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CP VIOLATION IN *B* DECAYS*

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ABSTRACT

I review how one can test the Standard Model predictions for *CP* violation. This test requires sufficient independent measurements to overconstrain the model parameters and thus be sensitive to possible beyond Standard Model contributions. I address the challenges for theory as well as for experiment to achieve such a test.

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1. Introduction

This review will not contain much that is new for those of you who have spent the past few years thinking about B-decays, of whom there are quite a few in this audience. It is mostly aimed at those who have been focussing on other aspects of CP violation physics. I will assume that this audience is familiar with the usual physics issues in CP violation studies, namely the role of tree and penguin diagrams and the notation for the three generation Standard Model matrix of weak couplings known as the Cabibbo Kobayashi Maskawa or CKM matrix. For a more pedagogical treatment of many of the topics addressed in this lecture see for example the review that I wrote with Yossi Nir.[1]

I will focus on the ability of proposed B factory experiments to confront theoretical predictions, and discuss particularly what experiments and what theoretical developments are needed if we are to truly test the Standard Model predictions. B physics offers us the opportunity to do just that, but in order to achieve such a test we must do more than just observe CP violation in the most readily accessible channel.

In the Standard Model mixing occurs for the neutral B system in much the same way as it does for the neutral Kaons—through box diagrams with two W bosons exchanged. The weak phase of this mixing amplitude is thus predicted from CKM matrix elements. Many models beyond the Standard Model introduce additional mixing diagrams which would in general destroy the relationship between the weak-mixing phase and the CKM matrix.

The weak phases of the decay amplitudes are also predictable in the Standard Model; or rather the phases of the tree and penguin contributions are separately predictable. Thus if either one class or the other dominates the decay there is a simple relationship between the measurable CP-violating asymmetry and the phases of the CKM matrix elements. If the two types of diagrams give comparable contributions more work is needed to relate measurable asymmetries to CKM parameters. I will discuss some examples of this later.

Tests of the Standard Model will be made by testing whether the results reflect the relationships among CKM matrix elements such as those required by the unitarity. The unitarity triangle is a simple geometrical representation of one such relationship:

$$\mathcal{U}_{db} = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{1}$$

The three complex quantities $V_{id}V_{ib}^*$ form a triangle in the complex plane. The three angles of this triangle are labelled

$$\begin{aligned} \alpha &\equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \\ \beta &\equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \\ \gamma &\equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \end{aligned} \tag{2}$$

The aim is to make enough independent measurements of the sides and angles so that this triangle is overdetermined and thereby check the validity of the Standard Model.

I now discuss the measurements that are needed to achieve this test. I will present them in what I consider to be the likely order of accuracy; the measurements for which both theoretical and experimental problems are easiest to control will be the first ones I treat.

2. Measuring V_{cb}

The parameter V_{cb} is best measured in the decay $B \to D^* \ell \nu$. Heavy quark effective theory provides exact predictions for the limit where both the b and c quark masses are taken very large compared to the scale set by QCD (i.e. the physical size of the heavy quark-light quark bound state, which I denote by Λ_{QCD}.) In that limit the bound-state wave function is independent of heavy-quark flavor and of quark spin orientations. Thus, at the kinematic limit point, where the D^* meson is at rest in the B meson rest frame, the wave-function overlap between the initial and final state mesons in this decay is unity. The leading corrections to the heavy quark limit, of order Λ_{QCD}/M_c, vanish for this process at this kinematic point,[2] and the coefficient of $(\Lambda_{QCD}/m_q)^2, q = c, b$ corrections can be estimated with the help of QCD sum rules and models. Corrections from QCD loops have been calculated at order α_s plus all leading logs.[3] All this provides a very accurate relationship between the kinematic-limit-point decay rate and V_{cb}. The accuracy of V_{cb} then depends on how close to the kinematic limit one can measure. Some model dependence creeps in to the extrapolation to the limit point from the data. A high luminosity source of B's should thus allow improved accuracy in the extraction of V_{cb}; already excellent results have been achieved with this method applied to data from CLEO.

