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Short-Distance Expansion of Heavy-Light Currents at Order $1/m_Q$

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Abstract

The short-distance expansion of the heavy-light currents $\bar{q}\gamma^\mu Q$ and $\bar{q}\gamma^\mu\gamma_5 Q$ is constructed to order $1/m_Q$, and to next-to-leading order in renormalization-group improved perturbation theory. It is shown that the 10×10 anomalous dimension matrix, which describes the scale dependence of the dimension-four effective current operators in the heavy quark effective theory, is to a large extent determined by the equations of motion, heavy quark symmetry, and reparameterization invariance. The next-to-leading order expressions for the Wilson coefficients at order $1/m_Q$ depend on only five unknown two-loop anomalous dimensions, among them that of the chromo-magnetic operator.

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1 Introduction

Over the last few years, the heavy quark effective theory (HQET) has been established as a convenient tool to analyze the properties of hadrons containing a heavy quark [1, 2, 3, 4, 5, 6, 7]. It provides a systematic expansion of hadronic matrix elements around the limit $\Lambda_{\text{QCD}}/m_Q \rightarrow 0$, in which one or more heavy quark masses tend to infinity and the effective low-energy Lagrangian of QCD exhibits a spin-flavor symmetry under the substitution of heavy quarks of different flavor or spin, but with the same velocity [8, 9, 10, 11]. The existence of such a symmetry limit, and the establishment of HQET to analyze the symmetry-breaking corrections to it, helped to remove much of the model dependence from the theoretical description of semileptonic weak decay processes of heavy mesons or baryons.

The construction of a systematic $1/m_Q$ expansion of hadronic matrix elements consists of two steps: One first constructs the $1/m_Q$ expansion of the weak currents and of the effective low-energy Lagrangian using the operator product expansion and renormalization group techniques. This part of the calculation involves short-distance physics only and, in particular, is independent of the external states. In a second step, hadronic matrix elements of the HQET operators are parameterized by universal, m_Q -independent form factors. For the case of transitions between two heavy quarks, the short-distance expansion is known to order $1/m_Q$, and the corresponding Wilson coefficients have been calculated to next-to-leading order in perturbation theory. The necessary hadronic matrix elements for meson and baryon decays have been investigated in detail up to order $1/m_Q^2$. These results build a solid theoretical basis for a model-independent extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{cb} from semileptonic B decays, with a theoretical uncertainty of only 5% (for a review, see Ref. [7] and references therein).

The $1/m_Q$ expansion is less developed for heavy-to-light transitions, which are, nevertheless, of great phenomenological importance: A thorough understanding of exclusive decays such as $\bar{B} \rightarrow \ell \bar{\nu}$ or $\bar{B} \rightarrow \pi \ell \bar{\nu}$ is necessary for a precise determination of V_{ub} . For the most interesting processes of this type, the constraints imposed by heavy quark symmetry have been analyzed in the $m_Q \rightarrow \infty$ limit (see, e.g., Refs. [10, 12, 13, 14, 15]), but little is known about the symmetry-breaking corrections. In particular, already at order $1/m_Q$ the short-distance expansion of the weak currents is only known in leading

logarithmic approximation [16], and the hadronic matrix elements appearing at that order have only been analyzed for the simplest case, namely for heavy meson decay constants [17]. In this paper we address the first problem and reconsider the short-distance expansion of heavy-light currents at order $1/m_Q$ beyond the leading logarithmic approximation. A detailed analysis of the hadronic matrix elements for $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decays at order $1/m_Q$ will be presented elsewhere [18].

After a brief review of the formalism of HQET relevant to our work, we calculate in Sec. 3 the Wilson coefficients appearing in the expansion of heavy-light currents at the one-loop level. This provides, in particular, the matching corrections between QCD and HQET at $\mu = m_Q$. In Sec. 4, we improve this calculation by using standard renormalization group techniques to sum the leading and next-to-leading logarithms $[\alpha_s \ln(m_Q/\mu)]^n$ and $\alpha_s[\alpha_s \ln(m_Q/\mu)]^n$ to all orders in perturbation theory. This is a complicated problem in that one has to deal with a set of ten operators that mix under renormalization, with the technical complication that the 10×10 one-loop anomalous dimension matrix cannot be fully diagonalized. For the complete next-to-leading order solution of the renormalization group equation, one needs the anomalous dimension matrix at two-loop order. Although we do not calculate this matrix explicitly, we are able to infer much of its structure by analyzing the constraints imposed by various symmetries of the effective theory, as expressed in its Feynman rules, the equations of motion, and an invariance under reparameterizations of the momentum operator. From these considerations, we can determine the 10×10 two-loop matrix up to only five unknown parameters. We then construct the exact next-to-leading order solution for the Wilson coefficients in terms of these parameters. In Sec. 5, we summarize our results and give a sample application.

2 Short-distance Expansion

In HQET, a heavy quark Q bound inside a hadron moving at velocity v is represented by a velocity-dependent field $h_v(x)$, which is related to the conventional spinor field $Q(x)$ by [3]

$$h_v(x) = \exp(im_Q v \cdot x) \frac{1 + \not{v}}{2} Q(x). \quad (1)$$

By means of the phase redefinition one removes the large part of the heavy quark momentum from the new field. When the total momentum is written as $p = m_Q v + k$, the field h_v carries the residual momentum k , which results from soft interactions of the heavy quark with light degrees of freedom and is typically of order Λ_{QCD} . The operator $\frac{1}{2}(1 + \not{v})$ projects out the heavy quark (rather than antiquark) components of the spinor. The antiquark components are integrated out to obtain the effective Lagrangian [1, 2, 3, 4, 6]

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \left[O_{\text{kin}} + C_{\text{mag}}(\mu) O_{\text{mag}} \right] + \mathcal{O}(1/m_Q^2), \quad (2)$$

where $D^\mu = \partial^\mu - ig_s T_a A_a^\mu$ is the gauge-covariant derivative. The operators appearing at order $1/m_Q$ are

$$O_{\text{kin}} = \bar{h}_v (iD)^2 h_v, \quad O_{\text{mag}} = \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v. \quad (3)$$

Here $G^{\mu\nu}$ is the gluon field strength tensor defined by $[iD^\mu, iD^\nu] = ig_s G^{\mu\nu}$. In the hadron's rest frame, it is readily seen that O_{kin} describes the kinetic energy resulting from the residual motion of the heavy quark, whereas O_{mag} describes the chromo-magnetic coupling of the heavy quark spin to the gluon field. One can show that, to all orders in perturbation theory, the kinetic operator O_{kin} is not renormalized [19]. The renormalization factor $C_{\text{mag}}(\mu)$ of the chromo-magnetic operator will be given later.

