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Influence of Kaonic Resonances on the CP Violation in $B \to K^*\gamma$ Like Processes*

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Abstract

We consider CP violating effects in decays of the type $B \to k_i\gamma \to K\pi\gamma$, $K^*\pi\gamma$ and $K\rho\gamma$, where k_i represents a strange meson resonance. We include in our calculations five of the low-lying resonances with quantum numbers (J^P) 1^-, 1^+ and 2^+. At the quark level these decays are driven by the penguin graph as well as tree graphs. CP violation arises in the Standard Model due to the difference in the CKM phase between these graphs. We model the final state interaction of the hadronic system using the low lying k_i resonances. Bound state effects are incorporated by using ideas based on the model of Grinstein et al and also in another bound state model (somewhat similar to the model of Wirbel et al) which we construct designed to take into account relativistic effects better. In these models we find that radiative decays of B mesons give rise to four of the five kaonic resonances at about 1 to 7% of the inclusive $b \to s\gamma$ rate. Furthermore, in both bound state models we find that the probability of formation of the other three higher resonances is roughly the same as that of $K^*(892)$, which was recently seen at CLEO. In addition to the partial rate asymmetry which arises due to interference between resonances of the same quantum numbers, we show how interference between resonances of the same parity, and also between resonances of the opposite parity, result in two different types of energy asymmetries. CP differential asymmetries at the level of a few percent seem possible. We thus obtain CP violating distributions which may be observed in a sample of about $10^9 B^\pm$ mesons. For concreteness in this paper we deal with only charged B's, neutral B's will be dealt with separately.

1 Introduction

The recent observation [1] of the long awaited process $B \to K^*\gamma$ gave another reminder of the possible richness of the physics which could be observed at B factories.

In this paper we will consider the generalizations of the above decay, *i.e.* we consider the decays $B \to k_i\gamma$ where the k_i denote excited K meson states or resonances. In particular, we wish to investigate if widths of resonances can be used to enhance CP violation effects in B-decays; their importance in the context of the top quark has been emphasized in the past few years[2, 3]. The key difference is that in the case of B-decays the resonances are strongly interacting so that width to mass ratio is much larger than was the case for the top decays wherein the resonances are electroweak in character.

In the case at hand, namely $B \to k_i\gamma$, k_i must have $J \geq 1$; so, in particular we will focus on the five lowest lying such states. We will denote these states k_0, k_1, k_2, k_3 and k_4. The first such state which we write k_0 is $K^*(892)$; it has quantum numbers 1^-. In a constituent quark model it is a $\bar{u}s$ (or $\bar{d}s$) quarkonium state $^3S_1$. The next two states that we are interested in we will denote as k_1 and k_2. They both have quantum numbers 1^+ and in the notation of [4] are written as $K_1(1270)$ and $K_1(1400)$. In the quark model they should correspond to mixtures of the states $^1P_1$ and $^3P_1$. For these pure quark model states we will use the notation $\hat{k}_1 = {}^1P_1$ and $\hat{k}_2 = {}^3P_1$ and so, following [5] the physical states are related to these by a mixing angle θ_{12}:

$$\begin{aligned} k_1 &= \cos\theta_{12}\ \hat{k}_1 - \sin\theta_{12}\ \hat{k}_2 \\ k_2 &= \sin\theta_{12}\ \hat{k}_1 + \cos\theta_{12}\ \hat{k}_2. \end{aligned} \tag{1}$$

Though strictly speaking the states $\hat{k}_1$ and $\hat{k}_2$ are not eigenstates of charge conjugation, in the literature they are sometimes referred to as 1^{+-} and 1^{++} states since they are related by $SU(3)$ to the $b_1(1235)$ and $a_1(1260)$ mesons with those quantum numbers. In fact the mixing here has been observed to be close to maximal; in [5] θ_{12} is experimentally determined to be $56 \pm 3°$.

The 2^3S_1 state $K^*(1410)$ will be denoted as k_3. In principle k_3, which is a radially excited 2^3S_1 state, could also mix with k_0 however since the masses are so far apart it is unlikely that this mixing is large. We will therefore ignore this mixing in our calculation. We also consider the $K_2(1430)$ state with quantum numbers 2^+ which will be designated k_4. D wave states have also been observed around 1700 MeV but we will not include these in our analysis. We have summarized some of the known properties [4] of these states in Table 1.

Consider the two decays $B \to k_i\gamma;\ k_j\gamma$ followed by decays of k_i and k_j decay to a common hadronic final state XY. If the two channels have different CP phases then CP violation could manifest in the momentum distribution of the mesons making up XY. Thus, for example, the energy distribution of one of the particles in XY may be different for B decay compared to that in $\bar{B}$ decay. If the quantum numbers of the two states are the same, as is the case for k_1 versus k_2 and k_0 versus k_3 then there also exists the possibility of a partial rate asymmetry (PRA) between B decay and $\bar{B}$ decay. Of course in the case of neutral B decay one must also consider these phenomena in the context of $B - \bar{B}$ oscillations [6].

We find that the largest effect occurs in the case of the final state $K^*\pi$. In this case we estimate that a difference between the distribution of the K^* in the decays of B^- versus B^+ may well be observable with about 5×10^8 $B^\pm$ decays. The final state $K\pi$ seems to require about 5×10^9 $B^\pm$ decays. In passing, we mention that, whereas we are focussing on influence of resonances to CP violation in radiative decays of B *mesons* to *exclusive* channels, CP violation in *inclusive*, radiative decays of the b *quark* has been examined by Soares [7]. The two approaches are therefore somewhat complementary.

The rest of the paper will proceed as follows: In section 2 we will explain how it could happen that the different k_i decays could acquire different CP phases, in particular in the Standard Model (SM); we will also estimate the magnitude these phases could have. In making these estimates, for incorporating bound states effects, we will use ideas based on references [8, 9]. In section 3 we will explain how these CP phases will give rise to CP violating kinematic distributions as well as partial rate asymmetries. In that section we also estimate the magnitude of various asymmetries to be expected.

2 Basic Processes

Any CP violating phase which enters into the process $B \rightarrow k_i\gamma$ must have its origin either in the electroweak physics which drives the process or in physics beyond the standard model. In the standard model, three classes of quark graphs may contribute as shown in Figure 1. Figure 1a shows the penguin graph for the quark level process $b \rightarrow s\gamma$ which has been studied extensively [10, 11]. Figure 1(b) shows an annihilation process which is operative only in the case of $B^\pm$. This process will give a γk_i state if the two quarks coalesce into the appropriate k_i state. Figure 1(c) shows a spectator process which could give rise to γk_i if the four quarks shown should coalesce into two mesonic states and thence to a k_i state.

Of course to have observable CP violating effects one need not only have a CP violating phase but there must be the interference of processes with different CP phases. In the standard model such a phase is given by the CKM matrix (V). Introducing the standard Wolfenstein [15] parameterization of the CKM matrix [16]:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\sigma e^{-i\delta} \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\sigma e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} \tag{2}$$

where $\lambda = sin\theta_{cabibbo} = 0.22$. The amplitude for the penguin graphs [10, 11] will be proportional to the quantity:

$$\sum_{i=u,c,t} V_{is}^* V_{ib} F(m_i) \tag{3}$$

where the form of F for the graph in Figure 1a is given in [10] and the QCD corrected form is given in [11]. Note that the above sum will be dominated by the $i = c$ and $i = t$ terms, hence from (2) we see that in this parameterization there is little phase in the penguin amplitude. Both the spectator and the annihilation graphs will, on the other hand, be proportional to the product

$$V_{us}^* V_{ub} = A\lambda^4\sigma e^{-i\delta} \tag{4}$$

which has a phase of precisely $-\delta$ in this notation. Thus interference between the penguin graph and either the spectator or the annihilation graphs potentially can produce observable CP violating effects.

To see how this comes about consider, for instance, the two decay channels $B \to k_2\gamma \to \pi K^*\gamma$ and $B \to k_4\gamma \to \pi K^*\gamma$. The final states are the same, hence there could be interference effects between them. If there is a difference in CP phase as well as a difference in the strong interaction phase (*i.e.* CP conserving phase) , there could be a difference in the energy or other distributions between B and $\bar{B}$ decay which would clearly violate CP. We will assume that near these resonances the strong interaction phase is dominated by Breit-Wigner forms for the k_i propagators.

In order to calculate the CP phase for the decay $B \to k_i\gamma$ we will thus need to estimate the relative contribution of each of the three classes of graphs to the five k_i resonances that we are considering.

In the case of the annihilation and spectator graphs it will be useful to compare these rates with Γ^0_{bus}, the tree level inclusive process $b \to u\bar{u}s$ given by:

$$\Gamma^0_{bus} = 3\frac{G_F^2 m_b^5}{192\pi^3}|V_{ub}V^*_{us}|^2 = 3f^{-1}(\frac{m_c^2}{m_b^2})\left|\frac{V_{ub}V^*_{us}}{V_{cb}}\right|^2 \Gamma(b \to e\nu c) \tag{5}$$

where

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x \tag{6}$$

is the phase space factor defined in [14]; in this case $f(m_c^2/m_b^2) = 0.46$, where we are using $m_c = 1.5GeV$ and $m_b = 4.6GeV$.

For a given B decay, via the spectator graph, to the final state X we define

$$r(X) = \frac{\Gamma(X)}{\Gamma^0_{bus}}. \tag{7}$$

2.1 Penguin Graph

The dominant graph as we shall see is likely to be the penguin process depicted in Figure 1(a). In this case the k_i system is formed by the merging of the spectator and the s quark. In order to calculate the probability of the formation of each of the resonances, one needs a model for the bound state effects involved. For this purpose we will consider potential models [8, 9, 17, 18].

In general such a model will relate meson level amplitude $\mathcal{M}_m$ to the quark level amplitude $\mathcal{M}_q$ according to the formula:

$$\mathcal{M}_m = \int \mathcal{M}_q \Phi^B(P)\Phi_i^k(\hat{P})dP \tag{8}$$

where the meson wave functions Φ^B and Φ_i^k are functions of quark momenta and spins with the correct quantum numbers to form the indicated mesons. Here P represents the momentum of the b quark in the rest frame of the B meson and $\hat{P}$ represents the momentum of the s quark in the frame of the k_i. The exact details of how the integral is constructed will depend on the specific model. We will construct two such models which we will designate A and B in order to get a feel for the accuracy of the predictions.

Model A is similar to the one used in [17] and is based on the quark model of Grinstein *et al.* [8] which has been quite successful in semileptonic charm and bottom decays. Model B is constructed to take relativistic effects better and is based roughly on the ideas of Wirbel *et al.* [9]. For model A, non-relativistic kinematics is used for the quarks and P is taken to be the 3 momentum of the b-quark in the B-meson rest frame and $\hat{P}$ is given by

$$\hat{P} = \vec{P} - x_u \vec{K} \tag{9}$$

where $x_u = m_u/(m_u + m_b)$ and $\vec{K}$ is the momentum of the k_i meson in the B rest frame.

