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ON THE MEASURABILITY OF ELECTROMAGNETIC FIELDS: A New Approach*

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ABSTRACT

We revive the Bohr-Rosenfeld discussion of the measurabily of electromagnetic fields by replacing their classical apparatus with NO-YES counter firings of fixed spacial (Δx) and temporal (Δt) resolution. This gives us *scale invariant* commutation relations bounded from below from which we can—following the Feynman-Dyson-Tanimura proof—derive both the free space Maxwell Equations and the Einstein gravitational geodesic equations for a *single* test particle with Lorentz invariant charge to mass and gravitational to inertial mass ratios. We mention briefly the new fundamental theory which led us to this simple analysis of the problem.

1. INTRODUCTION

In their classic paper, Bohr and Rosenfeld[1] (BR) showed that the restrictions on measurement accuracy implied by *non-relativistic* quantum mechanics are sufficient to allow one to derive the restrictions on the measurability of the electric and magnetic fields using *classical* measurement apparatus. They demonstrate that these restrictions are the same as those implied by *second quantization* of the classical Maxwell equations and the resulting commutation relations between $\vec{E}$ and $\vec{B}$ in free space.

The remarkable result that a Galilean invariant theory (i.e. non-relativistic quantum mechanics) has an intimate connection with a Lorentz invariant theory (i.e. Maxwell's equations and the related relativistic quantum field theory) is easy to understand using dimensional analysis. The only universal constants used by BR are $\hbar$ and c. Therefore BR are fully justified in using arbitrarily complicated arrangements of macroscopic, classical rigid rods, clocks, springs, charges, currents,... *within* the wavelength at which the theory is being probed. Scale invariance then allows them to extend the results up or down until a fundamental length, or time, or mass, or charge, or energy, or temperature, or ... is encountered. As I have noted elsewhere,[2] non-relativistic quantum mechanics for any detectable particle of *arbitrary* mass m is *also* scale invariant (so long as the mass ratio of any other system to that particle is measurable and has no universal significance). Under this caveat, non-relativistic quantum mechanics can use the same universal constant $\hbar$ as BR without breaking scale invariance.

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These pedagogical considerations have taken on contemporary relevance thanks to the resurrection by Dyson[3] of Feynman's 1948 proof of the free space Maxwell equations.[4] The proof requires as postulates only Newton's Second Law and the commutation relations of non-relativistic quantum mechanics. The subsequent steps in the proof are algebraic. Dyson remarks,

> ...The proof begins with assumptions invariant under Galilean transformations. How could this have happened? After all, it was the incompatibility between Galilean mechanics and Maxwell electrodynamics that led Einstein to special relativity in 1905. Yet here we find Galilean mechanics and Maxwell equations coexisting peacefully. Perhaps it was lucky that Einstein had not seen Feynman's proof when he started to think about relativity.

Or perhaps not. Recently Tanimura[5] has (a) provided a manifestly covariant version of the Feynman proof, (b) extended the proof to Einstein's geodesic equations for gravitation, and (c) showed that it also includes special relativistic non-Abelian gauge theories. If his claim is correct, and Dyson's hypothetical history had occurred, quantum gravity might now be an established theory instead of the perennial nightmare in which theorists are still enmeshed.

Recently we have tried to disentangle this situation by going back to Bridgman's operational approach[6] using insights from "deterministic chaos"[7] and a new fundamental theory.[8–10] In this paper we try to make the argument more compelling.

2. FINITE AND DISCRETE MEASUREMENT

The Bohr-Rosenfeld derivation approaches measurability in terms of the "Copenhagen Interpretation". This takes classical physics as the only way to describe the world of human experience, and limits measurement accuracy using the *formalism* of non-relativistic commutation relations. This leaves the discreteness of quantum phenomena as a brute fact which is not reconcilable with (is complementary to) classical physics. For a more detailed discussion of the *positivistic* nature of the Copenhagen interpretation, see the account by Henry Stapp,[12] which includes exchanges of correspondence with Heisenberg.

