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OPERATIONALISM REVISITED: Measurement Accuracy, Scale Invariance and the Combinatorial Hierarchy*

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Abstract

It is claimed here that by 1936 Bridgman had developed severe criticisms of orthodox quantum mechanics from the point of view of his operational philosophy. We try to meet these criticisms by a radical analysis of the measurement of the finite and discrete length and time intervals between *particulate events.* We show that a scale invariant *counter paradigm* based on two arbitrary finite and discrete measurement intervals $\Delta\ell$, Δt offers an alternative starting point for constructing relativistic particle mechanics. Using the scale invariant definitions

$$\Delta\ell/c\Delta t \equiv 1;\ \Delta\ell^2/\kappa\Delta t = 2\pi$$

for the Einstein limiting velocity c and for "Kepler's constant" κ— proportional to the area per unit time swept out by a particle moving with constant velocity past a center — we derive the finite and discrete Lorentz invariant bracket expression $[x_i, \dot{x}_j] = \kappa\delta_{ij}$ for rectangular coordinates and velocities in three dimensions. Defining field per unit charge as the force per unit mass acting on a test particle with this constraint, we find that these fields satisfy the free field Maxwell equations provided only charge per unit mass for this test particle is a Lorentz invariant, generalizing the Feynman-Dyson-Tanimura proof. This scale invariant theory for classical relativistic fields is broken by any measurement of length with measurement accuracy $\Delta\ell < \hbar/2m_ec$ because electron-positron pairs are produced with finite probability, violating the single test particle postulate. This allows us to recover our new fundamental theory based on the *combinatorial hierarchy* and *bit-string discrimination.* Recent mass ratios, coupling constants and cosmological parameters obtained by bit-string dynamics are quoted.

...the ultimately important thing about any theory is what it actually does, not what it says it does or what its author thinks it does, for these are often very different things indeed.

— P.W.Bridgman, *The Nature of Physical Theory*

1. INTRODUCTION

This paper is dedicated to the hope that we now know enough to start recasting modern elementary particle physics and physical cosmology in a form that would come closer to Bridgman's vision of physics. Although Bridgman's operational philosophy[1] contributed a great deal to the early discussions of the meaning of relativity and quantum theories, the "orthodox" Copenhagen interpretation of quantum mechanics as evolved by Bohr departs widely from what — in my opinion — Bridgman had in mind. He wished to base physical concepts on laboratory operations, actually carried out, or as a minimum on paradigms which are consistent as thought experiments suggested by actual laboratory practice. In later reflections he was prepared to extend his "operationalism" to sufficiently clear mathematical operations[2]; I suspect that the term "computable" would have found favor in his eyes.

In his earlier work (LMP) Bridgman criticized the concept of "light traveling". In NPT he grants that operational meaning might eventually be given to the concept of "light traveling" by measuring the scattering of light by light. Presumably he had in mind Furry's successful calculation of the finite cross section for this scattering process using the then available formulation of quantum electrodynamics (QED). But this calculation produces a finite result in QED only because of specific symmetries. Most processes predicted by the second quantized relativistic field theories are infinite, and therefore *not* related to experiment in any obvious way. It would be hard to give an operational gloss to such theories even after they are "renormalized", or restricted to the class of "non-Abelian gauge theories". I doubt that Bridgman would have found any of the theories currently used in high energy particle physics palatable. He also dismissed cosmology as operationally meaningless, a judgment that the rich body of data now available to the observational astronomer could have led him to revise.

We believe it is proving fruitful to revive Bridgman's dream using a finite and discrete reconstruction of relativistic quantum mechanics and physical cosmology. This new fundamental theory has taken shape gradually. The most easily identified starting point is the work by Bastin and Kilmister on the "Concept of Order".[3–9] The research program had its first dramatic success in the discovery of the *combinatorial hierarchy* by A.F.Parker-Rhodes in 1961.[10] An adequate, if somewhat unsystematic, survey of subsequent developments is available in the proceedings of some of the annual meetings of the Alternative Natural Philosophy Association (founded in 1979)[11] and of its western chapter.[12] We will not review this history here. Two up-to-date and systematic presentations of the theory from very different perspectives are in preparation.[13,14] The specific approach followed in this paper is adequately introduced by a short description[15] and three more technical papers.[16–18] Some additional recent work will be referenced below.

2. EXAMPLES of BRIDGMAN'S APPROACH

As an experimentalist, and a critic of the practice of theoretical physics, Bridgman had a healthy distrust of mathematics:

"[Mathematics] begins by being a most useful servant when dealing with phenomena of the ordinary scale of magnitude, but ends by dragging us by the scruff of the neck willy nilly into the inside of an electron where it forces us to repeat meaningless gibberish." (LMP, p. 149)

It is claimed in FDP[16] that McGoveran's *ordering operator calculus* provides a mathematics which could meet Bridgman's challenge. In fact, any formalism restricted to computability in McGoveran's sense might be utilized to meet that goal. For our purposes we adopt the following definition:

A physicist who claims that a problem is **computable** *must — if challenged — be*

able to produce an integer answer to that problem within a year using the computational facilities and research budget available to him.

If we could succeed in getting this requirement adopted in the discussions of theoretical physics, it could eliminate a lot of fruitless argument. Bridgman tried to meet basically the same difficulty by pleading with physicists to stop discussing "meaningless problems". His failure leaves me little hope that the current generation of physicists — or the next — will prove to be that rational. Of course he did not intend, nor do I, to hobble *speculation* once it is properly identified as such.

Initially, Bridgman hoped that his criticism had taken root in the "new" quantum mechanics:

"This section was written early in 1926 without access to the recent literature. Our attitude toward quantum phenomena has been so much changed since then by the "new" quantum mechanics, that a number of the following statements are superseded as a statement of present opinion. However it has seemed worth while to let the section stand as written, because many of the developments actually taken in the new mechanics follow the lines that it is here urged they ought to take, and so far afford interesting confirmation of the point of view of this essay." (LMP, p. 186, footnote 1)

But he was not satisfied with the shape that quantum mechanics actually took.

"I think there is significance in the difficulty which my theoretical friends find in suggesting what sort of apparatus they would set up in the laboratory in order to answer such questions as: 'Can e, or m, or h be measured separately with unlimited precision by a single experiment, or may they be measured simultaneously in a single experiment?' Or what is the apparatus in terms of which any arbitrary 'observable' of Dirac acquires its physical meaning? I think it will be granted by most theoretical physicists that there are situations of this sort which have not yet been thought completely through. Since we are now prepared to admit that the correspondence between mathematics and experience is never a one to one correspondence, so that because a mathematical theory accomplishes successfully one-half of what we would like to have it [accomplish] there is no certainty or even a high probability that it will accomplish the other half, I think we are justified in a certain amount of disquietude in the face of any situation that has not been thought through completely.