3. Some Formalism for B Decays

To discuss CP violating asymmetries we need to introduce the relevant formalism. The two mass eigenstates of the neutral B meson system can be written

$$|B_L\rangle = p\,|B_0\rangle + q\left|\overline{B}^0\right\rangle, \qquad |B_H\rangle = p\,|B_0\rangle - q\left|\overline{B}^0\right\rangle. \tag{3}$$

Here H and L stand for Heavy and Light, respectively. I write $M \equiv (M_H +$

$M_L)/2, \quad \Delta M \equiv M_H - M_L$. I neglect the tiny difference in width between B_H and B_L, $\Gamma_H = \Gamma_L \equiv \Gamma$. $\Delta\Gamma \ll \Gamma$ because it is produced by channels with branching ratios of $\mathcal{O}(10^{-3})$ which contribute with alternating signs.[4] In this approximation the mixing in the B_d system is

$$(q/p)_{B_d} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) = e^{i\phi_M} \ . \tag{4}$$

The amplitudes for decays into a CP eigenstate, which I denote by f_{CP}, are

$$A \equiv \langle f_{CP}|\mathcal{H}|B^0\rangle, \quad \overline{A} \equiv \left\langle f_{CP}|\mathcal{H}|\overline{B}^0\right\rangle. \tag{5}$$

Let us define

$$\lambda \equiv \frac{q}{p}\frac{\overline{A}}{A}. \tag{6}$$

The time-dependent rates for initially pure B^0 or $\overline{B}^0$ states to decay into a final CP eigenstate at time t can then be written

$$\begin{aligned}
\Gamma(B^0_{\rm phys}(t) \to f_{CP}) &= |A|^2 e^{-\Gamma t} \\
&\quad \times \left[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2}\cos(\Delta M t)\right] - {\rm Im}\lambda \sin(\Delta M t)\Big], \\
\Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP}) &= |A|^2 e^{-\Gamma t} \\
&\quad \times \left[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2}\cos(\Delta M t)\right] + {\rm Im}\lambda \sin(\Delta M t)\Big].
\end{aligned} \tag{7}$$

The time dependent CP asymmetry

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})}{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})} \tag{8}$$

is given by

$$a_{f_{CP}}(t) = \frac{(1-|\lambda|^2)\cos(\Delta M t) - 2\mathrm{Im}\lambda \sin(\Delta M t)}{1+|\lambda|^2}. \tag{9}$$

This analysis corresponds to a CP-even final state, for CP-odd states there is an additional minus sign in λ .

In an e^+e^- B factory the initial B and $\overline{B}$ are produced in a coherent state which remains $B^0\overline{B}^0$ until such time as one of the particles decays. If one B decays to a flavor-tagging mode while the other decays to a CP-study mode we have an event that can be used to reconstruct the time dependence of the asymmetry. The time that appears in the equations above is the time between the tagging decay and the CP-study-mode decay. The tagging decay may be the later decay, in which case the correct procedure is to assign a negative time to that event. Note that this makes the measurement of time dependence essential at such a machine, since the time-integrated CP asymmetry vanishes if $|\lambda| = 1$.

If all contributions to the decay amplitude have the same weak phase, ϕ_D, then $\overline{A}/A = e^{-2i\phi_D}$. In this case $\lambda = e^{-2i(\phi_M+\phi_D)}$ and the expression (9) simplifies to

$$a_{f_{CP}} = -Im(\lambda)\sin(\Delta M t). \tag{10}$$

While each of ϕ_M and ϕ_D is convention dependent, the sum $\phi_M + \phi_D$ is not; $\mathrm{Im}\lambda$ depends on convention independent combinations of CKM parameters only.

I now turn to a review of some experiments which can measure the angles β and α :

4. Measuring $\sin(2\beta)$ in $B \to \psi K_S$.

This is the easiest CP-violating B_d decay channel to tackle; both experimentally and theoretically it is very clean. The decay of the ψ to a pair of leptons

(e or μ) gives a readily recognized signature, even in a hadronic environment. Although there is a small penguin contribution to the decay amplitude it has (to a very good approximation, the same weak phase (mod π) as the tree contribution. Thus the extraction of the CKM phase from the experiment does not suffer from uncertainties due to the limitations of our ability to calculate the relative strength of tree and penguin contributions.

