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CP VIOLATION AND THE TOP QUARK

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Abstract

We consider signals of CP violation in semi-leptonic decay of the top quark. We show that the transverse polarization asymmetries of the τ-lepton in the decay $t \rightarrow b\tau\nu$ is extremely sensitive to CP violation. As an illustration we consider CP phases arising from the charged Higgs exchange in the Weinberg three higgs doublet model. Qualitatively, the polarization asymmetries are enhanced over rate or energy asymmetries by a factor of $O(m_t/m_\tau) \approx 100$ with a corresponding increase in sensitivity to CP violating parameters. We also examine τ polarization in b decays via $b \rightarrow c\nu\tau$ and find that it may also be very effective in constraining CP violating effects such as those that arise from an extended Higgs sector.

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1. Introduction

CP violation is a phenomena which has been known to exist for nearly 30 years, and yet, in spite of the best efforts of experimental physics, the known examples remain CP violating decays of the K_L^0 meson. The standard model is thought to explain these effects adequately through a phase in the KM-matrix and perhaps if similar effects are observed in the B^0 system this hypothesis will be bolstered. It is also possible however that other sources of CP-violation besides the KM phase exist in nature. In recent years, there has been much theoretical speculation about physics beyond the standard model. Such models, which often involve an additional particle spectrum above the electro-weak scale, tend to introduce new sources of CP violation. This will happen when the particle spectrum is sufficiently rich that complex couplings may be introduced that can not be transformed away by field redefinitions. Indeed, perhaps a further hint that there is CP violation beyond the KM matrix lies in the baryon asymmetry of the universe. Recent work[1] suggests that although the standard model does not provide enough CP violation to explain this, extensions to the the standard model may well.

As an illustration of an extension to the standard model which contains novel CP violation consider the the Weinberg model[2]. This is essentially the standard model higgs doublet necessary for symmetry breaking is extended to n higgs doublets. If one considers such models where there are no flavour changing neutral currents, when $n \geq 3$ the mass matrix for the higgs sector has sufficient degrees of freedom to allow complex CP violating phases much in the manner that 3 generations are necessary for CP violation in the KM matrix. This allows the possibility of CP violating effects[3,4,5,6].

In such extensions to the standard model it is natural to assume that the novel forms of CP violation which are introduced can be best observed at a high energy scale. Thus due to its large mass the top quark represents a unique probe for addressing these issues; in addition, standard model CP violating effects in the top quark are small[7]. While the FNAL Tevatron is expected to produce enough top quarks to establish the existence of the top quark, hadronic and $e^+ - e^-$ colliders under construction and being proposed should allow the study of possible CP violation. A careful scrutiny of how CP violation may be manifest is therefore warranted.

2. CP Violating Observables

It is useful to separate possible CP violating observables into two categories, those observables that do not require an absorbtive phase and those that do. In order to distinguish between these two cases it is useful to introduce the transformation T_N referred to as "naive time reversal". This transformation is simply the application of time reversal to all momenta and spins without interchanging initial and final states. Thus a CP violating observable which does not have an absorbtive phase will be CP odd and CPT_N even while if an absorbtive phase is required then the observable will CP odd and CPT_N odd. Of course in both cases observables considered will be CPT even.

As an illustration of a quantity which requires an absorbtive phase consider a partial rate asymmetry (PRA). This type of observable occurs when a particle A can decay to a state B and it happens that $\Gamma(A \to B) \neq \Gamma(\bar{A} \to \bar{B})$ motivating the definition:

$$PRA(A \to B) = \frac{\Gamma(A \to B) - \Gamma(\bar{A} \to \bar{B})}{\Gamma(A \to B) + \Gamma(\bar{A} \to \bar{B})}. \tag{1}$$

This observable clearly violates CP and CPT_N. It does not violate CPT since this merely guarantees that $\Gamma(A) = \Gamma(\bar{A})$. The equivalence of total rates between A and $\bar{A}$ under CPT implies that there exists a state C (or perhaps more than one state) such that $PRA(A \to B) + PRA(A \to C) = 0$. This may seem somewhat mysterious until one realizes that the absorbtive phase of $A \to B$ is given by rescattering through state C. This fact is often referred to as the CP-CPT connection[8].

