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INSTANTANEOUS INTERACTIONS OF HADRONS ON THE LIGHT CONE*

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ABSTRACT

Hadron wavefunctions are most naturally defined in the framework of light-cone quantization, a Hamiltonian formulation quantized at equal light-cone 'time' $\tau \equiv t + z$. One feature of the light-cone perturbation theory is the presence of instantaneous interactions, which complicate the consideration of processes involving bound states. We show that these interactions can be written in a simple and general form, parametrized by an instantaneous contribution $\tilde{\psi}$ to the hadronic wavefunction. We use the rotational invariance of Feynman diagrams to relate this instantaneous piece of the meson wavefunction to the propagating part, and to obtain constraints relating wavefunctions and quark fragmentation amplitudes.

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Light-cone quantization (LCQ) is a natural framework for the description of processes involving scattering into bound states; as such, it offers the most attractive basis for the description of hadrons in terms of their partonic constituents.

LCQ is a Hamiltonian theory, quantized at equal light-cone 'time' $\tau \equiv t + z$. As a result, its perturbation theory (LCPTh) shares several features with old-fashioned time-ordered perturbation theory, notably that internal particles propagate on mass shell, while the 'energy' $P^- \equiv P^0 - P^z$ is not conserved in intermediate states.

In perturbation theory, each Feynman diagram can be written as a sum of LCPTh diagrams over all possible τ-orderings of its vertices; the advantage of LCPTh is that the conserved momentum $p^+ \equiv p^0 + p^z$ is positive for each particle, so that vacuum-creation graphs do not appear in perturbative calculations, and the number of nonzero LCPTh diagrams corresponding to a single Feynman diagram is greatly reduced.

In LCQ, the formation of hadrons from underlying partonic processes is governed by process-independent light-cone wavefunctions [1]. In the consideration of exclusive processes at leading twist, only the projection of this wavefunction onto the valence Fock state need be considered. For a meson, the valence state is simply the $q\bar{Q}$ state with the flavor quantum numbers of the meson, and such processes are governed by the valence light-cone wavefunction

$$\psi_{h\to q\bar{Q}}(x, k_\perp),$$

which is the amplitude for the meson h to decompose into on-shell partons q and $\bar{Q}$, with the quark q carrying momentum

$$p_q = (xp_h^+, p_q^-, xp_{\perp,h} + k_\perp);$$

the on-shell condition requires $p_q^- = (p_{\perp,q}^2 + m_q^2)/p_q^+$.

By convention, this wavefunction includes the light-cone energy denominator $(p_h^- - p_q^- - p_{\bar{Q}}^- - i\epsilon)^{-1}$, as well as spinor normalization factors $\sqrt{p_q^+ p_{\bar{Q}}^+}$ which simplify the calculation of Dirac numerator factors. Thus, in perturbative calculations an incoming meson with momentum p should be represented by the factors

$$\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} \psi_{h\to\bar{Q}q}(x, k_\perp)\ \frac{u(\bar{x}p - k_\perp)}{\sqrt{\bar{x}p^+}}\ \frac{\bar{v}(xp + k_\perp)}{\sqrt{xp^+}}, \tag{1}$$

where we have introduced the notation $\bar{x} \equiv 1 - x$.

In the same way, the closely analogous *fragmentation amplitudes* are defined as

$$\psi_{Q\to hq}(x, k_\perp),$$

the amplitude for a quark Q to emit a meson h. Here the momenta and energy denominator are defined as above (but with $h \leftrightarrow Q$). The fragmentation amplitude contains the factor $\sqrt{p_q^+ p_Q^+}$, rather than $\sqrt{p_h^+ p_q^+}$; other than that, the definition of the fragmentation amplitude is entirely analogous to that of the wavefunction.

One feature of light-cone quantization is that, in addition to the familiar forward-going propagators of internal particles, there are additional 'instantaneous' contributions to the propagator (due to the fact that the quantization surface is not strictly spacelike) [2]. The light-cone Green functions can be derived from covariant Green functions by integrating over τ; instantaneous terms represent 'contact' interactions arising when vertices share the same coordinate τ.

