SLAC-PUB-6342
SLAC/SSRL-0055
August 1993
(I/SSRL-M)

An Elliptically-Polarizing Undulator with Phase Adjustable Energy and Polarization

Steve Lidia
Department of Physics, University of California, Davis, CA 95616
and
Roger Carr
Stanford Synchrotron Radiation Laboratory, Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

We present a planar helical undulator designed to produce elliptically polarized light. Helical magnetic fields may be produced by a variety of undulators with four parallel cassettes of magnets. In our design, all cassettes are mounted in two planes on slides so that they may be moved parallel to the electron beam. This allows us to produce x-rays of left- or right-handed elliptical or circular polarization as well as horizontal or vertical linear polarization.

In model calculations, we have found that by sliding the top pair of rows with respect to the bottom pair, or the left pair with respect to the right pair, we retain the polarization setting but change the magnetic field strength, and hence the x-ray energy. This allows us to select both energy and polarization by independent phase adjustments alone, without changing the gap between the rows. Such a design may be simpler to construct than an adjustable gap machine. We present calculations that model its operation and its effects on an electron beam.

1. Introduction

Experiments on magnetic and biological materials which exhibit circular dichroism are now being performed using circularly polarized soft x-rays. The most common sources of such radiation are electron storage ring bending magnets, where the different senses of polarization are obtained above and below the plane of the electron trajectory. For increased intensity, new sources are being built using insertion devices such as wigglers and undulators [1]. One class of undulator is the planar helical design, where rows of magnets are arrayed above and below the electron beam in two planar jaws; generally each jaw comprises two rows of magnets [2].

We are building a device of this type, which is a combination of a magnet design by Sasaki [3], and the adjustable phase undulator (APU) concept [4]. The elliptically-polarizing undulator (EPU) has four identical rows of Halbach sinusoidal pure permanent magnet blocks [5] arranged in the four quadrants about the axis of the electron beam. If the rows in quadrants 2 and 4 are moved longitudinally in the same direction by the same distance with respect to the rows in quadrants 1 and 3, a helical magnetic field is created. Electrons in this field execute a helical trajectory, and emit elliptically-polarized x-rays. By moving in the opposite direction, the other sense of helicity is obtained. We call this motion a "row phase" shift. One period (of length λ_u) of the EPU magnet lattice is shown in Fig. 1.

---

* Supported in part by the Department of Energy, Office of Basic Energy Sciences and Department of Energy contract DE-AC03-76SF00515.

Presented at the 8th National Conference on Synchrotron Radiation Instrumentation, Gaithersburg,. Maryland, August 23-26, 1993