The wave functions for the various meson states are approximated by harmonic oscillator functions. Thus the momentum dependent part of the wave functions are:

$$\begin{aligned}
\Phi_B &= \pi^{-\frac{3}{4}} \beta_B^{-\frac{3}{2}} e^{-\frac{P^2}{2\beta_B^2}} \\
\Phi_{1S} &= \pi^{-\frac{3}{4}} \beta_S^{-\frac{3}{2}} e^{-\frac{P^2}{2\beta_S^2}} \\
\Phi_{1P(0)} &= \pi^{-\frac{3}{4}} \beta_P^{-\frac{3}{2}} \frac{\sqrt{2} P_z}{\beta_P} e^{-\frac{P^2}{2\beta_P^2}} \\
\Phi_{1P(\pm 1)} &= \pi^{-\frac{3}{4}} \beta_P^{-\frac{3}{2}} \frac{(P_x \mp i P_y)}{\beta_P} e^{-\frac{P^2}{2\beta_P^2}} \\
\Phi_{2S} &= \sqrt{\frac{3}{2}} \pi^{-\frac{3}{4}} \beta_S^{-\frac{3}{2}} \left(\frac{2P^2}{3\beta_S^2} - 1\right) e^{-\frac{P^2}{2\beta_S^2}}
\end{aligned} \tag{10}$$

Here Φ_{nS} represents the nth excited S-wave state, $\Phi_{nP(m)}$ represents the n th excited P-wave state with angular momentum projection m and ϕ_B is the wave function for the B meson. The constants β_i are determined from the potential model using a variational method. The values obtained in references [17, 8] are

$$\beta_B = 0.41 \text{ GeV} \quad \beta_S = 0.34 \text{ GeV} \quad \beta_P = 0.30 \text{ GeV} \tag{11}$$

where they use the constituent masses

$$m_u = m_d = 0.33 \text{ GeV} \quad m_s = 0.55 \text{ GeV} \quad m_b = 5.12 \text{ GeV}. \tag{12}$$

With the use of the above momentum distributions above we can specify the spin-dependent part of the wave function for the B-meson and for the k_i with $J_z = -1$:

$$\begin{aligned}
|B> &= \frac{1}{\sqrt{2}}\Phi_B\,(b(\uparrow)\bar{u}(\downarrow) + b(\downarrow)\bar{u}(\uparrow)) \\
|k_0> &= \Phi_{1S}\ s(\downarrow)\bar{u}(\downarrow) \\
|\hat{k}_1> &= \frac{1}{\sqrt{2}}\Phi_{1P(-1)}\ (s(\uparrow)\bar{u}(\downarrow) + s(\downarrow)\bar{u}(\uparrow)) \\
|\hat{k}_2> &= \frac{1}{\sqrt{2}}\Phi_{1P(0)}\ s(\downarrow)\bar{u}(\downarrow) - \frac{1}{2}\Phi_{1P(-1)}\ (s(\downarrow)\bar{u}(\uparrow) - s(\uparrow)\bar{u}(\downarrow)) \\
|k_3> &= \Phi_{2S}\ s(\downarrow)\bar{u}(\downarrow) \\
|k_4> &= \frac{1}{\sqrt{2}}\Phi_{1P(0)}\ s(\downarrow)\bar{u}(\downarrow) + \frac{1}{2}\Phi_{1P(-1)}\ (s(\downarrow)\bar{u}(\uparrow) - s(\uparrow)\bar{u}(\downarrow))
\end{aligned} \tag{13}$$

Note that for the 1^+ states we use the quark model basis $\{\hat{k}_1, \hat{k}_2\}$.

At the quark level the $bs\gamma$ coupling is

$$\text{a}\ \bar{s}\ \sigma^{\mu\nu} q_\mu (P_R + \frac{m_s}{m_b} P_L)\ b \tag{14}$$

where $P_R = \frac{1}{2}(1+\gamma_5)$ and $P_L = \frac{1}{2}(1-\gamma_5)$ and a is given in [11]. We will concentrate on the proportional to P_R which is dominant. Note that this gives rise to *left* handed photons in the final state.

Let us take the z-axis in the direction of the hadronic momentum so the 4-momentum of the photon is

$$p_\gamma = \frac{m_B^2 - m_i^2}{2} \begin{bmatrix} +1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{15}$$

and we take the 3-momentum of the b-quark to be $\vec{P}$. Expanding the amplitude for small P_x, P_y we obtain:

$$\begin{aligned}
\mathcal{M}_{++} &= -\mathrm{a}\sqrt{8}(m_b - m_s)P_- \\
\mathcal{M}_{+-} &= 0 \\
\mathcal{M}_{-+} &= -\mathrm{a}\sqrt{8}(m_b^2 - m_s^2) \\
\mathcal{M}_{--} &= +\mathrm{a}\sqrt{2}m_b^{-1}(m_b^2 - m_s^2)P_-
\end{aligned} \tag{16}$$

where $\mathcal{M}_{ij}$ is the amplitude for b quark with spin projection $S_z = j/2$ going to s-quark with spin projection $S_z = i/2$. The quantity $P_\pm = P_x \pm iP_y$.

Using the above expansion we can obtain analytic expressions for the meson amplitudes:

$$\begin{aligned}
\mathcal{M}(k_0) &= -2\mathrm{a}\left(\frac{\hat{\beta}^2}{\beta_B\beta_S}\right)^{\frac{3}{2}}(m_b^2 - m_s^2)e^{-\Delta_S} \\
\mathcal{M}(\hat{k}_1) &= -\sqrt{\frac{1}{2}}\mathrm{a}\frac{\hat{\beta}^2}{m_b\beta_P}\left(\frac{\hat{\beta}^2}{\beta_B\beta_P}\right)^{\frac{3}{2}}(m_b - m_s)^2e^{-\Delta_P} \\
\mathcal{M}(\hat{k}_2) &= -\mathrm{a}\left[\left(\frac{\hat{\beta}^2}{\beta_B\beta_P}\right)^{\frac{5}{2}}\beta_B^{-1}x_u\left(\frac{m_B^2 - m_i^2}{2m_B}\right)(m_b^2 - m_s^2)\right. \\
&\quad \left. +\frac{1}{2}\left(\frac{\hat{\beta}^2}{\beta_B\beta_P}\right)^{\frac{3}{2}}\frac{\hat{\beta}^2}{\beta_P m_b}(m_b - m_s)(3m_b + m_s)\right]e^{-\Delta_P} \\
\mathcal{M}(k_3) &= -\mathrm{a}\left(\frac{2}{3}\right)^{\frac{1}{2}}\left(\frac{\hat{\beta}^2}{\beta_B\beta_S}\right)^{\frac{3}{2}}\frac{3(\beta_B^4 - \beta_S^4) + 2\beta_S^2(x_uK)^2}{(\beta_B^2 + \beta_S^2)^2}(m_b^2 - m_s^2)e^{-\Delta_S} \\
\mathcal{M}(k_4) &= -\mathrm{a}\left[\left(\frac{\hat{\beta}^2}{\beta_B\beta_P}\right)^{\frac{5}{2}}\beta_B^{-1}x_u\left(\frac{m_B^2 - m_i^2}{2m_B}\right)(m_b^2 - m_s^2)\right. \\
&\quad \left. -\frac{1}{2}\left(\frac{\hat{\beta}^2}{\beta_B\beta_P}\right)^{\frac{3}{2}}\frac{\hat{\beta}^2}{\beta_P m_b}(m_b - m_s)(3m_b + m_s)\right]e^{-\Delta_P}
\end{aligned} \tag{17}$$

where

$$\Delta = \frac{x_u^2(m_B^2 - m_i^2)^2}{2m_B^2(\beta_B^2 + \beta_i^2)} \qquad \hat{\beta}_i = \sqrt{\frac{2\beta_B^2\beta_i^2}{\beta_B^2 + \beta_i^2}} \tag{18}$$

We can now calculate the ratio

$$R_i = \frac{\Gamma(B \to k_i\gamma)}{\Gamma(b \to s\gamma)}. \tag{19}$$

The results are shown in Table 2. Note that our results are slightly different than those obtained in [17] because here we expanded the matrix element only to first order in $\vec{P}$.

One thing which is worrisome about the model constructed in this way is that the transformation from P to $\hat{P}$ is not relativistic. The velocity of the mesons however is, since $\frac{v}{c}$ ranges from .85 in the case of k_2 to .95 in the case of K^*. This motivates us to consider a modification which respects relativity.

In model B, we consider wave functions Ψ_i which are functions of 4-momentum and are related to Φ_i by

$$\Psi_i(P) = N_i\Phi_i(\vec{P})\sqrt{\frac{E(m_i - E)}{m_i}}\,e^{-\frac{(E-E_0)^2}{2\beta_i^2}}. \tag{20}$$

In this equation $P \equiv (E, \vec{P})$ is the 4-momentum of the quark in the meson i and

$$E_0 = \frac{m_i^2 + m_q^2 - m_{\bar{q}}^2}{2m_i} \tag{21}$$

m_q and $m_{\bar{q}}$ being the constituent masses. We define the wave function to be 0 if E is outside of the physical range $0 \leq E \leq m_i$ and the normalization constant N_i is determined by the condition

$$\int_{0\leq E\leq m_i} ||\Psi(P)||^2 d^4P \;=\; 1. \tag{22}$$

For the form of the E dependent part of the wave function we are motivated by the similar form in [9].

Thus, in the reaction $B \to k_i\gamma$ the relation between $\hat{P}$ and P becomes, $\hat{P} = L(P - P_\gamma)$ where

$$L = \begin{bmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \tag{23}$$

is the Lorentz boost into the rest frame of k_i. We still use the quark level amplitudes expanded in P_x and P_y in equation (16) and substitute them into equation (8). Thereby we obtain the results in Table 2 labeled model B. Note that these results are somewhat smaller, compared to those from model A, particularly in the case of k_2, k_3 and k_4.

2.2 Four Quark Hamiltonian for B Decays

The four quark couplings involved in $\Delta S = 1$ charmless $B^{\pm}$ decay is given at tree level by $W^{\pm}$ exchange. There are however potentially large QCD corrections which have been calculated using the renormalization group approach [19, 20]. Following these papers, let us establish a basis of $\Delta S = 1$ operators that may mix together:

$$
\begin{aligned}
O_1^{ij} &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha)(\bar{q}^j_\beta \gamma_\mu P_L q^i_\beta) \\
O_2^{ij} &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta)(\bar{q}^j_\beta \gamma_\mu P_L q^i_\alpha) \\
O_3 &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_k (\bar{q}^k_\beta \gamma_\mu P_L q^k_\beta) \\
O_4 &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_k (\bar{q}^k_\beta \gamma_\mu P_L q^k_\alpha) \\
O_5 &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_k (\bar{q}^k_\beta \gamma_\mu P_R q^k_\beta) \\
O_6 &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_k (\bar{q}^k_\beta \gamma_\mu P_R q^k_\alpha)
\end{aligned} \tag{24}
$$

where $i, j \in \{u, c\}$ and $k \in \{u, d, c, s, b\}$. α and β are color indices.

The effective Hamiltonian may be expanded in terms of these operators in the following way:

$$
\begin{aligned}
H_{eff} = 2^{\frac{3}{2}} G_F \Big[& V^*_{cs} V_{cb} (\sum_{i=1,2} c^c_i(\mu) O^{cc}_i + \sum_{i=3,\ldots,6} c^c_i(\mu) O_i) \\
& + V^*_{us} V_{ub} (\sum_{i=1,2} c^u_i(\mu) O^{uu}_i + \sum_{i=3,\ldots,6} c^u_i(\mu) O_i) \\
& + V^*_{us} V_{cb} (\sum_{i=1,2} c^{uc}_i(\mu) O^{uc}_i) + V^*_{cs} V_{ub} (\sum_{i=1,2} c^{cu}_i(\mu) O^{cu}_i) \Big]
\end{aligned} \tag{25}
$$

Following the treatment in [20] to next to leading order in QCD the operators satisfy the following evolution equations:

$$
\begin{aligned}
\mu\frac{d}{d\mu}c_i^q(\mu) &= \frac{\alpha_s(\mu)}{2\pi}\sum_{j=1,\ldots,6} c_j^q A_{j,i} \\
\mu\frac{d}{d\mu}c_i^r(\mu) &= \frac{\alpha_s(\mu)}{2\pi}\sum_{j=1,2} c_j^r B_{j,i}
\end{aligned}
\tag{26}
$$

where $q \in \{u, c\}$ and $r \in \{uc, cu\}$ and the one loop α_s is given by

$$
\alpha_s(\mu) = \frac{12\pi}{(33-3f)\log\frac{\mu}{\Lambda_5}}
\tag{27}
$$

where Λ_5 is the QCD scale for 5 flavors. The matrix A, to the lowest non-trivial loop order is [20]:

$$
A = \begin{pmatrix}
-\frac{3}{N} & 3 & 0 & 0 & 0 & 0 \\
3 & -\frac{3}{N} & -\frac{1}{3N} & \frac{1}{3} & -\frac{1}{3N} & \frac{1}{3} \\
0 & 0 & -\frac{11}{3N} & \frac{11}{3} & -\frac{2}{3N} & \frac{2}{3} \\
0 & 0 & 3-\frac{f}{3N} & \frac{f}{3}-\frac{3}{N} & -\frac{f}{3N} & \frac{f}{3} \\
0 & 0 & 0 & 0 & \frac{3}{N} & -3 \\
0 & 0 & -\frac{f}{3N} & \frac{f}{3} & -\frac{f}{3N} & -6c_F+\frac{f}{3}
\end{pmatrix}
\tag{28}
$$

and the matrix B is:

$$
A = \begin{pmatrix}
-\frac{3}{N} & 3 \\
3 & -\frac{3}{N}
\end{pmatrix}.
\tag{29}
$$

Here the QCD color factors $N = 3$ and $c_F = \frac{4}{3}$. The number of flavors $f = 5$ since we are interested in the evolution above m_b.