If particle A decays to a state $B = \{b_1, \ldots, b_n\}$ which contains at least 3 particles (ie $n \geq 3$), then CP violation with an absorbtive phase may also give rise to a difference between the energy spectrum of b_i as compared to that of $\bar{b}_i$. Denoting the energy spectrum of particle b_i by $f_i(E)\, dE$ and spectrum of $\bar{b}_i$ by $\bar{f}_i(E)\, dE$ then the energy spectrum asymmetry $\Delta_i(E) = f_i(E) - \bar{f}_i(E)$ is such a CP violating observable. Again this observable violates both CP and CPT_N but not necessarily CPT.

If no absorbtive phase is present a CP violating observable must involve a triple product correlation (ie involve a Levi-Civita ϵ). Thus if no polarizations are measured, at least four particles are required in the final state to construct such an observable; if some polarizations are observed correspondingly fewer particles in the final state are required. It is also important to note that in order to construct a truly CP violating observable, one must combine information from the decay of A and $\bar{A}$. A triple product observable from the decay of A violates T_N however this may be due to final state interaction effects with no CP violation; only if information from A and $\bar{A}$ decay are combine can the two cases be separated.

3. Semi-leptonic Top Quark Decay

Let us now specialize our discussion to the case of semi-leptonic top quark decay:

$$t \to bW^+ \to bl^+\nu_l. \tag{2}$$

Following the above discussion, if the polarization of the lepton is not measured only CP violating observables which involve an absorbtive part may be constructed. In particular we will consider the observable $PRA(t \to bl^+\nu_l)$ and the spectrum asymmetry $\Delta_l(E)$. The absorbtive phase required may, however be easily supplied by the fact that m_t is known to be larger than $m_W + m_b$ hence the W boson involved in this decay will be close to on-shell. Thus the W-propagator will have a large imaginary (absorbtive) component.

A particular model which can give rise to these quantities is the Weinberg model with 3 Higgs doublets. In particular, if the Yukawa couplings between the scalars and the fermions are expressed in terms of the mass eigenstates, the charged higgs bosons have CP violating couplings with fermions. Thus, one can expect CP violating observables will arise due to the interference of $t \to bW^+ \to bl^+\nu_l$ with $t \to bH^+ \to bl^+\nu_l$ (H^+ being the lightest charged Higgs boson).

Following ref. [9], the part of the lagrangian involving fermion H^+ couplings is

$$\mathbf{L} = \frac{g}{\sqrt{2}} H_2^+ (\bar{u}_i \frac{m_{dj}}{m_W} U_d P_R d_j V_{ij}^{KM} + \bar{u}_i \frac{m_{ui}}{m_W} U_u P_L d_j V_{ij}^{KM} + \bar{\nu}_i \frac{m_{li}}{m_W} U_l P_R l_j \delta_{ij}) + H.C. \tag{3}$$

where

$$U_l = -\frac{c_1 s_2 s_3 + c_2 c_3 e^{i\delta}}{s_1 s_2}, \quad U_u = \frac{c_1 c_2 s_3 - s_2 c_3 e^{i\delta}}{s_1 c_2}, \quad U_d = \frac{s_1 s_3}{c_1} \tag{4}$$

and V^{KM} is the Kobayashi-Maskawa matrix. We denote $c_i = \cos(\theta_i)$ and $s_i = \sin(\theta_i)$ where θ_i, $i = 1, 2, 3$ and δ are parameters of the Higgs potential. The CP violation which arises in the above case is proportional to $V_{ul} \equiv Im(U_u^* \ U_l)$.

The fact that the couplings are proportional to fermion mass results in only the $l = \tau$ decay mode being useful. It also turns out that the PRA is vanishingly small[4] due to the miss match between the spin 0 Higgs boson and the spin 1 W boson. The asymmetry in the energy spectrum Δ_τ is not subject to these difficulties and as suggested above it benefits from the resonance effects in the W boson propagator.