We propose a new approach to measurement using as our paradigm the non-classical device of a "counter" with linear dimensions Δx in three independent (and for convenience, "orthogonal") dimensions which does not fire (a NO event) or does fire (a YES event) with a time resolution Δt. Then to any NO-YES event we can assign coordinates $(x \pm \frac{1}{2}\Delta x, y \pm \frac{1}{2}\Delta x, z \pm \frac{1}{2}\Delta x; t \pm \frac{1}{2}\Delta t)$ relative to an orthogonal laboratory reference frame (e.g. one corner of the room, the two edges of the floor meeting in that corner, the vertical edge of the room upward from that corner) and a laboratory clock, using the Einstein convention to relate t for this event to that clock and standard laboratory protocol for measuring the distances. We can now state our measurement accuracy postulate, namely that there is *no way* to measure the linear position and time of any such event, either directly or indirectly to better than $\pm\frac{1}{2}\Delta x$ and $\pm\frac{1}{2}\Delta t$. To remove possible ambiguity in this definition, we require $\frac{x}{\Delta x}, \frac{y}{\Delta x}, \frac{z}{\Delta x}, \frac{t}{\Delta t}$ to be non-null *integers*. This means, in particular, that we are *not* allowed to use any "theory of errors" which specifies our measurements to an accuracy better than $\pm\frac{1}{2}$. It also implies that there is some maximum positive integer N_{max} which we can at best establish to $\pm\frac{1}{2}$ by measurement.

If we now try to stick to this understanding of "measurement accuracy" we find that it is not compatible with either rotational or Lorentz boost invariance of intervals between two events in the sense implicitly implied by conventional continuum theories. We therefore need to use some care in spelling out how we rotate and/or boost coordinate values so measured to another laboratory frame. In particular, usually we cannot find an integer $\frac{r}{\Delta x}$ such that r^2 taken equal to $x^2 + y^2 + z^2$ is the square of an integer times Δx^2. However, by taking our clues from the elementary treatment of angular momentum, for any r, z for which

$$r^{\pm}(r,z) \equiv r \mp z, \tag{1}$$

we find that

$$r^{\pm}r^{\mp} = r^{\mp}r^{\pm} = r^2 - z^2 \tag{2}$$

is invariant for rotations of the vector $\vec{r}(x,y,z)$ about the z-axis. In order to describe rotations which change z by the minimum amount allowed in our discrete theory, we now define

$$\begin{aligned} R^{\pm,\pm}(r,z) &\equiv r^{\pm}(r, z \pm \Delta x) \\ R^{\mp,\pm}(r,z) &\equiv r^{\pm}(r, z \mp \Delta x) \end{aligned} \tag{3}$$

and find that

$$\begin{aligned} R^{-,-}(r,z)r^{+}(r,z) &= r(r+\Delta x) - z^2 - z\Delta x \\ R^{+,+}(r,z)r^{-}(r,r_z) &= r(r+\Delta x) - z^2 + z\Delta x. \end{aligned} \tag{4}$$

Hence,

$$\frac{1}{2}[R^{+,+}r^{-} + R^{-,-}r^{+}] = r(r+\Delta z) - z^2 \tag{5}$$

is invariant under rotations about the z-axis. However, if we rotate the coordinates around any axis perpendicular to the z-axis through an angle which changes z by $+\Delta z$ and then return z to its initial value by a rotation about a different axis producing the reverse change($-\Delta z$), these two cancelling rotations in reverse order do not commute:

$$R^{+,+}r^{-} - R^{-,-}r^{+} = 2z\Delta x \tag{6}$$

Note that in any finite and discrete theory, "0" is not a value which can be obtained by measurement.