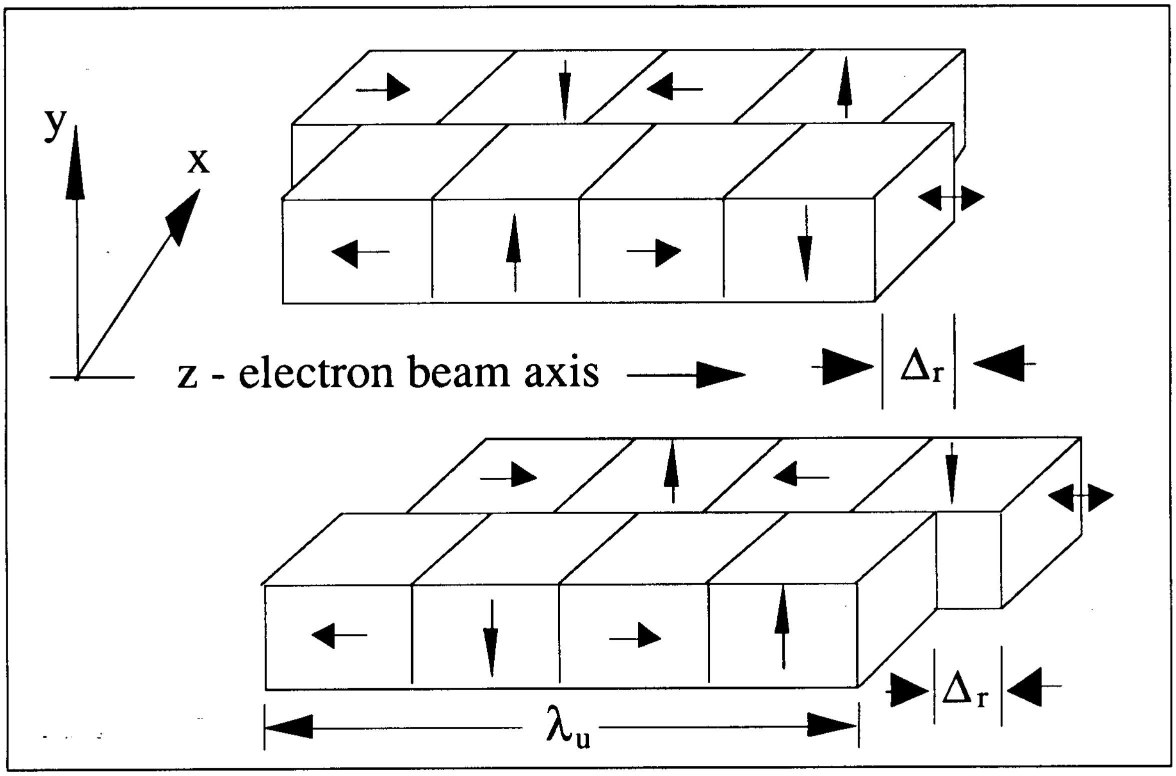


Figure 1: Schematic of one period of EPU magnets, with an arrow showing the direction of magnetization in each block. The forward row of the upper jaw (quadrant 2), and the rear row of the lower jaw (quadrant 4) are shifted by a row phase of Δ_r/λ_u.

An APU is a conventional Halbach undulator whose magnetic field strength is varied by changing the longitudinal position of one jaw with respect to the other. We call this motion a "jaw phase" shift. The effect is to vary the strength of the magnetic field, and hence the characteristic energy of the x-ray output. This function is performed conventionally by varying the gap between the jaws. We found, however, that the effect on horizontal steering and vertical focusing is much greater when the gap is varied than when the jaws are phased. Otherwise, the APU performs in the same way as in a device where the gap is adjusted [6].

The EPU has four rows of magnet blocks, and each row may be moved independently [7]. This allows us to obtain vertical or horizontal linear polarization as well as left- and right-hand circular polarization. We calculate that we may also vary the energy of x-rays, as with the APU, by adjusting the jaw phase. After a given state of polarization is selected with row phase, changing the jaw phase allows us to change the energy of the x-rays.

We also find that, after setting the row phase, we may slide the rows in quadrants 1 and 4 with respect to those in quadrants 2 and 3 to change the x-ray energy. We call this motion a "pair phase" shift. Pair phase and jaw phase motions have the same first-order effect on the electron beam and produce identical x-ray spectra on-axis, but produce different higher-order effects on the electron beam. Both of them require small adjustments to the row phase in order to maintain the same state of elliptical polarization.

In this paper we present further calculations of the expected performance of the EPU. We discuss the effect of jaw and pair phase motions on x-ray energy, polarization, and focusing, both horizontal and vertical. All calculations are based on a 26-period device of pure NdFeB with a remanent field B_r = 1.2 T. Within a row, each 6.5 cm-long period consists of four contiguous blocks of square cross section. The EPU will be installed in the Stanford Synchrotron Radiation Laboratory (SSRL) SPEAR storage ring on the beamline 5 undulator mover. We use SPEAR storage ring parameters where the electron beam energy is 3 GeV [8]. All of the numerical calculations in this paper were based on a finite element decomposition of the magnet blocks, and radiation codes based on direct integration at each point in the trajectory [9].

2. Row Phaseand Polarization

We selected a zero of row phase for which there is only a horizontal transverse magnetic field, and for which the EPU yields vertically-polarized x-rays. Our choice of the zero of row phase places the phase values for left- and right-hand circular polarization closer to the zero than if the zero were chosen for horizontal polarization. This allows for a smaller distance, and thus a shorter time, required to slew the rows in order to switch between helicities. Figure 2 shows the variation of polarization with row phase, for two values of EPU gap.