In order to construct a systematic $1/m_Q$ expansion, one works with the eigenstates of the leading-order term in the effective Lagrangian and treats the $1/m_Q$ corrections as a perturbation. The set of operators that appear at order $1/m_Q$ can be reduced by using the leading-order equation of motion $i v \cdot D h_v = 0$, since the physical matrix elements of operators which vanish by this equation are at least of order $1/m_Q^2$. We have used this freedom to omit in (2) the operator $\bar{h}_v (i v \cdot D)^2 h_v$.

The goal of this paper is the construction of the short-distance expansion for the heavy-light vector and axial vector currents $V^\mu = \bar{q} \gamma^\mu Q$ and $A^\mu = \bar{q} \gamma^\mu \gamma_5 Q$ in terms of operators of the effective theory, beyond the leading order in $1/m_Q$. We shall discuss the case of the vector current in detail. The general form of the expansion can be written as

$$V^\mu \cong \sum_i C_i(\mu) J_i + \frac{1}{2m_Q} \sum_j B_j(\mu) O_j + \frac{1}{2m_Q} \sum_k A_k(\mu) T_k + \mathcal{O}(1/m_Q^2). \quad (4)$$

The symbol \cong is used to indicate that this is an equation that holds on the level of matrix elements. The operators $\{J_i\}$ form a complete set of local dimension-three current operators with the same quantum numbers as the original vector current in the full theory. In HQET there are two such operators, namely

$$J_1 = \bar{q} \gamma^\mu h_v, \quad J_2 = \bar{q} v^\mu h_v. \quad (5)$$

Similarly, $\{O_j\}$ are a complete set of local dimension-four operators. It is convenient to use the background field method, which ensures that there is no mixing between gauge-invariant and gauge-dependent operators. Moreover, operators that vanish by the equation of motion are irrelevant at order $1/m_Q$. It is thus sufficient to consider gauge-invariant operators that do not vanish by the equation of motion. A convenient basis of such operators is [20]:

$$\begin{aligned} O_1 &= \bar{q} \gamma^\mu i \not{D} h_v, & O_4 &= \bar{q} (-iv \cdot \overleftarrow{D}) \gamma^\mu h_v, \\ O_2 &= \bar{q} v^\mu i \not{D} h_v, & O_5 &= \bar{q} (-iv \cdot \overleftarrow{D}) v^\mu h_v, \\ O_3 &= \bar{q} i D^\mu h_v, & O_6 &= \bar{q} (-i \overleftarrow{D}^\mu) h_v. \end{aligned} \quad (6)$$

For simplicity we consider here the limit where the light quark is massless, $m_q = 0$. Otherwise one would have to include two additional operators $O_7 = m_q J_1$ and $O_8 = m_q J_2$.

In (4) we have also included nonlocal operators T_k which arise from an insertion of a $1/m_Q$ correction to the effective Lagrangian into matrix elements of the leading-order currents:

$$\begin{aligned} T_1 &= i \int dy T \{J_1(0), O_{\text{kin}}(y)\}, \\ T_2 &= i \int dy T \{J_2(0), O_{\text{kin}}(y)\}, \\ T_3 &= i \int dy T \{J_1(0), O_{\text{mag}}(y)\}, \\ T_4 &= i \int dy T \{J_2(0), O_{\text{mag}}(y)\}. \end{aligned} \quad (7)$$

The Wilson coefficients of these time-ordered products are simply the products of the coefficients of their component operators, i.e.

$$\begin{aligned} A_1(\mu) &= C_1(\mu), \\ A_2(\mu) &= C_2(\mu), \\ A_3(\mu) &= C_1(\mu) C_{\text{mag}}(\mu), \\ A_4(\mu) &= C_2(\mu) C_{\text{mag}}(\mu). \end{aligned} \quad (8)$$

Nevertheless, since these nonlocal operators can mix into the local operators O_j under renormalization (but not vice versa), it is convenient to include them as parts of the effective currents [16].

Our goal is the calculation of the short-distance coefficients $B_j(\mu)$ in (4) beyond the leading logarithmic approximation. The coefficients $C_i(\mu)$ have been calculated at next-to-leading order in Ref. [21]. We shall perform the calculation using dimensional regularization with modified minimal subtraction ($\overline{\text{MS}}$) and anticommuting γ_5 . This scheme has the advantage that, to all orders in $1/m_Q$, the operator product expansion of the axial vector current A^μ can be simply obtained from (4) by replacing $\bar{q} \rightarrow -\bar{q}\gamma_5$ in the HQET operators. The Wilson coefficients remain unchanged [20]. The reason is that in any diagram the γ_5 from the current can be moved outside next to the light quark spinor. For $m_q = 0$, this operation always leads to a minus sign. Hence it is sufficient to consider the case of the vector current in detail.

Before we turn to explicit calculations, let us recall that there are non-trivial relations between some of the coefficients $B_j(\mu)$ and $C_i(\mu)$ imposed by a “hidden” symmetry of the effective theory, namely its invariance under reparameterizations of the heavy quark velocity and residual momentum which leave the total momentum unchanged. One can show that, as a consequence, operators with a covariant derivative acting on a heavy quark field must always appear in certain combinations with lower-dimension operators [19]. For the case at hand, there is a unique way in which the operators O_1 , O_2 , O_3 can be combined with J_1 , J_2 in a reparameterization invariant form, namely

$$\begin{aligned} \bar{q} \gamma^\mu \left(1 + \frac{i\not{D}}{2m_Q}\right) h_v + \dots &= J_1 + \frac{O_1}{2m_Q} + \dots, \\ \bar{q} \left(v^\mu + \frac{iD^\mu}{m_Q}\right) \left(1 + \frac{i\not{D}}{2m_Q}\right) h_v + \dots &= J_2 + \frac{O_2 + 2O_3}{2m_Q} + \dots. \end{aligned} \quad (9)$$

This implies that, to all orders in perturbation theory,

$$\begin{aligned} B_1(\mu) &= C_1(\mu), \\ B_2(\mu) &= \frac{1}{2} B_3(\mu) = C_2(\mu). \end{aligned} \quad (10)$$

This is an important constraint, which has to be obeyed by any explicit calculation.