Let us define the coefficients of these operators at $\mu = m_W$ to be the tree level values, thus

$$
c_2^u(m_W) = c_2^c(m_W) = c_2^{uc}(m_W) = c_2^{cu}(m_W) = 1
\tag{30}
$$

and all the other coeficients are 0. This implies that $c_i^u(\mu) = c_i^c(\mu) = c_i^{uc}(\mu) = c_i^{cu}(\mu) = c_i(\mu)$. In particular c_1 and c_2 may be solved in a simple form. If we write

$$
c_1 = \frac{1}{2}(c_+ - c_-) \qquad c_2 = \frac{1}{2}(c_- + c_+)
\tag{31}
$$

then the solutions for c_+ and c_- are:

$$c_+(\mu) = \left[\frac{\alpha_s(m_W^2)}{\alpha_s(\mu^2)}\right]^{\frac{6}{23}} \qquad c_-(\mu) = \left[\frac{\alpha_s(m_W^2)}{\alpha_s(\mu^2)}\right]^{-\frac{12}{23}} \tag{32}$$

The evolution equation (26) is readily integrated numerically. If we take $\Lambda_5 = 0.2GeV$ and $m_b = 4.7GeV$ then

$$\begin{array}{ll} c_+ = .846 & c_- = 1.397 \\ c_4 + c_3 = -0.016 & c_4 - c_3 = -0.041 \\ c_6 + c_5 = -0.027 & c_6 - c_5 = -0.043 \end{array} \tag{33}$$

2.3 Annihilation Diagram

In the case of $B^\pm$ decay it is possible to produce a γk_i state through the annihilation graphs such as the one depicted in Figure 1(b).

In general such annihilation graphs can be calculated by relating them to the weak decay constant f_B which we take to be 180 MeV [22], defined so that $f_\pi = 130MeV$. Following the calculation in [23] we assume that the annihilation takes place at 0 relative momentum. Thus if $\mathcal{U}$ is the amplitude for $b\bar{u}$ annihilation at 0 relative momentum where we take

$$p_b = x_b P_B \qquad p_u = x_u P_B \tag{34}$$

then the meson level amplitude can be written as

$$\mathcal{M} = \frac{1}{4} f_B Tr((\not{P}_B + m_B)\gamma_5 \mathcal{U}) \ \ \frac{1}{3}\delta_{ab} \tag{35}$$

where a and b are color indices.

Since our goal is to find contributions which interfere with the penguin graph we must take the photon from the annihilation graph also to be left handed. This photon polarization leads to the graph where the photon is radiated from the b-quark vanishing. In addition graphs where the photon is radiated from the final state vanish in the limit that the light quark mass goes to zero. In contrast the graph where the photon is radiated from the initial $\bar{u}$ quark is proportional to $x_u^{-1} f_B$ and so dominates. In fact in the case of 0 relative momentum this graph by itself is gauge invariant so it makes sense to consider it alone.

The four quark operator which we need to extract from the effective Hamiltonian is $(\bar{u}_\alpha \gamma_\mu P_L b_\alpha)(\bar{s}_\beta \gamma_\mu P_L u_\beta)$. Note that the color structure is fixed by the constraint that the initial $\bar{u}b$ system is a color singlet. Furthermore the CP phase is only present in the CKM product $V_{ub}V_{us}^*$. Since we obtain CP violating observables by interference of this process with the penguin graph, henceforth we will extract only the portion of the amplitude proportional to $V_{ub}V_{us}^*$. The total coeficient for the above operator after suitable Fierz transformation is $\mathcal{D} = 2^{\frac{3}{2}} G_F D V_{ub} V_{us}^*$ where

$$D = -(\frac{1}{3}c_1 + c_2 + \frac{1}{3}c_3 + c_4) \tag{36}$$

Using the numerical results in (33) we calculate $D = -1.029$. Operators with the current structure of O_5 and O_6 do not contribute to the photon emission from the initial $\bar{u}$ quark as may readily be verified by substitution into (35). Emission from the final legs is also suppressed as it does not go like $1/m_u$ or $1/m_s$.

Using equation (35) it is straightforward to calculate the amplitude to specific spin states which we will denote $\mathcal{N}_{\mathcal{S},m}$ where $|\mathcal{S}, m>$ is the angular momentum of the $\bar{u}s$ system quantized in the z-direction:

$$\begin{aligned}
\mathcal{N}_{1\ +1} &= \mathcal{Z} P_-^2 \\
\mathcal{N}_{1\ 0} &= -\frac{\mathcal{Z}}{\sqrt{2}}(\sqrt{s} + m_s + m_u + 2P_z)P_- \\
\mathcal{N}_{1\ -1} &= +\mathcal{Z}(E_u + m_u + P_z)(E_s + m_s + P_z) \\
\mathcal{N}_{0\ 0} &= -\mathcal{Z}\frac{(m_s - m_u)(m_s + m_u + \sqrt{s})}{\sqrt{2s}} P_-
\end{aligned} \tag{37}$$

where $s = (p_u + p_s)^2$; in the $\bar{u}s$ rest frame the 4-momentum of the s quark is

$$p_s = \begin{pmatrix} E_s \\ P_x \\ P_y \\ P_z \end{pmatrix} \tag{38}$$

The energy of the $\bar{u}$ is $E_u = \sqrt{s} - E_s$ and $P_\pm = P_x \pm i P_y$. The factor $\mathcal{Z}$ is:

$$\mathcal{Z} = \frac{G_F m_B f_B e_u e V_{ub} V_{us}^*}{m_u \sqrt{(E_u + m_u)(E_s + m_s)}} D \tag{39}$$

where $e_u e$ is the charge of the u quark and D is the coeficient defined in equation 36. We now consider two different methods for estimating the amplitude for $B \to \gamma k_i$ from this annihilation process. First of all we use the nonrelativistic quark model A above and then we will try to estimate the amplitude in a model independent way.

In terms of a nonrelativistic wave function, the meson level amplitude $\mathbf{N}_i$ is given by:

$$\mathbf{N}_i = \frac{1}{2}\pi^{-\frac{3}{2}} m_i^{-\frac{1}{2}} \int \mathcal{N} |k_i > d^3\vec{P} \tag{40}$$

where the wave functions $|k_i>$ are those given in equation (13) and $\vec{P}$ is the 3-momentum of the s quark in the k_i frame.

We may evaluate this integral analytically if we use the non-relativistic approximation $E_u \approx m_u$ and $E_s \approx m_s$.

Performing this integral the meson amplitudes thus obtained are:

$$\begin{aligned}
\mathbf{N}_0 &= +\mathcal{Z}\sqrt{2}\pi^{-\frac{3}{4}} m_i^{-\frac{1}{2}} \beta_S^{\frac{3}{2}} (4m_u m_s + \beta_s^2) \\
\hat{\mathbf{N}}_1 &= -4\mathcal{Z}\pi^{-\frac{3}{4}} m_i^{-\frac{1}{2}} \beta_P^{\frac{5}{2}} (m_s - m_u) \\
\hat{\mathbf{N}}_2 &= -2\sqrt{2}\mathcal{Z}\pi^{-\frac{3}{4}} m_i^{-\frac{1}{2}} \beta_P^{\frac{5}{2}} (m_s + m_u) \\
\mathbf{N}_3 &= +\mathcal{Z}\sqrt{3}\pi^{-\frac{3}{4}} m_i^{-\frac{1}{2}} \beta_S^{\frac{3}{2}} (4m_u m_s + \frac{7}{3}\beta_s^2) \\
\hat{\mathbf{N}}_4 &= 0
\end{aligned} \tag{41}$$

Using these equations and the values of parameters mentioned above we obtain the following values of r (defined in eqn (7)) for the contribution of the annihilation graphs to various channels:

$$\begin{array}{lclclcl}
r(\gamma k_0) &=& 5.3 \times 10^{-5} & & r(\gamma \hat{k}_1) &=& 1.1 \times 10^{-6} \\
r(\gamma \hat{k}_2) &=& 3.3 \times 10^{-5} & & r(\gamma k_3) &=& 6.6 \times 10^{-5} \\
r(\gamma k_4) &=& 0 & & & &
\end{array} \tag{42}$$

Our model independent attempt to estimate the value of r is based on projecting the component of the quark amplitude with the same quantum numbers as k_i and then converting these components to a decay rate as we shall discuss below.

With respect to the spin and angular degrees of freedom in the meson rest frame we can define the following eigenstates of angular momentum:

$$|1^- > \;=\; Y_0^0 |1 \; -1>_s$$

$$
\begin{aligned}
|1^{+-}> &= Y_1^{-1}|0\ 0>_s \\
|1^{++}> &= \frac{1}{\sqrt{2}}(Y_1^{-1}|1\ 0>_s - Y_1^0|1\ -1>_s) \\
|2^{+}> &= \frac{1}{\sqrt{2}}(Y_1^{-1}|1\ 0>_s + Y_1^0|1\ -1>_s).
\end{aligned} \tag{43}
$$

where $|sm>_s$ represents the spin state of the $\bar{u}s$ system.

For each of these states let $\mathcal{N}(|i>)$ be the corresponding amplitude. We may then construct the quantity $dr_{|i>}/ds$. We now estimate the value of $r(\gamma k_i)$ using

$$
r(\gamma k_i) = \int_{s_{min}}^{s_{max}} \frac{dr_{|i>}}{ds} ds \tag{44}
$$

for suitably chosen values of s_{min} and s_{max}.

For the four states $k_{1,\ldots,4}$ we note that the next k_i states above them is at $1.950GeV$ with width $.2GeV$ so it seems reasonable to chose $\sqrt{s_{max}} = 1.750GeV$. Intuitively a $\bar{u}s$ system with invariant mass below this threshold is forced to form the k_i state with the appropriate quantum numbers. The results for these states do not depend strongly on the value of s_{min} so we take $s_{min} = 0$. likewise for k_0 we take the threshold $s_{max} = 1.2GeV$ since the next similar state (k_3) has mass about $1.4GeV$ and width about $.2GeV$. Using these thresholds we obtain the following r values:

$$
\begin{aligned}
r(\gamma k_0) &= 1.2\times 10^{-6} & r(\gamma \hat{k}_1) &= 3.9\times 10^{-7} \\
r(\gamma \hat{k}_2) &= 3.8\times 10^{-5} & r(\gamma k_3) &= 6.5\times 10^{-5} \\
r(\gamma k_4) &= 0
\end{aligned} \tag{45}
$$

As is apparent, the values are similar to those above in (42) except for the case of k_0 which will not effect our results greatly as it is separated too far from the other resonances to interfere to any large extent.

2.4 Spectator Diagram

Let us now consider the decay rate which is generated by the spectator graph. For this purpose we first calculate the quark level process and then estimate the resultant meson formation. The quark level reaction at tree level proceeds through four diagrams similar to the one shown in figure 1c. For the purposes of our calculation we approximate the final state quarks as being massless and so it is convenient to calculate the helicity amplitudes for the processes.

Let us define a convention for spinors and photon polarizations similar to those used in [21]. Our conventions will be based on three arbitrary light-like reference vectors λ_0, λ_1 and λ_2. Let u_0 be a right handed spinor in the direction λ_0 so that

$$u_0 \bar{u}_0 = P_R \not{\lambda}_0. \tag{46}$$

Likewise we define the left handed spinor u_1 in direction λ_1 as

$$u_1 = \frac{\not{\lambda}_1 u_0}{\sqrt{2\lambda_1 \cdot \lambda_2}} \tag{47}$$

For a general light-like vector p let us define the left and right handed spinors u_- and u_+ respectively:

$$u_- = \frac{\not{p} u_0}{\sqrt{2p \cdot \lambda_0}} \qquad u_+ = \frac{\not{p} u_1}{\sqrt{2p \cdot \lambda_1}} \tag{48}$$

we will not include the color indices throughout.

For simplicity let us adopt the notation

$$[p_1, \ldots, p_n]_\pm = \bar{u}_\#(p_1) \not{p}_2 \ldots \not{p}_{n-1} u_\pm(p_n) \tag{49}$$

where $u_\#(p_1) = u_\pm(p_1)$ if n is odd and $u_\#(p_1) = u_\mp(p_1)$ if n is even. Here p_1 and p_n are assumed to be massless while $p_2 \ldots p_{n-1}$ need not be. Using the definitions of the spinors, we can expand this notation in terms of traces:

$$[p_1, \ldots, p_n]_+ = \begin{cases} \frac{Tr(\not{p}_1 \ldots \not{p}_n \not{\lambda}_1 P_R)}{\sqrt{4 \ \lambda_1 \cdot p_1 \ \lambda_1 \cdot p_n}} & \text{if } n \text{ is odd} \\ \frac{Tr(\not{\lambda}_0 \not{p}_1 \ldots \not{p}_n \not{\lambda}_1 P_R)}{\sqrt{4 \ \lambda_0 \cdot p_1 \ \lambda_1 \cdot p_n}} & \text{if } n \text{ is even} \end{cases} \tag{50}$$

and the corresponding expression for $[\ \ ]_-$ is obtained by changing $P_L \leftrightarrow P_R$ and $\lambda_0 \leftrightarrow \lambda_1$.