"The mere fact that such a debate is possible as that carried out on the one hand by Einstein, Podolsky and Rosen[2], [[2]A.Einstein, B.Podolsky and N. Rosen, *Phys. Rev.*, 47,777,1935.] and on the other hand by Bohr,[3] [[3]N.Bohr, *Phys. Rev.*,48,696,1935.] increases our disquietude...." (NPT, p.118)

With regard to this specific difficulty in orthodox quantum mechanics, we can claim that our computational point of view has much to offer. In any finite and discrete theory, there is necessarily a limiting velocity for the transfer of information, and also a finite and discrete meaning for the "Lorentz transformations" (FDP, ch. 4 pp 48-54.). At the same time, this implies "supraluminal" correlations *without* supraluminal signaling.[19,20] We have also claimed that the new approach goes far toward resolving the quantum mechanical "measurement problem",[21,22] a claim which we hope is strengthened by the preliminary abstract provided as the appendix to this paper. The basic fact which make this resolution possible is that at a deep level, which is explored further in this paper, we claim to have achieved a successful reconciliation of quantum mechanics with relativity.[23]

We hope to give further evidence for the usefulness of criticizing the current practice of theoretical physics by presenting this paper. When we speak of "criticism", we follow Bridgman's usage:

"The material for the physicist as critic is the body of physical theory, just as the material of the physicist as theorist is the body of experimental knowledge." (NPT, p.2)

In contrast to the open ended and rapidly expanding task of the theorist, he saw the task of the physicist as critic as one that could be accomplished, simply because it depended on the finite mind of the critic:

"In so far as we may assume that the human mind has approximately fixed and definite properties and is not in such a rapid state of evolution that it runs away with us during the discussion, we are not here confronted with unlimited possibilities of complexity, but the field is an essentially closed one." (NPT, p.2)

Unless or until we encounter extraterrestrial species, or the claims of strong AI are convincingly demonstrated in practice, I have no quarrel with this statement. However, I find his optimistic conclusion:

"...having acquired this amount of understanding we may then pass on, leaving criticism behind us as a well rounded and more or less definite discipline." (NPT, p.3) less convincing, simply because the task has to be undertaken again in our generation.

As already noted, the task itself has become possible because of the work of a large number of people, starting with the critique of Eddington's fundamental theory in the 50's by Bastin and Kilmister. I do not discuss here — let alone evaluate the significance of — the subsequent contributions by Fredrick Parker-Rhodes, Irving Stein, Michael Manthey, and David McGoveran all of which were vital to the process which led to the current state of our new fundamental theory. Instead, I will concentrate on a recent development that — in my opinion — can revolutionize our way of thinking about the connection between relativistic quantum mechanics and classical relativistic field theory.

3. MEASUREMENT ACCURACY and SCALE INVARIANT BRACKET EXPRESSIONS

3.1 The Counter Paradigm and the definition of "c"

In McGoveran's fundamental approach to finite and discrete theories based on the *ordering operator calculus* and *attribute distance* one is restricted to *derivates* (i.e. finite and discrete differences) and never encounters the limiting processes or the derivatives used in continuum mathematics. The exponentiation of the derivate operator allows him to construct a generalized commutation relation which can be given a discrete geometrical interpretation (FDP, Sec. 4.2, pp 63-69). This construction provides the bracket expressions needed to derive classical relativistic field theory from measurement accuracy, a derivation we present in the next chapter. Rather than follow McGoveran's rigorous discussion we provide a somewhat heuristic argument in the spirit of Bridgman.

We start our more limited discussion from the measurement of finite and discrete length and time intervals using methods whose accuracy is specified by some relevant technology. In the past we sometimes started our discussion of the new fundamental theory from what we called the *counter paradigm*, We had in mind the devices used in elementary physics which record whether or not some discrete "event" takes place within a finite and discrete spacial volume during a finite time interval. When tied to finite and discrete laboratory coordinates and a finite and discrete laboratory clock, we can then introduce the coupled concepts of *particle* and *event* by the informal descriptions:

A **particle** *is a conceptual carrier of conserved quantum numbers between* **events.**

An **event** *is a region which particles carring conserved quantum numbers enter and leave during a finite time interval. These quantum numbers can be iether positive or negative. The algebraic sum of the entering quantum numbers of a given type is equal to the algebraic sum of the quantum numbers of the same type which leave the region. In that sense quantum numbers are conserved across an event. However the number of particles need not be conserved across an event.*

Here the undefined term "region" can be given some content by requiring that its effective volume can be specified by three independent lengths whose magnitudes are three independent integers times a common finite and fixed "shortest length" $\Delta\ell$. The event occurs during some well specified time interval which is again an integer times a "shortest time" Δt. We postulate standard laboratory protocol for measuring these finite length and time intervals. The event itself is a NO-YES event, i.e. "NO" if it does not happen in the specified space-time volume, and "YES" if it does. The event is made manifest by the non-firing or firing of a recording counter activating some type of discrete "memory". In general it will take several such counter firings, and a fair amount of theory, to give precision to the concept of "conservation laws". We will refine earlier attempts to do so in the context of bit-string dynamics on another occasion.

What we have sometimes called the "counter paradigm" is two such YES events separated by a spacial interval L and a time interval T neither of which can be known to better accuracy that $\Delta\ell$ and Δt respectively. In high energy particle physics, this would be called a "counter telescope". It takes the physical realization of at least four such counter telescopes and considerable calibration and experimentation to start giving precision to the "energy-momentum" conservation laws. Other particulate conservation laws can be constructed from and tested using data so measured.[24]

Consider a counter telescope consistent with this definition, and a variety of "particle sources" with adjustable parameters more or less under our control. We pick those situations which consist of two sequential counts in the two detectors and which provide an approximately constant value for the measured ratio L/T. We find that all sources of "particles" so far explored in this way give a value which is less than or indistinguishable from a universal constant called "c", unique for any fixed units of length and time measurement. This fact has led the appropriate international committee to fix this ratio — by *definition* the velocity standard for SI units — as the integer

$$c \equiv 299\ 792\ 458\ m\ sec^{-1} \tag{3.1}$$

If there were *any* demonstrable situations in which this definition of c as a *dimensional standard* led to contradictions between the way "c" is used in theoretical formulae and the way these formulae are used to interpret experimental results, the committee would not have introduced this *integer definition* of c. For historical reasons this conventional constant is still sometimes called "the speed of light" despite the fact that is now simply a definitional dimensional standard. A better phrase from a modern point of view would be "the limiting speed for the transfer of information". Of course "supraluminal velocities" that describe correlations which *cannot* be used to transfer information — such as phase velocities — have a well defined meaning in both classical and quantum physics. In a finite and discrete theory they are in no way paradoxical, as we noted in Chapter 2.

Since we do not wish to specify our units of measurement in advance, we will build this fact into our approach by relating our minimum measurable distance and time intervals by the dimensionless constraint

$$\frac{\Delta \ell}{c\Delta t} \equiv 1 \tag{3.2}$$

3.2 FINITE AND DISCRETE LORENTZ BOOSTS

In a theory that depends on finite measurement accuracy, the measurement of the velocity of a particle — as in the counter paradigm — requires both the measurement of a finite space interval and the measurement of an ordered, finite time interval together with a discussion of the accuracy to which the ratio of these two intervals is determined by these separate measurements. We consider two YES events such as the sequential firing of the two counters in a counter telescope of length L with a time interval T, defining a velocity which we attribute to a particle. We call this a *quasi-local* measurement.

In this section we will use the two events at the ends of a counter telescope of length L with time interval T as a paradigm for the discrete measurement of the quasi-local coordinates x, t of a particle. In the next section we will "embed" this counter telescope measurement in a larger finite and discrete space-time in a way that allows us to represent finite and discrete rotations consistently with finite and discrete Lorentz boosts. In order to make that extension, it is convenient to pick some small reference region in the laboratory, which also has a standard clock associated with it and make the relation to the laboratory measurements explicit.

