SLAC-PUB-6442
February 1994
(A)

Calibration of the X-Ray Ring Quadrupoles, BPMs, and Orbit Correctors Using the Measured Orbit Response Matrix *

J.Safranek
National Synchrotron Light Source, Brookhaven National Laboratory,
Upton, New York 11973

M.Lee
Stanford Linear Accelerator Center, Stanford University
Stanford, California 94309

Abstract

The quadrupole strengths, beam position monitor (BPM) gains, and orbit correction magnet strengths were adjusted in a computer model of the NSLS X-Ray ring in order to best fit the model orbit response matrix to the measured matrix. The model matrix was fit to the 4320 data points in the measured matrix with an rms difference of only 2 to 3 microns, which is due primarily to noise in the BPM measurements. The strengths of the 56 individual quadrupoles in the X-Ray ring were determined to an accuracy of about 0.2%. The BPM and orbit corrector calibrations were also accurately determined. A thorough analysis of both random and systematic errors is included.

1 Introduction

At the NSLS, there are a number of different models of the X-Ray ring in use. The models differ by a couple percent in quadrupole family strengths, so it is hard to say what the real strengths of the quadrupoles are. For many of the accelerator physics projects, one needs to know the ring optics, so we have developed a computer code to calibrate the X-Ray ring quadrupoles using

Work supported by the Department of Energy, Contract DE-AC03-76SF00515

Presented at Orbit Correction and Analysis Workshop,
Upton, Long Island, New York, December 1-3, 1993

the measured orbit response matrix. The work we have done builds upon and borrows ideas from the computer codes CALIF [1] and RESOLVE [2].

The high accuracy BPMs at the NSLS yield very precise information about the ring optics.[3] In the X-Ray ring there are 51 horizontal correctors and 39 vertical correctors, and the closed orbit can be measured in both planes at 48 BPMs. When we measure the change in orbit at each BPM for a change in each corrector magnet, we have $(51+39)48 = 4320$ very accurate pieces of data describing the magnetic field gradient around the ring. With this data we are able to find all the quadrupole strengths in the ring as well as the BPM and corrector calibrations.

2 Method

We used the COMFORT [4] accelerator optics modeling program to calculate the model response matrix. The quadrupole, BPM, and corrector calibrations were varied in order to best fit the model matrix to the measured one. The orbit response matrix is defined by

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = M \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

where M is either the model or the measured matrix which gives the change in orbit $\mathbf{x}, \mathbf{y}$ with a change in corrector strengths $\boldsymbol{\theta}_{x,y}$. To minimize the difference between the model and measured matrices, we made a vector, $\mathbf{V}$, with the elements of $\mathbf{V}$ equal to the difference between the measured and model response matrices. $\mathbf{V}$ has 4320 elements, which is the number of horizontal and vertical correctors times the number of BPMs. Then the equation

$$\mathbf{V} = \frac{d\mathbf{V}}{dK_j}\Delta K_j + \frac{d\mathbf{V}}{d\theta_j}\Delta\theta_j + \frac{d\mathbf{V}}{dG_j}\Delta G_j + \frac{d\mathbf{V}}{d(\Delta p/p)_j}\Delta(\Delta p/p)_j \qquad (1)$$

was solved for changes in quadrupole strengths (K_j), corrector strengths (θ_j), the BPM gains (G_j), and $(\Delta p/p)_j$ in order to best fit the measured to model response matrices. The parameter $(\Delta p/p)_j$ is the electron energy shift that occurs when the j^{th} horizontal corrector strength is changed by θ_j. This energy shift causes a shift in orbit proportional to the dispersion that is just large enough to keep the total path length of the electron trajectory fixed. The elements of $\frac{d\mathbf{V}}{d(\Delta p/p)_j}$ are equal to the horizontal dispersion. This orbit shift can be large in the X-Ray ring, so it must be included to get a good fit between the measured and model response matrices.

In equation 1 we varied 57 K_j's for the 56 individual quadrupoles in the X-Ray ring plus the gradient in the dipoles. We varied 51 $(\Delta p/p)_j$'s for the 51 horizontal correctors, and we varied 96 G_j's for the 48 horizontal BPMs and the 48 vertical BPMs. We could not independently vary all the BPM G_j's and all the corrector θ_j's, because there would be a degeneracy in the solution. All the BPM gains could be increased while all the corrector θ_j's were decreased, and the model matrix would stay constant. To avoid this degeneracy, we assumed one horizontal corrector and one vertical corrector were calibrated correctly. We fixed these two corrector strengths, and calibrated all the other correctors and BPMs relative to these two correctors. Thus we varied 50 θ_j's for the 51 horizontal correctors and 38 θ_j's for the 39 vertical correctors. This gave us a total of 292 varied parameters to fit the 4320 measured data points. Equation 1 can be written as

$$V_i = \frac{dV_i}{dx_j} \Delta x_j \tag{2}$$

with the 292 parameters denoted by x_j's.