Circularly polarized photons may also be expressed in this notation. Thus for a photon with momentum q we may write

$$E_R^\mu = \frac{\bar{u}_+(\lambda_2) \gamma^\mu u_+(q)}{2\sqrt{\lambda_2 \cdot q}}. \tag{51}$$

Left handed polarized photons may be expressed as $E_L^\mu = E_R^{\mu *}$.

The spinor for a massive fermion with mass m momentum p and spin s can be expanded in terms of massless spinors as follows:

$$\begin{aligned} u(p,s) &= u_-(p_-) + \frac{1}{m}[p_+,p_-]_- u_+(p_+) \\ v(p,s) &= u_-(p_-) - \frac{1}{m}[p_+,p_-]_- u_+(p_+) \end{aligned} \tag{52}$$

where $p_\pm = (p \pm ms)/2$

Let us now define the function

$$\begin{aligned} \mathcal{L}(p_1,q,p_2,p_3,p_4,p_5,p_6) &= \\ & [\bar{u}_-(p_2)\gamma^\mu \not{q} \gamma^\nu u_-(p_1)]\ [\bar{u}_+(p_4)\gamma_\nu u_+(p_3)] \\ & [\bar{u}_-(p_6)\gamma_\mu u_-(p_5)] \\ &= -2\frac{[p_2p_6p_5]_-[p_6p_5qp_3]_+[p_4p_1]_-}{p_5\cdot p_6} \end{aligned} \tag{53}$$

We may now write the expression for the b spectator process $b(p_b) \to u(p_1)\bar{u}(p_2)s(p_3)\gamma(q)$ for a left polarized photon as:

$$\begin{aligned} \mathcal{M}(\gamma_L) &= \frac{e.g_W^2 V_{us}^* V_{ub}}{24 m_W^2 \sqrt{q\cdot\lambda_2}} \left[\frac{2}{q\cdot p_1}\mathcal{L}^*(p_1, q+p_3, p_{b-}, \lambda_2, q, p_3, p_2) \right. \\ & -\frac{1}{q\cdot p_3}\mathcal{L}^*(p_3,q,p_2,\lambda_2,q,p_1,p_{b-}) - \frac{2}{q\cdot p_2}\mathcal{L}(p_2,q,p_3,q,\lambda_2,p_{b-},p_1) \\ & +\frac{1}{q\cdot p_b}\mathcal{L}(p_{b-},p_{b-}-q,p_1,q,\lambda_2,p_2,p_3) \\ & \left. +\frac{1}{q\cdot p_b}(p_2\cdot p_3 \quad \lambda_2\cdot q)^{-1}[p_1p_3p_2]_-[\lambda_2 q p_{b+} p_{b-}]_-[p_3p_2q]_- \right]. \end{aligned} \tag{54}$$

where g_W is the weak coupling constant given in terms of the fermi coupling G_F by:

$$G_F = \frac{g_W^2}{\sqrt{32} m_W^2} \tag{55}$$

The amplitude for right polarized photons is given by interchanging $q \leftrightarrow \lambda_2$.

QCD corrections of course come into play, the value of r obtained from (54) need only be multiplied by a factor of:

$$F = \frac{1}{3}\left(2(c_+ + c_4 + c_3)^2 + (c_+ c_4 - c_3)^2 + 2(c_6 + c_5)^2 + (c_6 - c_5)^2\right) \quad (56)$$

from the results in equation (33) we see that $F = 1.073$ so in fact the tree level description seems to be reasonable.

Given the amplitudes for the tree graphs discussed in the last sections, we must now estimate the formation of various particular k_i resonances. We will do this using a "hand waving" argument along the lines of our cutoff method above. Thus, we will use the following two simplifying assumptions: (1) if a system of quarks has the correct quantum numbers to form a particular resonance and the invariant mass of the system is within a reasonable range of the resonance (which we shall define) then it will form the given resonance. (2) A system of quarks will couple most strongly to the lowest orbital angular momentum state possible.

Thus, in the case of the spectator graph we need to distinguish two separate cases (a) the b quark has spin $S_z(b) = -\frac{1}{2}$ and (b) the b quark has spin $S_z(b) = +\frac{1}{2}$. Note that this discussion applies equally to B^- or $\bar{B}^0$ decays.

In case (a) the spectator $\bar{u}$ quark must be polarized $S_z(\bar{u}_{spec}) = +\frac{1}{2}$. Since we are considering only the left handed photon, the spin projection of the hadronic system $J_z(h) = -1$. The left handed nature of the coupling implies that the spins of the quarks from the decay of the b quark are $S_z(s) = -\frac{1}{2}$, $S_z(u) = -\frac{1}{2}$ and $S_z(\bar{u}_{part}) = +\frac{1}{2}$. In total the spin of the quarks forming the hadron is $S_z(h) = 0$ hence $L_z(h) = -1$. From our assumption (2) it follows that $L(h) = 1$ and therefore the $J^P = 1^-$. Thus in this case the preferred final states are k_0 or k_3.

In case (b) the spectator $\bar{u}$ must be polarized $S_z(\bar{u}_{spec}) = -\frac{1}{2}$ while all the participant quarks have the same S_z as before. Thus for a left handed photon, we have $L_z(h) = 0$. and from assumption (2) therefore $L(h) = 0$ so that the total $J^P = 1^+$ and $\hat{k}_1$ and $\hat{k}_2$ should be the favored states.

We can be somewhat more specific by noticing that for example if the decay occurs through O_2 the spectator $\bar{u}$ quark and the u quark have the same color. If it should further happen that the remaining pair of quarks have a different color, then the $u\bar{u}$ pair have the quantum numbers of a ρ while the remaining pair have the quantum numbers of K. On the other hand, only in the situation where the two $\bar{u}$'s (the spectator and the one derived from the virtual W) which have opposite spins happened to have the same color could they form a system which had the quantum numbers as a $K^*\pi$. In fact the color part of the amplitude for the other pairing in the πK^* configuration is 3 times smaller than that of the $K\rho$. We assume that the production of the final state $\hat{k}_1$ and $\hat{k}_2$ are in proportion to the coupling to the $K^*\pi$ or $K\rho$ like configuration of the quarks times the coupling of these 1^+ states to the $K^*\pi$ or $K\rho$. The situation for O_1 is the same except the u and s quark are interchanged as are the $K^*\pi$ and $K\rho$ final states and so a fermionic $-$ sign must also be inserted.

Let us denote $\hat{b}_1^{\rho K}$, $\hat{b}_1^{\pi K*}$, $\hat{b}_2^{\rho K}$ and $\hat{b}_2^{\pi K*}$ to be respectively the coupling of ρK and πK^* to $\hat{k}_1$ and $\hat{k}_2$. From the above argument therefore the ratio between the spectator amplitudes $\hat{\mathcal{M}}_1^{spec}$ and $\hat{\mathcal{M}}_2^{spec}$ is

$$\hat{\mathcal{M}}_1^{spec} : \hat{\mathcal{M}}_2^{spec} = c_1\hat{b}_1^{\pi K*} - c_2\hat{b}_1^{\rho K} : c_1\hat{b}_2^{\pi K*} - c_2\hat{b}_2^{\rho K} \tag{57}$$

Using the values in equation (33) and the couplings derived from experiment derived in reference [5] the above ratio is

$$\hat{\mathcal{M}}_1^{spec} : \hat{\mathcal{M}}_2^{spec} = 14 : -1 \tag{58}$$

so most of the amplitude is in the $\hat{k}_1$ channel.

In order to use assumption (1) we need to decide what threshold to use. Following our discussion above we again pick the threshold $s_{max} = (1750\text{ MeV})^2$. In the case of the annihilation graph, the threshold applies to $s_h = (p_u+p_s)^2$ while in the case of the spectator graph $s_h = (p_1+p_2+p_3+p_4)^2$ where p_4 refers to the momentum of the spectator $\bar{u}$. In our formulation of the amplitude of the b-quark decay we can directly calculate the differential cross section in terms of the variable $s_3 = (p_1 + p_2 + p_3)^2$. If we assume that the spectator quark is roughly stationary then $p_4 = x_u P_B$ and the relation between the quantities is

$$s_h = (m_b + m_u)(m_b m_u + s_3)m_b^{-1} \tag{59}$$

thus $s_h \leq (1750\text{ MeV})^2$ translates to $s_3 \leq 1120$ MeV.

Therefore the values of r for the spectator graph is 5×10^{-7} for case (a) where $S_z(b) = -\frac{1}{2}$ and 1.1×10^{-5} for case (b) where $S_z(b) = +\frac{1}{2}$. Thus, from the spectator graph we obtain the following values:

$$\begin{array}{rclrcl} r(\gamma k_0) & = & 5 \times 10^{-7} & r(\gamma \hat{k}_1) & = & 1.1 \times 10^{-5} \\ r(\gamma \hat{k}_2) & = & 5.6 \times 10^{-8} & r(\gamma k_3) & = & 5 \times 10^{-7} \\ r(\gamma k_4) & = & 0 & & & \end{array} \tag{60}$$

2.5 Meson Couplings

In order to guide our calculations let us now estimate the CP phase and couplings for each of the five channels which we consider. Let us denote $\mathcal{B}_{pen}$ to be the inclusive branching ratio of $b \to s\gamma$ through the penguin graph and $\mathcal{B}_{bus}$ to be $\frac{\Gamma^0_{bus}}{\Gamma_B}$. In our numerical calculations, for concreteness we will take, $\mathcal{B}_{pen} = 2.5 \times 10^{-4}$ roughly corresponding to $m_t = 150 GeV$ [12]. If we take $|\frac{V_{ub}}{V_{cb}}| = .08$ [13], $V_{us} = .22$ and the leptonic branching ratio to be 0.107 then from equation (5) we obtain $\mathcal{B}_{bus} = 1 \times 10^{-4}$. (Of course, the rate for the process $b \to u\bar{s}u\gamma$ will be significantly smaller than this)

For a given channel k_i which decays to a final state XY we model the contribution to the decay process

$$B \to k_i\gamma \to XY\gamma \tag{61}$$

in two stages. Thus we define a coupling A_i governing the decay $B \to k_i\gamma$ and the coupling b_i governing $k_i \to XY$. The amplitude for the entire process (61) is thus $A_i\Pi_{ij}b_j$ where Π_{ij} is the propagator to be discussed later.

Our model for this process will be such that all the interaction phase is in Π_i while all the CP phase is in A_i. Thus A_i is a complex coupling which we may express as:

$$A_i = a_i e^{i\phi_i} \tag{62}$$

where ϕ_i is the CP phase and therefore flips sign under charge conjugation. Note that ϕ_i is related to the CKM phase parameter δ as explained below.

From the decay rates which we have estimated for the penguin and tree processes separately, we may determine the total amplitude, A_i:

$$\begin{aligned} A_i \;=\; & \sigma^i_p\sqrt{\frac{16\pi m_B^3\mathcal{B}_{pen}R_i\Gamma_B}{(m_B^2-m_i^2)}} + \sigma^i_{ann}\sqrt{\frac{16\pi m_B^3\mathcal{B}_{bus}r_i^{ann}\Gamma_B}{(m_B^2-m_i^2)}}e^{i\delta} \\ & +\sigma^i_{spec}\sqrt{\frac{16\pi m_B^3\mathcal{B}_{bus}r_i^{spec}\Gamma_B}{(m_B^2-m_i^2)}}e^{i\delta} \end{aligned} \tag{63}$$

where R is defined in equations (19), δ is the CP phase defined in equation (2) and r_i^{ann} and r_i^{spec} are the values from the annihilation and spectator graphs obtained above. σ^i_p, σ^i_{ann} and σ^i_{spec} are the relative signs between the three amplitudes and are thus either ± 1. In the case of σ_p^i equation 17 gives $\sigma_p^i = -1$ for each of the states; model B gives similar results. Equation 41 gives $\sigma_{ann}^i = -1$ for $i \in \{1,2\}$ and $+1$ for $i \in \{0,3\}$; the projection method gives the same results. In our model σ_{spec}^i is undetermined though it has very little effect on our final results as the spectator amplitude is too small compared to the annihilation or the penguin amplitudes. In our numerical calculations, we will assume in the first instance, that $\sigma_{spec}^i = \sigma_{ann}^i$. Later, in section 3.4, we will try to determine these relative signs between the amplitudes by using a simple model. Furthermore, we will also numerically investigate the effect of switching the signs. Note that the penguin graph is the dominant production mechanism for $B \to k_i\gamma$, hence the magnitude a_i is given to a good approximation by the first term in (63)

As we mentioned before the CP phase, which in our convention is δ, is contained in the tree processes. We can therefore estimate the total phase ϕ_i by:

$$\left|\frac{\sin\phi_i}{\sin\delta}\right| = \sqrt{\frac{\mathcal{B}_{bus} r_i}{\mathcal{B}_{pen} R_i}} \tag{64}$$

where r_i is defined in equation (7). The numerical results for the cases which we consider are compiled in Table 3.

