

## Identifying Unconventional $E_6$ Models at $e^+ e^-$ Colliders\*.

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### *Abstract*

Recently it was shown that, in the framework of superstring inspired  $E_6$  models, the presence of generation dependent discrete symmetries allows us to construct a phenomenologically viable class of models in which the three generations of fermions do not have the same embedding within the fundamental **27** dimensional representation of  $E_6$ . In this scenario, these different embeddings of the conventional fermions imply that the left-handed charged leptons and the right-handed  $d$ -type quarks are coupled in a non-universal way to the new neutral gauge bosons ( $Z_\theta$ ) present in these models. It was also shown that a unique signature for this scenario, would be a deviation from unity for the ratio of cross sections for the production of two different lepton species in  $e^+e^-$  annihilation. However, several different scenarios are possible, depending on the particular assignment chosen for  $e_L$ ,  $\mu_L$  and  $\tau_L$  and for the right-handed  $d$ -type quarks, as well as on the type of  $Z_\theta$  boson. Such scenarios can not be disentangled from one another by means of cross section measurements alone. In this paper we examine the possibility of identifying the pattern of embeddings through measurements of polarized and unpolarized asymmetries for fermion pair-production at the 500 GeV  $e^+e^-$  Next Linear Collider (NLC). We show that it will be possible to identify the different patterns of unconventional assignments for the left-handed leptons and for the  $b_R$  quark, for  $Z_\theta$  masses as large as  $\sim 1.5$  TeV.

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# I. INTRODUCTION

Extended electroweak models based on the group  $E_6$  are known to lead to interesting phenomenology. During the last few years, a thorough analysis of these models was carried out both from the theoretical and phenomenological point of view [1]. In particular, all the possible embeddings of the Standard Model (SM) gauge group in  $E_6$ , as well as various possibilities for the assignments of the matter fields to the fundamental representation of the group, were analyzed in ref. [2].

However, it has been recently shown [3] that in the framework of superstring derived  $E_6$  models it is possible to implement an unconventional scenario that has not been addressed by the previous investigations. In this scenario some of the known fermions of the three families are embedded in the fundamental **27** dimensional representation of the group in a generation dependent way, implying the possibility that corresponding fermions belonging to different generations could have different gauge interactions with respect to some subgroup of  $E_6$ .

The realization of models with Unconventional Assignments (UA) for the matter fields relies on the presence of generation dependent discrete symmetries [3]. Symmetries of this kind arise naturally in field theories derived from the superstring, and for this reason it can be argued [3] that the UA scenario should be considered as a natural alternative to the standard schemes.

Of course, experimentally we know that the  $SU(2) \times U(1)$  gauge interactions of the known fermions do respect universality with a high degree of precision. However, since the SM gauge group is rank 4 while  $E_6$  is rank 6, as many as two additional massive neutral gauge bosons ( $Z_\theta$ ) can be present, possibly with  $M_\theta \sim 1$  TeV or less, and the possibility that the corresponding new neutral interactions could violate universality is still phenomenologically viable.

The 27 dimensional representation of  $E_6$  contains, in addition to the standard model fermions, two new leptonic  $SU(2)$ -doublets, two  $SU(2)$ -singlet neutral states and two color-triplet  $SU(2)$ -singlet  $d$ -type quarks. The UA models are realised by identifying in a generation dependent way some of the known doublets of left-handed (L) leptons and/or singlets of right-handed (R)  $d$ -type quarks with the new multiplets having the correct  $SU(2) \times U(1)$  transformation properties, in order to ensure that the standard interactions are unmodified with respect to the SM [3].

For example, in the model described in Ref. [3] the L-handed lepton doublet “ $(\nu_\tau)_L$ ” and the R-handed quark singlet “ $b_R$ ” of the third generation are assigned to the additional  $SU(2)$  multiplets present in the **27**. This model was shown to be consistent with a large number of experimental constraints, ranging from the direct and cosmological limits on the neutrino masses, to the stringent limits on flavor changing neutral currents. Clearly, since the different  $SU(2)$  multiplets carry different  $U_\theta(1)$  quantum numbers, in this model the “ $\tau_L$ ” lepton and the “ $b_R$ ” quark have a different interaction with the new  $Z_\theta$  with respect to the corresponding states of the first two generations.