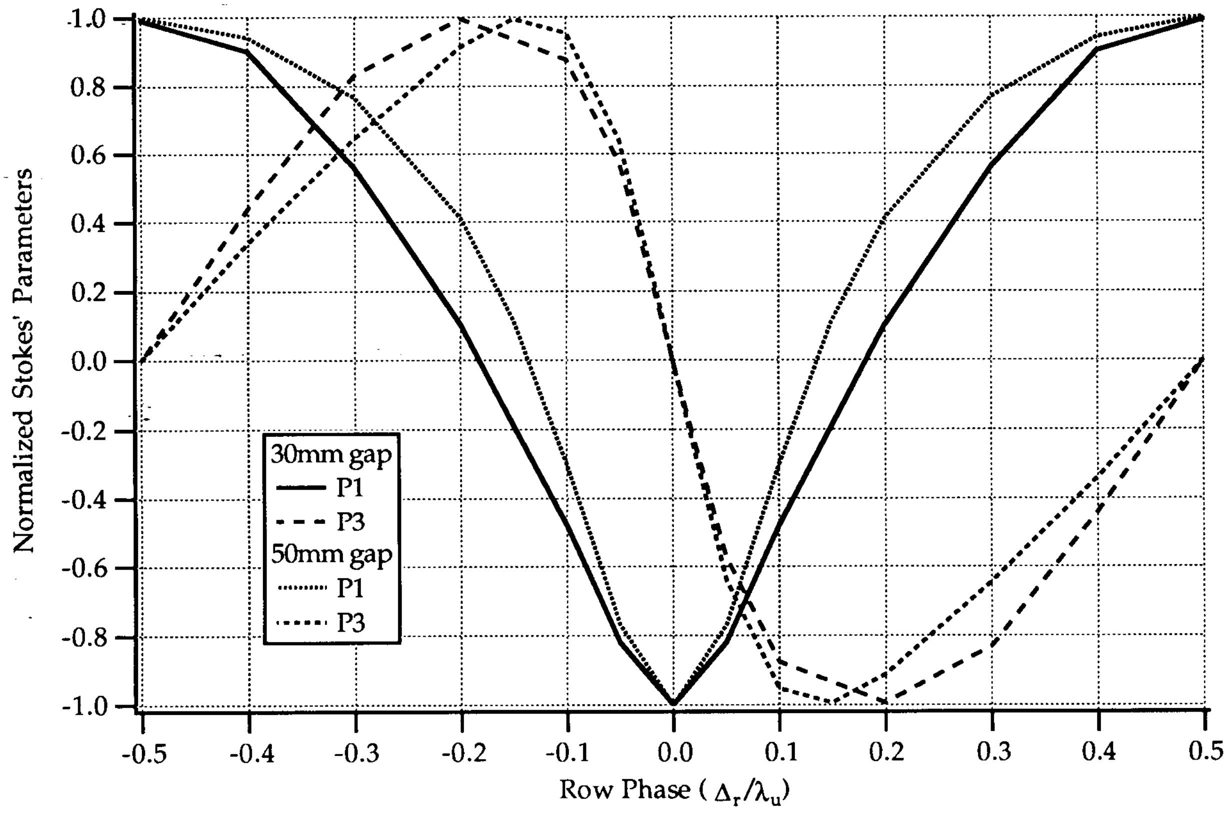


Figure 2: Linear (P1) and Circular (P3) polarization rates, derived from Stokes' parameters, at gaps of 30 mm and 50 mm, and zero jaw or pair phase. There is a slight variation with gap of the row phase required to achieve circular polarization. Circular polarization, P3 = ±1, occurs when the horizontal and vertical components of the magnetic field are equal in magnitude and 90° apart in phase.

The horizontal and vertical components of the magnetic field are always 90° apart in phase on-axis. This is an invariant condition due to the geometry of the magnetic blocks in the undulator lattice. The phase of the longitudinal magnetic field component varies in a complex manner with position off-axis. The row phase setting does not affect the relative phase between the horizontal and vertical field components, but does change their magnitudes. This allows us to obtain any ellipticity of the magnetic field, but with the major and minor axes of the ellipse oriented in the horizontal and vertical directions only. It is impossible to achieve a linear polarization direction out of these two planes.The slewing distance between left and right circularly polarizing states decreases faster as the gap is increased than as the jaw or pair phase is increased [7]. Scanning a given range of photon energy by changing phase will be faster than changing the gap.

3. Jaw and Pair Phase Effects

When the EPU's jaw or pair phase is shifted at any given value of row phase, the transverse magnetic field changes, which changes the characteristic x-ray energy. Figure 3 shows the variation of x-ray energy with jaw or pair phase for three polarization states.