3 One-loop Matching

The Wilson coefficients in (4) are defined by requiring that matrix elements of the vector current in the full theory agree, to any order in $1/m_Q$, with matrix elements calculated in HQET. Order by order in perturbation theory, these coefficients can be computed from a comparison of matrix elements in the two theories. Since the effective theory is constructed to reproduce correctly the low-energy behavior of the full theory, this “matching” procedure is independent of any long-distance physics such as infrared singularities, non-perturbative effects, and the properties of the external states. There is thus a freedom in the choice of the external states and the infrared regularization scheme, which can be exploited to simplify the calculations. We find it most convenient to perform the matching of QCD onto HQET using on-shell external quark states and dimensional regularization for both the ultraviolet and infrared singularities encountered in the evaluation of loop diagrams.¹ This scheme has the great advantage that all loop diagrams in the effective theory vanish, since there is no mass scale other than the renormalization point μ . This means that matrix elements in HQET are simply given by their tree-level expressions. We use momentum assignments such that the incoming heavy quark has momentum $p = m_Q v + k$ (with $2v \cdot k + k^2/m_Q = 0$), while the outgoing light quark carries momentum p' (with $p'^2 = 0$). Then, for instance,

$$\begin{aligned}\langle J_1 \rangle &= \bar{u}_q(p', s') \gamma^\mu u_h(v, s), \\ \langle O_1 \rangle &= \bar{u}_q(p', s') \gamma^\mu \not{k} u_h(v, s),\end{aligned}\tag{11}$$

etc., where $u_q(p', s')$ and $u_h(v, s)$ are on-shell spinors for a massless quark q and a heavy quark in HQET, respectively. They satisfy $\not{p}' u_q(p', s') = 0$ and $\not{v} u_h(v, s) = u_h(v, s)$. To complete the matching calculation, one computes in the full theory the vector current matrix element between on-shell quark states at one-loop order. Taking into account that the relation between the heavy quark spinors in QCD and in the effective theory is [4]

$$u_Q(p, s) = \left(1 + \frac{\not{k}}{2m_Q}\right) u_h(v, s) + \mathcal{O}(1/m_Q^2),\tag{12}$$

¹The usefulness of this scheme was pointed out by Eichten and Hill in the original calculation of the renormalized effective Lagrangian at order $1/m_Q$ [2].

we find at one-loop order

$$\begin{aligned}
\langle V^\mu \rangle_{\text{QCD}} = & \left\langle J_1 + \frac{O_1}{2m_Q} \right\rangle + \frac{\alpha_s}{3\pi} \frac{\Gamma(2-D/2)}{D-3} \left(\frac{m_Q^2}{4\pi\mu^2} \right)^{D/2-2} \\
& \times \left\{ \frac{D-7}{2} \left\langle J_1 + \frac{O_1}{2m_Q} \right\rangle - (D-4) \left\langle J_2 + \frac{O_2 + 2O_3}{2m_Q} \right\rangle \right. \\
& \left. + 2 \frac{D^2 - 9D + 23}{D-5} \left\langle \frac{O_4}{2m_Q} \right\rangle - 2(D-6) \left\langle \frac{O_5}{2m_Q} \right\rangle + 2 \left\langle \frac{O_6}{2m_Q} \right\rangle \right\} \\
& + \mathcal{O}(1/m_Q^2), \tag{13}
\end{aligned}$$

where D is the space-time dimension. The matrix elements on the right-hand side are evaluated in the effective theory. Note that the operators J_1 and O_1 , as well as J_2 , O_2 and O_3 , appear precisely in the combinations required by reparameterization invariance, cf. (9). Hence, our explicit one-loop calculation is in accordance with the general relations (10).

Next we expand the above expressions around $D = 4$ and subtract the poles in

$$\frac{1}{\epsilon} = \frac{2}{4-D} - \gamma_E + \ln 4\pi, \tag{14}$$

corresponding to an $\overline{\text{MS}}$ renormalization of the operators in the effective theory, to obtain the following one-loop expressions for the Wilson coefficients:

$$\begin{aligned}
C_1(\mu) = B_1(\mu) &= 1 + \frac{\alpha_s}{\pi} \left(\ln \frac{m_Q}{\mu} - \frac{4}{3} \right), \\
C_2(\mu) = B_2(\mu) &= \frac{1}{2} B_3(\mu) = \frac{2\alpha_s}{3\pi}, \\
B_4(\mu) &= \frac{4\alpha_s}{3\pi} \left(3 \ln \frac{m_Q}{\mu} - 1 \right), \\
B_5(\mu) &= -\frac{4\alpha_s}{3\pi} \left(2 \ln \frac{m_Q}{\mu} - 3 \right), \\
B_6(\mu) &= -\frac{4\alpha_s}{3\pi} \left(\ln \frac{m_Q}{\mu} - 1 \right). \tag{15}
\end{aligned}$$

The one-loop expressions for $C_i(\mu)$ have been first obtained in Ref. [21]. Our expressions for $B_j(\mu)$ are new.

4 Renormalization Group Improvement

4.1 Introduction

For $\mu \ll m_Q$, the calculation presented above becomes unsatisfactory, since the scale in the running coupling constant cannot be determined at one-loop order. Yet $\alpha_s(\mu)$ and $\alpha_s(m_Q)$ may differ substantially, and one would thus like to resolve the scale ambiguity problem by going beyond the leading order in perturbation theory. Furthermore, the perturbative expansion of the Wilson coefficients is known to contain large logarithms of the type $[\alpha_s \ln(m_Q/\mu)]^n$, which one should sum to all orders. Both goals can be achieved by using the renormalization group to improve the one-loop results derived in the previous section.