In the scattering of free particles, the form of the instantaneous propagator can be deduced from the Hamiltonian of the theory after integrating out dependent degrees of freedom. For fermions, the Dirac structure of these contributions is simply $\gamma^+ \equiv \gamma^0 + \gamma^3$. [3]. However, when a bound state scatters by exchange of an instantaneous particle, it is not immediately clear that a simple representation of the form of the interaction can be obtained.

For example, the simplest hadronic process imaginable is the photodissociation of a meson into a $q\bar{Q}$ pair. The standard methods of LCQ do not suffice to calculate the amplitude for this process, since the instantaneous interaction shown in Fig. 1 cannot be neglected. Thus, the applications of LCQ to wavefunction-controlled processes have largely been restricted to the computation of spacelike form factors, for which the evaluation of the +-component of the hadronic part of the amplitude is sufficient. Instantaneous terms do not affect such calculations, since $\gamma^+\gamma^+ = 0$.

In this paper, we show that the instantaneous interaction does indeed have a simple form, parametrized by a wavefunction analogous to those of Ref. [1]. We then use the rotational invariance of certain scattering amplitudes to write the instantaneous wavefunction in terms of the familiar propagating wavefunctions of Ref. [1] in the valence Fock state. In addition, we are able to obtain a sum rule constraining the behavior of the wavefunction and fragmentation amplitude. Sum rules for inelastic scattering at the probability level have been obtained by Gribov

Figure 1. Instantaneous interactions of a meson. Chapter I demonstrates that the interactions may indeed be written in the form suggested by this figure. Arrows indicate fermion flow; time flows from left to right.

and Lipatov [4], but the simple relations which prevail in exclusive processes at the amplitude level have not been elucidated.

I. THE INSTANTANEOUS WAVEFUNCTION

An example of an instantaneous interaction contributing to the photodissociation $\gamma h \to q\bar{Q}$ is shown in Fig. 1; however, this diagram does not as yet represent anything. To describe such interactions in a simple form, we must take one step further into the 'muck' of the quark-meson vertex, as shown in Fig. 2. In Fig. 3, a non-instantaneous diagram contributing to the photodissociation process, the propagating internal quark line is represented by the factor $D_{\text{prop}} = u_\lambda(q)/\sqrt{q^+}$, where $\lambda = \pm$ is the quark helicity.

We first consider the time-ordering shown in Fig. 2(c). The instantaneous quark is now represented by

$$D_{\text{inst}} = \frac{\gamma^+}{q^+} \, \tilde{\not{q}} \, \frac{u_\lambda(\tilde{p})}{\sqrt{\tilde{p}^+}}. \tag{2}$$

Figure 2. Underlying processes which contribute to interactions like that shown in Fig. 1. We must account for the possibility of the 'invisible' internal quark and gluon being either forward- or backward-moving.

Figure 3. Another diagram contributing to the photodissociation process, calculable using the methods of Ref. [1].

We will derive a more compact expression for eq. (2), depending only on the external momenta p_h, $p_{\bar{Q}}$, and q of Fig. 1. Once this is accomplished, the wavefunctions inside the muck of Fig. 2 may be integrated out, leaving a form similar to that of eq. (1). Fortuitously, the presence of the γ^+ acts as an 'information wall' which we now show serves to block out the dependence on the 'invisible' internal momentum $\tilde{p}$.

Since the light-cone wavefunctions must be quantized in the light-cone gauge $A^+ = 0$ [1], we substitute $\gamma^+ \tilde{\not\epsilon} \to -\gamma^+ \tilde{\epsilon}_\perp \cdot \gamma_\perp$. Then we can explicitly evaluate eq. (2), obtaining

$$D_{\rm inst} = \frac{\tilde{\epsilon}_1 + i\lambda\tilde{\epsilon}_2}{q^+}\ \zeta_\lambda, \tag{3}$$

where (in the Dirac representation of γ^μ)

$$\zeta_+ \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \zeta_- \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \tag{4}$$

the spinors $\zeta_\pm$ are related to the basis spinors χ of Ref. [1] by $\zeta_\pm = \gamma^0\gamma^1\chi_\pm = \pm\gamma^+\chi_\mp$.