We specify the two spacial positions for the two events relative to this "origin" by $x_1 = n_1\Delta\ell$, $x_2 = n_2\Delta\ell$ where we assume in this example that the event at x_2 *follows* the event at x_1. Then, clearly $L = |n_2 - n_1|\Delta\ell$. Since we have already introduced the limiting velocity for information transfer, the Einstein synchronization convention allows us to specify the times at which the two events occur, according to a clock located at the "origin", as $t_1 = n_1\Delta\ell/c$ and $t_2 = n_2\Delta\ell/c$. Clearly, $T = (n_1 + n_2)\Delta\ell/c$. Note that $t_1 + t_2$ is also the time it would take a light signal emitted in coincidence with event 1 in the direction of the "origin" and reflected there to arrive at position x_2 in coincidence with the second event. We have, in effect, assumed that $t_2 > t_1$ and hence that $n_2 > n_1$. Note that if we interpret these two events as "caused by a single particle", its velocity as measured by the counter telescope is

$$v_{12} \equiv \beta_{12} c = \frac{n_2 - n_1}{n_1 + n_2} c \tag{3.3}$$

Note further that the square of the invariant interval I between the two events isgiven by

$$I^2 \Delta\ell^2 = c^2(t_1 + t_2)^2 - (x_1 - x_2)^2 = 4 n_1 n_2 \Delta\ell^2 \tag{3.4}$$

One point to note here is that the maximum accuracy to which we can know the two distances is fixed by requiring n_1 and n_2 to be *positive definite integers.* If there were some way we could (even by indirect or statistical means) produce a set of reliable non-integral data for any such situation, this would violate the hypothesis that $\Delta\ell$ is the *shortest* interval to which we can give well defined experimental meaning. In other words our modern operational hypothesis about length and time measurements is that *THERE IS ALWAYS A SYSTEM OF UNITS IN TERMS OF WHICH LENGTHS AND TIMES ARE INTEGERS.*

Our means of relating the counter telescope — our paradigm for an "x, t" measurement — to the laboratory coordinate system makes use of the exchange of light signals in such a way that the standard Einstein clock synchronization convention can be used in our integer environment. This allows us to separate the problem of "quasi-local" Lorentz invariance from "event horizon limited" or "coherence limited" Lorentz invariance by the following construction. First note that our velocity $v_{12} = (n_2 - n_1)c/(n_1 + n_2)$ is invariant under the transformation $n_i \rightarrow N_T k_i$ with N_T a positive definite integer *if* k_1 and k_2 are positive, definite integers. Clearly this downward scale transformation is allowed only if n_1, n_2 have N_T as a common factor. For any particular empirical situation, measurement accuracy will limit such downward scale transformations by the quasi-local velocity resolution we can achieve. We call this Δv which, as a function of (k_1+k_2) is given by $c/(k_1+k_2)$, and the resulting limitation "scale invariance bounded from below".

For example, in high energy particle physics, we measure the momentum $p = (k_2 - k_1)\Delta mc/4k_1k_2$ of a charged track by its radius of curvature in a magnetic field and its energy $E = (k_1+k_2)\Delta mc^2/4k_1k_2$ calorimetrically defining the velocity as $v = pc/E = (k_2 - k_1)/(k_1 + k_2)$, in situations where we cannot measure the distance between two counters $(n_2-n_1)\Delta\ell$ to better than many orders of magnitude (i.e. $(n_2 - n_1) = L/\Delta\ell \approx N_L \pm \Delta N_L$, $\Delta N_L >> 1$). We have discussed on other occasions how the periodicity N_T implied by our finite and discrete measurement accuracy paradigm can lead to observable interference phenomena which *break* scale invariance. With this understood, we see that for fixed velocity resolution, (i.e., $k_1 + k_2 = K_v \in$ *positive definite integer*) we also can define a maximum coherence length $L_C = N_T^C K_v \Delta\ell$, with N_T^C the maximum number of coherent velocity periods we can show to interfere. In a "wave theory" this would be called a "coherent wave pulse".

In a finite and discrete theory, straight line motion cannot continue forever. As just discussed, we can bound it by defining the coherence length in terms of the velocity resolution K_v and the number of periods at that velocity resolution $N_{T_v}^C$ as $L_C = N_{T_v}K_v$. When we reach this limit, we can continue our coherent discussion to larger spaces by the trick of using periodic boundary conditions, or by breaking our trajectory into straight line segments and bending it around to form a closed "orbit" of length $2\pi L_C$. Taking this to be the perimeter of an orbit with discrete rotational symmetry will be discussed in the next section. The extension to elliptical orbits is straightforward (See Ref. 22).

We now return to our quasi-local position-time measurement by saying that the counter telescope measures a position $x = (k_2-k_1)\Delta\ell$, a time $t = (k_1+k_2)\Delta\ell/c$ and hence a velocity $\dot{x} = (k_2 - k_1)c/(k_2 + k_1)$. Clearly this places $x = 0$ half-way between the two counters in the counter telescope and $t = 0$ at a time $\frac{1}{2}(k_1+k_2)\Delta t$ after the firing of counter 1. This language implies that in a much larger context we can measure both x_1 and x_2 to an accuracy $\Delta\ell$, or in other words measure the discrete *phase* of this combined pair of events *within* the $x_2 - x_1$ length of the telescope. But this still does *not* allow us to assign an absolute meaning to both x and $\dot{x}$. Where ever we place x within this interval, we still cannot know its value to better than $\Delta\ell$. The best we can do is to assign it to some position which is ambiguous between $x_- \in [x_1, x_1+\Delta\ell, ...x_2-\Delta\ell]$ and $x_+ = x_- + \Delta\ell$. Between these two locations the velocity is, as measured locally, $+c$. Thus, using only quasi-local information, the product "$x\dot{x}$" is ambiguous depending on whether we use x_- or x_+. Defining the difference as the bracket expression, and using c for $\dot{x}$, we have that, for local measurements,

$$[x, \dot{x}] \equiv x_+\dot{x}(t) - x_-\dot{x}(t) = c\Delta\ell = -[\dot{x}, x] \tag{3.5}$$

It might be thought that by using a longer lever arm to improve our velocity resolution, we could improve on this limit. As we discuss in the next section, using periodic phenomena does allow us to reduce this constant to $\kappa = c\Delta\ell/2\pi$, but no further.

It remains to show that our definition of position and velocity in the larger space allows us to define finite and discrete Lorentz transformations in the single direction "$\vec{x}$" we have so far considered. For rational fraction velocities, this amounts to showing that transformation from $v = (k_2 - k_1)c/(k_2 + k_1)$ to $v' = (k_2' - k_1')c/(k_2' + k_1')$ can be obtained from the usual velocity addition law

$$v' = \frac{v + v''}{1 + vv''/c^2} \tag{3.6}$$

with some rational fraction velocity $v'' = (k_2'' - k_1'')c/(k_1'' + k_2'')$. It is trivial to show[25] that, in the case $k_2' = k_2$,

$$k_1'' = k_1'; \; k_2'' = k_1 \tag{3.7}$$

satisfies this requirement. The problem of showing that any Lorentz transformation needed in an experimental context can always be constructed in this way *without* producing contradiction with experiment will be discussed in detail elsewhere (eg Ref. 12).