The decay phase in the quark subprocess $b \to c\bar{c}s$ is

$$arg\frac{\overline{A}}{A} = arg\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}. \tag{11}$$

With a final kaon, one must also take into account the mixing phase in the K system, $(q/p)_K = (V_{cs}V_{cd}^*)/(V_{cs}^*V_{cd})$. Then, since ψK_S is a $CP = -1$ state

$$\lambda(B \to \psi K_S) = -\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right)\left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}\right).$$

We make the approximation of neglecting the tiny weak phase of V_{cs} which is of order $\theta^4_{\text{Cabibbo}}$, thus

$$\text{Im}\lambda = \sin(2\beta). \tag{12}$$

The branching ratio for this decay is known so we can quite reliably estimate the luminosity needed to measure the angle β. The result is that with $30 fb^{-1}$, (about one year of running at design luminosity), one can achieve a precision of about $\delta(\sin(2\beta)) = \pm 0.06$.[5] Estimated detector efficiencies for both this decay mode and for tagging modes to identify the flavor of the other B in the event have been included here. A couple of years of B factory running can almost certainly achieve a reliable measurement of this angle. Current measurements of related quantities already restrict $-1 \leq \sin(2\beta) \leq -0.08$ within the Standard Model. This mode is also the one that will be most accessible to hadron machines such as the upgraded Tevatron, or the LHC, since the decays $\psi \rightarrow \mu^+\mu^-$ gives a clean signature even in a hadronic environment. Preliminary estimates show that the accuracy obtainable with a year of running for example at the Tevatron is about $\delta(\sin(2\beta) = \pm 0.15$.[6]

A further measurement of $\sin(2\beta)$ can be made using the channel ψK^*. In this channel angular analysis is necessary to select the contribution of a definite CP since there are contributions from both even and odd relative angular momentum between the two particles and hence of both even and odd CP.[7] The branching ratio to this channel is somewhat bigger than that for ψK_S^0. Preliminary data suggest that the decay is dominated by a single CP. If this is so then angular analysis will not dilute the statistical significance. This mode may provide a more accurate constraint on $\sin(2\beta)$ than the simpler mode ψK_S.[8]

5. MEASURING $\sin(2\alpha)$ IN $b \rightarrow u\bar{u}d$.

Here the situation is somewhat more difficult both experimentally and theoretically. The branching ratios channel is not yet known; CLEO has events which could be either $\pi\pi$ or $K\pi$ (and are probably some of each), a branching ratio of order 2×10^{-5} is not unreasonable. Theoretically there are both tree and penguin amplitudes which contribute. The penguin contribution is expected to be small compared to the tree contribution but it depends on the CKM combination $V_{td}^* V_{tb}$ which has a phase different from that of the tree diagram. This destroys the simple relationship between the CP asymmetry and the CKM matrix elements.

If one ignores for the moment the complications due to penguin diagrams, the tree diagrams for such channels contribute an asymmetry (for a CP even final state) is given by (10) with

$$\mathrm{Im}\lambda = \sin(2\alpha). \tag{13}$$

For the $\pi^+\pi^-$ asymmetry the diagrams due to the penguin contribution can be eliminated using isospin analysis.[9] This will require good data for the full set of isospin related channels, including the more difficult to measure $\pi^0\pi^0$ mode. Only one asymmetry need be measured, that is time dependence needs to be reconstructed only in the $\pi^+\pi^-$ channel. This is fortunate because it is unlikely that one can reconstruct time-dependance in the $\pi^0\pi^0$ channel. Isospin analysis can be used to verify that the penguin contribution is small enough that, within experimental errors the measured asymmetry is directly related to $\sin(2\alpha)$, or to extract a corrected, but probably less accurately determined, value for α if this is not so.

A more likely way to get an accurate value for α is the study of the full set of channels $B^0 \to \rho\pi$. Again there are isospin relationships which limit the number of independent penguin amplitudes.

$$\begin{aligned}
\sqrt{2}A(B^+ \to \rho^+\pi^0) &= S_1 = T^{+0} + 2P_1 \\
\sqrt{2}A(B^+ \to \rho^0\pi^+) &= S_2 = T^{0+} - 2P_1 \\
A(B^0 \to \rho^+\pi^-) &= S_3 = T^{+-} + P_1 + P_0 \\
A(B^0 \to \rho^-\pi^+) &= S_4 = T^{-+} - P_1 + P_0 \\
2A(B^0 \to \rho^0\pi^0) &= S_5 = T^{+0} + T^{0+} - T^{+-} - T^{-+} - 2P_0 \ .
\end{aligned} \tag{14}$$

Similarly for the CP conjugate channels one can define the amplitudes $\overline{S}_i$,$\overline{T}^{ij}$, and $\overline{P}_i$ which differ from the original amplitudes only in the sign of the weak phase of each term.