We have almost accomplished our objective; the spinors $\zeta_\pm$ carry information about the spin of the invisible internal quark (as they must, since helicity is conserved for light quarks), but they do not depend on its momentum $\tilde{p}$ at all. The unwanted extra information has been blocked by the intervening γ^+.

The only remaining obstacle is the dependence on $\tilde{\epsilon}_\perp$. We use the light-cone gauge convention of Ref. [1], so that $\epsilon_\perp = (1, \pm i)/\sqrt{2}$. The gluon with spin $-\lambda$, opposite to the internal quark spin, contributes a factor $\sqrt{2}$; the gluon with spin $+\lambda$ does not contribute at all. Now we can write eq. (2) as

$$D_{\text{inst}} = \sqrt{2}\, \frac{\zeta_\lambda}{q^+}, \tag{5}$$

with the implicit constraint that the internal gluon of Fig. 2 has helicity $-\lambda$. Though we have as yet discovered nothing about $\tilde{\psi}$, the form of eq. (5), into which no momenta other than q enter, is sufficient to demonstrate its existence.

The wavefunctions inside the muck of Fig. 2 carry an extra unit of orbital angular momentum, which is not present in the wavefunction of Fig. 3; the difference serves to balance the angular momentum carried by the gluon which we have explicitly extracted.

We can now define the instantaneous wavefunction $\tilde{\psi}_{h(q_\lambda)\bar{Q}}(x, k_\perp)$ required for the evaluation of the amplitude shown in Fig. 1. The parentheses in the subscript denote the exchange of an instantaneous particle; the arguments x and $k_\perp$ are defined by

$$x \equiv \frac{p^+_{\bar{Q}}}{p^+_h} \qquad \text{and} \qquad k_\perp \equiv p_{\perp,\bar{Q}} - x p_{\perp,h}.$$

To ensure that the instantaneous wavefunction has the same spin properties as the propagating wavefunction $\psi_{h\to q\bar{Q}}$, we rewrite eq. (5) as

$$D_{\text{inst}} = \left[\frac{\sqrt{2}\; p^+_{\bar{Q}}}{q^+(k_1 + i\lambda k_2)}\right]\Big(\frac{k_1 + i\lambda k_2}{p^+_{\bar{Q}}}\Big)\zeta_\lambda, \tag{6}$$

and absorb the factor in square brackets into the definition of the wavefunction $\tilde{\psi}_{h(q_\lambda)\bar{Q}}$.

With this result, we can represent instantaneous interactions of the sort shown in Fig. 1 by replacing the incoming meson with the factor

$$\int_0^\infty \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} \tilde{\psi}_{h(q_\lambda)\bar{Q}}(x, k_\perp)\Big(\frac{k_1 + i\lambda k_2}{p^+_{\bar{Q}}}\Big)\,\zeta_\lambda\;\frac{\bar{v}(xp + k_\perp)}{\sqrt{xp^+}}; \tag{7}$$

this should be compared to eq. (1), the standard expression, which appears in the evaluation of the propagating amplitude shown in Fig. 3. The new terms in eq. 7, $(k_1 + i\lambda k_2)\zeta_\lambda/p^+_{\bar{Q}}$, combine to mimic the properties under boosts and rotations about $\hat{z}$ of the corresponding term $u_\lambda/\sqrt{p^+_q}$ in eq. (1); thus the two wavefunctions behave identically under those transformations.

Though we have constructed this result for only one of the time-orderings of Fig. 2, the proof in the other cases proceeds in exactly the same manner except for the substitutions $u_\lambda \to v_{-\lambda}$ or $\tilde{\epsilon} \to \tilde{\epsilon}^*$, which do not affect the result.

Figure 4. A process involving the instantaneous wavefunction $\tilde{\psi}(x < 0)$. Again, arrows indicate the direction of fermion flow.

For diagrams like that shown in Fig. 1, we must allow $x \in (0, \infty)$ since the momenta p_h^+ and $p_{\bar{Q}}^+$ can take any positive value, and q^+ may have either sign. Another class of diagrams, like that shown in Fig. 4, requires determination of the instantaneous wavefunctions for $x \in (-\infty, 0)$; we will not need to consider this case in the present work.