A very clean signature for the UA models would then be the detection of deviations from universality induced by  $Z_\theta$  exchange in neutral current (NC) processes. If UA are present in the leptonic sector the experimentally clearest signature would be a deviation from unity for the ratio of production cross sections for two different lepton species [4]. A first study of the discovery limits for lepton universality violation of this type in  $e^+e^-$  annihilation at the Large Electron Positron collider with 180 GeV c.m. energy (LEP-2) and at a 500 GeV  $e^+e^-$  Next Linear Collider (NLC) [4] showed that such a signature could be detected for a  $Z_\theta$  mass as large as  $\sim 800$  GeV (LEP-2) and  $\sim 2$  TeV (NLC) [4].

In fact, several different scenarios are possible, depending on the particular assignment chosen for the  $e$ ,  $\mu$  and  $\tau$  leptons, and on the type of  $Z_\theta$  boson, and also UA could be present in the R-handed  $d$ -type quark sector. While it is relatively easy to detect possible deviations from universality induced by the UA, it would

be impossible to disentangle the different scenarios by comparing the total cross sections alone.

In this paper we examine the possibility of identifying the pattern of embeddings for the leptons and for the  $b$  quarks through measurements of polarized and unpolarized asymmetries for the process  $e^+e^- \rightarrow f\bar{f}$ . In particular we will analyse the discovery potential of the NLC assuming a center of mass energy of 500 GeV.

In Sec. 2 we will briefly outline the main features of the  $E_6$  models based on the UA scenario, and establish our conventions and notations. A more complete discussion of the theoretical framework can be found in Ref. [3].

In Sec. 3 we will investigate the phenomenology of UA models at future  $e^+e^-$  machines. We will give the relevant expressions for the various polarized and unpolarized asymmetries and we will discuss our results. We stress that the large amount of data collected at the  $Z_0$  resonance by the LEP collaborations are not effective in probing for these kind of effects. In fact, as we have already mentioned, the UA for the known fermions would not affect the couplings to the standard  $Z_0$ , while the contributions of  $Z_\theta$ - $Z_0$  interference and of pure  $Z_\theta$  exchange to the various cross sections and asymmetries measured at LEP-1 are too small to be measured at the peak. Some effects could still be detected, however, if the  $Z_0$  had a sizeable mixing with the  $Z_\theta$  so that the universality of the couplings of the  $Z_0$  to the usual fermions would be indirectly affected by this mixing. However, the existing bounds on the  $Z_0$ - $Z_\theta$  mixing angle are relatively tight [5] so that this possibility seems unlikely, and we will disregard it throughout this paper. Finally in Sec. 4 we will summarize our results and draw our conclusions.

We note that other authors have considered the possibility of using the NLC to explore  $Z'$  couplings for masses in excess of the collider's center of mass energy [6][7][8] within a more general context. In this work we will focus on the specific problem of the ambiguity in the generation dependent fermion quantum number assignments which can arise in  $E_6$  models.

## II. UNCONVENTIONAL ASSIGNMENTS IN $E_6$ MODELS

In  $E_6$  grand unified theories, as many as two new neutral gauge bosons can be present, corresponding to the two additional Cartan generators that are not present in the SM gauge group. Here we will consider the embedding of the SM gauge group

$$\mathcal{G}_{\text{SM}} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \quad (2.1)$$

in  $E_6$  through the maximal subalgebras chain:

$$\begin{array}{ccc} E_6 & \longrightarrow & U(1)_\psi \times SO(10) \\ & & \searrow \\ & & U(1)_\chi \times SU(5) \\ & & \searrow \\ & & \mathcal{G}_{\text{SM}}. \end{array} \quad (2.2)$$

In general the two additional neutral gauge boson will correspond to some linear combinations of the  $U_\chi(1)$  and  $U_\psi(1)$  generators that we will parametrize in term of an angle  $\theta$  as

$$\begin{aligned} Z'_\theta &= Z_\psi \cos \theta - Z_\chi \sin \theta \\ Z''_\theta &= Z_\psi \sin \theta + Z_\chi \cos \theta, \end{aligned} \quad (2.3)$$

The angle  $\theta$  is a model dependent parameter whose value is determined by the details of the breaking of the gauge symmetry. In the following we will denote the lightest of the two new gauge bosons as  $Z_\theta$ .