Next we deal with the couplings of the strong decays of the resonances leading to the final states. Their couplings b_i for $i = 0$, 3 and 4 may be obtained from the meson decay widths:

$$b_i = \sigma_d^i \sqrt{\frac{16\pi m_i^3 Br(k_i \to XY)\Gamma_i}{\lambda^{\frac{1}{2}}(m_i^2, m_X^2, m_Y^2)}} \tag{65}$$

Here Γ_i is the total width of k_i and

$$\lambda(u,v,w) = u^2 + v^2 + w^2 - 2uv - 2vw - 2wu. \tag{66}$$

Again σ_d^i is ± 1. In the next section we will discuss in more detail how this sign may be determined.

In [5] the couplings to the physical states k_1 and k_2 are parameterized in terms of θ_{12}, γ_+ and γ_-:

$$b_1^{K*\pi} = -\frac{1}{2}\gamma_+ \sin\theta_{12} + \sqrt{\frac{9}{20}}\gamma_- \cos\theta_{12}$$

$$
\begin{aligned}
b_2^{K*\pi} &= +\frac{1}{2}\gamma_+ \cos\theta_{12} + \sqrt{\frac{9}{20}}\gamma_- \sin\theta_{12} \\
b_1^{K\rho} &= -\frac{1}{2}\gamma_+ \sin\theta_{12} - \sqrt{\frac{9}{20}}\gamma_- \cos\theta_{12} \\
b_2^{K\rho} &= +\frac{1}{2}\gamma_+ \cos\theta_{12} - \sqrt{\frac{9}{20}}\gamma_- \sin\theta_{12} \qquad (67)
\end{aligned}
$$

where the observed values of these $\gamma's$ are[5]:

$$
\gamma_+ = 0.82 \quad \gamma_- = 0.59 \quad \theta_{12} = 56^\circ \qquad (68)
$$

3 How CP Violation May be Detected

Let us consider the various two body decay modes of the k_i states listed in Table 1. Our basic strategy will be to consider the process $B \to \gamma k_i \to \gamma XY$ where more than one possible intermediate state k_i may occur. If the different states have different quantum numbers although the final states are the same, the angular distributions will be different. In addition if there are different interaction phases and CP phases associated with each k_i state there will be a difference in the angular distribution between the decay products of B and $\bar{B}$ which therefore signal CP violation. Indeed if the quantum numbers are the same, as in the case of k_0 vs. k_3 and k_1 vs. k_2, the interference could lead to a partial rate asymmetry.

For such two body decays, let us define s_h to be the invariant mass of the k_i state, $s_h = (P_X + P_Y)^2$ and let θ be the angle between the boost axis and the momentum of the strange particle (K or K^*) in the k_i rest frame and let ϕ be the azimuthal angle. Denoting

$$z = \cos\theta \tag{69}$$

the energy of X in the B rest frame, is given b:

$$E_X = \frac{(m_B^2 + s_h)(s_h + m_X^2 - m_Y^2) - (-1)^{\mathbf{s}_X} z (m_B^2 - s_h)\lambda^{\frac{1}{2}}(s_h, m_X^2, m_Y^2)}{4 m_B s_h} \tag{70}$$

where $\mathbf{s}_X$ is the strangeness of X. Using these variables we denote the decay distributions

$$G(s_h, z) = \frac{d^2}{ds_h dz}\Gamma(B \to \gamma XY) \qquad \bar{G}(s_h, z) = \frac{d^2}{ds_h dz}\bar{\Gamma}(B \to \gamma XY) \tag{71}$$

Of interest to us are the sum and the difference of these quantities:

$$\Delta(s_h, z) = G(s_h, z) - \bar{G}(s_h, z) \qquad \Sigma(s_h, z) = G(s_h, z) + \bar{G}(s_h, z) \tag{72}$$

A non-zero value of Δ is clearly CP violating.

Examining Table 1 it is evident that there are many cases where different channels can lead to the same final state and therefore CP violating energy distributions may be possible. In particular there are two classes of final states which we will consider; $k_i \to XY$ where X is a vector and Y is a pseudoscalar (ie either ρK or πK^*) and the case $k_i \to UV$ where both U and V are pseudoscalars (ie. $K\pi$).

3.1 Interference Between k_i Resonances

All of the observables which we consider here are based on the distribution Δ being non-zero. In the case of the interference between the two k states with different quantum numbers, Δ will arise due to the diagram in Figure 2a. The blobs in this diagram indicate the rescattering which produces an imaginary part of the propagator.

In the case of the interference between k_i and k_j where these two states have the same quantum numbers, there is the additional possible graph in Figure 2b where one state rescatters to the other. Indeed these graphs are very important since the CPT theorem implies that the total decay rate of B and $\bar{B}$ must be the same [24]. Hence for a particular final state f which has a partial rate asymmetry (PRA) then there must be some other final state g which has a compensating PRA. To see how this is implemented in the two diagrams consider, for example, the final state being f in figure 2a. A contribution to the PRA of f arises from the rescattering of state k_i through state g. This is related to the contribution of figure 2b to the PRA of g where k_j rescatters to k_i through state f. In fact these two can be shown to be opposite through the Cutkosky relations and thus will exactly cancel.

In order to understand this properly let us consider the instance of two interfering states k_i and k_j giving rise to the PRA of state f_l. We can break down the amplitude in this instance into three parts. The decay $B \to \gamma k_i$, the propagation of k_i and the decay of $k_i \to f_l$.

The decay $B \to \gamma k_i$, γk_j may be described by the amplitudes

$$A = \begin{pmatrix} A_i \\ A_j \end{pmatrix}. \tag{73}$$

which may contain CP phases. We can write the propagator for the two k states as a matrix

$$\Pi = \begin{pmatrix} \Pi_{ii} & \Pi_{ij} \\ \Pi_{ji} & \Pi_{jj} \end{pmatrix} \tag{74}$$

and we represent the strong interaction decay of the k states by the amplitudes

$$b^l = \begin{pmatrix} b_i^l \\ b_j^l \end{pmatrix}. \tag{75}$$

which are real since there is no CP phase in this instance and we assume that the absorbtive phase is contained in Π.

The amplitudes for B and $\bar{B}$ decays are thus:

$$\mathcal{M}_l = A^T \Pi b^l; \qquad \bar{\mathcal{M}_l} = A^\dagger \Pi b^l \tag{76}$$

Δ is thus related to

$$\begin{aligned} \Delta \mathcal{M}_l^2 &= |\mathcal{M}_l|^2 - |\bar{\mathcal{M}_l}|^2 \\ &= \mathrm{Tr}\left((A^* A^T - A A^\dagger)\Pi b_l b_l^T \Pi^\dagger\right). \end{aligned} \tag{77}$$

Let us consider now what the structure of Π is. For the optical theorem to be true for any possible initial state, Π must satisfy the Cutkosky relation:

$$-Im(\Pi) = \Pi \epsilon \Pi^\dagger \tag{78}$$

where for the matrix Π, $Im(\Pi) = \frac{1}{2i}(\Pi - \Pi^\dagger)$ and ϵ is the rescattering matrix defined by:

$$\epsilon_{st} = \sum_l \int b_s^l b_t^{l\dagger} d\phi_l. \tag{79}$$

Here the sum is over all possible final states and the integral is over the appropriate phase space ϕ_l for the final state l.

We can thus rearrange equation (78) to

$$Im(\Pi^{-1}) = \epsilon \tag{80}$$

Note that ϵ is real since b is real. Since ϵ is also hermitian, therefore it is symmetric too, and so is $Im(\Pi^{-1})$. Let us write

$$Re(\Pi^{-1}) = s_h - M \tag{81}$$

where M is a mass matrix. We choose the basis of the k states so that M is diagonal. Thus

$$M = \begin{pmatrix} m_i^2 & 0 \\ 0 & m_j^2 \end{pmatrix} \qquad (82)$$

and consequently $Re(\Pi^{-1})$ is also symmetric. Since both $Im(\Pi^{-1})$ and $Re(\Pi^{-1})$ are symmetric, so is Π.

Returning now to equation (77) we can verify the demand of CPT that the sum of all PRA's must vanish. To see this we note that if we sum over all final states l and integrate over the phase space of the final state the second factor becomes, after application of equation (78), $Im(\Pi)$. Thus

$$\Delta\mathcal{M}_l^2 = \mathrm{Tr}\left[\left((A^*A^T) - (A^*A^T)^T\right) Im\Pi\right]. \qquad (83)$$

which vanishes as $Im\Pi$ is symmetric whereas its coefficient is anti-symmetric. The requirement of CPT is therefore confirmed.

If we apply this formalism to the more general case where the k_i states of distinct quantum numbers are present, then we may also include components of the distribution Δ which do not contribute to the PRA but nonetheless are CP violating.

Furthermore, if a particular state is the only one with a given set of quantum numbers contributing to the final state the above formalism gives the standard Breit-Wigner form:

$$\Pi_i = \frac{1}{s - m_i^2 + i\Gamma_i m_i} \qquad (84)$$

Note that with respect to the 1^+ states in equation (77) we have worked in the mass basis $\{k_1, k_2\}$. The calculation of the production is however most naturally carried out in the quark model basis $\{\hat{k}_1, \hat{k}_2\}$. If we denote $\hat{A}$ the production amplitude in the quark model basis and $\mathcal{F}$ the suitable mixing matrix then we can relate A to $\hat{A}$ by $A = \mathcal{F}\hat{A}$ in equation (77).

3.2 Vector Pseudoscalar Case

First let us consider the case where $k_i \to XY$; X is a vector and Y is a pseudoscalar. From Table 1 we see that this happens for the resonances $\{k_1, k_2, k_3, k_4\}$. Consider first the case of 1^+. The quantum numbers dictate that the decay may proceed through $L = 0$ or $L = 2$. Recalling that the k_i

has $J_z = -1$ (as mentioned before the z axis is antiparallel to $\vec{p_\gamma}$), thus the decay distribution is proportional to

$$Y_0^0 X_{-1} \tag{85}$$

for the $L = 0$ channel where X_i means vector X with polarization i and $Y_j^i(\theta, \phi)$ is the spherical harmonic. For the $L = 2$ channel the corresponding amplitude is

$$\sqrt{\frac{1}{10}} Y_2^0 X_{-1} - \sqrt{\frac{3}{10}} Y_2^{-1} X_0 + \sqrt{\frac{3}{5}} Y_2^{-2} X_{+1} \tag{86}$$

In the case of a 2^+ channel, $L = 2$ and so the decay distribution is proportional to

$$\sqrt{\frac{1}{2}} Y_2^0 X_{-1} - \sqrt{\frac{1}{6}} Y_2^{-1} X_0 - \sqrt{\frac{1}{3}} Y_2^{-2} X_{+1}. \tag{87}$$

Finally, in the case of a 1^- channel, $L = 1$ and hence the decay distribution is proportional to

$$\sqrt{\frac{1}{2}} \left(Y_1^0 X_{-1} - Y_1^{-1} X_0 \right). \tag{88}$$

Expanding these amplitudes in terms of z and ϕ we get

$$\begin{aligned}
\mathcal{M}_1 &= b_1 \sqrt{\frac{1}{4\pi}} X_{-1} \\
\mathcal{M}_2 &= b_2 \sqrt{\frac{1}{4\pi}} X_{-1} \\
\mathcal{M}_3 &= b_3 \sqrt{\frac{3}{8\pi}} \left(z X_{-1} - \sqrt{\frac{1}{2}} \sqrt{1 - z^2} e^{-i\phi} X_0 \right) \\
\mathcal{M}_4 &= b_4 \sqrt{\frac{5}{32\pi}} \Big((3z^2 - 1) X_{-1} \\
&\quad - \sqrt{2} z \sqrt{1 - z^2} e^{-i\phi} X_0 - (1 - z^2) e^{-2i\phi} X_{+1} \Big)
\end{aligned} \tag{89}$$

where we define the couplings between k_i and XY to be b_i. Thus the matrix U defined above is given by $U_{ij} = b_i b_j \mathcal{R}_{ij}$ where

$$\mathcal{R}_{(XY)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \sqrt{\frac{3}{8}}\, z & \sqrt{\frac{5}{32}}(3z^2-1) \\ 0 & \frac{1}{2} & \frac{1}{2} & \sqrt{\frac{3}{8}}\, z & \sqrt{\frac{5}{32}}(3z^2-1) \\ 0 & \sqrt{\frac{3}{8}}\, z & \sqrt{\frac{3}{8}}\, z & \frac{3}{8}(1+z^2) & \sqrt{\frac{15}{16}}z^3 \\ 0 & \sqrt{\frac{5}{32}}(3z^2-1) & \sqrt{\frac{5}{32}}(3z^2-1) & \sqrt{\frac{15}{16}}z^3 & \frac{5}{8}(4z^4-3z^2+1) \end{pmatrix} \tag{90}$$

