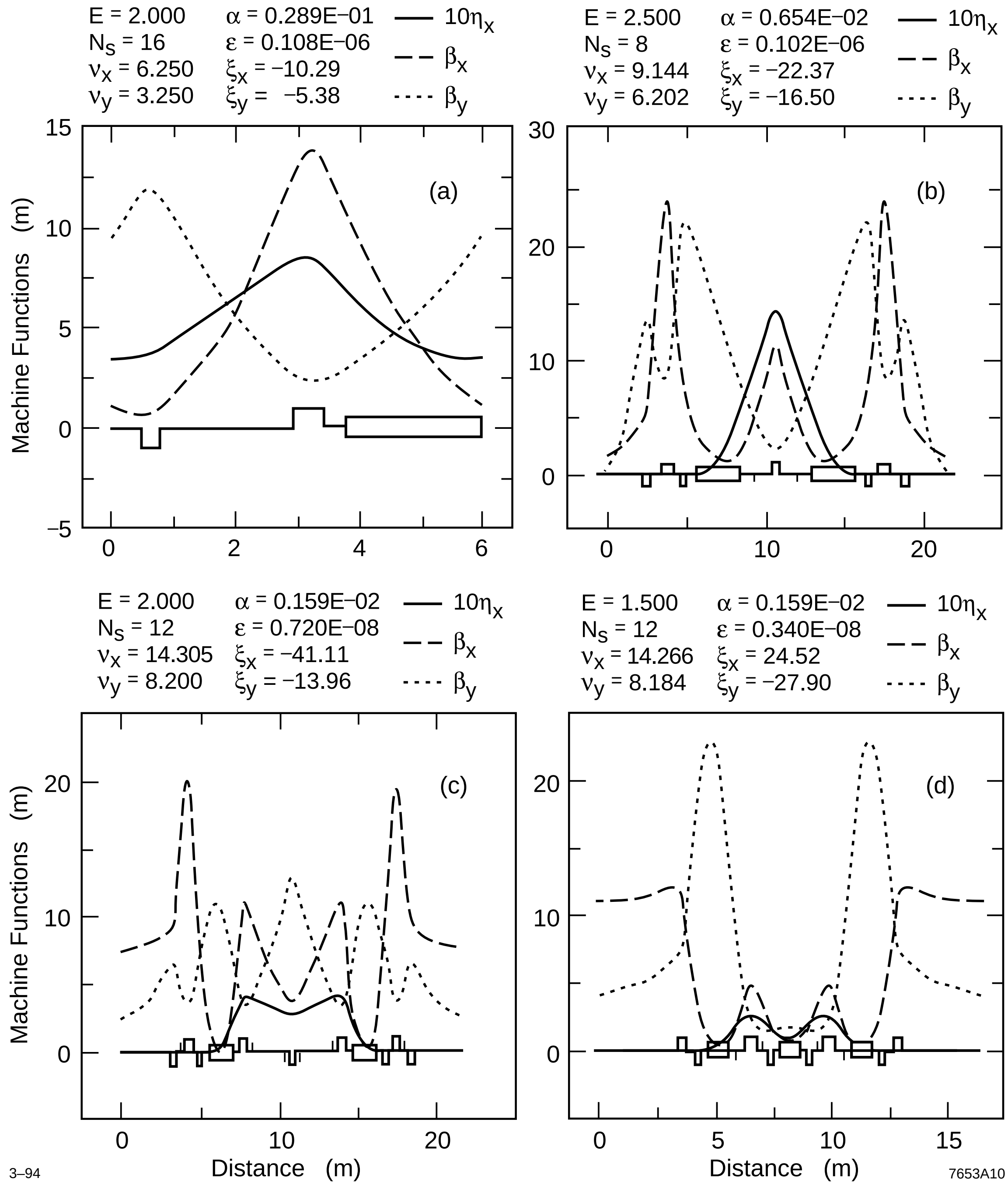

Fig.10 Types of lattices at various facilities. The symbols signify: E = particle energy (GeV), N_S = number of cells, ν_x = horizontal tune, ν_y = vertical tune, α = momentum compaction factor, ε = horizontal emittance (meter-radian), ξ_x = horizontal chromaticity, ξ_y = vertical chromaticity, η_x = dispersion function (m), β_x = horizontal β function (m), β_y = vertical β function (m).

The problem of the compactness of the Chasman-Green cell and the restricted area where sextupoles can be placed may be overcome by enlarging the region between the two dipoles (Fig. 10c, ELETTRA version). Typically, three to four quadrupoles between the dipoles (instead of one or two as in a simple Chasman-Green layout) give higher dispersion and more decoupling between horizontal and vertical β-functions. The benefits are weaker sextupoles (as implied by Eq. (22)), an improved dynamic aperture, more space for diagnostics, and an overall more flexible lattice. This cell is known as "Enlarged Chasman-Green," although the distinction between a simple Chasman-Green and an enlarged version is often a matter of subjective definition.