Figure 5 shows a configuration in which it is not clear which wavefunction we should use. This process may be considered either as an instantaneous process like those of Fig. 1, or as a higher-order correction to the tree-level diagram of Fig. 3. How do we avoid double-counting such contributions?

The answer depends on the choice of separation scale μ. Define the internal perpendicular momentum $k_\perp \equiv q_\perp - (q^+/p_{\bar{Q}}^+)p_{\perp,\bar{Q}}$. If $|k_\perp| > \mu$, we must consider the process shown in Fig. 5 as a higher-order correction to the propagating amplitude of Fig. 3; for $|k_\perp| < \mu$, the same amplitude is already accounted for as part of the amplitude corresponding to Fig. 2(a). Thus the instantaneous wavefunctions, like the propagating ones, are dependent on the factorization scale; their μ-dependence is determined by diagrams like that of Fig. 5. Consideration of this evolution is outside the scope of the present work; we will fix the same factorization scale μ

Figure 5. A diagram which may or may not be considered instantaneous, depending on the momentum transfer $k_\perp$.

for the instantaneous and propagating wavefunctions, and compare the resulting quantities. The discussion, however, should highlight the fact that at small momentum transfer, the quark is not the simple object which enters into perturbative calculations, but has all the complexity usually associated with composite hadrons.

In sum, we have constructed a rule, eq. (7), for the calculation of instantaneous scattering from mesons; its use is exactly analogous to that of the familiar non-instantaneous wavefunctions of Ref. [1]. The power of this result is its independence of the internal dynamics of the muck, demonstrated by eq. (5); the internal momenta do not affect the form of the interaction.

It must be pointed out that the theoretical stature of the instantaneous wavefunction is not on a par with that of the more familiar two-particle wavefunction. The latter is simply the projection onto a $q\bar{q}$ basis Fock state of an eigenstate of the light-cone Hamiltonian, while the former incorporates the sum and integration over more populous Fock states represented in Fig. 2. Thus the wavefunction entering into eq. 7 is, in terms of the expansion of the meson wavefunction over the

Fock state basis, only an effective wavefunction entering into processes like that shown in Fig. 1.

Equation (7) ensures that the properties of the instantaneous wavefunction under rotations about $\hat{z}$ are the same as those of the propagating wavefunction with the same meson and parton helicities; thus the two contributions may readily be combined in the calculation of scattering amplitudes. Finally, we note that time-reversal invariance requires

$$\tilde{\psi}_{\bar{Q}(q)h}(x, k_\perp) = \tilde{\psi}^*_{h(q)\bar{Q}}(x^{-1}, x^{-1}k_\perp). \tag{8}$$

We next turn to the problem of relating the instantaneous contributions so defined to the conventional wavefunction.

II. CONSTRAINTS FROM ROTATIONAL INVARIANCE

With the introduction of the wavefunctions $\tilde{\psi}$, we are finally able to calculate entire hadronic amplitudes, rather than only their +-components. The simplest such process is the photodissociation of a meson; for definiteness, we will consider the process $\gamma K^*_\uparrow \to \bar{d}_+ s_+$, where the subscripts denote particle helicities.

We neglect all quark mass terms in the following analysis; thus our results will suffer from corrections of order m/Q, where $Q^2 = -t$ is the momentum transfer in the scattering process. This enables us not only to probe the projections of the wavefunctions onto a state with definite helicities, but also to prepare the fermion spinors in the spin-projection eigenstates of Ref. [1] without spoiling the rotational invariance of the amplitude.

Other analyses of this sort [5] have been hampered by the fact that the amplitudes corresponding to single Feynman diagrams are not in general Lorentz invariant; thus one often is forced to deal with a complicated sum of such amplitudes, greatly reducing the power of the resulting constraints.

Armed with the result of eq. (7), however, we can now calculate all of the components of the hadronic part H^μ of the amplitude, rather than only H^+. Thus it is possible to circumvent the lack of gauge invariance of single Feynman diagrams by specializing immediately to Coulomb gauge and working only in center-of-momentum frames. While individual Feynman graphs lack the gauge invariance which is prerequisite to full Lorentz invariance, in this case they will be invariant (up to at most a phase) under rotations, though not under boosts.