3.3 Pseudoscalar-Pseudoscalar case

Now let us consider the case where $k_i \to UV$ and U, V are pseudoscalars; in particular, π, K. In this instance the only states involved are k_0, k_3 and k_4. The relevant amplitudes are as follows:

$$\begin{aligned} \mathcal{M}_0 &= b_0\sqrt{\frac{3}{8\pi}}\sqrt{1-z^2}e^{-i\phi} \\ \mathcal{M}_3 &= b_3\sqrt{\frac{3}{8\pi}}\sqrt{1-z^2}e^{-i\phi} \\ \mathcal{M}_4 &= b_4\sqrt{\frac{15}{8\pi}}z\sqrt{1-z^2}e^{-i\phi} \end{aligned} \tag{91}$$

Hence the corresponding matrix $\mathcal{R}$ is given by:

$$\mathcal{R}_{(UV)} = \begin{pmatrix} \frac{3}{4}(1-z^2) & 0 & 0 & \frac{3}{4}(1-z^2) & \sqrt{\frac{45}{16}}z(1-z^2) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4}(1-z^2) & 0 & 0 & \frac{3}{4}(1-z^2) & \sqrt{\frac{45}{16}}z(1-z^2) \\ \sqrt{\frac{45}{16}}z(1-z^2) & 0 & 0 & \sqrt{\frac{45}{16}}z(1-z^2) & \frac{15}{4}z^2(1-z^2) \end{pmatrix} \tag{92}$$

3.4 Signs of Decay Amplitudes

The convention which we used for the angular variables θ is constructed such that if all the amplitudes are positive then constructive interference occurs if the final strange meson (K or K^*) is in the forward ($+z$) direction. Bearing this convention in mind, let us consider a crude model, based loosely on the idea of "vacuum dominance", which we will use, for the sole purpose of suggesting the signs of the decay amplitudes. As an illustration, let us consider the $K^*\pi$ final state. The full reaction is thus:

$$B \to \gamma k_i \to \gamma \pi K^* \tag{93}$$

The contributing k_i are then k_0, k_3 and k_4 (see Table 1). Let us concentrate on the case when the $B \to \gamma k_i$ decay takes place via the penguin graph. Then the vacuum saturation representation is

$$\mathcal{A} = Tr\,(\Pi_b \gamma_5 \Pi_{u1} \Gamma_i \Pi_{s1} \sigma^{\mu\nu} P_R)\, Tr\,(\Pi_{s2} \Gamma_i \Pi_{u2} \gamma_5 \Pi_d \not{\mathcal{E}}) \tag{94}$$

where Π_b, Π_{u1} and Π_{s1} are propagators of the b, $\bar{u}$ and s quarks in the $B \to k_i\gamma$ decay; Π_d, Π_{u2} and Π_{s2} are propagators of the d, $\bar{u}$ and s quarks in the $k_i \to K^*\pi$ decay and Γ_i is the appropriate gamma matrix insertion for the state k_i and $\not{\mathcal{E}}$ is the polarization of the K^*.

For $i = 0$, 3 we take $\Gamma_i = \gamma^\mu E_\mu$ while for $i = \hat{2}$ we take $\Gamma_i = \gamma^\mu \gamma_5 E_\mu$. In the case of $i = \hat{1}$ the relative sign with $i = \hat{2}$ is determined from [5] as described in equation (67). For the spin 2^+ case $i = 4$ we take $\Gamma_i = \gamma^\mu E_{\mu\nu} \vec{P}^\nu$ where $E_{\mu\nu}$ is the spin 2 polarization tensor and $\vec{P}$ is the momentum of the s-quark in the k_4 frame.

Let us now consider the configuration where $\theta = 0$ and $\mathcal{E}$ is left handed. Further let us take $p_b = x_b p_B$ and $p_{s2} = x_s p_{K*}$ where $x_b = m_b/(m_b + m_u)$ and $x_s = m_s/(m_s + m_u)$ so that all the other quark momenta are determined by momentum conservation. We thus find that the signs in this model are given by: $\sigma_{d0} = \hat{\sigma}_{d2} = \sigma_{d3} = \sigma_{d4} = +1$. Applying a similar analysis to the ρK final state we find the same signs hold: $\sigma_{d0} = \hat{\sigma}_{d2} = \sigma_{d3} = \sigma_{d4} = +1$ as well as for the $K\pi$ final state $\sigma_{d0} = \sigma_{d3} = \sigma_{d4} = +1$. Although, in our numerical work, for definiteness we will use signs as given by this simple model, later we will comment on possible effects due to the signs being different from those given by this model.

3.5 Observables

In order to observe a component of the asymmetry, it is useful to form an observable with the same symmetry as the component we wish to observe. Thus we can take any function $w_i(s_h, z)$ and form the quantity

$$< w_i >_B - < w_i >_{\bar{B}} \tag{95}$$

which is CP violating. The effectiveness of this observable to statistically extract the signal from the background can be parameterized by the quantity:

$$\mathcal{E}_i = \frac{\int w_i(s_h, z)\Delta ds_h dz}{\sqrt{\int w_i^2 \Sigma ds_h dz \ \int \Sigma ds_h dz}} \tag{96}$$

where Σ and Δ are defined in equation (72) and z is defined in equation (69).

The meaning of this quantity is that given N events of the specified form the effect may be distinguished with a significance of $\mathcal{S} = \mathcal{E}\sqrt{N}$. Thus the total number N_B of B mesons (including both B and $\bar{B}$) needed to observe the effect at $1 - \sigma$ is

$$N_B = \frac{1}{\mathrm{Br}\ \mathcal{E}^2} \tag{97}$$

where Br is the total branching ratio for (61). Clearly we would like $\mathcal{E}$ to be as large as possible. In fact the function which maximizes $\mathcal{E}$ is [25]

$$w_{opt} = \frac{\Delta}{\Sigma} \tag{98}$$

Using this observable the expression for $\mathcal{E}$ simplifies to

$$\mathcal{E}_{opt} = \left(\frac{\int \frac{\Delta^2}{\Sigma}\ ds_h\ dz}{\int \Sigma\ ds_h\ dz} \right)^{\frac{1}{2}} . \tag{99}$$

Another form of observables that we consider are asymmetries where $w = \pm 1$ at all points in phase space. In this case the definition of $\mathcal{E}$ simplifies to:

$$\mathcal{E} = \frac{\int w(s_h, z)\Delta ds_h dz}{\int \Sigma ds_h dz} \tag{100}$$

corresponding to the usual definition of an asymmetry.

Let us now consider three specific asymmetries:

$$w_0 = 1 \quad w_1 = \mathrm{sign}(z) \quad w_2 = \mathrm{sign}(|z| - \frac{1}{2}) \tag{101}$$

In Figure 3 we plot the differential asymmetries $d\mathcal{E}_i/d\sqrt{s_h}$ together with $d\Sigma/d\sqrt{s_h}$. Note that $i = 0$ corresponds to PRA and arises when resonant states with identical quantum contribute to the same final state; $i = 1$ and $i = 2$ correspond to asymmetries in the energy distributions. These arise when contributing resonance states have the opposite parity or have the same parity, respectively.

Finally, we note that, in order to enhance the asymmetry observed it may also be useful to modify the above as follows:

$$w_i' = \mathrm{sign}(d\mathcal{E}_i/ds_h) w_i(z) \tag{102}$$

thus flipping the sign according to the expected sign changes as a function of s_h. However, in the specific cases that we consider asymmetries do not seem to switch signs as s_h changes so that this sort of multiplication by the sign turns out not to be useful.

It is instructive at this point to consider how $\mathcal{E}$ and N_B scale with B_{pen}. Consider changing $B_{pen} \to \lambda B_{pen}$. In the above formalism then $\Sigma \to \lambda\Sigma$ and $\Delta \to \lambda^{\frac{1}{2}}\Delta$ thus $\mathcal{E} \to \lambda^{-\frac{1}{2}}\mathcal{E}$ but since $\mathrm{Br} \to \lambda\mathrm{Br}$ $N_B \to N_B$. Thus N_B is independent of the exact normalization of the penguin rates. Furthermore this implies that N_B is also relatively independent of the efficiency of forming k_i states from the penguin process.

3.6 Numerical Results

For the purposes of numerical results we take the CKM phase δ to be $\frac{\pi}{2}$ and the signs of the amplitudes as described in section 2.5 and 3.4.

For resonance formation from the annihilation graph we use the potential model results given in (42). Our key results are shown in Table 4 and Figure 3. In Table 4 we use this method to calculate $\mathcal{E}_i$ for the above observables as well as the optimal observable give by equation (98) for each of the final states. By using equation (97) and the corresponding branching ratios, we also calculate the number of B's necessary to observe a statistical effect at the 1 sigma level. The resulting numbers are also shown in Table 4; note that these are given in units of 10^8.

Figure 3a shows $\frac{1}{\Sigma}\frac{d\Sigma}{d\sqrt{s_h}}$ as a function of $\sqrt{s_h}$. Here the solid line represents the decay to the ρK final state, the dotted line to the $K^*\pi$ final state and the dashed line for the $K\pi$ final state. Note that the peaks correspond to the resonances in Table 1 which are indicated in the graph by the bars. In Figure 3b $\frac{1}{\Sigma}\frac{d\mathcal{E}_0}{d\sqrt{s_h}}$ is shown. In the ρK and $K^*\pi$ modes the effects are due to the interference of k_1 and k_2. The resultant values of $\mathcal{E}_0$ are also shown in Table 4. Note that since the resonances k_0 and k_3 are so far apart the value of $\mathcal{E}_0$ is negligibly small for the $K\pi$ final state. Likewise Figure 3c shows $\frac{1}{\Sigma}\frac{d\mathcal{E}_1}{d\sqrt{s_h}}$ for each of the final states. For the $K^*\pi$ and $K\rho$ final states, there is a complex structure since contributions result from the interference of any pair of resonances with opposite parity. This also accounts for the large values in $\mathcal{E}_1$. Similar comments are valid for the $K\pi$ state except the k_1 and k_2 states are not involved while the k_0 is. With the resonances that we consider the $K\pi$ final state does not contribute to $\mathcal{E}_2$. For the other final states, the curves in Figure 3d are due to the interference of various positive parity states with each other. The optimal $\mathcal{E}_{opt}$ and the corresponding values of N_{opt} given in Table 4 show that these effects may be seen with about 10^9 B's .

It is apparent that there is considerable uncertainty in these results; some of which should be reduced in the future. First of all consider the ratio $R(k_i\gamma)$. The theoretical prediction based, at the moment, on potential models, are rather unreliable as our calculations show. However, this source of uncertainty will get substantially under control as experimentally samples of about 10^7 B's (i.e. well before the 10^8 or 10^9 needed for CP studies) become available, as then the rates for different resonant channels will be experimentally measured. So by the time 10^8 B mesons have been accumulated one might anticipate that many of these ratios will be well determined. Furthermore, detail fits to the CP-conserving distributions, e.g. $\frac{1}{\Sigma}\frac{d\Sigma}{ds_h}$ as a function of $\sqrt{s_h}$ (see Fig. 3a), of the data that becomes available at that time should also allow the determination of the ambiguities in the signs of the amplitudes as well as a more careful determination of the strong phase than the approximations considered here.

It is important to note that if the penguin graphs are rescaled by a constant amount the resultant value of N_B, needed for the CP asymmetry to be observed, is unaffected. To see this suppose that the amplitude for the penguin is multiplied by a factor of λ. Since the penguin dominates the production, the total branching ratio to a final state varies like λ^2. The asymmetry due to interference with the tree graphs will therefore vary like λ^{-1}; following equation (97) N_B will therefore be independent of λ.

Although we have attempted to determine the signs of the decay amplitude, it is instructive to consider the uncertainty in N_B introduced for other possible sign combinations. Consider N_{opt} in model A for the $K^*\pi$ final state. The value with the sign choice indicated above is 2.2×10^8 as given in Table 4. If however one checks all the possible sign combinations one finds that this quantity varies between 1.5×10^8 and 4.0×10^8. So once again, our estimate for the required number of B's is not greatly effected from this source of uncertainty either.

4 Conclusions and Summary

Despite the fact that there are appreciable uncertainties in our estimates it seems likely that the asymmetries considered here may be observable in the case of $B^{\pm}$ at a B factory capable of producing about 10^9 B mesons. In our estimates we do not consider what happens for s_h well above the k_i states. Note especially that the tree graphs tend to increase rapidly with s_h so that larger CP phases may be available, though the strong rescattering phases and the branching ratios are likely to become somewhat smaller. Thus our estimates may well be underestimates. Of course CP violation may also arise from physics beyond the standard model, and then too larger asymmetries are possible.