Consider now the isosceles triangle with height r, base Δx and vertex at the origin of coordinates. If after $2\pi r/\Delta x$ minimal rotations in the same sense about an axis through the origin and perpendicular to the plane of the triangle, this triangle does not return to its initial position, the departure from that position would give us a "vernier" that on repeated rotations, assumed countable and recorded, would enable us to measure changes in length to some arbitrarily small fraction of Δx, contrary to our initial assumption. That is, in addition to the rotational invariance of $r(r+\Delta x)$, we must also require the rotational *symmetry* of equal sided polygons if we are to keep our measurement postulate intact. This does tell us, however, that the minimal distance we can measure using rotations specified in this way is $\Delta x/2\pi$ in contrast to our linear resolution using a single rectangular counter of linear spacial resolution Δx. For more detail, see our discussion of quantized conic sections.[13]

We conclude that, if we want to introduce rotational invariance in this way, we can specify an integer coordinate $z/\Delta x = n_z$ relative to some fixed direction and a radial parameter $r/\Delta x = n_r$, with $n_z \in -n_r, -n_r+2, \ldots, n_r-2+n_r$, such that finite rotations about this axis leave $n_r(n_r+1)-n_z^2$ invariant, but the minimum finite rotations about an axis perpendicular to that direction which first increase z and then decrease it do not commute with those for which the order is reversed.

Our treatment of Lorentz boosts in the z direction is analagous. Let the velocity be $\vec{v} = \vec{\beta}c$ with c the limiting velocity for information transfer and $|\vec{\beta}|^2 = \beta^2 < 1$. Define $\gamma^2\beta^2 \equiv \gamma^2 - 1$ and the four-velocity $\mathbf{u} = (\gamma, \gamma\vec{\beta})$. Then for rotations keeping $u(u+\Delta u) - u_z^2$ invariant, we can construct the same quantities we used in discussing positions. For boosts keeping $u_\perp^2 = u(u+\Delta u) - u_z^2$ invariant we simply require that $u'(u'+\Delta u) - (u'_z)^2 = u(u+\Delta u) - u_z^2$ and that $u'_z - u_z \in -U_z, -U_z + 2\Delta u, \ldots, U_z - 2\Delta u, +U_z$, where U_z is the maximum value of the vector component of the four-velocity in the z-direction that our context allows us to consider. Then any finite and discrete Lorentz transformation can be constructed from an integer description of the position and four-velocity of the particle using finite and discrete rotations and boosts. We have discussed some of the physical considerations in this construction elsewhere.[14] In the next chapter, we show how to use this formalism to describe the calculation of fields, given a piecewise continuous trajectory for the particle, or vice versa.

3. THE CONNECTION BETWEEN A SINGLE PARTICLE TRAJECTORY AND FIELDS

The formulation of the Feynman theorem as reconstructed by Dyson is simple. In Tanimura's notation:
Given

A single particle trajectory $x(t)$ in terms of three mutually perpendicular coordinates $x_i(t)$, $i,j,k \in 1,2,3$ subject to the constraints

$$[x_i, x_j] = 0;\ m[x_i, \dot{x}_j] = i\hbar\delta_{ij};\ m\ddot{x}_k = F_k(x, \dot{x}; t) \tag{7}$$

then

the force components $F_k(x,\dot{x};t)$ can be expressed in terms of two functions, $E(x,t)$ and $B(x,t)$, which depend only on the coordinate components x_i and the time t and not on the velocity components $\dot{x}_j$; these functions are related to the force by the component equation

$$F_i(x,\dot{x};t) = E_i(x,t) + \epsilon_{ijk} < \dot{x}_j B_k(x,t) >, \tag{8}$$

and E and B satisfy the equations

$$div\ B = 0;\ \partial B/\partial t + rot\ E = 0. \tag{9}$$

Here the Weyl ordering $<>$ is defined by

$$< ab > \equiv \frac{1}{2}[ab + ba];\ < abc > \equiv \frac{1}{6}[abc + bca + cab + acb + cba + bac],\ etc. \tag{10}$$

The postulates can be made even simpler by invoking scale invariance. The Feynman postulates are independent of or linear in m. Therefore they can be replaced by the *scale invariant* postulates

$$f_k(x,\dot{x};t) = \ddot{x}_k;\ [x_i, x_j] = 0;\ [x_i, \dot{x}_j] = \kappa\delta_{ij}, \tag{11}$$

where κ is any fixed constant with dimensions of area over time $[L^2/T]$, and f_k has the dimensions of acceleration $[L/T^2]$. Keeping these postulates consistent with the scale parameter c as the limiting velocity for information transfer can clearly be done without breaking scale invariance. This removes the apparent paradox noted by Dyson of being able to derive Lorentz invariant equations from the Galilean invariant, non-relativistic commutation relations. In fact this "paradox" is already implicit in the BR discussion, as already noted.