In the kind of models we are considering here, each generation of matter fields belong to one fundamental  $\mathbf{27}$  representation of the group. The  $\mathbf{27}$  branches to the  $\mathbf{1} + \mathbf{10} + \mathbf{16}$  representations of  $SO(10)$ . The known particles of the three generations, together with an  $SU(2)$  singlet neutrino “ $\nu^c$ ”, are usually assigned to the  $\mathbf{16}$  of  $SO(10)$ , that in turn branches to  $\mathbf{1}_{16} + \bar{\mathbf{5}}_{16} + \mathbf{10}_{16}$  of  $SU(5)$ . Giving in parenthesis the Abelian charges  $Q_\psi$  and  $Q_\chi$  for the different  $SU(5)$  multiplets, we have

$$\begin{aligned}
[\mathbf{1}_{16}] (1c_\psi) (-5c_\chi) &= [\nu^c] \\
[\bar{\mathbf{5}}_{16}] (1c_\psi) (3c_\chi) &= [L = \begin{pmatrix} \nu \\ e \end{pmatrix}, d^c] \\
[\mathbf{10}_{16}] (1c_\psi) (-1c_\chi) &= [Q = \begin{pmatrix} u \\ d \end{pmatrix}, u^c, e^c]
\end{aligned} \tag{2.4}$$

The  $\mathbf{10}$  of  $SO(10)$  that branches to  $\mathbf{5}_{10} + \bar{\mathbf{5}}_{10}$  of  $SU(5)$  contains the fields

$$\begin{aligned}
[\mathbf{5}_{10}] (-2c_\psi) (-2c_\chi) &= [H = \begin{pmatrix} N \\ E \end{pmatrix}, h^c] \\
[\bar{\mathbf{5}}_{10}] (-2c_\psi) (2c_\chi) &= [H^c = \begin{pmatrix} E^c \\ N^c \end{pmatrix}, h].
\end{aligned} \tag{2.5}$$

Finally the singlet  $\mathbf{1}$  of  $SO(10)$  corresponds to

$$[\mathbf{1}_1] (4c_\psi) (0c_\chi) = [S^c]. \tag{2.6}$$

According to the normalization  $\sum_{f=1}^{27} (Q_{\psi,\chi}^f)^2 = \sum_{f=1}^{27} (\frac{1}{2}Y^f)^2 = 5$ , in (2.4)-(2.6) we have respectively  $c_\psi = \frac{1}{6}\sqrt{\frac{5}{2}}$  and  $c_\chi = \frac{1}{6}\sqrt{\frac{2}{3}}$ . Matter fields will couple for example to the  $Z'_\theta$  boson through the charge

$$Q_\theta = Q_\psi \cos \theta - Q_\chi \sin \theta. \tag{2.7}$$

As it is clear from the second lines in (2.4) and (2.5), there is an ambiguity in assigning the known states to the  $\mathbf{27}$  representation, since under the SM gauge group the  $\bar{\mathbf{5}}_{10}$  in the  $\mathbf{10}$  of  $SO(10)$  has the same field content as the  $\bar{\mathbf{5}}_{16}$  in the  $\mathbf{16}$ . The same ambiguity is also present for the two  $\mathcal{G}_{SM}$  singlets, namely  $\mathbf{1}_1$  and  $\mathbf{1}_{16}$ . This ambiguity has no physical consequences as long as the same assignments are chosen for *all* the three generations of SM fermions, that we will collectively denote as  $\{\psi_{SM}\}$ . In fact, it is easy to verify that transforming the model parameter  $\theta$  according to

$$\theta \rightarrow \theta' = \tan^{-1}(\sqrt{15}) - \theta \tag{2.8}$$

induces on the charges in (2.7) the transformations  $Q_\theta(\bar{\mathbf{5}}_{16}) \rightarrow Q'_\theta(\bar{\mathbf{5}}_{16}) = Q_\theta(\bar{\mathbf{5}}_{10})$  and  $Q_\theta(\mathbf{1}_{16}) \rightarrow Q'_\theta(\mathbf{1}_{16}) = Q_\theta(\mathbf{1}_{10})$  while at the same time the charges for the

$\mathbf{5}_{10}$  and for the  $\mathbf{10}_{16}$  are left invariant. This means that a model defined by a particular value of  $\theta$  and by a choice of the assignments for the three generations of SM fermions  $\{Z_\theta, \{\psi_{SM}\} \subset \bar{\mathbf{5}}_{16}\}$  is physically equivalent to the model defined by  $\{Z_{\theta'}, \{\psi_{SM}\} \subset \bar{\mathbf{5}}_{10}\}$ .