Let us introduce a compact matrix notation where we collect the renormalized operators O_j and T_k , as well as the coefficient functions $B_j(\mu)$ and $A_k(\mu)$, into 10-component vectors

$$\begin{aligned}\vec{O} &= (O_1, \dots, O_6, T_1, \dots, T_4), \\ \vec{B} &= (B_1, \dots, B_6, A_1, \dots, A_4).\end{aligned}\tag{16}$$

The Wilson coefficients obey a renormalization group equation

$$\left(\mu \frac{d}{d\mu} - \hat{\gamma}^t\right) \vec{B}(\mu) = 0,\tag{17}$$

where $\hat{\gamma}^t$ is the transposed 10×10 anomalous dimension matrix. It is defined in terms of the matrix \hat{Z} that relates the “bare” operators to the renormalized ones: $\vec{O}_{\text{bare}} = \hat{Z} \vec{O}$. In the $\overline{\text{MS}}$ scheme, \hat{Z} obeys an expansion

$$\hat{Z} = \hat{1} + \sum_k \frac{1}{\epsilon^k} \hat{Z}_k(g_s),\tag{18}$$

with coefficients $\hat{Z}_k(g_s)$ that depend on the renormalized coupling constant g_s . Given this expansion, one can show that [22]

$$\hat{\gamma} = -g_s \frac{\partial}{\partial g_s} \hat{Z}_1(g_s),\tag{19}$$

meaning that the anomalous dimension matrix can be computed from the $1/\epsilon$ poles in \hat{Z} .

The formal solution of the renormalization group equation reads

$$\vec{B}(\mu) = \hat{U}(\mu, m_Q) \vec{B}(m_Q), \quad (20)$$

with the evolution matrix [23, 24]

$$\hat{U}(\mu, m_Q) = T_g \exp \int_{g_s(m_Q)}^{g_s(\mu)} dg \frac{\hat{\gamma}^t(g)}{\beta(g)}. \quad (21)$$

The β -function $\beta(g_s) = \mu dg_s/d\mu$ describes the scale-dependence of the renormalized coupling constant in QCD. The symbol “ T_g ” means an ordering in the coupling constant such that the couplings increase from right to left (for $\mu < m_Q$). This is necessary since, in general, the anomalous dimension matrices at different values of g do not commute: $[\hat{\gamma}(g_1), \hat{\gamma}(g_2)] \neq 0$. Eq. (21) can be solved perturbatively by expanding the β -function and the anomalous dimension matrix in the renormalized coupling constant:

$$\begin{aligned} \beta(g) &= -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} + \dots, \\ \hat{\gamma}(g) &= \hat{\gamma}_0 \frac{g^2}{16\pi^2} + \hat{\gamma}_1 \left(\frac{g^2}{16\pi^2} \right)^2 + \dots \end{aligned} \quad (22)$$

Here

$$\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta_1 = 102 - \frac{38}{3} n_f, \quad (23)$$

where n_f denotes the number of light quark flavors with mass below m_Q . At next-to-leading order in renormalization-group improved perturbation theory, the evolution matrix can be written in the form [24]

$$\hat{U}(\mu, m_Q) = \left\{ 1 - \frac{\alpha_s(\mu)}{4\pi} \hat{S} \right\} \hat{U}_0(\mu, m_Q) \left\{ 1 + \frac{\alpha_s(m_Q)}{4\pi} \hat{S} \right\} + \dots, \quad (24)$$

where

$$\hat{U}_0(\mu, m_Q) = \exp \left\{ \frac{\hat{\gamma}_0^t}{2\beta_0} \ln \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right\} \quad (25)$$

describes the evolution in leading logarithmic approximation, and \hat{S} contains the next-to-leading corrections. This matrix is defined by the algebraic equation

$$\hat{S} + \frac{1}{2\beta_0} [\hat{\gamma}_0^t, \hat{S}] = \frac{1}{2\beta_0} \hat{\gamma}_1^t - \frac{\beta_1}{2\beta_0^2} \hat{\gamma}_0^t. \quad (26)$$

Finally, we expand the initial values of the coefficient functions at $\mu = m_Q$ as

$$\vec{B}(m_Q) = \vec{B}_0 + \frac{\alpha_s(m_Q)}{4\pi} \vec{B}_1 + \dots \quad (27)$$

From our one-loop results in (15) we obtain

$$\begin{aligned} \vec{B}_0 &= (1, 0, 0, 0, 0, 0, 1, 0, 1, 0), \\ \vec{B}_1 &= \frac{2}{3}(-8, 4, 8, -8, 24, 8, -8, 4, 5, 4), \end{aligned} \quad (28)$$

where we have used that [2]

$$C_{\text{mag}}(m_Q) = 1 + \frac{26}{3} \frac{\alpha_s(m_Q)}{4\pi}. \quad (29)$$

Putting everything together, we obtain from (20)

$$\begin{aligned} \vec{B}(\mu) &= \hat{U}_0(\mu, m_Q) \left[\vec{B}_0 + \frac{\alpha_s(m_Q)}{4\pi} (\vec{B}_1 + \hat{S} \vec{B}_0) \right] \\ &\quad - \frac{\alpha_s(\mu)}{4\pi} \hat{S} \hat{U}_0(\mu, m_Q) \vec{B}_0 + \dots \end{aligned} \quad (30)$$

What remains to be calculated are the matrices \hat{U}_0 and \hat{S} . To this end, we need to know the 10×10 anomalous dimension matrix $\hat{\gamma}$ at two-loop order. Note that the leading-log evolution matrix, as well as the coefficient $(\vec{B}_1 + \hat{S} \vec{B}_0)$ that multiplies $\alpha_s(m_Q)$, are independent of the renormalization scheme [23]. The matrix \hat{S} that multiplies $\alpha_s(\mu)$, on the other hand, is scheme-dependent. Only when the μ -dependent terms in $\vec{B}(\mu)$ are combined with the μ -dependent matrix elements of the renormalized HQET operators does one obtain a renormalization-scheme invariant result.