The remaining physical point that needs to be made clear is that the "fields" referred to in classical relativistic field theory are *defined* in terms of their action on a *single* test particle. Thus, if we measure the *acceleration* of that particle in a Lorentz invariant way (force per unit rest mass) *and* the force per unit charge is also defined by acceleration *and* the charge per unit rest mass of the test particle is *also* a Lorentz invariant *then* our electromagnetic field theory itself becomes an LT (length-time) scale invariant theory. That is, once we replace the Feynman postulates by Eq. 11 and define $\mathcal{E}(x,t) = E/Q = F_E/m$ and $\mathcal{B}(x,t) = B/Q = F_B/m$, we need only derive the scale invariant versions of Eqs. 8 and 9 obtained by the obvious notational change $F_i \to f_i$, $E_i \to \mathcal{E}_i$, $B_i \to \mathcal{B}_i$. Extension to gravitation makes more use of the concept of path and requires that the ratio of gravitational to inertial mass of the test particle also be Lorentz invariant.

Another way of seeing that these postulates are Lorentz and scale invariant is to define

$$\frac{\Delta x}{c\Delta t} = 1;\quad \frac{\Delta x^2}{\kappa\Delta t} = 2\pi. \tag{12}$$

Replacing $\dot{x}$ by the vector four-velocity $\vec{u}$, note that we can define

$$K_k \equiv x_i u_j - x_j u_i = n_k \kappa, \tag{13}$$

where κ is a finite constant fixed by our measurement accuracy and n_k an integer or half-integer measuring angular momentum per unit mass. For constant velocity segments between events along a particle trajectory, this is simply a scale invariant and (thanks to the use of four-velocity) Lorentz invariant quantization of Kepler's second law. Similarly, "Newton's second law" in our context is simply the requirement that an acceleration be a function only of a position and a velocity, and the "Lorentz force law" simply the resolution of such an acceleration into two functions which depend only on position and time, one of which must be perpendicular to the velocity.

In a continuum relativistic theory, the concept of "acceleration" is difficult to define consistently and the concept of "forces on an extended rigid body " hopeless. Here, by using piecewise continuous trajectories— just as Newton did in deriving gravitation from Kepler's laws — we can define acceleration at a "point" as the finite change in velocity between the two segments. To make this covariant, we need three distinct points along the trajectory, a fourth distinct point for the position of the reference clock, and must use the relativistic velocity difference formula (or equivalently the change in the vector components of four-velocity) to define acceleration. In effect this makes the electric and magnetic field components non-commutative and reproduces Bohr and Rosenfeld in reverse by showing that our equations are a quantized version of the Maxwell equations rather than vice versa. To get the $g_{\mu\nu}$ of the geodesic equations, we need the connectivity between four points and a center, which increases the non-locality of the theory beyond that encountered in electromagnetism and gives us (weak field) quantum gravity. We will spell out details on another occasion. [15]

Note that in both cases we must separate source and sink in order to avoid the problem of radiation reaction. We can treat the field as given and calculate the trajectory or treat the trajectory as given and calculate the field, but not both at once. That would require a finite-particle-number relativistic quantum scattering theory.