However, UA models realize a scenario in which the assignments of the SM fermions to the two  $\bar{\mathbf{5}}$ 's are generation dependent. For example in the model analyzed in [3] what we call “ $\tau_L$ ” corresponds in fact to the charged component of the  $H_3$  weak doublet belonging to  $\bar{\mathbf{5}}_{10}$ , while the “ $e_L$ ” and the “ $\mu_L$ ” leptons are as usual assigned to the  $\bar{\mathbf{5}}_{16}$ . Since these fermions might not correspond to the entries as listed in (2.4) we use quotation marks to denote the known states with their conventional labels, while labels not enclosed within quotation marks will always refer to the  $SU(2)$  multiplets in (2.3)–(2.5). This model was realized by imposing on the superpotential a particular family-non-blind  $Z_2 \times Z_3$  discrete symmetry. As a result of such a symmetry, the masses of the known (light) chiral leptons are generated by vacuum expectation values (VEVs) of Higgs doublets, through the terms  $m_\tau E_{3L} e_{3R}$  (with  $m_\tau \sim \langle \tilde{L}_3 \rangle_0$ ) and  $m_{\alpha\beta} e_{\alpha L} e_{\beta R}$  (with  $m_{\alpha\beta} \sim \langle \tilde{H}_2 \rangle_0$  and  $\alpha, \beta = 1, 2$ ). The remaining charged leptons  $e_{3L}$ ,  $E_{3R}$ ,  $E_{\alpha L}$ ,  $E_{\alpha R}$  are vectorlike, and acquire large masses from VEVs of Higgs singlets.

Clearly, also in the UA models of this kind it is not physically meaningful to ask if the L-doublet of leptons of the first generation sits in the  $\mathbf{16}$  or in the  $\mathbf{10}$  of  $SO(10)$  since these two choices are equivalent under the transformation (2.8). However, experiments could tell us if for example the L-doublet of leptons of the second generation sits in the *same*  $SO(10)$  multiplet or in a *different* one. That is, experiment can tell us whether or not the transformation properties of the three generations of fermions under the extended gauge group are the same, provided the scale associated with the breaking of this extended group is not too high.

In the present paper we will concentrate on the phenomenological consequences of having different assignments for the standard L leptons and for the “ $b_R$ ” quark. We will present the theoretical predictions for several quantities (cross sections and asymmetries) measurable in the process  $e^+e^- \rightarrow f\bar{f}$ . Refer-

ring to the initial and final states, for each observable  $\mathcal{A}$  we will label the two physically different possibilities as  $\mathcal{A}_{\mathbf{16}\rightarrow\mathbf{16}}$  and  $\mathcal{A}_{\mathbf{16}\rightarrow\mathbf{10}}$ . However, according to the previous discussion, it should be kept in mind that when the model parameter  $\theta$  is transformed according to (2.8) these observables are found to directly correspond to  $\mathcal{A}_{\mathbf{10}\rightarrow\mathbf{10}}$  and  $\mathcal{A}_{\mathbf{10}\rightarrow\mathbf{16}}$ . Thus these latter observables are not truly independent. That this is true can be seen most clearly in the case of charm pair production since the charm couplings to the  $Z_\theta$  are invariant under (2.8) (as charm always lies in the  $\mathbf{16}$ ) and thus this reaction can potentially only probe whether  $e_L^-$  is placed in the  $\mathbf{16}$  or  $\mathbf{10}$ . Fig. 1 shows the production cross section for charm pairs at the NLC for a  $Z_\theta$  mass of 1 TeV as a function of the parameter  $\theta$ . The two curves show how the cross section apparently depends on the choice of the  $e_L^-$  assignment. However, we see under close examination that the two curves are actually the *same* in that one can be obtained from the other merely by making the transformation (2.8). From this we learn explicitly that  $\mathcal{A}_{\mathbf{10}\rightarrow\mathbf{16}}$  and  $\mathcal{A}_{\mathbf{16}\rightarrow\mathbf{16}}$  are simply related through (2.8). The same sort of relation can also be shown to exist between  $\mathcal{A}_{\mathbf{16}\rightarrow\mathbf{10}}$  and  $\mathcal{A}_{\mathbf{10}\rightarrow\mathbf{10}}$  through identical arguments.

