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High Energy Parton-Parton Elastic Scattering in QCD*

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ABSTRACT

We show that the high energy limit of quark-quark, or gluon-gluon, elastic scattering is calculable in terms of the BFKL pomeron when $-t \gg \Lambda^2_{QCD}$. Surprisingly, this on-shell amplitudes does not have infrared divergences in the high energy limit.

1. Introduction

The high energy parton-parton elastic scattering through an exchange of Balitsky-Fadin-Kuraev-Lipatov (BFKL)[1-4] pomeron is a very interesting process[5]. First, it is the QCD background to the Higgs hunting by looking for events where a gap is produced in the central rapidity region[6,7]. As pomeron is colorless, exchange of pomeron between partons also leads to a final state which, at the parton level , contains two jets with a rapidity gap between them[8,9]. Second, BFKL pomeron plays an important role in small x physics. It leads to the rapid rise in the parton density. At extermely small values of x the corrections to the BFKL pomeron become important and lead to the idea of partonic saturation[10] over small spatial regions in the target and beam, hot spots[11-14].

In this talk, I will study the process in the kinematic region $\hat{s} \gg -t \gg \Lambda^2_{QCD}$ with $\hat{s}$ the center of mass energy squared of the partons and t the invariant momentum transfer squared. $\hat{s}$ is taken to be large but not too large so that unitarity still holds and multiple pomeron exchange can be neglected. In order to separate the soft physics, the momentum transfer should be taken to be much larger than Λ^2_{QCD} so that the diffusion[14] in transverse momentum of the BFKL equation still remains in the perturbative region and so PQCD and thus BFKL pomeron can be applied. Therefore, a complete calculation can be done for parton-parton scattering including all normalization factors, though the normalization is not expected to be reliable until higher correction are carried out.

At first sight it is a little surprising that one can calculate high energy, on-shell, quark-quark scattering without encountering infrared divergences. Indeed, there are infrared divergences in on-shell quark-quark scattering amplitudes, however these infrared divergences disappear, at least for the color singlet exchange, where the high energy limit is taken. This is a general phenomenon in that solutions to the BFKL equation are less infrared singular, by a full power of transverse momentum, than would be guessed from pertubative theory. This follows form the fact

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that the anomalous dimension $\gamma_n = 1/2$ when $n = \alpha_P$, where $\alpha_P = 1 + 4\ln 2 C_A \alpha(-t)/\pi$ is the trajectory of the pomeron, while $\gamma_n = 0$ in free field theory.

2. BFKL Pomeron

We first consider the elastic scattering of two colorless particles through an exchange of BFKL pomeron. The amplitude of this process can be written as

$$A(s,t) = i|s| \int \frac{d\omega}{2\pi i} (\frac{s}{-t})^{\omega} f_\omega(q_\perp^2), \tag{1}$$

with $q_\perp^2 = -t$ and

$$f_\omega(q_\perp^2) = 2 \int \frac{d^2k_\perp d^2k'_\perp}{(2\pi)^2} \Phi_1(k_\perp, q_\perp) \Phi_2(k'_\perp, q_\perp) f_\omega(k_\perp, k'_\perp, q_\perp), \tag{2}$$

where Φ_i are the impact factors of the colorless objects. By color current conservation, one can obtain

$$\begin{aligned} \Phi_1(k_\perp, q_\perp)\, |_{k_\perp = 0} &= \Phi_1(k_\perp, q_\perp)\, |_{k_\perp - q_\perp = 0} \\ &= 0, \end{aligned} \tag{3}$$

and similar equations hold for Φ_2. The partial wave amplitude $f_\omega(k_\perp, k'_\perp, q_\perp)$, which represents the propagation of a BFKL pomeron in transverse momentum plane, in the leading logarithmic approximation, satisfies the following integral equation

$$\begin{aligned} \omega k_\perp^2 (q_\perp - k_\perp)^2 f_\omega(k_\perp, k'_\perp, q_\perp) &= \delta^2(k_\perp - k'_\perp) \\ &+ \frac{C_A \alpha}{2\pi^2} \int d^2k_{1\perp} \mathcal{K}(k_{1\perp}, k_\perp, k'_\perp, q_\perp) f_\omega(k_{1\perp}, k'_\perp, q_\perp). \end{aligned} \tag{4}$$