For massless particles, the most general form for the initial- and final-state four-momenta $(k^+, k^-, k_\perp)$ satisfying $P_\perp = 0$ is

$$\begin{aligned} p_{K^*} &= (yP^+, \bar{y}P^-, l_\perp), \\ p_\gamma &= (\bar{y}P^+, yP^-, -l_\perp), \\ p_s &= (\bar{x}P^+, xP^-, -k_\perp), \text{ and} \\ p_{\bar{d}} &= (xP^+, \bar{x}P^-, k_\perp). \end{aligned} \tag{9}$$

Here we have introduced the notation $\bar{a} \equiv 1 - a$; the requirement that all particles be on mass shell means that $k_\perp^2 = x\bar{x}P^+P^-$ and $l_\perp^2 = y\bar{y}P^+P^-$.

In order to obtain rotationally invariant quantities, we must work in center-of-mass frames, where $P^+ = P^- = E_{\rm cm}$; since there is only one energy scale in the problem, we set $E_{\rm cm} = 1$ for convenience. Then the three-momenta corresponding to the definitions of eq. (9) are

$$\begin{aligned} p_{K^*} &= (l_\perp, y - \frac{1}{2}), \\ p_\gamma &= (-l_\perp, \frac{1}{2} - y), \\ p_s &= (k_\perp, x - \frac{1}{2}), \text{ and} \\ p_{\bar{d}} &= (-k_\perp, \frac{1}{2} - x). \end{aligned} \tag{10}$$

We will use three-vectors, with the notation $\vec{v} = (v_\perp, v_z)$, in the remainder of this work.

In Coulomb gauge, $\epsilon^0 = 0$, and the photon polarization three-vectors are

$$\vec{\epsilon}_\uparrow = \vec{\epsilon}^{\,*}_\downarrow = \sqrt{2y\bar{y}}\Big(\frac{l_{\scriptscriptstyle L}}{y}\hat{\epsilon}_{\scriptscriptstyle R} - \frac{l_{\scriptscriptstyle R}}{\bar{y}}\hat{\epsilon}_{\scriptscriptstyle L}, 1\Big), \tag{11}$$

where for the sake of brevity we have introduced the notations

$$\hat{\epsilon}_{\scriptscriptstyle R} \equiv \frac{1}{\sqrt{2}}(1, i), \qquad \hat{\epsilon}_{\scriptscriptstyle L} \equiv \frac{1}{\sqrt{2}}(1, -i), \qquad \text{and} \qquad l_{\scriptscriptstyle R(L)} \equiv l_\perp \cdot \hat{\epsilon}_{\scriptscriptstyle R(L)}.$$

As a first example, we calculate the s-channel amplitude for Compton scattering $e\gamma \to e\gamma$, given these restrictions. When the electron helicity is positive, the only nonzero contribution is that in which both photon helicities are negative. If we let p_K and $p_{\bar{d}}$ above represent the incoming and outgoing electron momenta, the s-channel contribution to the full Compton amplitude is

$$e^2\Big[\sqrt{\bar{x}\bar{y}} + \frac{2l_{\scriptscriptstyle L}k_{\scriptscriptstyle R}}{\sqrt{\bar{x}\bar{y}}}\Big] = e^2\cos\frac{\theta_{\rm cm}}{2}e^{i\phi},$$

where ϕ is a pure phase [6].

This is indeed rotationally invariant, except for a phase factor from our spinor conventions, due to the fact that the scattering plane is not parallel to the $\hat{z}$-axis. If we require $k_\perp \parallel \pm l_\perp$, the amplitude is purely real. We will impose this additional constraint in our later calculations. The kinematically allowed region of the xy-plane is shown (with and without this restriction) in Fig. 6.

Now we can implement the program of using the requirement of rotational invariance to constrain the meson wavefunction. The first step is to calculate the

Figure 6. The kinematically allowed region of the xy-plane for some values of t/s. In each case, the part of the boundary given by the heavy solid line is allowed when $k_\perp \parallel l_\perp$.