4.2 Anomalous dimension matrix

Let us now analyze the structure of the anomalous dimension matrix in more detail. We will see that, to a large extent, the texture of this matrix can be determined without an explicit calculation. From (8) and (10) we know that the operators O_1 , O_2 , O_3 and T_k renormalize multiplicatively. They can mix

into O_4, O_5, O_6 , but not vice versa. Therefore, the anomalous dimension matrix must be of the form

$$\hat{\gamma} = \begin{pmatrix} \hat{\gamma}_{\text{hl}} & \hat{\gamma}_A & 0 \\ 0 & \hat{\gamma}_B & 0 \\ 0 & \hat{\gamma}_C & \hat{\gamma}_D \end{pmatrix}, \quad (31)$$

with diagonal matrices

$$\begin{aligned} \hat{\gamma}_{\text{hl}} &= \text{diag}(\gamma^{\text{hl}}, \gamma^{\text{hl}}, \gamma^{\text{hl}}), \\ \hat{\gamma}_D &= \text{diag}(\gamma^{\text{hl}}, \gamma^{\text{hl}}, \gamma^{\text{hl}} + \gamma^{\text{mag}}, \gamma^{\text{hl}} + \gamma^{\text{mag}}). \end{aligned} \quad (32)$$

Here γ^{hl} is the universal anomalous dimension of dimension-three heavy-light current operators in HQET, and γ^{mag} is the anomalous dimension of the chromo-magnetic operator in (3). The form of $\hat{\gamma}_{\text{hl}}$ is a consequence of reparameterization invariance [cf. (10)], whereas the structure of $\hat{\gamma}_D$ follows from (8). The one-loop coefficients of the anomalous dimensions are [10, 6]

$$\gamma_0^{\text{hl}} = -4, \quad \gamma_0^{\text{mag}} = 6. \quad (33)$$

The two-loop coefficient $\gamma_1^{\text{hl}} = -\frac{254}{9} - \frac{56}{27}\pi^2 + \frac{20}{9}n_f$ has been calculated in Refs. [21, 25], whereas γ_1^{mag} is not yet known.

The 3×3 submatrix $\hat{\gamma}_B$ in (31) may be constructed by noting that the equation of motion $iv \cdot D h_v = 0$ can be used to rewrite

$$\begin{aligned} O_4 &= -iv \cdot \partial J_1, \\ O_5 &= -iv \cdot \partial J_2, \\ O_6 - O_3 &= -i\partial^\mu [\bar{q} h_v]. \end{aligned} \quad (34)$$

The total derivatives of the currents on the right-hand side renormalize multiplicatively and in the same way as the dimension-three operators J_i . The additional power of the external momentum carried by the current does not affect the divergences of loop diagrams. It follows that

$$(\hat{\gamma}_B)_{ij} = \gamma^{\text{hl}} \delta_{ij} + \delta_{i3} (\hat{\gamma}_A)_{3j}. \quad (35)$$

Let us then focus on the 3×3 submatrix $\hat{\gamma}_A$, which describes the mixing of O_1, O_2, O_3 into O_4, O_5, O_6 . To compute $\hat{\gamma}_A$, one has to calculate in the

effective theory the ultraviolet divergent $1/\epsilon$ poles of the p' -dependent terms in the matrix element of the generic operator $\bar{q}\Gamma iD_\alpha h_v$, where p' denotes the external momentum of the light quark. At the end of the calculation, one substitutes $\Gamma = \gamma^\mu\gamma^\alpha, v^\mu\gamma^\alpha, g^{\mu\alpha}$ for O_1, O_2, O_3 , respectively. The structure of these terms is strongly constrained by the form of the Feynman rules of the effective theory, and by the equations of motion. In HQET, a heavy quark couples to gluons proportional to its velocity [3, 5], and hence there do not appear any Dirac matrices on the right-hand side of Γ . Furthermore, since we consider the limit of a massless light quark, there can only be an even number of γ -matrices on the left-hand side of Γ , since vertices and propagators come in pairs. Moreover, the equation of motion for the heavy quark, $iv\cdot D h_v = 0$, requires that the matrix elements must vanish upon contraction with v^α . Likewise, the equation of motion for the light quark, $i\not{D}q = 0$, implies that the p' -dependent terms must vanish when one substitutes $\Gamma = \gamma^\alpha\Gamma'$. Using these constraints, we find that, to all orders in perturbation theory, the ultraviolet divergent pole in the matrix element must be of the form

$$\begin{aligned} \langle \bar{q}\Gamma iD_\alpha h_v \rangle \Big|_{\text{pole}} &= \frac{f_1(g_s)}{3\epsilon} \bar{u}_q(p') \left[3p'_\alpha - 2v \cdot p' v_\alpha - v \cdot p' \gamma_\alpha \not{v} \right] \Gamma u_h(v) \\ &\quad + p'\text{-independent terms,} \end{aligned} \quad (36)$$

where the function $f_1(g_s)$ has a perturbative expansion in powers of the coupling constant. From this, it is straightforward to derive that

$$\hat{\gamma}_A = \gamma^a \begin{pmatrix} -\frac{2}{3} & -\frac{4}{3} & 2 \\ 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{2}{3} & 1 \end{pmatrix}, \quad (37)$$

where

$$\gamma^a = -g_s \frac{\partial f_1}{\partial g_s}. \quad (38)$$

We can use similar arguments to impose some restrictions on the 4×3 submatrix $\hat{\gamma}_D$, which describes the mixing of the nonlocal operators T_k into O_4, O_5, O_6 . The Feynman rules of HQET imply that

$$\langle T_{1,2} \rangle \Big|_{\text{pole}} = \frac{f_2(g_s)}{\epsilon} v \cdot p' \bar{u}_q(p') \Gamma u_h(v) + p'\text{-independent terms,}$$