A jaw or pair phase shift can sweep the entire range of photon energies available from the EPU in a sinusoidal fashion, as shown in Fig. 3. This is in contrast to a conventional gap-tunable undulator which approaches its limiting energy asymptotically as the gap is increased. However,for a given jaw or pair phase setting, the energy of the x-rays will change as one changes their polarization state. That is, for a given jaw or pair phase, vertically polarized photons have the highest energy,

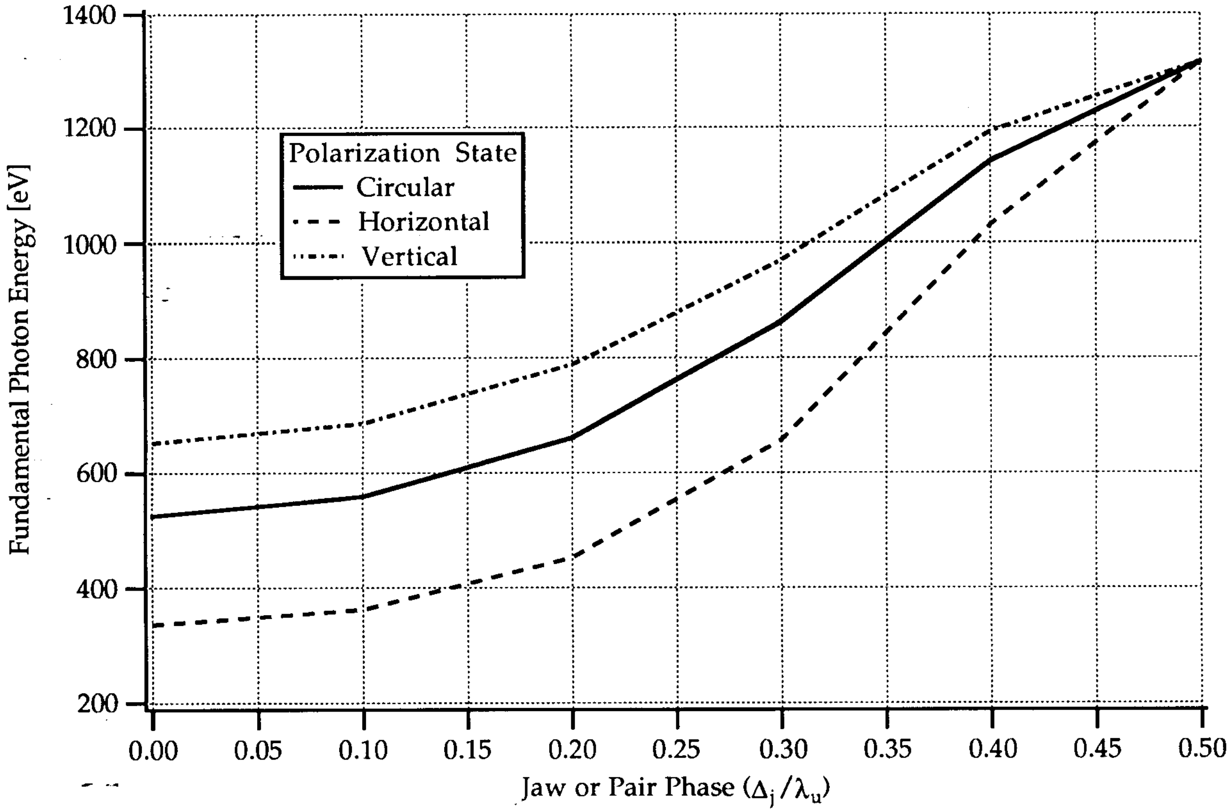


Figure 3: X-ray fundamental peak energy versus jaw or pair phase (Dj/lu) for circular, vertical, and horizontal polarization. A constant 30 mm gap was assumed.

followed by circularly polarized photons and, finally, by horizontally polarized photons with the lowest energy. This reflects the fact that a Halbach-style geometry for a pure permanent magnet, planar lattice gives the highest on-axis field strengths, and thus the lowest photon energies. From flux considerations, an upper limit on the jaw or pair phase would probably be about 0.375 ($K \approx 1/2$). However, this easily enables our EPU to scan over a range of photon energies from 550eV to 950eV with circular polarized photons of either helicity.

4. Focusing and Higher-Order Electron Beam Effects

To tune the energy of on-axis x-rays, changing either gap, or jaw or pair phase is equivalent. To take emittance into account, we consider off-axis electrons and trajectory slope errors. For the APU we showed that the focusing strength is constant with phase, while it varies with gap [4]. For the EPU, we wish to know the effect of row, jaw, and pair phase on the focusing strength.