Figure 7. Part of the K^* photodissociation amplitude. (a) shows the Feynman diagram, (b) the associated LCPTh diagrams.

t-channel contribution, shown in Fig. 7, to the amplitude for $\gamma_\uparrow K^*_\uparrow \to s_+ \bar{d}_+$ in the massless approximation:

$$\mathcal{M} = -eq_s\sqrt{p_s^+ p_{\bar{d}}^+}\Big\{\Big[\frac{\bar{u}_+(p_s)}{\sqrt{\bar{x}}}\vec{\gamma}\cdot\vec{\epsilon}\frac{u_+(p_s-p_\gamma)}{\sqrt{y-x}}\Big]\theta(y-x)\psi_{K^*_\uparrow\to\bar{d}_+ s_+}\big(\frac{x}{y},-\frac{x}{y}t\big)$$
$$+\Big[\frac{\bar{u}_+(p_s)}{\sqrt{\bar{x}}}\vec{\gamma}\cdot\vec{\epsilon}\frac{v_-(p_\gamma-p_s)}{\sqrt{x-y}}\Big]\theta(x-y)\psi^*_{\bar{d}_+\to K^*_\uparrow\bar{s}_-}\big(\frac{y}{x},-\frac{y}{x}t\big)$$
$$+\Big[\frac{\bar{u}_+(p_s)}{\sqrt{\bar{x}}}\vec{\gamma}\cdot\vec{\epsilon}\zeta_+\Big]\Big(\frac{p_{\bar{d}R}}{x}-\frac{p_{K^*R}}{y}\Big)\tilde{\psi}_{K^*_\uparrow(s_+)\bar{d}_+}\big(\frac{x}{y},-\frac{x}{y}t\big)\Big\};$$

the three terms in braces give the contributions from the wavefunction, fragmentation amplitude, and instantaneous exchange amplitude, respectively. Our notation is conventional, except that we have used $k_\perp^2$ rather than $k_\perp$ as the argument for the wavefunctions, for the sake of brevity. Note that the definition of $k_\perp$ in the fragmentation amplitude differs from that used in the wavefunctions.

Inserting the explicit representation of eq. (11) and the spinors of Ref. [1], we can evaluate this expression:

$$
\begin{aligned}
\mathcal{M} = &-2eq_s\sqrt{2x\bar{x}y\bar{y}}\Big(\frac{l_L}{y}\hat{\epsilon}_R - \frac{l_R}{\bar{y}}\hat{\epsilon}_L, 1\Big) \\
&\cdot\Bigg\{\Big(\frac{l_R - k_R}{y-x}\hat{\epsilon}_L - \frac{k_L}{\bar{x}}\hat{\epsilon}_R, \frac{1}{2} + \frac{k_L(l_R-k_R)}{\bar{x}(y-x)}\Big)\theta(y-x)\psi_{K^*_\uparrow\to\bar{d}_+s_+}\big(\frac{x}{y}, -\frac{x}{y}t\big) \\
&\quad + \Big(\frac{k_R - l_R}{x-y}\hat{\epsilon}_L - \frac{k_L}{\bar{x}}\hat{\epsilon}_R, \frac{1}{2} - \frac{k_L(l_R-k_R)}{\bar{x}(x-y)}\Big)\theta(x-y)\psi^*_{\bar{d}_+\to K^*_\uparrow\bar{s}_-}\big(\frac{y}{x}, -\frac{y}{x}t\big) \\
&\quad + \Big(\hat{\epsilon}_L, \frac{k_L}{\bar{x}}\Big)\Big(\frac{k_R}{x} - \frac{l_R}{y}\Big)\tilde{\psi}_{K^*_\uparrow(s_+)\bar{d}_+}\big(\frac{x}{y}, -\frac{x}{y}t\big)\Bigg\} \\
= &-eq_s\sqrt{2}\Big\{(x\bar{x}y + x^2\bar{y} - 2x\sqrt{x\bar{x}y\bar{y}})\frac{\theta(y-x)}{y-x}\psi_{K^*_\uparrow\to\bar{d}_+s_+}\big(\frac{x}{y}, -\frac{x}{y}t\big) \\
&\quad + (2x\sqrt{x\bar{x}y\bar{y}} - x\bar{x}y - x^2\bar{y})\frac{\theta(x-y)}{x-y}\psi^*_{\bar{d}_+\to K^*_\uparrow\bar{s}_-}\big(\frac{y}{x}, -\frac{y}{x}t\big) \\
&\quad + (2\sqrt{x\bar{x}y\bar{y}} - \bar{x}y - x\bar{y})\tilde{\psi}_{K^*_\uparrow(s_+)\bar{d}_+}\big(\frac{x}{y}, -\frac{x}{y}t\big)\Big\}.
\end{aligned} \tag{12}
$$