$$\begin{aligned} \langle T_{3,4} \rangle \Big|_{\text{pole}} &= -\frac{1}{4\epsilon} \bar{u}_q(p') \left[f_3(g_s) v \cdot p' \sigma_{\alpha\beta} + i f_4(g_s) \gamma_\alpha p'_\beta \not{p} \right] \Gamma (1 + \not{p}) \sigma^{\alpha\beta} u_h(v) \\ &+ p'\text{-independent terms,} \end{aligned} \quad (39)$$

where $\Gamma = \gamma^\mu$ for T_1 and T_3 , whereas $\Gamma = v^\mu$ for T_2 and T_4 . It follows that

$$\hat{\gamma}_D = \begin{pmatrix} \gamma^b & 0 & 0 \\ 0 & \gamma^b & 0 \\ \gamma^c & -4\gamma^c - 2\gamma^d & \gamma^d \\ 0 & -3\gamma^c - \gamma^d & 0 \end{pmatrix}, \quad (40)$$

where

$$\gamma^b = -g_s \frac{\partial f_2}{\partial g_s}, \quad \gamma^c = -g_s \frac{\partial f_3}{\partial g_s}, \quad \gamma^d = -g_s \frac{\partial f_4}{\partial g_s}. \quad (41)$$

We have calculated the anomalous dimension matrix $\hat{\gamma}$ at the one-loop level by computing the ultraviolet divergences of the bare operators \vec{O}_{bare} . Our results are in agreement with the general relations derived above. Expanding the anomalous dimensions $\gamma^a, \dots, \gamma^d$ as in (22), we find the one-loop coefficients

$$\gamma_0^a = 4, \quad \gamma_0^b = -\frac{32}{3}, \quad \gamma_0^c = \gamma_0^d = -\frac{8}{3}. \quad (42)$$

We note that the first row in $\hat{\gamma}_A$, as well as the first and third rows in $\hat{\gamma}_C$, can be derived from a previous one-loop calculation by Falk and Grinstein [16]. We confirm their results.

4.3 Leading-log solution

Given the one-loop anomalous dimension matrix, one can calculate the evolution matrix in leading logarithmic approximation from (25). The evaluation of this equation would be straightforward if there were a matrix \hat{V} such that $\hat{V}^{-1} \hat{\gamma}_0^t \hat{V}$ were diagonal [24]. However, such a matrix does not exist in the present case. The best one can achieve is to construct a matrix \hat{W} which brings $\hat{\gamma}_0^t$ into Jordan form:

$$\hat{W}^{-1} \hat{\gamma}_0^t \hat{W} = \hat{\gamma}_J. \quad (43)$$

A matrix which accomplishes this is:

$$\hat{W} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & 0 & -\frac{4}{27} & -\frac{32}{3} & \frac{16}{27} & \frac{1}{3} & -\frac{64}{27} & \frac{2}{3} \\ -\frac{2}{3} & \frac{16}{9} & -\frac{32}{3} & -\frac{16}{9} & \frac{8}{9} & \frac{128}{9} & 0 & \frac{26}{3} & -\frac{32}{27} & \frac{52}{3} \\ 1 & 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (44)$$

A Jordan matrix is convenient enough for an exponentiation in closed form. In our case, we only need that

$$\exp \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} = \exp(a) \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}. \quad (45)$$

It is then straightforward to compute the evolution matrix from

$$\hat{U}_0(\mu, m_Q) = \hat{W} \exp \left\{ \frac{\hat{\gamma}_J}{2\beta_0} \ln \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right\} \hat{W}^{-1}. \quad (46)$$

Multiplying \hat{U}_0 with the tree-level matching condition as encoded in \vec{B}_0 in (28), we recover the leading-log results of Refs. [16, 20]:

$$\begin{aligned} C_1(\mu) &= B_1(\mu) = x^{2/\beta_0}, \\ C_2(\mu) &= B_2(\mu) = B_3(\mu) = 0, \\ B_4(\mu) &= \frac{34}{27} x^{2/\beta_0} - \frac{4}{27} x^{-1/\beta_0} - \frac{10}{9} + \frac{16}{3\beta_0} x^{2/\beta_0} \ln x, \\ B_5(\mu) &= -\frac{28}{27} x^{2/\beta_0} + \frac{88}{27} x^{-1/\beta_0} - \frac{20}{9}, \\ B_6(\mu) &= -2x^{2/\beta_0} - \frac{4}{3} x^{-1/\beta_0} + \frac{10}{3}, \end{aligned} \quad (47)$$

with

$$x = \frac{\alpha_s(\mu)}{\alpha_s(m_Q)}. \quad (48)$$

For $m_Q \approx \mu$, these expressions can be expanded using

$$x \approx 1 + 2\beta_0 \frac{\alpha_s}{4\pi} \ln \frac{m_Q}{\mu}, \quad (49)$$

and one readily recovers the logarithmic terms in the one-loop coefficients given in (15).

At this point, it is worthwhile to compare the the leading logarithmic approximation with the one-loop results. For the purpose of illustration, we use $\alpha_s(m_Q) = 0.20$ and $\alpha_s(\mu) = 0.36$ (corresponding to $m_Q \approx m_b$ and $\mu \approx 1$ GeV), as well as $n_f = 4$. From (47), we then obtain: $B_1^{\text{LL}} = 1.15$, $B_2^{\text{LL}} = B_3^{\text{LL}} = 0$, $B_4^{\text{LL}} = 0.63$, $B_5^{\text{LL}} = -0.38$, $B_6^{\text{LL}} = -0.21$. In the one-loop expressions (15), we use the average value $\alpha_s = 0.28$, as well as $m_Q/\mu = 4.8$. This gives: $B_1^{(1)} = 1.02$, $B_2^{(1)} = 0.06$, $B_3^{(1)} = 0.12$, $B_4^{(1)} = 0.44$, $B_5^{(1)} = -0.02$, $B_6^{(1)} = -0.07$. Obviously, there are significant differences between these two approximation schemes, which can only be resolved by going beyond the leading order.