In the general case of the EPU, the symmetries of the fields about the horizontal and vertical mid planes are broken. Symmetry can be retained for only a few cases. To analyze the focusing, it is useful to expand the field components in a power series about the origin. This analysis describes the fields within a distance of about one centimeter from the beam axis, before they begin to suffer finite width effects. Expanding the field about its on-axis value, we obtain for the general case:

$$\begin{aligned} B_x &\cong (B_{x0}+B_{x1}x+B_{x2}y+B_{x3}x^2+B_{x4}xy+B_{x5}y^2)\sin(kz+\phi_x) \\ B_y &\cong (B_{y0}+B_{y1}x+B_{y2}y+B_{y3}x^2+B_{y4}xy+B_{y5}y^2)\sin(kz+\phi_y) \\ B_z &\cong (B_{z0}+B_{z1}x+B_{z2}y+B_{z3}x^2+B_{z4}xy+B_{z5}y^2)\sin(kz+\phi_z) \end{aligned} \qquad (1)$$

where $k = 2\pi / \lambda_u$.

We average the Lorentz force on the electron beam over an integral number of periods to obtain:

$$\frac{d^2\bar{x}}{dz^2} = D_x + K_{xx}\bar{x} + K_{xy}\bar{y}, \quad \frac{d^2\bar{y}}{dz^2} = D_y + K_{yx}\bar{x} + K_{yy}\bar{y} \qquad (2)$$

$$D_{x,y} = -\mu_{x,y} B_{x,y0} B_{z0}, \quad \mu_{x,y} = \frac{c}{2k}\left(\frac{e}{\gamma m_0 c}\right)^2 \sin(\phi_z-\phi_{x,y}) \qquad (3),\ (4)$$

$$\begin{aligned} K_{xx} &= -\mu_x(B_{x0}B_{z1}+B_{x1}B_{z0}), \quad K_{xy} = -\mu_x(B_{x0}B_{z2}+B_{x2}B_{z0}), \\ K_{yx} &= -\mu_y(B_{y0}B_{z1}+B_{y1}B_{z0}), \quad K_{yy} = -\mu_y(B_{y0}B_{z2}+B_{y2}B_{z0}). \end{aligned} \qquad (5)$$

Here, $\bar{x}$ and $\bar{y}$ are the averaged horizontal and vertical positions of the electron beam in the EPU, and γ is the relativistic energy of the electrons divided by their rest energy. $D_{x,y}$ give the net deflection of the beam. The K_{ij} terms are the focusing strengths, with the linear focusing terms, K_{xx} and K_{yy}, dominating the linear coupling terms, K_{xy} and K_{yx}.

The first integral of $D_{x,y}$ along the z-axis gives a quantity proportional to the first longitudinal integrals of the transverse magnetic fields, and likewise for the second integrals of $D_{x,y}$. The variation of the first and second field integrals with row phase is shown in Figs. 4 and 5. The magnitudes of the integrals shown are well within the tolerances of 50 G cm for the first integrals, and 250 kG cm^2 for the second integrals, required for SPEAR compatibility.

When the jaw or pair phase has a finite value, the magnetic fields have slightly different transverse field profiles for opposite values of row phase. This results in slightly different averaged