### III. IDENTIFYING THE UNCONVENTIONAL ASSIGNMENTS AT $e^+ e^-$ COLLIDERS

We will consider now the cross sections and the asymmetries in the presence of additional neutral gauge bosons for the process  $e^+(p_L^+)e^-(p_L^-) \rightarrow f\bar{f}$  ( $f \neq e$ ), where  $p_L^\pm$  represent the longitudinal polarization of  $e^\pm$ . We will henceforth assume that one of the two new bosons in (2.3) is heavy enough so that its effects on the low energy physics are negligible. Then we will use the subscripts  $i, j = 0, 1, 2$  which correspond respectively to the  $\gamma$ ,  $Z_0$  and  $Z_\theta$  bosons. Denoting by  $\mathcal{P} \equiv (p_L^+ - p_L^-)/(1 - p_L^+ p_L^-)$  the overall polarization of the initial electron-positron system,



the cross section and the forward backward asymmetries for massless fermion pairs can be written in terms of

$$\sigma_{TOT}^f(s, p_L^+, p_L^-) = \frac{4}{3} \frac{\pi\alpha^2}{s} (1 - p_L^+ p_L^-) [T_1(s) + \mathcal{P}T_2(s)] \quad (3.1)$$

$$\sigma_{FB}^f(s, p_L^+, p_L^-) = \frac{\pi\alpha^2}{s} (1 - p_L^+ p_L^-) [T_3(s) + \mathcal{P}T_4(s)] \quad (3.2)$$

$$\begin{aligned} T_1(s) &= \sum_{i,j=0}^2 C_{ij}(e) C_{ij}(f) \chi_i(s) \chi_j^*(s) \\ T_2(s) &= \sum_{i,j=0}^2 C_{ij}(e) C'_{ij}(f) \chi_i(s) \chi_j^*(s) \\ T_3(s) &= \sum_{i,j=0}^2 C'_{ij}(e) C'_{ij}(f) \chi_i(s) \chi_j^*(s) \\ T_4(s) &= \sum_{i,j=0}^2 C'_{ij}(e) C_{ij}(f) \chi_i(s) \chi_j^*(s) \\ C_{ij} &= [v_i v_j + a_i a_j] \\ C'_{ij} &= [v_i a_j + a_i v_j] \end{aligned} \quad (3.3)$$

$$\chi_i(s) = \frac{g_i^2}{4\pi\alpha} \frac{s}{s - M_i^2 - iM_i\Gamma_i}. \quad (3.4)$$

where the labels  $e$  and  $f$  in  $T_1, \dots, T_4$  refer to the couplings to the  $Z_{i,j}$  bosons of the electron and of the fermions in the final state. The couplings in (3.3) and (3.4) are given by

$$\begin{aligned} g_0 &= e & v_0(f) &= Q_{\text{em}}^f & a_0(f) &= 0 \\ g_1 &= (\sqrt{2}G_\mu M_Z^2)^{\frac{1}{2}} & v_1(f) &= T_{3L}^f - 2Q_{\text{em}}^f s_w^2 & a_1(f) &= T_{3L}^f \\ g_2 &= s_w g_1 & v_2(f) &= Q_\theta^f - Q_\theta^{f^c} & a_2(f) &= Q_\theta^f + Q_\theta^{f^c} \end{aligned} \quad (3.5)$$

where  $Q_{\text{em}}^f$  is the electric charge,  $T_{3L}^f$  is the third component of the weak isospin, and  $s_w \equiv \sin \theta_w$  with  $\theta_w$  being the weak mixing angle. For numerical purposes we