It is important to realize why we have been able to, in effect, construct a relativistic quantum theory *without* mentioning Planck's constant. The reason is simply that our measurement accuracy postulate refers to measurement of position, velocity, and change in velocity of a *single* particle. If we try to measure distances below $\Delta x_{crit} = \hbar/2m_e c$ we will always have a finite probability of producing electron-positron pairs, and the concept of a particle (and hence of a classical field) becomes inapplicable. This shows that our reasoning is operationally sound and self-consistent. Any phenomenon which, directly or indirectly, allows us to measure Planck's constant in elementary particle mass and/or energy units *breaks* scale invariance.

4. A NEW FUNDAMENTAL THEORY

The discussion above is presented without explicitly exhibiting the route by which this author reached his conclusions. Recognizing that the Feynman-Dyson-Tanimura proof could be re-grounded in the work of Bridgman, Bohr, and Rosenfeld would not have been possible without much prior work by many people (see Chapter 5, "Historical Acknowledgements"), The new, alternative fundamental theory developed by this author in collaboration with various members of the Alternative Natural Philosophy Association[9−11,16] opens up further exciting possibilities in the discussion of the relationship between classical physics and quantum mechanics.

Define *particles* as the *conceptual* carriers of conserved quantum numbers between events and *events* as regions across which quantum numbers are conserved. Take as the basic paradigm for two events the sequential firing of two counters separated by distance L and time interval T, where the clocks recording the firings are synchronized using the Einstein convention. Define the velocity of the "particle" connecting these two events as $v = \beta c = L/T$, where c is the limiting velocity for the transfer of *information*. Given a beam of particles of this velocity selected by a collimator and counter telescope incident on two slits a distance w apart we find a double slit interference pattern at a detector array a distance D behind the slits whose maxima are separated by a distance s. Define the *deBroglie wavelength* $\lambda = ws/D$ using laboratory units of length. If a different source producing particles with the same velocity incident on the same arrangement gives a fringe spacing s', define the mass ratio $m'/m = s/s'$. Introduce Planck's constant h by the definition $\lambda = h/p$ where $\beta = pc/E$, $E^2 - p^2c^2 = m^2c^4$. *Postulate* that two events mediated by a particle of mass m and velocity βc can, but need not, take place *only* when they are separated by an integer number of deBroglie wavelengths.

Consider a particle bound to a center a distance r away which receives an impulsive force toward the center each time it has moved a deBroglie wavelength. Assume that the area swept out per unit time by the radial distance to the particle is constant for each step (Kepler's Second Law) and that the polygon closes after j steps. If we take $2\pi r = j\lambda$, and compute the square of the quantized angular momentum consistent with this correspondence limit we find it equal to $(j^2 - \frac{1}{4})\hbar^2 = \ell(\ell+1)\hbar^2$ where we have defined $\ell = j - \frac{1}{2}$. Assuming that the probability of the impulsive force occurring after one *Compton* wavelength is $1/137(\ell+1)$, we obtain[10,11] Bohr's relativistic formula $(\frac{m-\epsilon_\ell}{m})^2[1 + (\frac{1}{137(\ell+1)})^2] = 1$ for the levels of the hydrogen atom[17] in the approximation $e^2/\hbar c \approx 1/137$, and hence *his* correspondence limit. Adding a second degree of freedom gives us the Sommerfeld formula and an improvement of four significant figures[11] in our value for $e^2/\hbar c$. After deriving the commutation relations, we can invoke[18] Feynman's proof of the Maxwell Equations[4] to show that we also have the correct classical fields in the appropriate correspondence limit. For gravitational orbits about a center containing N particles of mass m, orbital velocity reaches c when $\ell = 0$ and $N = M_{Planck}/m$, where $M_{Planck} = (\frac{\hbar c}{G})^{\frac{1}{2}}$ is the Planck mass. Consequently the shortest

Table 1. Coupling constants and mass ratios predicted by the finite and discrete unification of quantum mechanics and relativity. Empirical Input: $c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990.