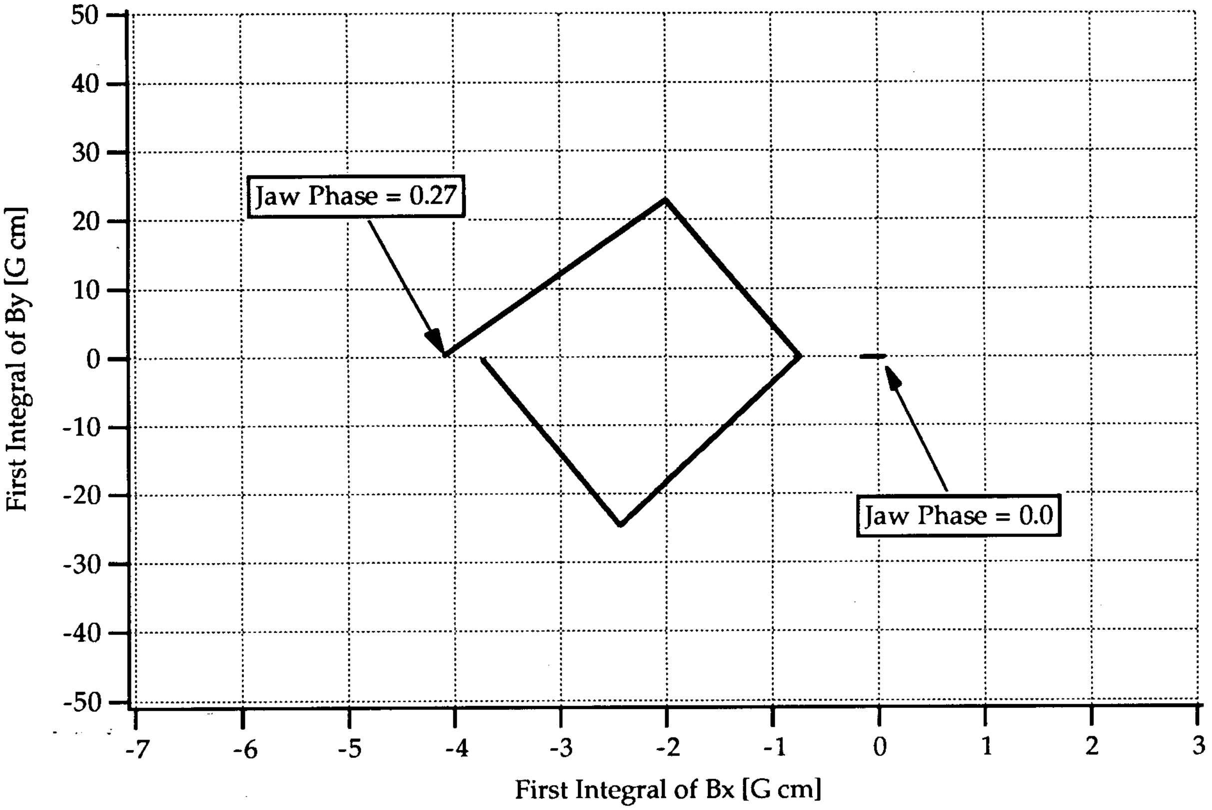
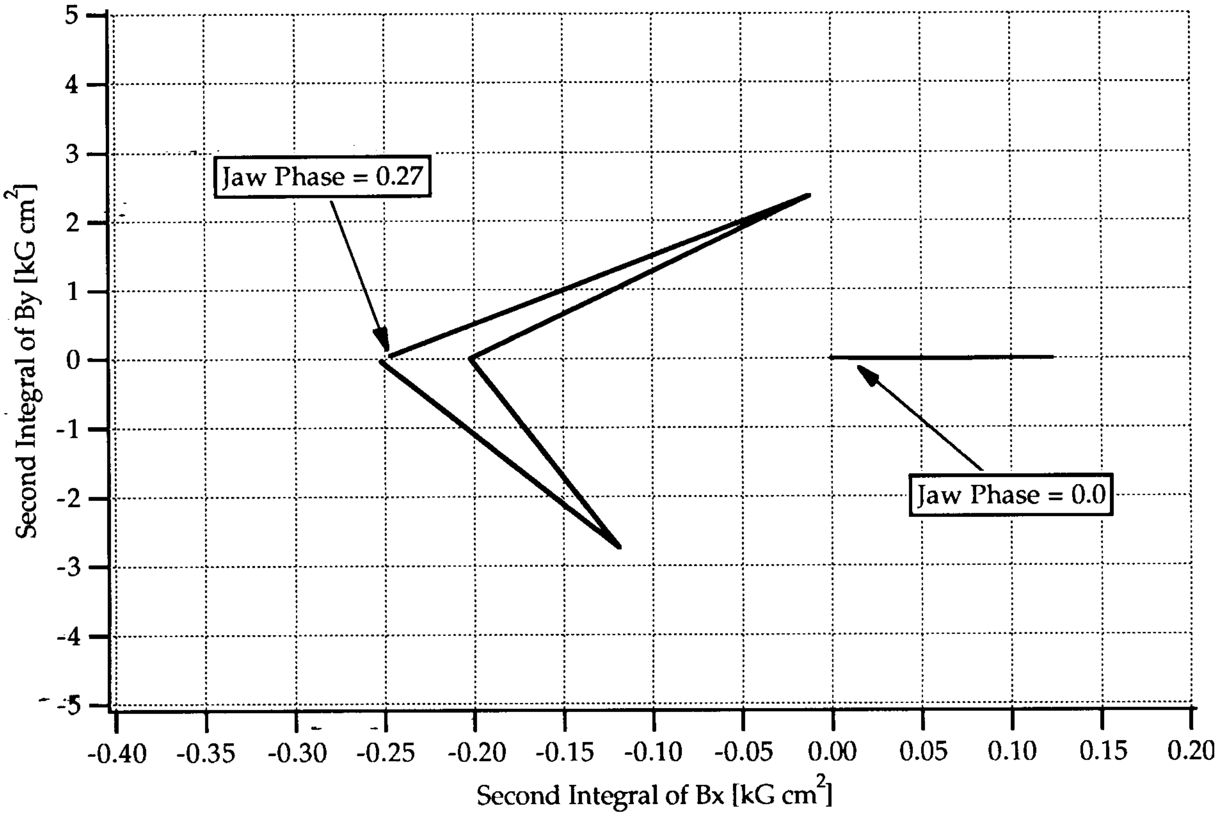