3.3 FINITE AND DISCRETE LORENTZ ROTATIONS

In order to extend our analysis from one to two spacial dimensions, it suffices initially to consider three counter firings F_1, F_2, F_3 at three fixed locations which form a triangle. [23] We assume that the three counters are at rest in the laboratory frame, and that the laboratory clock (using the usual Einstein synchronization convention) records the firings in the order 1,2,3. In order to insure that the three distances $s_{ij}\Delta\ell$ satisfy the triangle inequalities $|s_{ij} - s_{jk}| \leq s_{ki} \leq s_{ij} + s_{jk}$ it suffices to pick three positive definite times $t_i\Delta\ell/c$ with all three t_i *integers.* We then define the sums $s_{ij}\Delta\ell \equiv (t_i + t_j)\Delta\ell/c$ with the consequence that

$$|s_{ij} - s_{jk}| = |t_i - t_k| \leq s_{ki} = t_i + t_k \leq s_{ij} + s_{jk} = t_i + t_k + 2t_j \tag{3.8}$$

We now take the counter at position 2 as the origin of coordinates, and assume that a light signal $1 \rightarrow 2 \rightarrow 3$ emitted in coincidence with F_1 arrives in coincidence with F_3. Then a counter telescope $1 - 3$ measures a velocity

$$v_{13} = \frac{(t_1 + t_3)c}{t_1 + 2t_2 + t_3} \equiv \beta c \tag{3.9}$$

Noting that the radial distance to C1 (i.e. to counter 1, where the first event, F_1, occurs) is $s_{12}\Delta\ell$ and that the radial distance to C3 is $s_{23}\Delta\ell$, the radial velocity v_r is given by

$$v_r = \frac{(t_3 - t_1)}{t_1 + t_3 + 2t_2} \equiv \beta_r c \tag{3.10}$$

The square of the area of the triangle is given by

$$A^2 = (t_1 + t_2 + t_3)t_1 t_2 t_3 \Delta\ell^4 = \frac{1}{16}(t_1 + 2t_2 + t_3)^4(1 - \beta^2)(\beta^2 - \beta_r^2)\Delta\ell^4 \tag{3.11}$$

We now assume that if we make additional velocity measurements anywhere along the line defined by the counter telescope we obtain the same constant velocity

v. Of course the shape of the triangle and the radial velocity change with time, but the area per unit time is constant, as we now prove. This is, of course just Kepler's Second Law for straight line motion past a center which exerts negligible force on the particle. Consider first the symmetric case $r_1(-t) = r_3(+t) = r(t)$ for which $v_r = 0$. The distance of closest approach to the center on this straight line, constant velocity trajectory is called the *impact parameter*. With F_2 the origin of rectangular coordinates, we take the impact parameter to have magnitude $x\Delta\ell$ in the positive x direction. Then $v = \dot{y}$ is perpendicular to it and $|t| = r(t)/c$. Since the area swept out by $r(t)$ in time $2|t|$ is $A(t) = x\Delta\ell\dot{y}|t|$, we have that

$$A(t)/|t| = \beta c x \Delta\ell = const. \tag{3.12}$$

If we now consider the general case with s_{13} in the y direction, the area of the triangle formed when the particle moves from F_1 to F_3 in time $t_{13}\Delta t = (s_{12} + s_{23})\Delta\ell/c$ with velocity $v = \dot{y} = s_{13}c/t_{13}$ divided by $t_{13}\Delta\ell$, using the half-base times altitude rule, is again $\beta c x\Delta\ell$. This proves Kepler's Second Law for straight line motion with constant velocity past a center in our discrete, relativistic model.

We have seen that we can construct our integer version of Kepler's Second Law from an arbitrary integer triangle. To demonstrate rotational invariance, all we need to do is to define transformations which keep the three sides fixed. If our triangle is to return to the same position after a finite number of planar rotations, we must use some care. For instance, we can start by fixing the impact parameter as a constant $x\Delta\ell$ and the line from the origin we have called $r(t)$ to a constant $R\Delta\ell$ in a particular, symmetric case for which the distance from C1 to C3 is $2y\Delta\ell = \dot{y}t\Delta t > 0$. Then, taking the x-axis outward from C2 along the impact parameter, in terms of dimensionless coordinates x, y, t we see that F_1 has coordinates $(x, -y; -t)$, F_2 has $(0, 0; 0)$ and F_3 has $(x, y; t)$. If we require that $R^2 = x^2 + y^2$, we must face the dilemma encountered by Pythagoras two and a half millennia ago that we cannot in general satisfy this relation in integers, and must make some decision if our theory is to remain finite and discrete. We choose to take R and y integer, and then have two choices for x, namely $x_{\pm} = R \pm y$ which insure that $x_+ x_- = R^2 - y^2$.[26]

We now pick our length scale in such a way that $R/y \equiv 2j$ is integer and note that this allows us to construct a regular polygon with $2(2j+1)$ sides composed of isosceles triangles with base $2y$, slant height R and $x^2 = R^2 - y^2 = 4y^2(j^2 - \frac{1}{4})$ Clearly this construction insures rotational invariance under $2(2j+1)$ finite and discrete rotations. The circle circumscribing this polygon has perimeter $2\pi R \Delta \ell = j(2y)$, and we find that our requirement of rotational invariance in our finite and discrete context allows us to conclude that we can always pick units such that

$$(x\dot{y})^2 = (j^2 - \frac{1}{4})\kappa^2 = l_\odot(l_\odot + 1)\kappa^2 \tag{3.13}$$

with

$$l_\odot \equiv j - \frac{1}{2}; \ \kappa \equiv \frac{\Delta \ell^2}{2\pi \Delta t} \tag{3.14}$$

and $2j$ odd.

It may seem peculiar that we have arrived at the "quantum mechanical" result $\ell_\odot(\ell_\odot + 1)$ for the quantum numbers for the square of circular orbital angular momentum correctly related to "spin 1/2" by this essentially classical construction. This has a deep significance which we will pursue on another occasion. Note that our derivation of "angular momentum per unit mass" with $\kappa \equiv \hbar/m$ requires only that space and time measurement accuracy be scale invariant but bounded from below. Physically, this is an obvious move. The single particle assumption built into our analysis breaks down if we try to measure any linear distance to an accuracy $\Delta \ell < h/2m_e c$ because at that point we produce electron-positron pairs with finite probability. That we get a "classical" version of "spin" in units of $\kappa/2$ should not be too startling if one studies our treatment of the fine structure of hydrogen (Ref. 16) and consults the discussion by L.C.Biedenharn on the "Sommerfeld Paradox" which we cite in that paper.

The extension from two to three dimensions is straightforward in terms of the finite rotations implied in our regular polygon paradigm used above. We define a "pseudovector" perpendicular to the rotational plane to represent the orbital angular momentum per unit mass with a chirality convention that (for counter-clockwise rotations and a right-handed coordinate system) makes it the z axis. Then we can make the sign convention and definition

$$l_z = x\dot{y} - y\dot{x} \tag{3.15}$$

invariant for rotations in the $x - y$ plane under any choice of axes in that plane consistent with our polygonal symmetry. Since there are only $2(2j + 1)$ finite rotations defined, only this number of choices is allowed, without further injection of information into the model. As in the case of position-velocity measurement, if we include a larger system which allows us to go down to the $2y$ possible positions along the chord by measuring interference phenomena, we can define relative phases down to $\delta\phi \approx \Delta\ell/R_{max}$ but, as in conventional quantum mechanics, *absolute* phase refers to no known experimental phenomena and still eludes us.