In order to quantify this result it is useful to extract a single number from Δ_τ. Thus we introduce the quantity

$$\alpha_E = \frac{< E_\tau^+ > - < E_\tau^- >}{< E_\tau^+ > + < E_\tau^- >}. \tag{5}$$

Where $< E_\tau^+ >$ is the average energy of the τ^+ from t decay and $< E_\tau^- >$ is the average energy of the τ^- from $\bar{t}$ decay. In this model this quantity is given by[4]

$$\alpha_E = \frac{9\sqrt{2}}{4\pi} \frac{G_f m_\tau^2 r_{WH}(1 - r_{Wt}) V_{ul}}{(3r_{Wt}^2 + 2r_{Wt} + 1)(1 - r_{WH}) Br(W \to \tau\nu)} \tag{6}$$

where $r_{WH} = m_W^2/m_H^2$ and $r_{Wt} = m_W^2/m_t^2$. For example, given an ideal detector if $m_t = 150GeV$ $m_H = 200GeV$ and supposing that one had 10^6 $t\bar{t}$ events, one could place the 1 sigma bound of $|V_{ul}| \leq 1000$ (see also [5]). This is similar to the bound one can obtain on this parameter presently due to the current data on $b \to s\gamma$; The leptonic branching fraction of τ decay and the need for the Yukawa couplings to be small enough for perturbation theory to be sensible.

Let us now consider the additional observables which will be available if one is able to measure the polarization of the τ. In the rest frame of the τ lepton, let us define a reference frame where the momentum of the top quark is in the $-x$ direction, the y direction is defined to be in the decay plane such that the y component of the b momentum is positive and the z axis is defined by the right hand rule. In the limit that the τ mass is small, the Weinberg model can give rise to two kinds of CP violating polarization asymmetries[10]. These are

$$A_Y = \frac{\tau^+(\uparrow) - \tau^+(\downarrow) + \tau^-(\uparrow) - \tau^-(\downarrow)}{\tau^+(\uparrow) + \tau^+(\downarrow) + \tau^-(\uparrow) + \tau^-(\downarrow)} \quad A_Z = \frac{\tau^+(\uparrow) - \tau^+(\downarrow) - \tau^-(\uparrow) + \tau^-(\downarrow)}{\tau^+(\uparrow) + \tau^+(\downarrow) + \tau^-(\uparrow) + \tau^-(\downarrow)} \tag{7}$$

where for A_Y (A_Z) the arrows indicate the spin up or down in the direction y (z). While both A_Y and A_Z are CP odd, A_Y is CPT_N odd and thus needs an interaction phase while A_Z is CPT_N even.

One may enhance A_Z somewhat by noting that since it is CPT_N even it is proportional to the real part of the W propagator. Since this switches sign as $s = (p_\tau + p_\nu)^2$ passes through m_W^2 let us define A'_Z to be A_Z if $s \leq m_W^2$ and $-A_Z$ otherwise.

Of course the polarization of the τ is not directly observable and must be inferred from the decay distributions of the τ. Let us define the sensitivity S of a method of measuring the polarization of the τ so that given N_τ τ leptons, the error in the measurement of the polarization, ΔP, is given by $\Delta P = (S\sqrt{N})^{-1}$. In a study [11, 12] the decay modes $\pi\nu$, $2\pi\nu$, $3\pi\nu$, $e\nu\bar{\nu}$ and $\mu\nu\bar{\nu}$ are considered as polarization analyzers. The sensitivity which could be obtained in an ideal detector is found to be $S = 0.25$ if one considers only the mode $2\pi\nu$ while if one combines information from all four decay modes, $S = 0.35$. Thus, the error in measuring a polarization asymmetry A, given N_t top quarks, is given by

$$\Delta A = (S\sqrt{NBr(t \to \tau\nu b)})^{-1}. \tag{8}$$

Again assuming 10^6 $t\bar{t}$ pairs with $m_t = 150Gev$ and $m_H = 200GeV$, and taking into account the above efficiency for measuring τ polarization, assuming only the $2\pi\nu$, the 1 sigma limits^{10} that can be placed on $|V_{ul}|$ is 74 from observing A_y and 22 from observing A'_Z.

4. Back to b-Basics

Although e^+e^- colliders which can produce a large number of $t\bar{t}$ pairs may be somewhat in the future, colliders which can produce large numbers of $b\bar{b}$ pairs should be in operation much sooner. It is therefore worthwhile to consider the analogous observables in $b \to c\tau\nu$. In this case the W width effects are small hence only A_Z is likely to be useful. In the case of the Weinberg model A_Z will be proportional to the quantity $V_{dl} \equiv Im(U_d^* \ U_l)$. Supposing that one has 10^8 $b\bar{b}$ pairs at the 1 sigma level one may place a restriction^{10} of $|V_{dl}| \leq 6$.

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