Figure 4: First transverse field integrals as the row phase is varied from -0.5 to 0.5, for jaw phase settings of 0.0 and 0.27.

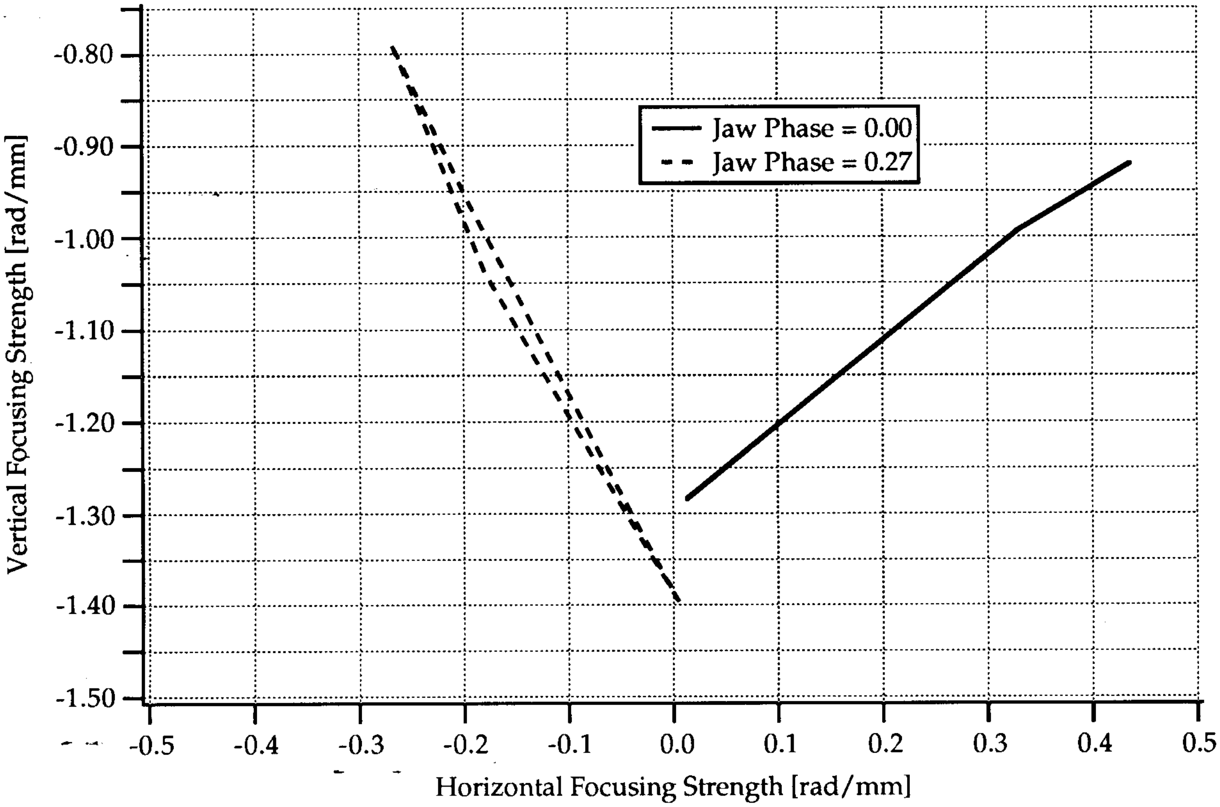


trajectories for opposite polarization states, and is reflected in the unsymmetrical plots in Figs. 4 and 5 when the jaw phase equals 0.27.