The further articulation of the commutation relations for these finite rotations need not detain us long here. For instance, with orbital angular momenta per unit mass measured in units of κ, we can define

$$l^z_\pm(l_\odot, l_z) \equiv l_\odot \mp l_z \tag{3.16}$$

$$l_{+-} \equiv l^z_+(l_\odot, l_z - 1) l^z_-(l_\odot, l_z) = l_\odot(l_\odot + 1) - l_z^2 + l_z \tag{3.17}$$

$$l_{-+} \equiv l^z_-(l_\odot, l_z + 1) l^z_+(l_\odot, l_z) = l_\odot(l_\odot + 1) - l_z^2 - l_z \tag{3.18}$$

$$< l^z_+, l^z_- > \equiv \frac{1}{2}(l+- + l_{-+}) = l_\odot(l_\odot + 1) - l_z^2 \tag{3.19}$$

$$[l^z_+, l^z_-] \equiv \frac{1}{2}(l_{+-} - l_{-+}) = l_z \tag{3.20}$$

and insure rotational invariance by the invariance of

$$< l_+^z, l_-^z > + l_z^2 = l_\odot(l_\odot + 1) \equiv l_x^2 + l_y^2 + l_z^2 \tag{3.21}$$

With

$$l_x = y\dot{z} - z\dot{y};\; l_y = y\dot{z} - z\dot{y} \tag{3.22}$$

the commutation relation for finite rotations is fully consistent with the commutation relation for rational fraction Lorentz boosts, (3.5) derived above. In fact, given either, the other follows (cf FDP, pp 85-86, and the reference to T.F.Jordan there cited). We conclude that the bracket expression (3.5) can be extended to the three dimensional result we need:

$$[x_i, \dot{x}_j] = \kappa\delta_{ij} = -[\dot{x}_j, x_i] \tag{3.23}$$

where $i, j, k \in 1, 2, 3$.

Since any Lorentz transformation is equivalent to a rotation and a boost, and we have now constructed integer rotations and integer boosts, we can now put the two together by the *scale invariant* definition of two constants c and κ,

$$\frac{\Delta\ell}{c\Delta t} \equiv 1;\; \frac{c\Delta\ell}{\kappa} = 2\pi \tag{3.24}$$

3.4 CONSEQUENCES OF OUR BRACKET EXPRESSIONS

As already noted, the fundamental development in FDP derives general commutation relations from *attribute distance* and the exponentiation of the derivate operator. These bracket expressions can easily be shown to have the properties used in the next chapter to generalize the Feynman proof of the Maxwell Equations in our finite measurement accuracy context. Since we have not invoked that background, this section is devoted to showing that our more physical way of viewing the bracket expressions has the needed algebraic consequences which go beyond (3.23).

In the derivations given above we have in effect assumed that positions and velocities of a single particle in three orthogonal directions can be specified independently, subject to the constraints of invariance under finite and discrete Lorentz boosts and rotations. These constraints can be summarized by the bracket expression (3.23), where the anti-symmetry is explicitly noted. We did not explicitly record the independence of the three coordinate positions, corresponding to our free choice of counter positions in the laboratory:

$$[x_i, x_j] = 0 \tag{3.25}$$

Subject to our requirement of not going beyond the finite limits to which our measurements can refer, the fact that the x_i can be represented by integers, and the $\dot{x}_i$ by rational fractions, allows us to assume that, for λ, μ *constants* subject to the same restrictions and $A, B, C \in x_i, \dot{x}_i$, $i \in 1, 2, 3$, the bracket expression has the properties

$$[\lambda A + \mu B, C] = \lambda[A, C] + \mu[B, C]$$

$$[A, \lambda B + \mu C] = \lambda[A, B] + \mu[A, C] \tag{3.26}$$

$$[A, \mu] = 0$$

In the next chapter we will be concerned with functions $g(x, t)$ which are not functions of $\dot{x}$ and accelerations $\ddot{x}(x, \dot{x}, t)$ which are functions of velocity as well as position, but of no "higher derivatives". Since these are also subject to our finite integer and rational fraction restrictions, we can assume that they are polynomials whose powers have context sensitive restrictions. If n is the highest power of x which is allowed to occur, then for any component

$$[\dot{x}_i, x_j^n] = \delta_{ij}[\dot{x}_i, x_i^n] = \delta_{ij}(-\kappa x_i^{n-1} + x_i[\dot{x}_i, x_i^{n-1}]) = -n\kappa x_i^{n-1}\delta_{ij} \tag{3.27}$$

This allows us to identify the usual symbol $\partial g(x, t)/\partial x_k$ for all such functions we consider by the equality

$$[\dot{x}_k, g(x,t)] = -\kappa \partial g/\partial x_k \tag{3.28}$$

Note also that

$$[x_i, g(x,t)] = 0 \Rightarrow g(x,t) \textit{ independent of } \dot{x} \tag{3.29}$$

It remains to define the symbols $[\dot{x}_i, \dot{x}_j]$ and $\ddot{x}(x, \dot{x}, t)$ in our context. Since (within the restriction to polynomials mentioned above) we are now talking about *functions* of $x, \dot{x}$, and t, we can introduce the concept of a *path*

$$x(t) = (x_i(t), x_j(t), x_k(t); t) \tag{3.30}$$

for the single particle we are considering. Then the bracket expression we derived above is equivalent to the definitions

$$\begin{aligned} x_i(t+\Delta t) &\equiv x_i(t) + \dot{x}_i(t)\Delta t \\ [x_i, \dot{x}_j] &\equiv [x_i(t+\Delta t)\dot{x}_j(t) - \dot{x}_j(t+\Delta t)x_i(t)] \\ &= [x_i\dot{x}_j(t) - x_j\dot{x}_i(t)] \equiv \kappa\delta_{ij} \end{aligned} \tag{3.31}$$

Taking the obvious step of saying that if time changes by Δt, then

$$\dot{x}_j(t+\Delta t) \equiv \dot{x}_j(t) + \ddot{x}\Delta t \tag{3.32}$$

and defining

$$[\dot{x}_i, \dot{x}_j] \equiv \dot{x}_i(t+\Delta t)\dot{x}_j(t) - \dot{x}_j(t+\Delta t)\dot{x}_i(t)$$

we have that

$$[\dot{x}_i, \dot{x}_j] + [x_i, \ddot{x}_j] = 0 \tag{3.33}$$

We could have derived this directly when we were discussing the polygons needed for rotational invariance in the last section by noting that if x_i is the end of the chord, where the direction but not the magnitude of the velocity changes, and the change is due to the reversal of the component along that direction, while the component perpendicular to that remains unchanged, the acceleration must lie along x_j with the magnitude and sign given in the last equation above.

Now that we know what we mean by $[\dot{x}_i, \dot{x}_j]$ it is straightforward to establish the Jacobi identity

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0 \tag{3.34}$$

for the symbols $A, B, C \in x_i, \dot{x}_i$. The same type of argument makes it easy to establish the fact that, in our context

$$g_k(x,t) = \kappa^{-1}\epsilon_{ijk}[\dot{x}_i, \dot{x}_j] \Rightarrow$$

$$\partial g_k/\partial t + [\dot{x}_j, \partial g_k/\partial x_j] = \kappa^{-1}\epsilon_{klm}[\dot{x}_l, \ddot{x}_m] \tag{3.35}$$

a result which we will need in the next chapter.