take  $\alpha(M_Z)^{-1} = 127.9$ ,  $M_1 = 91.187$  GeV,  $\Gamma_1 = 2.489$  GeV, and  $\sin^2 \theta = 0.2325$  in evaluating the above expressions[9]. In addition, for simplicity we will also assume that  $\Gamma_2 = 0.01M_2$  for all values of the parameter  $\theta$ ; our results are insensitive to this assumption. The couplings  $Q_\theta^{f,f^c} = Q_\psi^{f,f^c} \cos \theta - Q_\chi^{f,f^c} \sin \theta$  to the  $Z_\theta$  boson have been defined in (2.7), and the new charges  $Q_\psi^{f,f^c}$ ,  $Q_\chi^{f,f^c}$  are given in parenthesis in (2.4)-(2.6). In addition, in (3.5) we have assumed for the abelian coupling,  $g_2$ , a renormalization group evolution down to the electroweak scale similar to that of the hypercharge coupling  $g_Y \simeq s_w g_1$ .

Since we are neglecting a possible  $Z_0$ - $Z_\theta$  mixing, the vector and axial-vector couplings,  $v_{0,1}(f)$  and  $a_{0,1}(f)$  in (3.5) do not depend on the specific assignments, and are unmodified with respect to the SM. In contrast,  $v_2(f)$  and  $a_2(f)$  do depend on the particular assignments of the  $f$  fermion to the **16** or to the **10** representations of  $SO(10)$ . Here we are interested in the cases  $f = e_L, \mu_L, \tau_L, b_R$ .

The total cross section, the forward-backward, the polarized forward-backward and the left-right asymmetries are defined in terms of the above quantities by

$$\sigma^f = \frac{4}{3} \frac{\pi \alpha^2}{s} T_1 \quad (3.6)$$

$$A_{FB}^f = \frac{3}{4} \frac{T_3}{T_1} \quad (3.7)$$

$$A_{FB}^{pol f} = \frac{3}{4} \frac{T_2}{T_1} \quad (3.8)$$

$$A_{LR}^f = \frac{T_4}{T_1} \quad (3.9)$$

In the case of  $c, b$ -quark pair production, we include for numerical purposes the leading order QCD corrections to the cross sections and asymmetries taking  $\alpha_s(M_Z) = 0.123$  [9] and employing the standard three-loop renormalization group equations to control the running to the 500 GeV mass scale.

In order to give some feeling for the typical precision obtainable for the measurement of the quantities defined above we will assume an integrated luminosity of  $\mathcal{L} = 50 fb^{-1}$ , which corresponds to approximately 2 years of running at the

NLC, and a single beam polarization of  $\mathcal{P} = 90\%$ . Furthermore, we will assume both these values are rather precisely determined:  $\delta\mathcal{L}/\mathcal{L} = 0.6\%$  and  $\delta\mathcal{P}/\mathcal{P} = 1\%$ ; in addition, we use a  $\mu$ - and  $\tau$ -tagging efficiency of 100% and a b-tagging efficiency of 80%.

Figs. 2 and 3 show the predicted values for the quantities (3.6)-(3.9) as functions of the parameter  $\theta$  for lepton and  $b$ -quark pair production respectively. Also shown in each case as a ‘data point’ is the SM value and error assuming the values above for  $\mathcal{L}$ ,  $\mathcal{P}$ , etc. The light(dark) shaded region in each case corresponds to the transition  $\mathbf{10} \rightarrow \mathbf{16}(\mathbf{16} \rightarrow \mathbf{16})$  with the ‘inner’(‘outer’) edge of each region corresponding to a  $Z_\theta$  mass of 1.5 (1.0) TeV. The shaded regions thus show not only the anticipated deviation from the predictions of the SM but also how sensitive these are to the  $Z_\theta$  mass. (Presumably, by the time such experiments are performed at the NLC, the mass of any new neutral gauge boson will have already been well determined at the LHC.) Clearly if the  $Z_\theta$  mass were much larger than 1.5 TeV it would be quite difficult to distinguish between models without increased integrated luminosity and a reduction of systematic errors. In Fig. 2, we see that the  $\mathbf{10} \rightarrow \mathbf{16}$  and  $\mathbf{16} \rightarrow \mathbf{16}$  cases are easily separable for the 1 TeV possibility as long as the value of  $\theta$  is not too close to  $\sin^{-1} \sqrt{\frac{3}{8}} \simeq 37.76^\circ$ . For this particular value the  $Z_\theta$  couplings are *independent* of the choice  $\mathbf{10} \rightarrow \mathbf{16}$  or  $\mathbf{16} \rightarrow \mathbf{16}$  since this value of  $\theta$  maps into itself under the transformation (2.8). It is also clear from these figures that the total cross-section,  $\sigma^l$ , and the left-right polarization asymmetry,  $A_{LR}^l$ , play the dominant rôle in identifying the model, while for positive values of  $\theta$  the forward-backward asymmetry  $A_{FB}^l$  displays a large overlap between the two regions corresponding to the different assignments. From this figure we can see that our results are in general agreement, where they overlap, with those already existing in the literature which have considered more general models for  $Z'$ s interacting universally with the fermions [6][7][8].