In the last step, we have made explicit the assumption that $k_\perp \parallel l_\perp$. The requirement that the physics of this scattering process be rotationally invariant implies that $\mathcal{M}$ is a function of the Mandelstam invariants t and u only, for any x and y in the kinematically allowed region

$$(ys + \bar{x}t + xu)^2 \leq 4utx\bar{x}.$$

The restriction $k_\perp \parallel l_\perp$ restricts us to the boundary of the allowed region; along this boundary, the phase and magnitude of $\mathcal{M}$ are constant.

The first step in extracting the consequences of this independence is to consider the two limits $x \to 1$ (which requires $y \to -u/s$) and $y \to 1$ (whence $x \to -u/s$). The equality of the amplitude in these two cases yields the requirement

$$
\begin{aligned}
-\hat{u}\psi_{K^*_\uparrow \to \bar{d}_+ s_+}(\hat{u}, \hat{u}\hat{t}s) + \hat{t}\tilde{\psi}_{K^*_\uparrow (s_+)\bar{d}_+}(\hat{u}, \hat{u}\hat{t}s) \\
= \psi^*_{\bar{d}_+ \to K^*_\uparrow \bar{s}_-}(\hat{u}, \hat{u}\hat{t}s) + \hat{t}\tilde{\psi}_{K^*_\uparrow (s_+)\bar{d}_+}(\hat{u}^{-1}, \hat{u}^{-1}\hat{t}s),
\end{aligned}
\tag{13}
$$

where we have defined $\hat{u} = -u/s$ and $\hat{t} = -t/s$.

To obtain another, similar constraint, we consider the process $\gamma_\downarrow K^*_\uparrow \to s_+ \bar{d}_+$. The calculation proceeds in identical manner, and we obtain the result

$$
\hat{u}\psi_{K^*_\uparrow \to \bar{d}_+ s_+}(\hat{u}, \hat{u}\hat{t}s) = -\hat{t}\tilde{\psi}_{K^*_\uparrow (s_+)\bar{d}_+}(\hat{u}^{-1}, \hat{u}^{-1}\hat{t}s). \tag{14}
$$

Combined with eq. (13), this implies that

$$
\hat{t}\tilde{\psi}_{K^*_\uparrow (s_+)\bar{d}_+}(\hat{u}, \hat{u}\hat{t}s) = \psi^*_{\bar{d}_+ \to K^*_\uparrow \bar{s}_-}(\hat{u}, \hat{u}\hat{t}s). \tag{15}
$$

We might hope to obtain an additional constraint by considering the process $K^*_\uparrow \phi \to s_- \bar{d}_+$ for a scalar 'photon' ϕ [7]. However, the constraint thus derived is merely eq. (15); the calculation provides a check of our results, but yields no new information.

We can now substitute eqs. (14) and (15) back into eq. (12) to eliminate the dependence of $\mathcal{M}$ on $\tilde{\psi}$. Ignoring an overall factor of $-eq_s\sqrt{2}$, we now have

$$
\begin{aligned}
\mathcal{M} = x\big(\sqrt{\bar{x}y} - \sqrt{x\bar{y}}\big)^2 \, \frac{\theta(y-x)}{y-x} \Big[\psi_{K^*_\uparrow \to \bar{d}_+ s_+}\big(\frac{x}{y}, -\frac{x}{y}t\big) - \frac{y}{x}\psi^*_{\bar{d}_+ \to K^*_\uparrow \bar{s}_-}\big(\frac{x}{y}, -\frac{x}{y}t\big)\Big] \\
+ (x \leftrightarrow y).
\end{aligned}
\tag{16}
$$