4. SCALE INVARIANT GENERALIZATION OF THE FEYNMAN-DYSON-TANIMURA PROOF

4.1 Historical Remarks

The development of a new fundamental theory is a tedious process, as the fact that the current effort started at least as early as 1951 clearly attests. Creating the climate of opinion which allows the professional community to make the requisite "paradigm shift" that leads to acceptance of the new theory can be even more tedious, and can be subject to various poorly understood historical delays. We discuss a possible example here. A long buried piece of work by Feynman created in 1948 (only three years earlier than Bastin and Kilmister's first publication in 1951) has recently come to light,[27] — thanks to its reconstruction by Dyson.[28] Had this been available in the 1950's, physics might have taken a different course. We recognized as soon as we saw it that Feynman's "paradoxical" proof of Maxwell's Equations from Newton's Second Law and the non-relativistic quantum mechanical commutation relations makes eminently good sense in our finite and discrete reconstruction of relativistic quantum mechanics.[29] Unfortunately, our attempt to spell this out in the regular literature failed.[30]

Fortunately, an analysis of the proof which makes some of the same points I had already noted has now been published by Tanimura.[31] We hope that this will give us another chance to attempt to get our views before a larger audience.[32] Even without our proposed generalization, Tanimura's claims are already quite startling. I quote his complete abstract:

"R.P.Feynman showed F.J.Dyson a proof of the Lorentz force law and the homogeneous Maxwell equations, which he obtained starting from Newton's law of motion and the commutation relations between position and velocity for a single nonrelativistic particle. We formulate both a special relativistic and a general relativistic versions [sic] of Feynman's derivation. Especially in the general relativistic version we prove that the only possible fields that can consistently act on a quantum mechanical particle are scalar, gauge and gravitational fields. We also extend Feynman's scheme to the case of non-Abelian gauge theory in the special relativistic context."

In Tanimura's notation, the formulation of the theorem is simple:

Given

A single particle trajectory $x(t)$ in terms of three rectangular coordinates $x_i(t)$, $i \in 1,2,3$ subject to the constraints

$$[x_i, x_j] = 0; \;\; m[x_i, \dot{x}_j] = i\hbar\delta_{ij}; \;\; m\ddot{x}_k = F_k(x, \dot{x}; t) \tag{4.1}$$

then

the force components $F_k(x,\dot{x};t)$ can be expressed in terms of two functions $E(x,t)$, $B(x,t)$ which depend only the coordinate components x_i and the time t and not on the velocities $\dot{x}_j$; these functions are related to the force by the component equations

$$F_i(x,\dot{x};t) = E_i(x,t) + \epsilon_{ijk} < \dot{x}_j B_k(x,t) > \tag{4.2}$$

and E, B satisfy the equations

$$div\ B = 0;\ \partial B/\partial t + rot\ E = 0 \tag{4.3}$$

Here the Weyl ordering $<>$ is defined by

$$< ab > \equiv \frac{1}{2}[ab + ba];\ < abc > \equiv \frac{1}{6}[abc + bca + cab + acb + cba + bac],\ etc. \tag{4.4}$$

4.2 SCALE INVARIANT POSTULATES

As was noted recently, the postulates can be made even simpler once one invokes scale invariance.[33] The Feynman postulates are independent of or linear in m. Therefore they can be replaced by the *scale invariant* postulates

$$f_k(x,\dot{x};t) = \ddot{x}_k;\ [x_i, x_j] = 0;\ [x_i, \dot{x}_j] = \kappa\delta_{ij} \tag{4.5}$$

where κ is any fixed constant with dimensions of area over time $[L^2/T]$ and f_k has the dimensions of acceleration $[L/T^2]$. This step is suggested by Mach's conclusion[34] that it is Newton's Third Law which allows mass *ratios* to be measured, while Newton's Second Law is simply a *definition* of force. Hence in a theory which contains only "mass points" and known mass ratios, the scale invariance of classical MLT physics reduces to a purely kinematical LT theory. Breaking scale invariance in such a theory requires not only some unique specification of a particulate mass standard, but also the requirement that this particle have some *absolute* significance.

Relativity need not change this situation. Specify c in a scale invariant way as both the maximum speed at which information can be transferred (limiting group velocity) and the minimum speed for supraluminal correlation *without* information transfer [limiting phase velocity =(coherence length)/(coherence time)]. If the unit of length is $\Delta\ell$ and the unit of time is Δt, then the equation $(\Delta\ell/c\Delta t) = 1$ has a *scale invariant* significance. Further, the interval I specified by the equation $c^2\Delta T^2 - \Delta L^2 = I^2$ can be given a *Lorentz invariant* significance. We can extend this analysis to include the scale invariant definition $\Delta E/c\Delta P = 1$ and the Lorentz invariant interval in energy-momentum space $(\Delta E^2/c^2) - \Delta P^2 = \Delta m^2$ provided we require that $\frac{\Delta P \Delta L}{\Delta m} = \frac{\Delta E \Delta T}{\Delta m}$. Then, given any arbitrary particulate mass standard, mass ratios can be measured using a Lorentz invariant *and* scale invariant LT theory. We trust that this dimensional analysis of the postulates used in the Feynman proof already removes part of the mystery about why it works.

The remaining physical point that needs to be made clear is that the "fields" referred to in classical relativistic field theory are *defined* in terms of their action on a *single* test particle. Thus, if we measure the *acceleration* of that particle in a Lorentz invariant way (force per unit rest mass) *and* the force per unit charge is also defined by acceleration *and* the charge per unit rest mass of the test particle is *also* a Lorentz invariant our electromagnetic field theory itself becomes an LT scale invariant theory. That is, once we replace the Feynman postulates by (4.5) and define $\mathcal{E}(x,t) = E/Q = F_E/m$ and $\mathcal{B}(x,t) = B/Q = F_B/m$, we need only derive the scale invariant version of equations (4.2), (4.3) obtained by the obvious notational change $F_i \rightarrow f_i$, $E_i \rightarrow \mathcal{E}_i$, $B_i \rightarrow \mathcal{B}_i$. We make a few remarks later on about the extension to gravitation, where the obvious physical postulate is that the ratio of gravitational to inertial mass of our test particle is also a Lorentz invariant.

4.3 THE PROOF

Here we essentially repeat Tanimura's version of the Feynman-Dyson proof of the Maxwell equations using our scale invariant results derived from measurement accuracy in the last chapter. There we proved that for any function g(x,t) we need consider in our finite and discrete theory (by which notation we mean that the function does not depend on $\dot{x}$ or higher derivates)

$$[x_i, x_j] = 0 \Rightarrow [x_i, g(x,t)] = 0 \tag{4.6}$$

Hence, in order to prove that a function is in this class we need only prove that it commutes with all the components x_i. For any acceleration $\ddot{x} = f(x, \dot{x}, t)$ which depends only on position, velocity and time — which Newton's second law defines as a "force per unit mass" — the result from finite measurement accuracy is that

$$[\dot{x}_i, \dot{x}_j] + [x_i, \ddot{x}_j] = 0 \Rightarrow [\dot{x}_i, \dot{x}_j] + [x_i, f_j] = 0 \tag{4.7}$$