4.4 Next-to-leading order solution

Let us then discuss the construction of the complete next-to-leading order solution of the renormalization group equation. According to (30), we need to construct the matrix \hat{S} that satisfies the algebraic equation (26). To this end, it is convenient to define $\hat{T} = \hat{W}^{-1} \hat{S} \hat{W}$, where \hat{W} has been given in (44). The algebraic equation which determines \hat{T} reads

$$\hat{T} + \frac{1}{2\beta_0} [\hat{\gamma}_J, \hat{T}] = \frac{1}{2\beta_0} \hat{W}^{-1} \hat{\gamma}_1^t \hat{W} - \frac{\beta_1}{2\beta_0^2} \hat{\gamma}_J. \quad (50)$$

We have solved this equation by modifying the Jordan matrix, $\hat{\gamma}_J \rightarrow \hat{\gamma}_J(\eta)$, such that $\hat{\gamma}_J(0) \equiv \hat{\gamma}_J$, but $\hat{\gamma}_J(\eta)$ can be diagonalized for $\eta \neq 0$. It is then straightforward to construct a matrix $\hat{T}(\eta)$ which satisfies the above equation with $\hat{\gamma}_J$ replaced by $\hat{\gamma}_J(\eta)$ [24]. The desired matrix \hat{T} is obtained by taking the limit $\eta \rightarrow 0$. We find that most of the entries in \hat{T} vanish. Among the nonvanishing components T_{ij} are:

$$\begin{aligned} T_{ii} &= S_{\text{hl}} + S_{\text{mag}}; & i &= 2, 5, \\ T_{ii} &= S_{\text{hl}}; & i &= 3, 4, 6, 7, 8, 9, 10, \end{aligned} \quad (51)$$

where

$$S_{\text{hl}} = \frac{\gamma_1^{\text{hl}}}{2\beta_0} - \frac{\beta_1 \gamma_0^{\text{hl}}}{2\beta_0^2}, \quad S_{\text{mag}} = \frac{\gamma_1^{\text{mag}}}{2\beta_0} - \frac{\beta_1 \gamma_0^{\text{mag}}}{2\beta_0^2}. \quad (52)$$

These are nothing but the next-to-leading corrections to the Wilson coefficients of the dimension-three heavy-light currents and of the chromomagnetic operator, which are given by [21, 2, 6]:

$$\begin{aligned} C_1(\mu) &= x^{2/\beta_0} \left\{ 1 + \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} S_{\text{hl}} - \frac{4}{3} \frac{\alpha_s(m_Q)}{\pi} \right\}, \\ C_2(\mu) &= \frac{2}{3} x^{2/\beta_0} \frac{\alpha_s(m_Q)}{\pi}, \\ C_{\text{mag}}(\mu) &= x^{-3/\beta_0} \left\{ 1 + \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} S_{\text{mag}} + \frac{13}{6} \frac{\alpha_s(m_Q)}{\pi} \right\}. \end{aligned} \quad (53)$$

As previously, $x = \alpha_s(\mu)/\alpha_s(m_Q)$. The remaining nonvanishing components of \hat{T} are:

$$\begin{aligned} T_{11} &= \frac{\gamma_1^{\text{hl}} + \gamma_1^a}{2\beta_0}, \\ T_{15} &= \frac{4\gamma_1^{\text{mag}} - 4\gamma_1^a + 3\gamma_1^d}{6(\beta_0 - 1)}, \\ T_{32} &= -\frac{9}{4} T_{35} = -3 T_{65} = \frac{16\gamma_1^{\text{mag}} + 27\gamma_1^c + 9\gamma_1^d}{192(\beta_0 - 3)}, \\ T_{34} &= T_{67} = -\frac{3\gamma_1^b}{64\beta_0} - \frac{\beta_1}{2\beta_0^2}. \end{aligned} \quad (54)$$

Given these results, one can compute the Wilson coefficients from (30). For the coefficients $A_k(\mu)$ of the nonlocal operators T_k , and for the first three of the coefficients $B_j(\mu)$, we confirm the general relations (8) and (10) at next-to-leading order. The exact next-to-leading order results for the remaining coefficients are rather complicated. We present them in the form of three independent combinations:

$$\begin{aligned} 4B_4 + B_5 + 2B_6 &= \frac{128}{3} C_1 \left\{ \frac{1}{2\beta_0} \ln x - T_{34} \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} \right\} \\ &\quad + \frac{2}{9} C_2 \left\{ 1 + 8 C_{\text{mag}} + \frac{24}{\beta_0} \ln x \right\}, \end{aligned}$$

$$\begin{aligned}
3B_5 + 2B_6 &= -\frac{64}{9} C_1 \left\{ \left[1 + 6 T_{32} \frac{\alpha_s(m_Q)}{4\pi} \right] - C_{\text{mag}} \left[1 + 6 T_{32} \frac{\alpha_s(\mu)}{4\pi} \right] \right\} \\
&\quad + \frac{16}{3} C_2 \left\{ C_{\text{mag}} - \frac{29}{24} + \frac{3}{\beta_0} \ln x \right\}, \\
B_6 &= -2 C_1 - 2 C_2 - \frac{4}{3} C_1 C_{\text{mag}} \left\{ 1 + \frac{3}{4} T_{15} \frac{\alpha_s(\mu)}{4\pi} \right\} + T_{15} \frac{\alpha_s(m_Q)}{4\pi} \\
&\quad + \frac{10}{3} \left\{ 1 + \frac{\alpha_s(m_Q)}{3\pi} + T_{11} \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} \right\}, \tag{55}
\end{aligned}$$

where we have omitted the μ -dependence of the coefficient functions for simplicity. The coefficients $C_i(\mu)$ have been given in (53).

5 Summary and an Application

We have presented a detailed analysis of the short-distance expansion of the heavy-light currents $\bar{q}\gamma^\mu Q$ and $\bar{q}\gamma^\mu\gamma_5 Q$ to order $1/m_Q$ in the heavy quark expansion, and to next-to-leading order in renormalization-group improved perturbation theory. Our main result is that the 10×10 anomalous dimension matrix $\hat{\gamma}$, which describes the scale dependence of the dimension-four effective current operators in the heavy quark effective theory, can be determined to a large extent from symmetry considerations. By evaluating the constraints that arise from reparameterization invariance, the equations of motion, and heavy quark symmetry, we have shown that, to all orders in perturbation theory, $\hat{\gamma}$ can be expressed in terms of the universal anomalous dimension of the leading order (dimension-three) currents, the anomalous dimension of the chromo-magnetic operator, and four functions $\gamma^i(g_s)$ of the coupling constant. We have calculated these functions to one-loop order, and have treated their two-loop coefficients, as well as the yet unknown two-loop anomalous dimension of the chromo-magnetic operator, as free parameters in the exact next-to-leading order solution of the renormalization group equation. Our final expressions for the Wilson coefficients appearing at order $1/m_Q$ in the expansion of the currents are given in (8), (10), and (55).