Actually the equation we solved was a slightly modified version of equation 2. Different BPMs in the ring have different noise levels associated with their orbit measurements. We measured the noise level for each BPM by measuring the orbit many times in succession without changing any corrector magnet strengths. The rms orbit shift between successive orbits for the k^{th} BPM, σ_k, gave the noise level associated with that BPM. The rms noise levels ranged from 1.1 μm to 5.1 μm, with a typical noise level of about 2 μm. We gave greater weight to those BPMs with lower noise by solving

$$V_i/\sigma_k = \frac{dV_i/\sigma_k}{dx_j} \Delta x_j. \tag{3}$$

In this way we were minimizing the χ^2 deviation of the model from the measurements, where

$$\chi^2 = \sum_{i=1}^{4320} \frac{V_i^2}{\sigma_k^2}. \tag{4}$$

The change in the model matrix with quadrupole strengths is nonlinear, so equation 3 was solved iteratively. The parameter changes from the first iteration were put into COMFORT, and a new model matrix was calculated. Then equation 3 was solved again, and so on until the solution converged to the minimum χ^2. After convergence, the rms difference between the model and measured matrices was 2.7 μm which is very close to the BPM noise level of 2.0 μm. Figure 1 shows the very good agreement between the measured and model response from one of the vertical correctors.

3 Error Analysis

Once the algorithm had converged to minimize χ^2, we had calibrations for the quadrupoles, BPMs, and correctors that gave a very good fit to the measured data. Then we had to determine if these magnet strengths and BPM calibrations were really the ones in the ring. We had to address both random and systematic errors.

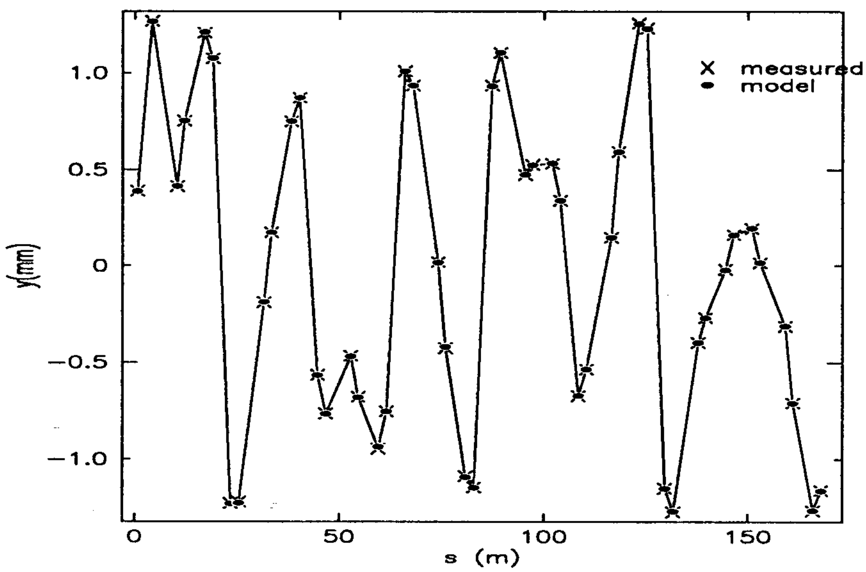

Figure 1: The measured and model response from one of the vertical correctors at the 48 BPMs.

3.1 random errors

The easiest way to determine how much the fitted parameters vary due to random errors in the measurements is simply to take many data sets, analyze each one separately, and see how much variation there is between fitted parameters for the different data sets. We measured the response matrix ten times, and fitted a model to each response matrix. Then for each of the parameters we took the average over the ten data sets and calculated the rms difference from this average. For example, table 1 shows the average quadrupole gradient over the ten data sets for the 16 quadrupoles in the QA family of the X-Ray ring: The rms deviations are about .05%; the differences between the parameter sets for the ten models were small. This means that the solution is unique (to within .05%). There is only one model that fits the data. As far as random errors are concerned, we can be confident that the model we have calculated gives the true quadrupole gradients of the ring. The rms deviations over the ten models for the calibrations of the BPMs and correctors were .17%, so random errors give a contribution of .17% to the error bars on our model BPM and corrector calibrations.