We modeled the EPU as a thin lens, and numerically calculated trajectories to determine the focusing strengths, betatron tune-shifts, and closed-orbit displacement for different gap, jaw phase, and row phase settings.

In the horizontal plane, the EPU will either focus or defocus slightly depending upon its particular settings. In the vertical plane, the EPU focuses for any setting of the gap, jaw or pair phase, and row phase. The vertical focusing strength in the case of horizontal polarization and zero jaw or pair phase is the same as for a conventional Halbach undulator [10]. Figure 6 shows the vertical and horizontal focusing strengths, at two values of jaw phase, as the row phase varies from -0.5 to +0.5. The slightly unequal focusing for opposite settings of the row phase when the jaw phase equals 0.27 again reflects the lack of symmetry of the transverse field distributions when the row phase changes sign.

Vertical and horizontal betatron tune-shifts are directly proportional to the corresponding focusing strengths. The maximum vertical tune-shift that has been calculated for the EPU has a magnitude of approximately 2.0×10^{-4}, well below the 0.001 maximum allowable tune-shift for devices on SPEAR. The vertical tune-shift remains nearly constant as we switch between positive and negative helicities. Adjusting the jaw phase from 0.0 to 0.27 while maintaining circular polarization changes the vertical tune-shift by approximately 5.0×10^{-6}, while adjusting the gap to cover a comparable range of photon energies changes the vertical tune-shift by approximately 9.0×10^{-5}.



The maximum closed orbit displacement [11] of the electron beam downstream can vary up to 6% of the horizontal beam size (1.64 mm) and 1.5% of the vertical beam size (0.07 mm) when the EPU switches helicities. This displacement is within the acceptable tolerances for the SPEAR ring; the horizontal displacement is easily remedied with trim coils.

5. Summary

The EPU can produce both left- and right-hand circular polarization, and both horizontal and vertical polarization by adjusting the row phase. The energy of the fundamental harmonic can be changed purely by adjusting the jaw or pair phase, while at a constant gap setting. This allows us to tune both the polarization state and energy of the emitted x-rays by combinations phase shifts between the four rows.

The EPU focuses in the vertical for all settings of gap or phase. It focuses or defocuses in the horizontal depending on its phase settings. The vertical tune-shifts remain stable under both row phase and jaw or pair phase.

This device may prove to be an attractive source of both circularly- and linearly-polarized photons without the added complexity of gap adjustment.

6. Acknowledgments

The authors wish to thank Brian Kincaid and Kwang-Je Kim of Lawrence Berkeley Laboratory for their generous support of this work, and Heinz-Dieter Nuhn and Helmut Wiedemann of SSRL for valuable technical discussions. This work was supported by the Materials Sciences Division of the Office of Basic Energy Sciences of the US. Department of Energy.

7. References

[1] K.-J. Kim, SPIE Proceedings 1345, 116 (1990).

[2] R. Carr, Proceedings of the 1993 IEEE Particle Accelerator Conference, Washington, D.C.

[3] S. Sasaki, K. Miyata, and T. Takada, Jpn. J. Appl. Phys. 31, L1794 (1992) .

[4] R. Carr, Nucl. Inst. & Methods A306, 391 (1991).

[5] K. Halbach, Nucl. Inst. & Methods 187, 109 (1981).

[6] R. Carr, H.-D. Nuhn, and J. Corbett, Proceedings of the 3rd European Particle Accelerator Conference (1992).

[7] R. Carr and S. Lidia, Proceedings of the 1993 SPIE Conference, San Diego, CA.

[8] H.-D. Nuhn, SSRL ACD-NOTE 118 (Revised, 1992).

[9] See, for example, J.D. Jackson, Classical Electrodynamics, Wiley, 1975, p. 675.

[10] R.P. Walker, Nucl. Inst. & Methods 214, 497 (1983).

[11] See "Synchrotron Light Source Data Book," J.B. Murphy, ed. BNL 42333 (Revised, 1990) p. 7.

[12] PATPET is a variation of PATRICIA and PETROS. See L. Emery, H. Wiedemann, and J. Safranek, SSRL ACD-NOTE 36 (Revised, 1988).