The delta function reproduces the lowest order diagram and the kernel $\mathcal{K}$ is conformal invariant in the impact parameter representation. However, the source term (delta function) is not invariant under conformal transformation. Lipatov[3,4] modifies the source term to make it conformal invariant so that the integral equation can be solved by the eigenfunctions of the conformal group. The modification is at $k_\perp = k_\perp - q_\perp = 0$ which does not change the ampltiudes of colorless objects in view of equation (3). The kernel is infared safe but the source term is infared divergent, as expected, before the modification. Lipatov's modification renders the source term infared safe. Thus, we should expect the solution to be free of infared divergences. According to Lipatov[3,4], the solutions are

$$\begin{aligned} f_\omega(k_\perp, k'_\perp, q_\perp) &= \frac{1}{(2\pi)^4} \int d^2\rho_\perp d^2\rho'_\perp \exp[i(k_\perp - \frac{1}{2}q_\perp)\cdot\rho_\perp - i(k'_\perp - \frac{1}{2}q_\perp)\cdot\rho'_\perp] \\ & \hat{f}_\omega^{q_\perp}(\rho_\perp, \rho'_\perp), \end{aligned} \tag{5}$$

with

$$\hat{f}_\omega^{q_\perp}(\rho_\perp, \rho'_\perp) = \frac{|\rho_\perp \rho'_\perp|}{16} \sum_{n=-\infty}^{\infty} \int d\nu \frac{E_{q_\perp}^{n,\nu *}(\rho'_\perp) E_{q_\perp}^{n,\nu}(\rho_\perp)}{[\nu^2 + (\frac{n-1}{2})^2][\nu^2 + (\frac{n+1}{2})^2][\omega - \omega(\nu, n)]}, \tag{6}$$

where $E^{n,\nu}_{q\perp}(\rho_\perp)$ are the conformal eigenfunctions and $\omega(\nu,n)$, the eigenvalues, are the pomeron trajectories. The leading trajectory corresponds to $n=0$.

$$
\begin{aligned}
E^{n,\nu}_{q\perp}(\rho_\perp) &= \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^{\frac{1-n}{2}+i\nu}\left(\frac{\rho^*_{12}}{\rho^*_{10}\rho^*_{20}}\right)^{\frac{1+n}{2}+i\nu} \\
\omega(\nu,n) &= \frac{2C_A\alpha}{\pi}Re[\psi(1)-\psi\left(\frac{1+\mid n\mid}{2}+i\nu\right)],
\end{aligned}
\tag{7}
$$

with ψ the standard logarithmic derivative of the Gamma function.

3. Parton-parton elastic scattering

For explicitness, we consider elastic quark-quark scattering with pomeron exchange in the t channel. The formula for the cross section is

$$
\frac{d\hat{\sigma}}{dt} = \left(\frac{\alpha C_F}{\pi}\right)^4 \frac{\pi^3}{(N_c^2-1)^2}\left|\int d^2k_\perp d^2k'_\perp f^{q_\perp}(k_\perp,k'_\perp,y)\right|^2, \tag{8}
$$

where $y=\ln(\hat{s}/-t)$ is the rapidity interval between the partons. $f^{q_\perp}(k_\perp,k'_\perp,y)$ is related to $f_\omega(k_\perp,k'_\perp,q_\perp)$ by a Mellin transformation:

$$
f^{q_\perp}(k_\perp,k'_\perp,y) = \int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\exp[\omega y]f_\omega(k_\perp,k'_\perp,q_\perp). \tag{9}
$$

In the lowest order,

$$
f^{q_\perp}(k_\perp,k'_\perp,y) = \frac{\delta^2(k_\perp-k'_\perp)}{k_\perp^2(q_\perp-k_\perp)^2}, \tag{10}
$$

which is the source term in equation (4). As shown in the previous section, Lipatov does not actually evaluate the Feynman diagrams but rather a conformally invariant scattering amplitude equivalent to the Feynman graphs for colorless external particles. Since our external particles are not colorless we must modify the expressions (5) and (6). We shall show how to carry out that modification after examining the Lipatov's result. According to equations (5) and (9),

$$
\int d^2k_\perp d^2k'_\perp f^{q_\perp}(k_\perp,k'_\perp,y) = \int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\exp[\omega y]\hat{f}^{q_\perp}_\omega(0,0) \tag{11}
$$

As $E^{n,\nu}_{q\perp}(\rho_\perp)$ is finite when $\rho_\perp\to 0$, by equation (6), $\hat{f}^{q_\perp}_\omega(0,0)=0$. Thus it leads to the conclusion that $d\hat{\sigma}/dt=0$, however, as mentioned earlier $f^{q_\perp}$ does not actually refer to any set of Feynman diagrams. Our task now is to determine exactly what amplitude does give the leading logarithms for Feynman diagrams.