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136]\times[1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 31\ldots\times 10^{38}$	$[1.69358(21)\times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\ \ \overline{2}]^{-1}\times[1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots\times 10^{-5}$	$[1.02\ 682(2)\times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	$[0.2259(46)]$
$\alpha^{-1}(m_e)$	$137\times[1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	$[137.0359\ 895(61)]$
$G^2_{\pi N\bar{N}}$	$[(\frac{2M_N}{m_\pi})^2-1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	$[13, 3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\ \frac{4}{5}} = 1836.15\ 1497\ldots$	$[1836.15\ 2701(37)]$
$m_\pi^\pm/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	$[273.12\ 67(4)]$
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 143\ldots$	$[264.1\ 373(6)]$
m_μ/m_e	$3\cdot 7\cdot 10[1-\frac{3}{3\cdot 7\cdot 10}] = 207$	$[206.768\ 26(13)]$

COSMOLOGICAL PARAMETERS

Parameter	Calculated	Observed
N_B/N_γ	$\frac{1}{256^4} = 2.328....\times 10^{-10}$	$\approx 2\times 10^{-10}$
M_{dark}/M_{vis}	≈ 12.7	$M_{dark} > 10M_{vis}$
$N_B - N_{\bar{B}}$	$(2^{127}+136)^2 = 2.89...\times 10^{78}$	*compatible*
ρ/ρ_{crit}	$\approx \frac{4\times 10^{79}m_p}{M_{crit}}$	$.05 < \rho/\rho_{crit} < 4$

distance (between *two* events!) in the theory is the Planck length $h/M_{Planck}c$. Thanks to the fact that our Lorentz-invariant (for finite and discrete boosts and rotations!) theory predicts both the (quantized) Newtonian interaction and spin 2 gravitons, it meets the three classical tests of general relativity.[19]

The first approximations $\hbar c/e^2 \approx 137$, $\hbar c/Gm_p^2 \approx 1.7\times 10^{38}$, and $\hbar c/G_F m_p^2 \approx \overline{2}(256)^2$ came initially from the *combinatorial hierarchy* of Parker-Rhodes (cf. Chapter 5), and the original electron-proton mass ratio calculation was also due to him; alternative derivations now exist. The proton, viewed as a charged, rotating black hole is stabilized against decay due to Hawking radiation by charge, spin, and baryon number conservation.[20] Equating the electromagnetic mass of the electron to its weak-interaction mass provides weak-electromagnetic unification at the tree level. Corrections to the first approximations are made in a uniform way, and are mainly due to McGoveran. Representing the first three levels of the combinatorial hierarchy by bit-strings of length 16 conserving lepton number, baryon number, charge and the z-component of weak isospin in the usual way provides the quantum numbers of all the individual particles of the first three generations of the standard model of quarks and leptons and defines no other elementary particles. A simple algorithm generating the bit-strings provides a good first order cosmology. Quantitative results are given in Table I.

5. HISTORICAL ACKNOWLEDGMENTS

The idea that the way we *measure* physical quantities specifies what we *mean* by these numbers was distilled by Bridgman[21,22] out of his critical examination of the *practice* of physics. That the way we *think* about physics and *calculate* physical quantities might have consequences that are naively interpreted as observed rather than *predictable* was an insight of Eddington's.[23] Bastin and Kilmister found Eddington's articulation of this insight flawed, and launched a research program which was intended to put this approach on a firmer foundation.[24–29] They were joined by John Amson, Gordon Pask, and Fredrick Parker-Rhodes. In 1961 Parker-Rhodes invented the *combinatorial hierarchy*[30] and in subsequent work crystallized his concept of *indistinguishables*.[31] My encounter with these ideas[32,33] led, eventually, to the foundation of the *Alternative Natural Philosophy Association* (ANPA) by Amson, Bastin, Kilmister, Noyes, and Parker-Rhodes in 1979.

Subsequent work is somewhat unevenly documented in the proceedings of the annual international meetings of ANPA (1979 —) and ANPA WEST (1985 —). Contributions to the research program by I. Stein, D.O. McGoveran, M.J. Manthey, and C. Gefwert deserve particular mention. I am especially indebted to V.A. Karmanov for bringing Tanimura's paper to my attention.

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