Further the Standard Model predicts for the penguin amplitude the decay weak phase cancels the weak phase of the mixing amplitude, so the only unknown weak phase in the problem is the sum of the tree weak phase and the mixing phase, which is precisely α. Art Snyder and I did a simulation[10] of these channels which showed that a multi-parameter maximum-likelihood fit to the time-dependant Dalitz plots for B^0 or $\overline{B}^0$ decaying to $\pi^+\pi^-\pi^0$ can be used to extract the quantity α. The data from the $\rho\pi$ channels can be parameterized in terms of a sum of products of B weak decay amplitudes for the various channels with the appropriate Breit Wigner function for the decay of the ρ.

Let us denote the Breit Wigner kinematic-distribution functions for the pions produced in the decay of the ρ as f^+, f^-, and f^0 where the superscript denotes the charge of the decaying ρ. The amplitude for $B^0 \rightarrow \pi^+\pi^-\pi^0$ can then be written, ignoring non-resonant contributions, as

$$A(B^0) = f^+ S_3 + f^- S_4 + f^0 S_5/2 \tag{15}$$

while that for the CP conjugate channel is given by

$$A(\overline{B}^0) = f^- \overline{S}_3 + f^+ \overline{S}_4 + f^0 \overline{S}_5/2 \ . \tag{16}$$

Interference between the different ρ charge channels gives a structure to the Dalitz plot distribution that contains information beyond that obtained by considering only the total rates for each channel. The interference effects are large because the angular distribution of the zero-helicity ρ decay throws many events into the corners of the Dalitz plot where ρ bands overlap. Our simulation suggest that 1000 $B \rightarrow \rho\pi$ events are sufficient to allow us to fit all the parameters and thus extract α. If the penguin contributions are small it is also sufficient to resolve the ambiguity between α and $90 - \alpha$ since terms proportional to $\cos(2\alpha)$ occur in the interference regions. Our analysis did not include background from non-resonant channels, but that should not present a problem as its distribution over the Dalitz plot is quite different from the resonant terms which populate only the ρ bands. The branching ratio for B to $\rho\pi$ is not known but it is expected to be higher than that for $\pi\pi$ so this channel will certainly be an interesting one to study.

These three measurements, along with the already well known elements of the CKM matrix (the Cabibbo angle) are enough to determine the CKM triangle fully. In the language of the Wolfenstein parameterization of the CKM matrix they are sufficient to fix the four parameters $\lambda = \sin(\theta_C), A, \rho$ and η. In fact there could be conflicts with the standard model even at this stage because of constraints on these parameters from the value of ϵ measured in K decay. However any additional measurement, either of the side proportional to V_{ub} or of the remaining angle γ will clearly overconstrain the parameters and thus provide a strong test of the standard model.

6. Measuring V_{ub}

For V_{ub} there are both theoretical and experimental challenges. The semi-leptonic inclusive decay rate for B to charm-free final states are proportional to $V_{ub}m_b^5$ times calculable factors. The total rate cannot be measured; one must make kinematic cuts to eliminate the region with possible charm decays. So a spectrum calculation is needed and this introduces the usual uncertainties of hadronization. Further even the total rate calculation is subject to the uncertainties in the value of the heavy quark mass, which can give large corrections because that mass appears to the fifth power in the rate.

Another approach, which eventually may give more accurate results would be to compare the rates for $B \to (\pi \text{ or } \rho)e\nu$ and those for similar D decays. (One can in principle use angular analysis to isolate a particular form factor in $B(D) \to \rho e\nu$ decays. However this will require very high statistics, even the branching ratio $D \to \rho e\nu$ is yet to be observed.)

Assuming the experiment can be done there are model-dependent Λ_{QCD}/m_c corrections to the relationship between the measured ratio and V_{ub}. The challenge here is to calculate these corrections accurately. Naive estimates say these can be as large as 20% effects and they are quite model-dependent. Lattice methods should be able to reduce the uncertainties. Since V_{ub} gives the length of one side of the unitarity triangle it is important to push the accuracy of these estimates as far as can be. If the 20% corrections can be calculated even to an accuracy of 20% that translates into a 4% theoretical error on the extraction of V_{ub}. This is probably an optimistic estimate but a 10% result should be achievable. The lattice calculators are becoming steadily more confident of their ability to treat such heavy quark systems accurately.