TABLE 1: Calculated quadrupole gradients averaged over ten data sets.

QUADRUPOLE	<K>(1/m**2)	rms deviation
QA1	-1.5074	.0009
QA2	-1.5099	.0009
QA3	-1.5073	.0006
QA4	-1.5098	.0008
QA5	-1.5091	.0007
QA6	-1.5068	.0009
QA7	-1.5052	.0005
QA8	-1.5118	.0008
QA9	-1.5076	.0008
QA10	-1.5109	.0010
QA11	-1.5082	.0011
QA12	-1.5086	.0009
QA13	-1.5047	.0005
QA14	-1.5111	.0007
QA15	-1.5088	.0005
QA16	-1.5087	.0008

There is another way to show that the model is uniquely determined by the response matrix data. If the matrix $\frac{dV_i/\sigma_k}{dx_j}$ in equation 3 were degenerate, then it would not have a unique inverse. In such a case there would not be a unique solution to equation 3. In the method section we pointed out that we had avoided one potential degeneracy related to the relative scaling of the BPM and corrector calibrations. This degeneracy was recognized and removed from the problem, but we need to ascertain if there are less obvious degeneracies that could lead to singularities in $\frac{dV_i/\sigma_k}{dx_j}$.

When we solved equation 3, we used singular value decomposition (SVD) [5] to invert $\frac{dV_i/\sigma_k}{dx_j}$. As part of inverting a matrix, SVD calculates the eigenvalues and eigenvectors associated with the matrix. The eigenvalues give a good diagnostic for finding degeneracies in the matrix: a degeneracy shows up as a zero eigenvalue. If we hadn't removed the BPM-corrector scaling degeneracy and had varied all corrector and BPM calibrations, then SVD would have given two zero (to within roundoff error) eigenvalues, one for each plane. A zero eigenvalue means that there is an unbounded region of parameter space in which all the parameter sets fit the data equally well. An eigenvalue close to zero means that there is a large area in parameter space in which χ^2 does not change much from its minimum value. Within measurement accuracy we cannot distinguish between the parameter sets in this area.

Ultimately we can find out if there are problems with degeneracies in solving for the model by fitting multiple data sets as we described above. There are no degeneracies if the parameter sets for different data sets come out nearly the same. Keeping track of the size of the eigenvalues, however, is a good guide while setting up the program to help avoid the pitfall of including too many parameters in the fit. As new parameters are added to the fit to improve the agreement between the model and the measured matrices, one should make sure no very small eigenvalues appear. If they do appear, this implies that the new parameters cannot be uniquely determined by the data.

3.2 systematic errors

In order to get the model to fit the measurements to close to the noise level of the BPMs, we had to work hard to reduce systematic errors. The following is a list of the more important systematic errors we considered:

1. Sextupoles
   (a) gradients from orbit offset
   (b) nonlinearities
2. Coupling
3. Longitudinal position of BPMs, correctors, and quadrupoles

The sextupoles produce two types of systematic errors. A sextupole has a field gradient when the orbit does not go through the center of the sextupole. In principle, these gradients could be fit as seperate parameters, but this does not work in practice with the X-Ray ring. In the X-Ray ring we have two focusing sextupoles per superperiod, one on each side of the quadrupole in the dispersive straight sections. The sextupoles are too close to the quadrupole to independently fit the gradient in all three magnets. Sextupoles also add nonlinearity. This nonlinearity could be accounted for in the model, but the present fitting program uses only the linear response matrix.

To avoid the systematic errors from the sextupoles, we simply turned them off. We can store 50 mA in the X-Ray ring without any sextupoles.

The program COMFORT assumes that the horizontal and vertical planes were completely decoupled for the model response matrix calculation. The decoupling in the X-Ray ring is very good [6], so this should put little error in our fitting. It would be interesting in the future, however, to include coupling in the fitting. We could try to derive the skew gradient distribution around the ring, as well as the normal gradient.

The model response matrix changes as the longitudinal positions of the elements are changed. For example, if a BPM has a longitudinal position error of

Δs and the orbit shift for some corrector at the BPM has an angle x', then the model response matrix element will have an error of $x'\Delta s$. For the orbit shifts such as the one shown in figure 1, the angle at the BPMs is as large as $0.8mrad$. A one centimeter Δs would give an $8\mu m$ error in the model matrix. We found that many of the BPMs and correctors were off by a centimeter or more in our model. To correct this we measured each of the positions with a ruler, and put the new positions in our model. The accuracy of our position measurements was probably a millimeter or two, so there remains some systematic error due to BPM and corrector alignment in our model.