The Enlarged Chasman-Green lattice is particularly suitable for hard x-ray facilities, and is used in the ESRF, APS, and SPRING-8 facilities. It was also chosen for SuperAco, ELETTRA, BESSY II, MAX II, and the proposed SOLEIL and SIBERIA 2 designs.

5.3.3. The Triple Bend Achromat (TBA)

In the Triple Bend Achromat, a third bending magnet is symmetrically placed between two outer ones. The addition of the third dipole has the advantage that the bending radius may be increased, since more magnets contribute to the total bending. This reduces the dispersion in the dipoles and makes it easier to achieve a low emittance. This lattice, shown in Fig. 10d in the ALS version, has good tunability, particularly if a pair of quadrupoles (rather than one) is placed on each side of the center dipole. The TBA was the choice of the BESSY I and Hefei facilities and, later, was adopted by the ALS and SRRC. In the ALS and the SSRC the dipoles have a vertically focusing gradient superimposed on the bending field. This "combined function" magnet helps to reinforce the vertical focusing and also reduces the horizontal emittance by decreasing the horizontal damping time.[26] The latter can be shown by analyzing the emission of radiation in a superimposed quadrupole-dipole field, where the orbit of an electron with an excess of energy is in a lower bending field and therefore radiates less.[27] ELETTRA, a double-bend-achromat structure, also utilizes combined function dipoles.

Higher energy rings (ESRF, APS) have avoided using combined function magnets, mostly because the strong gradient that is required makes the construction difficult.

5.3.4. Other Types of Lattice

This brief discussion does not presume to cover the whole variety of lattices for synchrotron radiation sources.[28]

A few interesting new concepts have emerged in recent times. The proposed SLS facility at PSI[29] (Villigen, Switzerland) introduces two very long straight sections (18 m) for long undulators, in addition to four, 7-m-long ones. The SLS lattice layout departs from the more conventional types described above. An interesting new feature is the incorporation of six superconducting magnets that act as wavelength shifters, extending the versatility of the facility (1.5–2.1 GeV energy) to the hard x-ray region of the spectrum (up to 50 keV photons).

SINBAD, a 700 MeV proposed ring at Daresbury, also introduces two very long (14 m) straight sections for novel insertion devices. The other Darebury facility (3 GeV) under consideration for construction is DIAMOND, a Triple-Bend-Achromat in which the central dipole is superconducting.

The lattice for the proposed 3 GeV Light Source ROSY[30] in Rossendorf (Germany) also departs from the conventional FODO, DBA, and TBA. The basic cell consists of five bending magnets (three with 20^{o} and two with 15^{o} bending angles). The bending magnets are of the combined function type (vertically focusing), allowing a more compact machine and further reduction of emittance. The ring consists of four achromats.

One of the concepts proposed to overcome the problem of the small dispersion, strong sextupoles, and dynamic aperture of low emittance light sources utilizes combined function magnets that include bending, quadrupole, and sextupole components for chromaticity correction.[20] With distributed sextupole fields a very small dispersion is permissible, while still maintaining a large dynamic aperture. The small dispersion is achieved using long magnets with large bending radius. A remarkably small emittance of 0.7 nanometer-radians at 6 GeV was computed for a ring consisting of 262 FODO-like cells using only combined function magnets. The length of each cell is about 1.8 m.

The possibility of creating and sustaining very short (sub-picosecond) bunches in storage rings has received attention recently. A storage ring operating in this manner would be very attractive for high-energy physics colliders, and there are also experiments in synchrotron radiation that would greatly benefit from such a machine. The theoretical feasibility and design criteria for such lattice have been investigated,[31] and the results are encouraging. This lattice is designed such that particles of different momenta have the same (or nearly the same) revolution period, leading to very short bunches.

Finally, we conclude this brief review by remarking that not all synchrotron radiation sources need to be large or of high energy. In addition to the compact machine designed for lithography applications (not included in this chapter) the 240 MeV SURF II machine at the National Bureau of Standards is a small, 5-m-circumference storage ring. It consists of a single magnet that bends and focuses the beam. It was converted into a storage ring in 1973 from a 180 MeV synchrotron.[32]

References

1. In the narrative, the word electron is used to signify electron or positron. The dynamics of a single particle are not affected by the sign of its charge when the magnetic field direction is changed accordingly.
2. The reader may be confused by the use of the word orbit and the accompanying adjectives. In this chapter, orbit always refers to an electron path that closes on itself after one revolution around the accelerator, thus, it is always a “closed” orbit. When this orbit describes the motion in an ideal lattice without magnetic imperfections or misalignment, this closed orbit is also the “ideal” orbit. The word trajectory is used to describe an oscillation around the closed orbit, called a betatron oscillation, which does not close after one revolution.
3. Defined by the azimuthal and horizontal/vertical axis.
4. Dipole and quadrupole fields may co-exist in a single *combined function magnet.* These magnets have found applications in the most recent third generation light sources, like the ALS, ELETTRA, and the SRRC light source. For the sake of clarity, in this description we treat the dipole and quadrupole magnets separately. The function of combined function magnets will be discussed in Section 5.3.3.