Another point we wish to emphasize briefly has to do with the formation of the higher resonances in radiative B transitions. In both bound state models that we studied, we found (as shown in Table 2) that, except for the k_1, the other three states are produced roughly with the same branching ratio as the k_0 (i.e. $K^*(892)$) which was recently seen experimentally[1]. Experimental searches of all of these states are vitally needed.

It should be clear that the essential idea proposed in this work is that resonances can have interesting and, perhaps, even a dramatic influence on the CP violating observables. In the case of neutral $B's$ leading to self-conjugate final states, effects arising from interference between the initial B and $\bar{B}$ are well known[26]. What is being demonstrated here is that charged B meson decays leading to common final states via resonances can also lead to important interference effects. Indeed, since the underlying theory[27] involves quarks (not mesons) it is difficult to confine CP violation just to neutral or just to charged mesons; resonance enhancement through such considerations should be possible both for charged as well as neutral B's. For concreteness we have, in this paper, only addressed to the radiative decays of charged B's. Clearly similar effects on neutral B's need to be investigated. Furthermore effects of non-standard physics needs to be ascertained. We will return to some of these issues in subsequent publications [6, 28].

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Figure Captions

Figure 1

Figure 1a: Example of a penguin graph for the subprocess for $b \to s\gamma$.

Figure 1b: Example of the annihilation subprocess $b\bar{u} \to s\gamma\bar{u}$.

Figure 1c: Example of the spectator subprocess $b \to s\gamma u\bar{u}$.

Figure 2

Figure 2a: A typical instance of two diagrams contributing to the partial rate asymmetry of a state f. The intermediate state g is shown giving a contribution to the imaginary part of the propagator.

Figure 2b: The two diagrams which give the compensating partial rate asymmetry to the final state g so that CPT is preserved.

Figure 3

Figure 3a: A plot of $\frac{1}{\Sigma}\frac{d\Sigma}{d\sqrt{s_h}}$ as a function of $\sqrt{s_h}$ for the ρK state (solid line); πK^* state (dotted line) and πK state (dashed line). The bars indicate the positions of the five resonances considered.

Figure 3b: A plot of $\frac{1}{\Sigma}\frac{d\mathcal{E}_0}{d\sqrt{s_h}}$ as a function of $\sqrt{s_h}$ for the same three final states.

Figure 3c: A plot of $\frac{1}{\Sigma}\frac{d\mathcal{E}_1}{d\sqrt{s_h}}$ as a function of $\sqrt{s_h}$ for the same three final states.

Figure 3d: A plot of $\frac{1}{\Sigma}\frac{d\mathcal{E}_2}{d\sqrt{s_h}}$ as a function of $\sqrt{s_h}$ for the ρK and πK^* final states.

Table 1

State			Mass (Mev)	Width (Mev)	$^{2S+1}L_J$	Selected Decays	
k_0	$K^*(890)$	$[1^-]$	892 ($\pm$), 896(0)	50	$^3S_1$	$K\pi$	100%
k_1	$K_1(1270)$	$[1^+]$	1270	90	$^1P_1$	$K\rho$	42%
						$K^*\pi$	16%
						$K\omega$	11%
k_2	$K_1(1400)$	$[1^+]$	1402	174	$^3P_1$	$K\rho$	3%
						$K^*\pi$	94%
						$K\omega$	1%
k_3	$K^*(1410)$	$[1^-]$	1412	227	$^3S_1$	$K\rho$	$< 7\%$
						$K^*\pi$	$> 40\%$
						$K\pi$	7%
k_4	$K_2(1430)$	$[2^+]$	1425($\pm$), 1432(0)	98($\pm$), 109(0)	$^3P_2$	$K\rho$	9%
						$K^*\pi$	25%
						$K\omega$	3%
						$K\pi$	50%

Table 1: Some of the properties of the k_i states are shown [4]. Branching fractions of the k_i states to various final states are given in the last column. In our computation, for definiteness, we used the branching ratios $Br(k_3 \to K\rho) = 7\%$ and $Br(k_3 \to K^*\pi) = 86\%$.

Table 2

k_i	Model A	Model B
k_0	2.5%	1.6%
$\hat{k}_1$	2.2×10^{-5}	1.4×10^{-5}
$\hat{k}_2$	6.5%	1.3%
k_3	3.2%	1.3%
k_4	5.4%	0.9%

Table 2: The calculated branching fraction for $B \to k_i\gamma$ in the two models considered.

Table 3

k_i	Model A	Model B
k_0	2.5×10^{-3}	3.2×10^{-3}
$\hat{k}_1$	$-.85$	$-.96$
$\hat{k}_2$	22×10^{-3}	49×10^{-3}
k_3	2.2×10^{-3}	3.5×10^{-3}
k_4	32×10^{-3}	80×10^{-3}

Table 3: The resulting ratio of CP phases, $\frac{\sin\phi}{\sin\delta}$, in the two models.

Table 4

Model A

	ρK	$K^*\pi$	$K\pi$
$\mathcal{E}_{opt}$	1.0%	1.2%	0.7%
$\mathcal{E}'_0$	0.3%	0.3%	1.7×10^{-4}
$\mathcal{E}'_1$	0.7%	0.6%	0.4%
$\mathcal{E}'_2$	0.4%	0.2 %	
N_{opt}	33	2.2	12
N'_0	460	40	2×10^4
N'_1	70	9	32
N'_2	280	60	

Model B

	ρK	$K^*\pi$	$K\pi$
$\mathcal{E}_{opt}$	2.4%	2.0%	0.9%
$\mathcal{E}'_0$	0.6%	0.5%	1×10^{-4}
$\mathcal{E}'_1$	1.7%	1.1%	0.4%
$\mathcal{E}'_2$	0.7%	0.3%	
N_{opt}	27	3	20
N'_0	460	47	1×10^5
N'_1	53	10	95
N'_2	310	110	

Table 4: Using model A and B we calculate the asymmetries $\mathcal{E}_{opt}$ and $\mathcal{E}'_i$ as well as the corresponding numbers of $B^{\pm}$ in units of 10^8. For the annihilation graph we have used the ISGW model[8] to calculate resonance formation.