Once we had done all we could to reduce systematic errors, we needed a way to determine if the remaining systematic errors were large enough to require including them when determining the error bars on the model parameters. To do so we needed to look at χ^2_{min} which is the value of χ^2 for the best fit model. If the only errors in the fitting are normally distributed random errors, then χ^2_{min} should be about equal to the number of degrees of freedom, $N-M$, where N is the number of data points (4320), and M is the number of fit parameters (292). More precisely, if there were only normally distributed random errors, and we took many data sets, solving for χ^2_{min} for each data set, then the distribution of χ^2_{min}'s would be centered at $N-M = 4028$ and would have a standard deviation of $\sqrt{2(N-M)} = 90$ [5].

For the ten data sets we fit, we found χ^2_{min} averaged about 7500, which is many standard deviations above 4028. A value of χ^2_{min} one or two standard deviations above 4028 could be explained by the fact that our orbit measurement errors were not normally distributed, but a χ^2_{min} of 7500 can only mean that the systematic errors, though small, are not small enough to be neglected in determining the error bars on our fit parameters.

One way we can gain confidence that our fit parameters are correct despite systematic errors is to look at other measured data from the storage ring that was not used in the model fitting and see if it agrees with the model. We found that the measured tunes agreed with the model tunes to within measurement accuracy. The measured dispersion also agreed with the model dispersion. (The measured and model dispersions did not agree perfectly, because the model does not include dipole terms in quadrupoles and orbit corrector magnets.)

Another way we can gain confidence in the model is look at the variation in calibrations of the quadrupoles that were supposed to have identical calibrations. For example, all the quadrupoles in the QA family shown in table 1 were designed to have identical gradients. However, nothing in the fitting program constrained them to be the same; each gradient was varied independently. The QA family gradients came out very close to the same, but there is more variation in the gradients than can be accounted for with the noise in the orbit measurements. The additional variation is either real or caused by systematic errors in the model. Averaging the gradient over the 16 QA quadrupoles and computing the rms deviation from this average gives an upper bound on the QA gradient error bars due to systematic errors. Similar calculations for the other three quadrupole families yields the following table for the upper bounds on the error bars from systematic errors for quadrupoles in each family:

TABLE 2: Calculated quadrupole gradients averaged over ten data sets, then averaged over all the quadrupoles in a family.

QUADRUPOLE FAMILY	<<K>>(1/m**2)	rms deviation
QA	-1.5085	.0019
QB	1.3585	.0018
QC	-1.4236	.0047
QD	1.3376	.0016

The same analysis of variation in the calibration of identical correctors gives an upper bound on the systematic error bars for correctors of 1.5%. Much of this variation may be real variation in the corrector calibration due to construction tolerances, so the actual error contribution from systematic errors may be smaller than 1.5%.

Each BPM has different electronics with varying gain, so there are no groups of identical BPMs from which error bars could be determined for the BPMs. The BPM and corrector strengths, however, come into the response matrix in the same way, so the 1.5% error bar of the correctors is valid for the BPMs as well.

4 Conclusion

We have shown that it is possible to accurately determine the individual quadrupole gradients, the corrector calibrations, and the BPM calibrations of a circular storage ring using the measured response matrix. In the future we hope to extend this algorithm to find the orbit offsets in sextupoles as well as skew quadrupole gradient distribution in the X-Ray ring.

5 Acknowledgements

The authors wish to acknowledge many useful discussions with Jeff Corbett. Sam Krinsky also gave helpful guidance. We would like to thank Julie Leader for her expert editing.

References

[1] W.J. Corbett, M.J. Lee and V. Ziemann, "A Fast Model-Calibration Procedure for Storage Rings," SLAC-PUB-6111, May, 1993.

[2] M.J. Lee, Y. Zambre, W.J. Corbett, "Accelerator Simulation Using Computers," SLAC-PUB-5701A, 1991.

[3] R. Biscardi, J.W. Bittner, "Switched Detector for Beam Position Monitor," Proceedings 1989 IEEE Part. Accel. Conf. Chicago, IL, p. 1516.

[4] M.D. Woodley et al., "Control of Machine Functions, or Transport Systems," SLAC-PUB-3086, March, 1983.

[5] W. Press, B. Flannery, S. Teukolsky, W. Vetterling, Numerical Recipes, Cambridge, 1990.

[6] J. Safranek, S. Krinsky, "Plans to Increase Source Brightness of NSLS X-Ray Ring," Proceedings 1993 IEEE Part. Accel. Conf. Washington, D.C., p. 1491.