7. CAN WE MEASURE $\sin(2\gamma)$?

The early studies for B factories suggested that the remaining angle of the unitarity triangle could be measured using the channel $B_s \to \rho K_S$. However production of the B_s requires that the accelerator be run at the $\Upsilon(5S)$ which is a smaller resonance than the $\Upsilon(4S)$. Furthermore decays to $B_s\overline{B}_s$ are only a fraction of the decays of this resonance. The net effect is that with present machine designs one cannot achieve a sufficient rate of $B_s\overline{B}_s$ production to measure the asymmetry of this mode and extract a value of γ in this way.

A second interesting possibility for studying γ has been suggested[11,12] namely looking at B^+ (or B_d) decays to $D^0 K$. Here CP-violation may be observable in the D^0 decays to a CP eigenstate mode such as $\pi^+\pi^-$. The interference is between the D^0 and $\overline{D}^0$ contributions. This experiment can possibly be done if branching ratio to this channel is somewhat larger than present models suggest. The extraction of γ from the measurement also requires accurate knowledge of the branching ratios of the D^0 and $\overline{D}^0$ decays. Measurements of flavor tagging D decay modes can be used to extract these quantities as long as the D branching fractions to the tagging modes are also well measured. Detailed modelling of all these measurements is needed to be able to estimate the accuracy one could achieve with this method.

My own feeling is that at present it does not seem likely that we will achieve a measurement of γ of sufficient accuracy to test the unitarity triangle on the basis of the sum of the angles. This makes it very important for theoretical work on the corrections to the extraction of V_{ub} to be pursued, as this seems the most likely path to a true confrontation of model with data.

8. Charged B Decays

With the exception of the $D^0 K$ modes mentioned above, CP asymmetries in charged B decays occur only because of interference between tree and penguin contributions in the Standard Model. The observation of any CP asymmetries of this type would be proof that direct CP violation occurs, equivalent to that given by a non-zero measurement of ϵ' in K decays. As for that quantity, the calculations of Standard Model predictions of CP-violating asymmetries in charged B decays contain many uncertainties (see for example Refs. 13, 14, or 15), so it will be difficult to interpret results. However asymmetries are expected only a few percent level; asymmetries much larger than expected would indicate beyond Standard

Model effects.

9. What will we learn from these measurements?

We all tend to take a shortcut and call these tests of the CKM matrix relationships tests of unitarity. Of course that does not mean that we expect theories beyond the standard model to violate unitarity. The theories are all unitary, but they contain physics beyond the Standard Model. Either the additional mixing contributions destroy the relationship between the mixing phase and the CKM matrix phases, or additional quarks require a larger quark coupling matrix fow which three-generation submatrix has no reason to be unitary. In either case the predicted relationships can break down; violations of these relationships would be evidence for physics beyond the standard model. Unfortunately the converse is not generally true. If results consistent with the standard model are observed then many of the models beyond the standard model, particularly those that give additional mixing mechanisms, are not ruled out. We do not even get new bounds on the masses of new particles. This is because the additional contributions have additional arbitrary phases which can always be accidentally aligned with the phase of the standard model contributions. While this does not seem a natural or likely situation, it does make it very difficult to convert a any experimental agreement with the standard model into lower bounds for the masses of new particles; however the parameter space of such models would be severely constrained.

Of course the more exciting possibility is that the Standard Model relationships will not be maintained, in which case we will have some clues as to the type of beyond Standard Model additions needed to accommodate the results. By exploring as many different processes as possible, not just the channels discussed here, we can begin to discover whether the differences from the Standard Model all arise because there is some additional mixing mechanism contributing to $B\overline{B}$ mixing or whether it is a new direct decay mechanism that is giving the non-Standard Model effects. Either way there will be a variety of models to explore.

Other tests of Standard Model predictions that can be made should not be forgotten. For example there are a number of channels such as $B_d \rightarrow K_s K_s$ or $B_s \rightarrow \psi\phi$ (a good one for hadronic machines) where the Standard Model predicts zero CP asymmetry because of the cancellation of decay and mixing weak phases. Bounds on the asymmetries in such channels therefore translate into bounds on contributions to the mixing with phases different from the Standard Model contribution. Non-zero asymmetries observed in such a channel would require either new mixing or new decay contributions so again would be clues to physics beyond the Standard Model.

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