Hence for any scale invariant "force" which has the Lorentz form

$$f_i(x, \dot{x}; t) = \mathcal{E}_i + \epsilon_{ijk} < \dot{x}_j \mathcal{B}_k > \tag{4.8}$$

the finite measurement accuracy result for the commutator has the implication

$$[x_i, \dot{x}_j] = \kappa\delta_{ij} \Rightarrow [\dot{x}_i, \dot{x}_j] = -[x_i, f_j] = \kappa\epsilon_{ijk}\mathcal{B}_k \tag{4.9}$$

This, in turn, allows us to define the scale invariant field $\mathcal{B}$ by

$$\mathcal{B}_k = \kappa^{-1}\epsilon_{ijk}[\dot{x}_i, \dot{x}_j] \tag{4.10}$$

The field $\mathcal{B}$ could still depend on velocity as well as position and time, but the fact that our bracket expression for position and velocity satisfies the Jacobi identity

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0 \tag{4.11}$$

and our central result in (4.1) allows us to show that the commutator of definition (4.10) with any coordinate vanishes, establishing the required property. Then the fact that our formalism for measurement accuracy implies that

$$[\dot{x}_k, g(x,t)] = -\kappa \partial g/\partial x_k \tag{4.12}$$

with $\partial/\partial x_k$ interpreted as the partial derivate (finite difference) rather than the partial derivative allows us to infer that

$$[\dot{x}_k, \mathcal{B}(x,t)] = -\kappa \partial \mathcal{B}/\partial x_k \tag{4.13}$$

But then our definition of $\mathcal{B}$ in (4.10) as proportional to the anti-symmetrized commutator together with the Jacobi identity establish the first Maxwell Equation

$$\Sigma_k \partial \mathcal{B}_k/\partial x_k \equiv div\mathcal{B} = 0 \tag{4.14}$$

Now that we know what we mean by the magnetic field per unit charge, we can use the Lorentz "force" equation (4.8) as the definition of the electric field per unit charge. Then, taking the commutator with any component and using the definition of the Weyl ordering (4.4) together with the fact we proved above that $[x_i, \mathcal{B}_k] = 0$ we find that $[x_i, \mathcal{E}_k] = 0$ establishing that the scale invariant electric field so defined is also only a function of x and t. The final step requires us to define what we mean by "partial and total derivatives" with respect to time in the finite measurement accuracy context in concert with the space connectivity given by (4.12). The necessary result we need is noted in the last chapter and is that

$$g_k(x,t) = \kappa^{-1}\epsilon_{ijk}[\dot{x}_i, \dot{x}_j] \Rightarrow$$

$$(d/dt)g_k(x,t) = \partial g_k/\partial t + [\dot{x}_j, \partial g_k/\partial x_j] = \kappa^{-1}\epsilon_{klm}[\dot{x}_l, \ddot{x}_m] \tag{4.15}$$

Then a sequence of algebraic steps given in Dyson's paper (Ref. 26) and summarized in Tanimura's Eq. 2.18 lead to the second Maxwell Equation

$$\partial \mathcal{B}/\partial t + rot\ \mathcal{E} = 0 \tag{4.16}$$

We wish to emphasize that the essential formal work has already been accomplished by Tanimura. This allows us to omit algebraic details in the above rewriting of his proof, and refer the interested reader to his paper if he wishes to check them. The *physical* point that Tanimura does not mention is that the proof can be made scale invariant, and does not depend on either Planck's constant or on the bracket expression having an imaginary value. Thus it is more general that its historical "quantum mechanical" genesis might suggest.

Again, we are greatly indebted to Tanimura by analysing so carefully what properties of the bracket expression he needs. He points out that bracket expression [,] needed for the proof is not necessarily an *operator* expression. It suffices that it have the five algebraic properties

bilinearity,

$$[\lambda A + \mu B, C] = \lambda[A, C] + \mu[B, C],$$

$$[A, \lambda B + \mu C] = \lambda[A, B] + \mu[A, C];$$

anti-symmetry,

$$[A, B] = -[B, A];$$

the Jacobi identity,

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0;$$

Leibniz rule I,

$$[A, BC]] = [A, B]C + B[A, C]];$$

and Leibniz rule II,

$$\frac{d}{dt}[A, B] = [\frac{dA}{dt}, B] + [B, \frac{dA}{dt}].$$

He further notes that "*It is one of the virtues of Feynman's proof that there is no need for a priori existence of Hamiltonian, Lagrangian, canonical equation, or Heisenberg equation.*"

From the point of view of a finite and discrete theory, the most critical steps in the proof are those which depend on "derivatives". We derived all of these from measurement accuracy in the last chapter and have pointed out where they occur in our summary of Tanimura's proof.

4.4 EXTENSION TO GRAVITATION

In order to make his result manifestly covariant, Tanimura finds that he has to introduce an ordering parameter τ which is *not* the proper time. This idea has an old history going back at least to Stueckelberg, which we learned of from E.O.Alt. Alt's interest stemmed from the difficulty of formulating a relativistic quantum mechanical few body scattering theory without introducing the infinite number of degrees of freedom required in any "second quantized" formalism. We will pursue this idea elsewhere in our own context. Here we simply note that our new fundamental theory comes to us with this ordering parameter (bit-string length) built in, as will be seen on consulting the *program universe* method for generating bit-stings in the fundamental papers. Thus, as before, Tanimura's formal steps go over into our fundamental theory practically unchanged. We leave this for detailed exploration at a later stage in the development of the theory. Our further discussion of gravitation in this paper is better left until we have discussed the *combinatorial hierarchy.*

5. SUMMARY AND CONNECTION TO THE COMBINATORIAL HIERARCHY

Our new fundamental theory [13–16] models the process of "measurement" as the *counting* of finite and discrete bits of information acquired using an understood laboratory protocol. Any such theory can be modeled by the *bit strings* of contemporary computer practice, rules for combining them, and rules for relating them unambiguously (although possibly statistically) to that laboratory protocol.

Define *particles* as the *conceptual* carriers of conserved quantum numbers between events and *events* as regions across which quantum numbers are conserved. Take as the basic paradigm for two events the sequential firing of two counters separated by distance L and time interval T, where the clocks recording the firings are synchronized using the Einstein convention. Define the velocity of the "particle" connecting these two events as $v = \beta c = L/T$ where c is the limiting velocity for the transfer of *information.* If our unit of length is ΔL and our unit of time ΔT, then $c \equiv \Delta L/\Delta T$ has the same significance in any system of units; this definition is *scale invariant.* Similarly, we can define *Kepler's Constant* with the dimensions of area over time by $\kappa \equiv \Delta L^2/2\pi\Delta T = c\Delta L/2\pi$ while retaining scale invariance. In these units an event at $x = (n_1 - n_2)\Delta L$, $t = (n_1 + n_2)\Delta T$ is at a Lorentz invariant interval from the origin given by $I^2 = c^2t^2 - x^2 = 4n_1n_2\Delta L^2$. n_1, n_2 are simultaneously Lorentz invariant and scale invariant.

We assume that "fields" are to be measured by the acceleration of a "test particle" which belongs to a class of particles whose ratios of charge to mass and gravitational to inertial mass are Lorentz invariant. We relate space and time derivatives of functions of $x, \dot{x}, t$ to measurement accuracy by *deriving* the bracket expression $[x_i, \dot{x}_j] \equiv \kappa\delta_{ij}$ from an analysis of measurement accuracy and Kepler's Second Law for motion with constant velocity past a center. This also allows us to give meaning to finite and discrete accelerations through the relation $[\dot{x}_i, \dot{x}_j] + [x_i, \ddot{x}_j] = 0$ and to replace partial derivatives with finite "derivates". Then we show that Tanimura's proof [29] that the only fields which can act on such particles are structurally indistinguishable from electromagnetic and gravitational fields is a consequence of measurement accuracy alone, and does not depend on any specific assumptions about quantum mechanics.