We believe that the fact that our results depend on only five unknown two-loop anomalous dimensions (as compared to the à priori 100 unknown entries in the two-loop anomalous dimension matrix) makes it feasible to obtain the complete next-to-leading order solution for the Wilson coefficients in the near future. This would be desirable since, as we have pointed out,

there are substantial numerical differences in the results obtained in leading logarithmic approximation as compared to those obtained from a one-loop calculation, which we have presented in Sec. 3. These differences can only be resolved at next-to-leading order.

The short-distance expansion of heavy-light currents considered in this paper plays an important role in applications of the heavy quark effective theory to decay processes such as $\bar{B} \rightarrow \ell \bar{\nu}$ or $\bar{B} \rightarrow X \ell \bar{\nu}$, where X is a light meson ($X = \pi, \rho$, etc.). As an example, we briefly discuss the case of meson decay constants, following the analysis of Ref. [17]. To order $1/m_Q$ in the heavy quark expansion, the decay constants of pseudoscalar and vector mesons can be written as

$$f_M \sqrt{m_M} = \left[C_1(\mu) + \frac{(1+d_M)}{4} C_2(\mu) \right] F(\mu) \quad (56)$$

$$\times \left\{ 1 + \frac{1}{m_Q} \left[G_1(\mu) - \frac{\bar{\Lambda}}{6} b(\mu) \right] + \frac{2d_M}{m_Q} \left[C_{\text{mag}}(\mu) G_2(\mu) - \frac{\bar{\Lambda}}{12} B(\mu) \right] \right\},$$

where $d_M = 3$ if M is a pseudoscalar meson, and $d_M = -1$ if M is a vector meson. $F(\mu)$, $G_i(\mu)$, and $\bar{\Lambda}$ are m_Q -independent low-energy parameters of the effective theory. The coefficients $b(\mu)$ and $B(\mu)$ contain combinations of the Wilson coefficients $B_j(\mu)$, which we have calculated in this paper. One obtains [17]

$$b = \frac{3}{4(C_1 + C_2)} [B_1 - B_2 + B_4 + B_5 + B_6] - \frac{3}{4C_1} [B_1 - B_3 - 3B_4 - B_6], \quad (57)$$

$$B = \frac{3}{4(C_1 + C_2)} [B_1 - B_2 + B_4 + B_5 + B_6] + \frac{1}{4C_1} [B_1 - B_3 - 3B_4 - B_6].$$

Using the next-to-leading order expressions for the Wilson coefficients given in (55), we find that

$$b(\mu) = \frac{16}{\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} + \frac{\alpha_s(m_Q)}{\pi} - 8 T_{34} \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{\pi}, \quad (58)$$

$$B(\mu) = \frac{16}{9} C_{\text{mag}}(\mu) \left\{ 1 + \frac{3}{2} T_{32} \frac{\alpha_s(\mu)}{\pi} \right\} - \frac{7}{9} - \left(\frac{41}{27} + \frac{8}{3} T_{32} \right) \frac{\alpha_s(m_Q)}{\pi}.$$

From the fact that the physical decay constants f_M in (56) must be μ -independent, one can deduce the scale dependence of the low-energy parameters. At leading order in $1/m_Q$, we recover the well-known result that $C_1(\mu) F(\mu)$ is μ -independent. At order $1/m_Q$, the scale independent combinations are $G_1(\mu) - \frac{1}{6}\bar{\Lambda} b(\mu)$ and $C_{\text{mag}}(\mu) G_2(\mu) - \frac{1}{12}\bar{\Lambda} B(\mu)$. Note that the structure of the coefficients $b(\mu)$ and $B(\mu)$ is consistent with this requirement, i.e., the μ -dependent terms in $b(\mu)$ are independent of m_Q , and those in $B(\mu)$ are proportional to $C_{\text{mag}}(\mu)$. This provides a nontrivial test of our results.² As a consequence, we can define renormalization-group invariant, μ - and m_Q -independent low-energy parameters by

$$G_1^{\text{ren}} = G_1(\mu) - \frac{\bar{\Lambda}}{6} \left\{ \frac{16}{\beta_0} \ln \alpha_s(\mu) + 8 T_{34} \frac{\alpha_s(\mu)}{\pi} \right\}, \quad (59)$$

$$G_2^{\text{ren}} = [\alpha_s(\mu)]^{-3/\beta_0} \left\{ 1 - S_{\text{mag}} \frac{\alpha_s(\mu)}{4\pi} \right\} \left\{ G_2(\mu) - \frac{4\bar{\Lambda}}{27} \left[1 + \frac{3}{2} T_{32} \frac{\alpha_s(\mu)}{\pi} \right] \right\}.$$

In terms of these parameters, one obtains, for instance,

$$G_1(\mu) - \frac{\bar{\Lambda}}{6} b(\mu) = G_1^{\text{ren}} + \frac{\bar{\Lambda}}{6} \left\{ \frac{16}{\beta_0} \ln \alpha_s(m_Q) - (1 - 8 T_{34}) \frac{\alpha_s(m_Q)}{\pi} \right\}, \quad (60)$$

and a similar relation for the invariant combination involving $G_2(\mu)$. These relations show explicitly the nonanalytic m_Q -dependence of the $1/m_Q$ corrections to meson decay constants.

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²We have checked that this ‘‘consistency’’ of our short-distance analysis with the requirement of renormalizability of hadronic matrix elements is also obeyed for $\bar{B} \rightarrow \pi \ell \bar{\nu}$ transitions, as well as for the decay constants of excited mesons.

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