5. E. D. Courant, H. S. Courant, and Snyder, “Theory of the Alternating-Gradient Synchrotron,” Annals of Physics **3**, 1–48 (1958). The principle of strong focusing opened the way to a new generation of synchrotrons and storage rings in which the beam size, and thus magnet aperture and cost, could be kept much smaller than in the previous, weak focusing type, accelerators.
6. Recall that we are using the ultra-relativistic approximation $E \approx c|P|$, where P is the particle momentum.
7. See, for instance, K.G. Steffen, “High Energy Beam Optics,” by Interscience Publishers, a division of John Wiley & Sons, New York.
8. Additional factors, as we shall see later on, also contribute to the beam size
9. See, for instance, G. Guignard, Physics of Particle Accelerators, Chapter on “Nonlinear Dynamics” pp. 820–890, AIP Conference Proceedings 184, Melvin Month & Margaret Dienes Editors.
10. Defined as the relative variation of the field with respect to the ideal value.
11. Reproduced from the ALS Conceptual Design Report, courtesy of Alan Jackson.
12. L. Smith, “Effects of Wigglers and Undulators on Beam Dynamics,” 1986, Proceedings of the International Particle Accelerator Conference, Novosibirsk, USSR.
13. K. Halbach, Nucl. Inst. & Meth. **169**, p. 1, 1980 and Nucl. Inst. & Meth. **107**, p. 109, 1981.
14. Equation (14) includes a single harmonic in the azimuth s. This is appropriate for closely spaced magnets. Otherwise, more harmonics must be included.
15. The term linear implies that the tune shift does not depend on the betatron amplitude and is driven by fields of the gradient, or quadrupole, type (see Eq. (1)).
16. Other effects that, under certain circumstances, contribute to increasing the beam emittance (collective instabilities, intra-beam scattering, ion trapping) are discussed in Chapter 12.
17. M. Sands, “The Physics of Electron Storage Rings—An Introduction,” SLAC Report 121.
18. Reproduced from the ALS Conceptual Design Report, courtesy of Alan Jackson.
19. H. Wiedemann, “Particle Accelerator Physics”, Springer-Verlag Editors, p. 355.
20. W.D. Klotz and G. Mulhaupt, “A Novel Low Emittance Lattice for a High Brilliance Electron Beam,” Proceedings of the Workshop on Fourth Generation Light Sources, Feb. 24–27, 1992, SSRL report 92–02.
21. For instance, the design MAX II (Lund) storage ring accepts a small (13 cm) dispersion in the center of the insertion device straight section.
22. The cell layouts of Fig. 10 are taken from J. Murphy, “Synchrotron Light Source Data Book,” BNL Report 42333, Version 3.0, by courtesy of the author.
23. R. Chasman, K. Green, E. Rowe, IEEE Trans. Nucl. Sci., 22, 1765 (1975). This type of achromat was originally proposed by P. Panofski for utilization as a spectrometer. Private communication in K. Steffen, “High Energy Beam Optics” Interscience Publishers, 1965, p. 113.

24. In decoupled β-functions $\beta_y << \beta_x$, where the sextupoles controlling the horizontal chromaticity are placed, and $\beta_y >> \beta_x$ in the vertical chromaticity sextupoles.

25. The tunability may be loosely defined as the capacity to change the tunes and/or lattice functions, while preserving the beam stability without appreciable changes of the source size and divergence.

26. G. Vignola, "Preliminary Design of a Dedicated 6 GeV Synchrotron Radiation Storage Ring," Nucl. Inst. Meth. Phys. Res. **A236**, 414 (1985).

27. H. Wiedemann, "Particle Accelerator Physics," Springer-Verlag, Editors, p. 347.

28. For a complete list, see J. Murphy, "Synchrotron Light Source Data Book", BNL report 42333.

29. Conceptual Design of the Swiss Synchrotron Light Source, Paul Scherre Institut, CH–232 Villigen PSI, Switzerland.

30. D. Einfeld and M. Plesko, "A Modified QBA Optics for Low Emittance Storage Rings", Nucl. Inst. and Meth. **A335** (1993) 402–416.

31. C. Pellegrini and D, Robin, "Quasi-Isochronous Storage Rings," NIM A301 (1991), pp. 27–36.

32. E.M. Rowe, M.A. Green, W.S. Trzeciak and W.R. Winter, Jr. "The Conversion of the NBS 180 MeV Electron Synchrotron to a 240 MeV Electron Storage Ring for Synchrotron Radiation Research," Proceedings of the 9th International Conference of Particle Accelerators, SLAC, 1965, p. 689.