The situation is similar but a bit more difficult in the  $b$ -quark case, as is shown in Fig. 3, the reason being the far greater overlap of the  $\mathbf{10} \rightarrow \mathbf{16}$  and  $\mathbf{16} \rightarrow \mathbf{16}$  shaded regions. Unlike the leptonic case, where the overlap between

the two sets of predictions occurs only near  $37.76^\circ$ , most of the quantities in the  $b$ -quark case display 2 overlap regions, and generally lie somewhat closer to the various SM predictions. Here we see that in contrast to the leptonic case the quantity  $A_{FB}^b$  will play a very dominant rôle in distinguishing the two scenarios. Combining all four observables allows the two embedding scenarios to be separated for all  $\theta$  values not too close to  $37.76^\circ$ , provided that  $Z_\theta$  is not far above 1.5 TeV.

Clearly, for large  $Z_\theta$  masses, separating the various scenarios within  $E_6$  becomes much more difficult. From the figures, however, we see that the *relative* assignments of the various leptons and the  $b$ -quark can be determined for masses as large as 1.5 TeV.

## IV. SUMMARY AND CONCLUSIONS

In  $E_6$  models it is possible that the three generations may have different embeddings into the fundamental **27** representation while still maintaining universality as far as SM interactions are concerned. The various embeddings can thus only be distinguished based upon probes of the model's  $Z_\theta$  couplings. Although we anticipate that such particles are more massive than 500 GeV, the NLC allows us to indirectly probe the  $Z_\theta$ 's couplings to leptons,  $b$ - and  $c$ -quarks thus allowing us to distinguish between various embedding scenarios. In order to do this we need large integrated luminosities to reduce statistical uncertainties as well as good control of experimental systematics. The results of our analysis show that for most of the value of the model dependent parameter  $\theta$ , the various embedding schemes can be clearly separated for  $Z_\theta$  bosons as heavy as 1.5 TeV. Only in a small region of the  $\theta$  axis around  $\theta = 37.76^\circ$  the disentanglement of the different embeddings is not possible, and this is due to the fact that for this value of  $\theta$  the different choices are physically equivalent. Clearly, use of both total cross section

and the various asymmetry measurements are needed to perform this analysis for leptons and  $b$ -quarks simultaneously, as no one piece of data alone is sufficient to separate the various embedding schemes.

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## Figure captions

FIG. 1.

The total charm pair production cross section at the NLC as a function of the  $E_6$  parameter  $\theta$  for the cases  $\mathbf{16} \rightarrow \mathbf{16}$  (dashes) and  $\mathbf{16} \rightarrow \mathbf{10}$  (dash-dots). The picture shows that the curves overlap when one of them is reflected with respect to the point  $\theta = 37.76^\circ$ . A  $Z_\theta$  with a mass of 1 TeV has been assumed.

FIG. 2.

Predicted values of the leptonic observables at the NLC as functions of the  $E_6$  parameter  $\theta$ . In each case the data point represents the SM prediction and anticipated error. The  $\mathbf{16} \rightarrow \mathbf{16}$  case is represented by the heavier shading while the  $\mathbf{16} \rightarrow \mathbf{10}$  case has a lighter shading. The inner (outer) boundaries in all cases correspond to a  $Z_\theta$  mass of 1.5 (1.0) TeV. (a) the total lepton pair production cross section, (b) the forward-backward asymmetry, (c) the left-right asymmetry, and (d) the polarized forward-backward asymmetry.

FIG. 3.

Same as Fig. 2, but for  $b$ -quark production at the NLC.