Consider a particle bound to a center a distance r away which receives an impulsive force toward the center each time it has moved a distance λ whose square is $4n_1n_2\Delta L^2/(n_1 - n_2)^2$. If we take $2\pi r = j\lambda$, the area swept out per unit time by the radial distance to the particle is $(j^2 - \frac{1}{4})\kappa^2 = \ell(\ell+1)\kappa^2$ where we have defined $\ell = j - \frac{1}{2}$. Assuming that the probability of the impulsive force occurring after one step of length $\Delta L = h/m_e c$ is $1/137(\ell + 1)$ we obtain[16] Bohr's relativistic formula $(\frac{m-\epsilon_\ell}{m})^2[1 + (\frac{1}{137(\ell+1)})^2] = 1$ for the levels of the hydrogen atom[35] in the approximation $e^2/\hbar c \approx 1/137$, and hence *his* correspondence limit. Adding a second degree of freedom gives us the Sommerfeld formula and an improvement of four significant figures[16] in our value for $e^2/\hbar c$. Our scale invariant theory is the proper correspondence limit for any relativistic particle theory which breaks scale invariance by taking $m\kappa = \hbar$. [31] Note that $h/2mc$ is the longest threshold distance for the non-classical process of particle-antiparticle pair creation. For gravitational orbits of a particle of mass m about a center containing N particles of mass m, orbital velocity reaches c when $N = (\kappa c/Gm)^{\frac{1}{2}}$. Consequently the shortest distance (between *two* events!) in the theory is $\Delta L/N$. This is the "black hole radius" for mass Nm. Thanks to the fact that our Lorentz-invariant theory predicts both the (quantized) Newtonian interaction and spin 2 gravitons, it meets the three classical tests of general relativity.[36]

The numbers n_1, n_2 as integer descriptors of velocity and two more integers for angular momentum provide quantized Mandelstam variables for four-leg diagrams and discrete conservation laws. This allows a bit-string representation of particulate events. We label these "space-time descriptors" or *content* bit-strings by bit-strings of length 16. These *label* bit-strings combine by XOR (addition, mod 2) and are organized into the first three levels of the *combinatorial hierarchy*: (1) $3 = 2^2 - 1$; (2) $10 = 3 + (2^3 - 1 = 7)$; (3) $137 = 10 + (2^7 - 1 = 127) \approx \hbar c/e^2$. These 16 bits in the string specify the conserved quantum numbers of the standard model of quarks and leptons (charge, baryon number, lepton number, weak hypercharge, color) with confined color, weak-electromagnetic unification, and precisely the three observed generations.

The fourth level uses labels of length 256 and closes with $2^{127} + 136 \approx 1.7 \times 10^{38} \approx \hbar c/Gm_p^2$ possible ways they can combine pair-wise. This specifies m_p as the unit of mass. Then, since our labels conserve baryon number, lepton number and charge, any gravitating system with spin 1/2, unit charge and lepton or baryon number one collapses by emitting Hawking radiation to become a *stable* charged, rotating black hole. The number of bits of information lost in the collapse of $\hbar c/Gm^2$ is equal to the area of the event horizon in Planck areas.[37] This stabilizes the proton and electron. Then most of the electron mass is generated either electromagnetically or by the Fermi interaction. Self consistency of the electron mass calculated using either $e^2/\hbar c = 1/137$ or $G_f m_p^2/\hbar c = 1/\sqrt{2}(256)^2$ of this calculation gives weak-electromagnetic unification at the tree level. Since the labels must be generated before space-time can be constructed and takes on meaning, closure of level four plus baryon number conservation implies about $(2^{127})^2$ baryons in the universe. The resulting cosmology is good to at least first order. Current results are summarized in the following table.

Table I. **Coupling constants and mass ratios** predicted by the finite and discrete unification of quantum mechanics and relativity. Empirical Input: $c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990.

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136] \times [1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 31\ldots \times 10^{38}$	$[1.69358(21) \times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\sqrt{2}]^{-1} \times [1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots \times 10^{-5}$	$[1.02\ 682(2) \times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	$[0.2259(46)]$
$\alpha^{-1}(m_e)$	$137 \times [1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	$[137.0359\ 895(61)]$
$G^2_{\pi N\bar{N}}$	$[(\frac{2M_N}{m_\pi})^2-1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	$[13, 3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$	$[1836.15\ 2701(37)]$
$m_\pi^{\pm}/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	$[273.12\ 67(4)]$
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 143\ldots$	$[264.1\ 373(6)]$
m_μ/m_e	$3\cdot 7\cdot 10[1-\frac{3}{3\cdot 7\cdot 10}] = 207$	$[206.768\ 26(13)]$

COSMOLOGICAL PARAMETERS

Parameter	Calculated	Observed
N_B/N_γ	$\frac{1}{256^4} = 2.328.... \times 10^{-10}$	$\approx 2 \times 10^{-10}$
M_{dark}/M_{vis}	≈ 12.7	$M_{dark} > 10M_{vis}$
$N_B - N_{\bar{B}}$	$(2^{127}+136)^2 = 2.89... \times 10^{78}$	*compatible*
ρ/ρ_{crit}	$\approx \frac{4\times 10^{79} m_p}{M_{crit}}$	$.05 < \rho/\rho_{crit} < 4$

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Appendix: COHERENCE, DETERMINISM and CHAOS

Abstract of contribution to ANPA 15, September 9-12, 1993

We assume that "fields" are to be measured by the acceleration of a "test particle" which belongs to a class of particles whose ratios of charge to mass and gravitational to inertial mass are Lorentz invariant. We relate the measurement accuracy in space, Δx, and in time, Δt, by the scale invariant definition of two constants c, and κ: $\frac{\Delta x}{c\Delta t} \equiv 1$; $\frac{\Delta x^2}{\kappa \Delta t} \equiv 2\pi$. Taking the experimental velocity resolution $\Delta v_x = \Delta x/T\Delta t = N\Delta x/NT\Delta t$ we derive the bracket expression $[x, v_x] = \kappa$ where $x = N\Delta x$. Then it is a *deductive* consequence that the only fields which can act on such particles are structurally indistinguishable from electromagnetic and gravitational fields in the sense that they satisfy the free space Maxwell Equations and Einstein geodesic equations. Such a scale invariant theory becomes the proper correspondence limit for any relativistic particle theory which breaks scale invariance by taking $m_e\kappa = \hbar$. Here we use m_e because it defines the threshold distance for position measurement, $h/2m_ec$, below which the non-classical process of electron-positron pair creation is observed, and above which that phenomenon cannot be *directly* observed. The coherence length $L = NT\Delta x$ specifies the maximum distance within which quantum mechanical interference effects can be observed. For non-overlapping "wave packets" of this length, the *deterministic* classical equations with particulate sources and sinks apply. But the characterization of a deterministic system as chaotic requires a specification of boundary conditions to a precision which violates the constraint due to measurement accuracy or electron-positron pair creation. Hence the number of degrees of freedom used in a model fixes whether the system is quantum coherent or classically decoherent but (approximately) deterministic and limits the applicability of chaos theory, removing certain paradoxes.

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