Taking the leading trajectory, we may rewrite

$$
f^{q_\perp}(k_\perp,k'_\perp,y) = \frac{1}{64}\int d\nu\frac{\nu^2\exp[\omega(\nu)y]}{\pi^6(\nu^2+\frac{1}{4})^2}I^*_\nu(k'_\perp,q_\perp)I_\nu(k_\perp,q_\perp), \tag{12}
$$

with

$$
I_\nu(k_\perp,q_\perp) = \int d^2\rho_\perp d^2\rho'_\perp\left(\frac{(\rho_\perp-\rho'_\perp)^2}{\rho_\perp^2\rho'^2_\perp}\right)^{\frac{1}{2}+i\nu}\exp[ik_\perp\rho_\perp+i(k_\perp-q_\perp)\rho'_\perp], \tag{13}
$$

and $\omega(\nu) = \omega(\nu, 0)$. Clearly, $I_\nu(k_\perp, q_\perp)$ has $\delta^2(k_\perp)$ and $\delta^2(k_\perp - q_\perp)$ terms as given by (13). But $\delta^2(k_\perp)$ terms do not arise from Feynman graphs. By assumming that, (1) Feynman amplitude is an analytic function of $k_\perp$ and $k_\perp - q_\perp$, (2) Lipatov's solution can be applied to color object when $k_\perp \neq 0$ and $k_\perp - q_\perp \neq 0$, one need to remove the delta functions in our application. If we define $I'_\nu(k_\perp, q_\perp)$ as in (13) but with the replacement

$$\left(\frac{(\rho_\perp - \rho'_\perp)^2}{\rho_\perp^2 \rho_\perp'^2}\right)^{\frac{1}{2}+i\nu}$$
$$\rightarrow \left(\frac{(\rho_\perp - \rho'_\perp)^2}{\rho_\perp^2 \rho_\perp'^2}\right)^{\frac{1}{2}+i\nu} - \left(\frac{1}{\rho_\perp^2}\right)^{\frac{1}{2}+i\nu} - \left(\frac{1}{\rho_\perp'^2}\right)^{\frac{1}{2}+i\nu}, \tag{14}$$

then $I'_\nu(k_\perp, q_\perp)$ is analytic in $k_\perp^2$ and $(k_\perp - q_\perp)^2$. Thus, the f' corresponding to Feynman diagrams is as in (12) but I_ν replaced by I'_ν. It is now straightforward to show that

$$\int d^2k_\perp d^2k'_\perp f'^{q_\perp}(k_\perp, k'_\perp, y) = \frac{4}{q_\perp^2} \int d\nu \frac{\nu^2}{(\nu^2 + \frac{1}{4})^2} \exp[\omega(\nu) y], \tag{15}$$

which leads to

$$\int d^2k_\perp d^2k'_\perp f'^{q_\perp}(k_\perp, k'_\perp, y) = \frac{(2\pi)^2}{q_\perp^2} \frac{\exp[(\alpha_P - 1)y]}{[\frac{7}{2}\alpha C_A \zeta(3) y]^{3/2}} \tag{16}$$

with $\alpha = \alpha(q_\perp^2)$ and $\alpha_P = 1 + 4\ln 2 C_A \alpha/\pi$. Using f' rather than f in (8) gives

$$\frac{d\hat{\sigma}}{dt} = (\alpha C_F)^4 \frac{\pi^3}{4t^2} \frac{\exp[2(\alpha_P - 1)y]}{[\frac{7}{2}\alpha C_A \zeta(3) y]^3}, \tag{17}$$

as our final formula for the elastic quark-quark scattering at high energy for color singlet quantum numbers in the t channel.

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