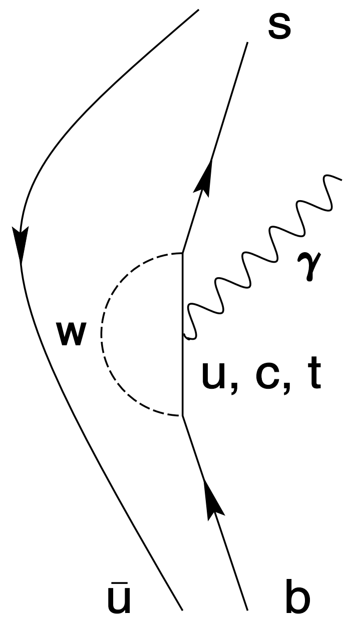


Figure 1a

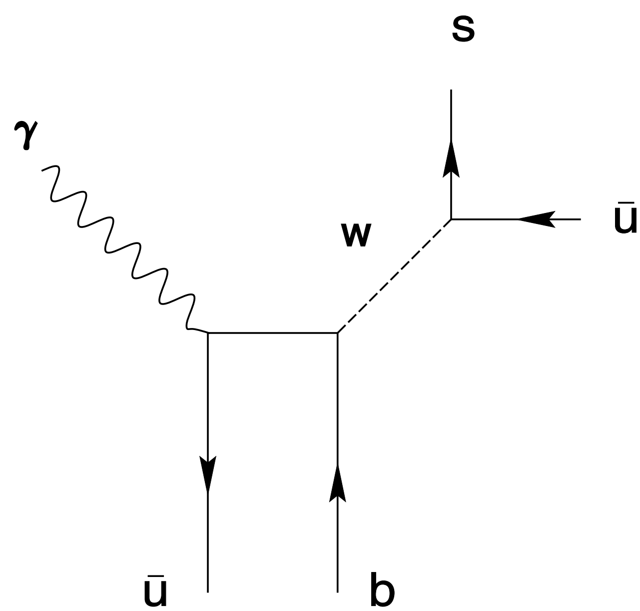


Figure 1b

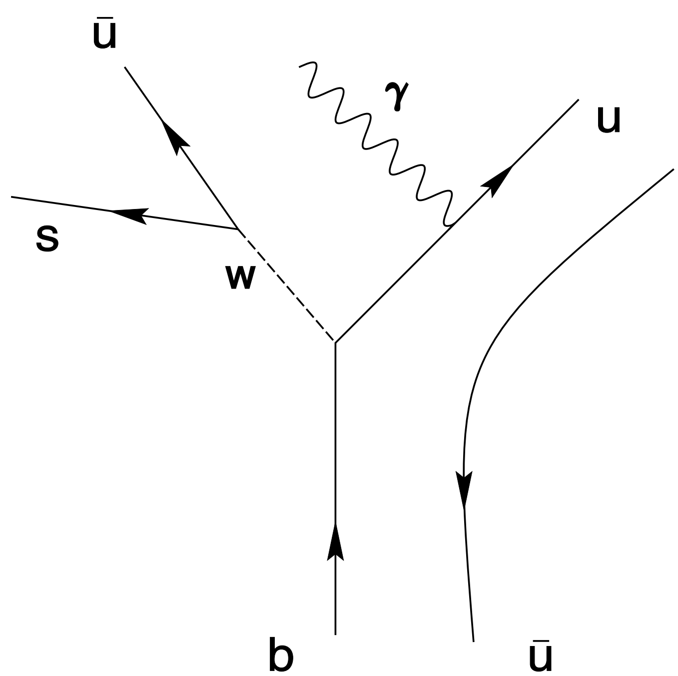


Figure 1c

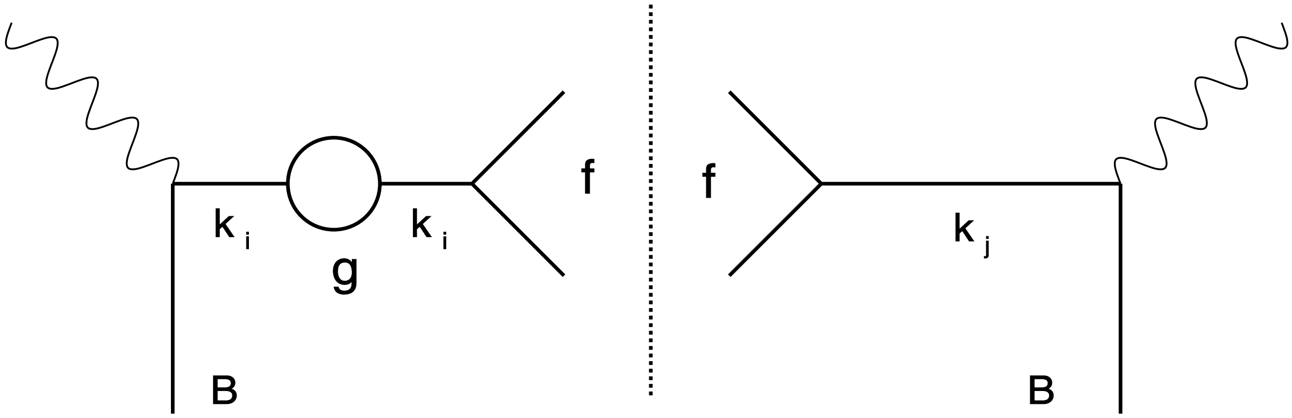


Figure 2a

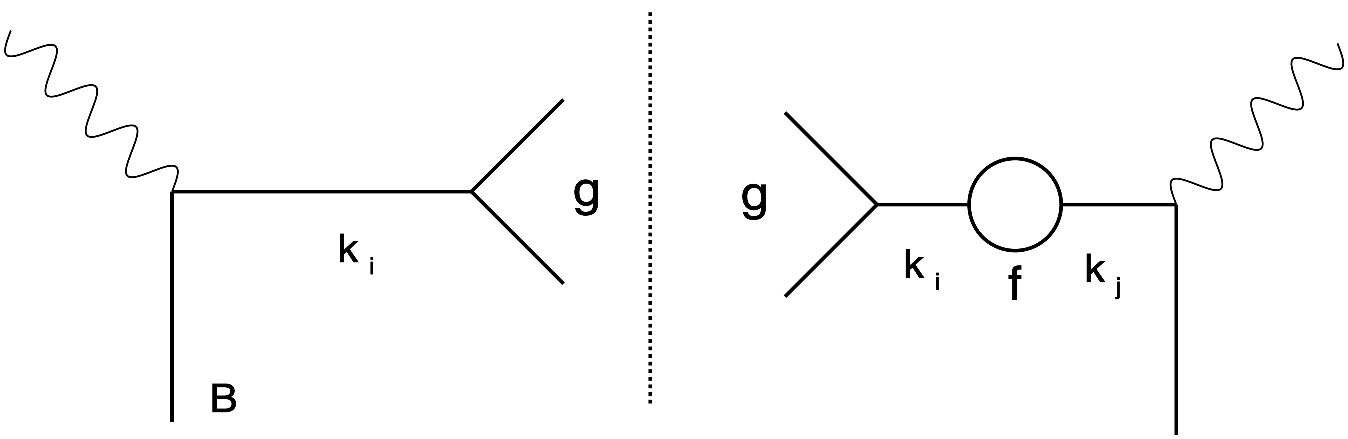


Figure 2b

Figure 3a

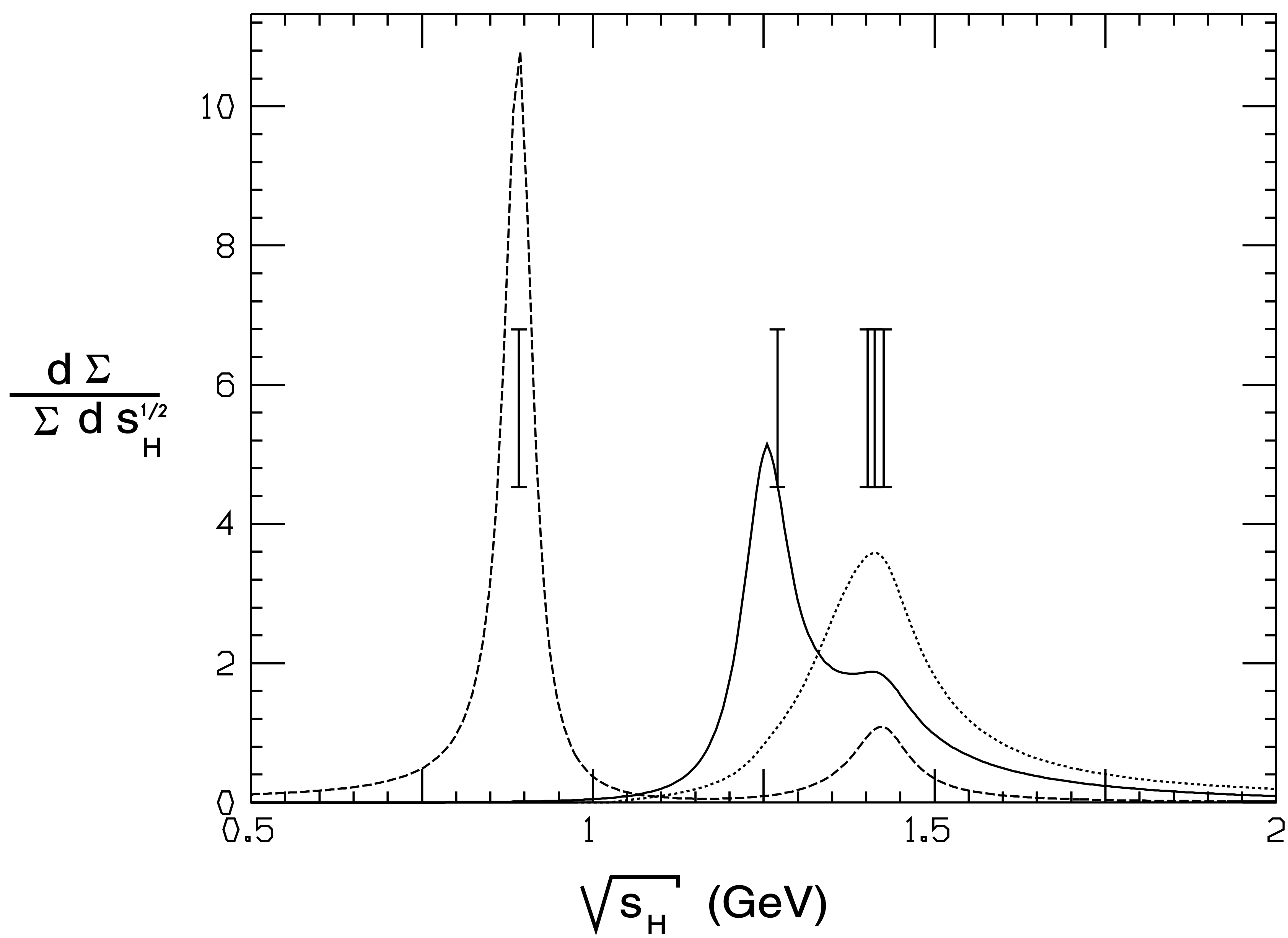


Figure 3b

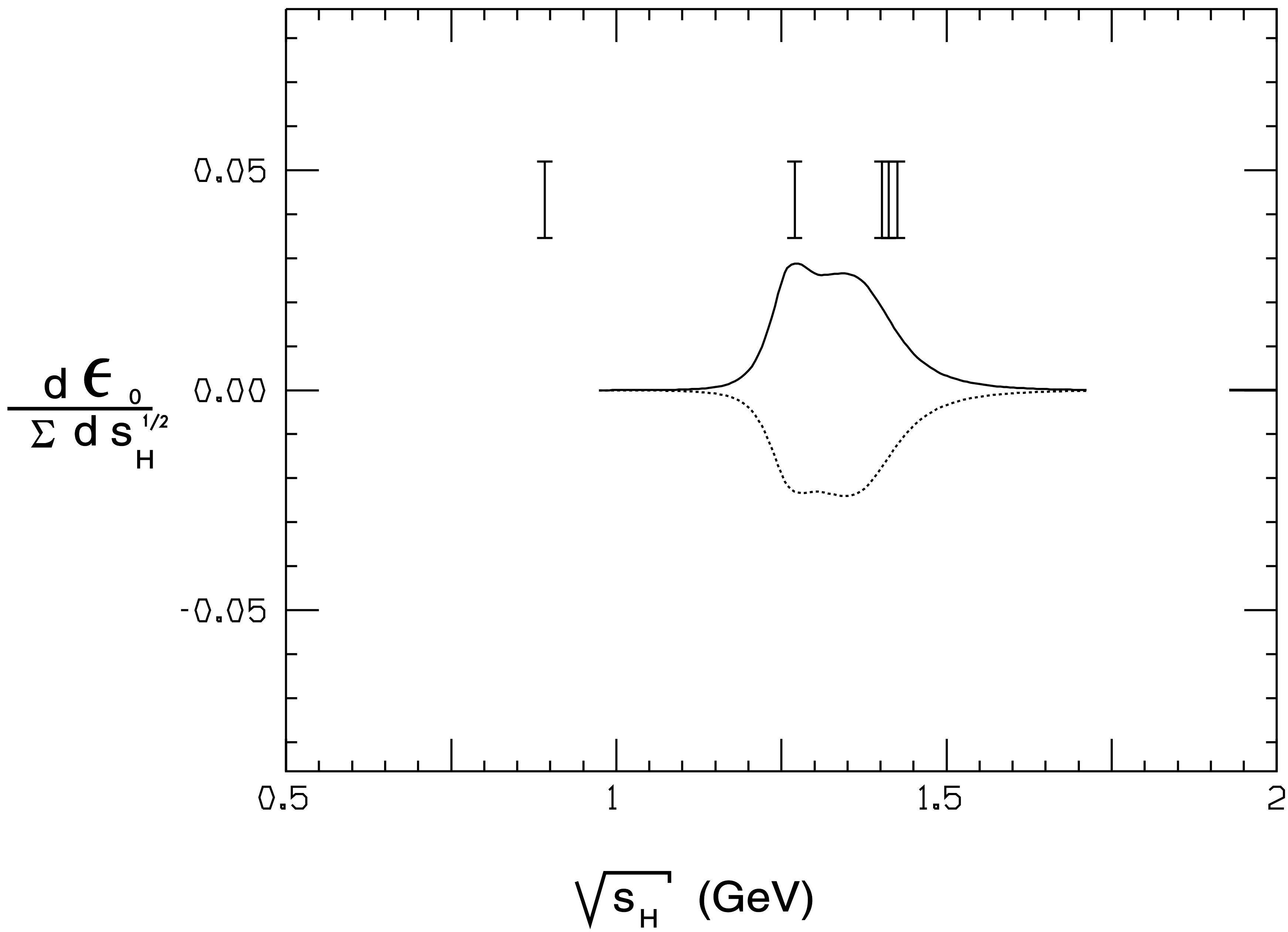
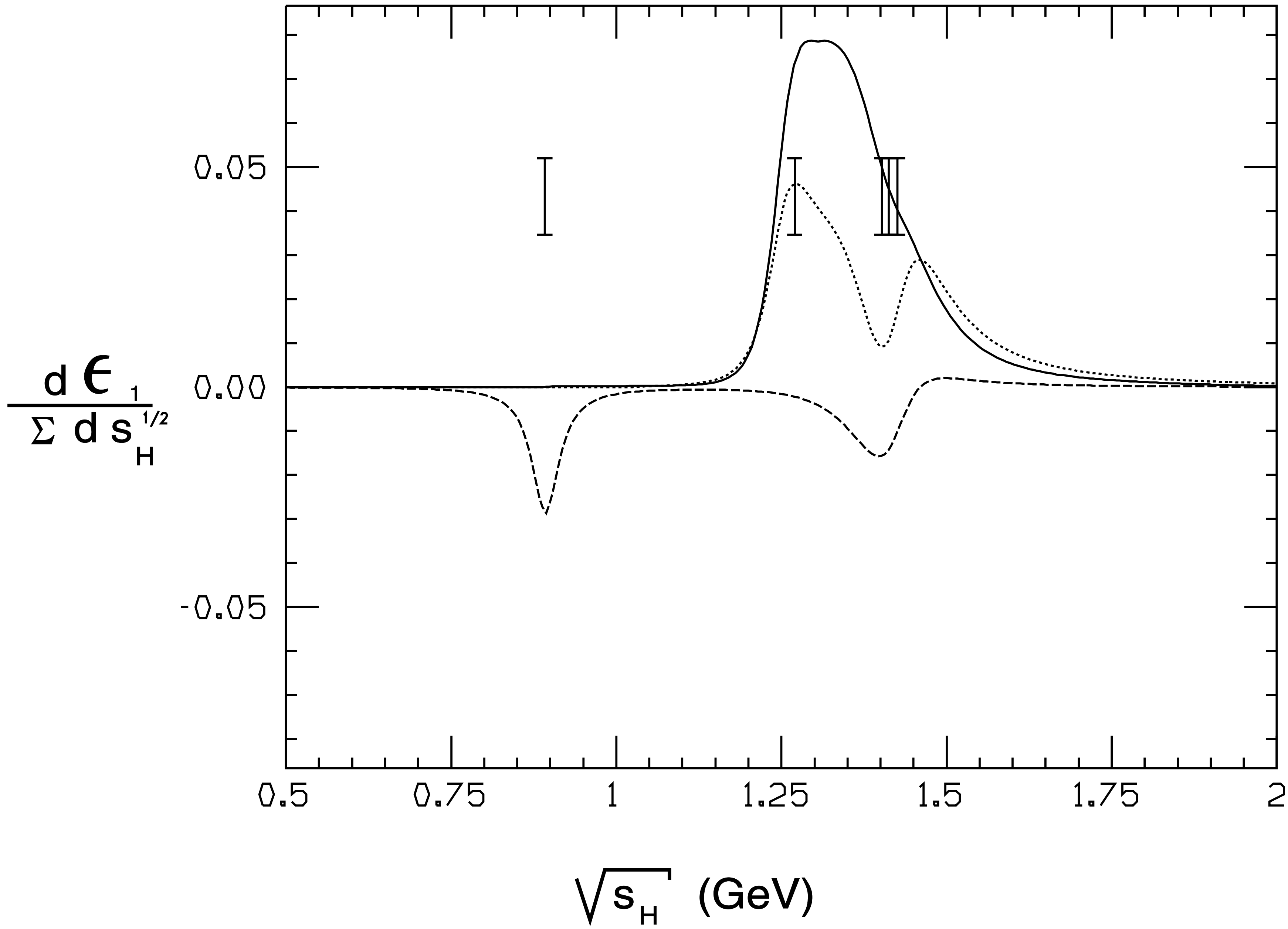


Figure 3c



0.05
0.00
-0.05
$\frac{d\epsilon_1}{\Sigma\, d s_H^{1/2}}$
0.5
0.75
1
1.25
1.5
1.75
2
$\sqrt{s_H}$ (GeV)

Figure 3d