Since this form is manifestly symmetric under $x \leftrightarrow y$, eqs. (13)–(15) encapsulate all of the consequences of the symmetry of $\mathcal{M}$ under $x \leftrightarrow y$. However, much more information is contained in eq. (12). To make eq. (16) clearer, we parametrize the momenta by $\hat{u}$, $k_\perp \equiv \sqrt{-t}$, and $z \equiv x/y$; for the moment we will assume $z < \hat{u}$. Then the constraint $k_\perp \parallel l_\perp$ requires

$$y = \frac{1-\hat{u}}{1+z-2\sqrt{\hat{u}z}} \qquad \Rightarrow \quad \frac{x}{y-x}\left(\sqrt{\bar{x}y} - \sqrt{x\bar{y}}\right)^2 = \frac{z}{1-z}(1-\hat{u}).$$

Inserting this result into eq. (16), we obtain a sum rule relating the wavefunction and fragmentation amplitude in the region $z < \hat{u}$. We can repeat the preceding analysis with $k_\perp \parallel -l_\perp$ to probe the region $z > \hat{u}$. In either case, we obtain the same result:

$$\frac{z\psi_{K^*_\uparrow \to \bar{d}_+ s_+}(z, \sqrt{z}k_\perp) - \psi^*_{\bar{d}_+ \to K^*_\uparrow \bar{s}_-}(z, \sqrt{z}k_\perp)}{1-z} = \frac{\mathcal{M}}{1-\hat{u}} = F(k_\perp), \qquad (17)$$

where F does not depend on z or $\hat{u}$ [8], and z can have any value in the allowed region $0 < z < 1$.

While our choice of the K^* meson gave us a concrete example with which to work, our results in no way depend on the nature of the meson in question. In addition, the above computation yields the same results regardless of the helicity of the struck quark. The only dependence on the spin properties of the particles arises from the fact that the argument of transverse momentum $k_\perp$ in the fragmentation amplitude $\psi^*_{\bar{q}}$ is antiparallel to that used in the wavefunctions. Taking this into account, we obtain the final result

$$\begin{aligned} \tilde{\psi}_{h_\lambda(Q_s)\bar{q}_{s'}}(z, k_\perp) = & \frac{\theta(1-z)}{1-z}(-1)^{\lambda-s-s'}\psi^*_{\bar{q}_{s'} \to h_\lambda \bar{Q}_{-s}}(z, k_\perp) \\ & - \frac{\theta(z-1)}{z-1}\psi_{h_\lambda \to \bar{q}_{s'} Q_s}(z^{-1}, z^{-1}k_\perp), \end{aligned} \qquad (18)$$

valid for $z > 0$.

Thus the instantaneous wavefunction for any meson in any spin state is entirely determined in terms of the ordinary wavefunction and the fragmentation amplitude. This simplification should greatly advance the calculation of scattering processes which mix free and bound states, in which instantaneous contributions cannot be ignored.

One noteworthy feature of eq. (18) is that the instantaneous contributions do not vanish as $z \to 1$. This is sensible, since the vanishing of the conventional wavefunction is due to the divergent energy denominator $k_\perp^2/z\bar{z}$; no such denominator appears in the instantaneous interaction.

The constraint (17) is equally general; we have the result

$$\frac{z\psi_{h_\lambda \to \bar{q}_{s'} Q_s}(z, \sqrt{z}k_\perp) - (-1)^{\lambda - s - s'} \psi^*_{\bar{q}_{s'} \to h_\lambda \bar{Q}_{-s}}(z, \sqrt{z}k_\perp)}{1-z} = F(k_\perp). \tag{19}$$

One should keep in mind that eqs. (18) and (19) are subject to errors on the order of $\mu/|k_\perp|$, where μ is a typical mass for the particles in question; thus they are best applicable to light mesons at large momentum transfer. For example, the relationship between distribution amplitudes

$$\frac{z\phi_{h \to \bar{q}Q}(z) - \phi^*_{\bar{q} \to h\bar{Q}}(z)}{z(1-z)} = \text{constant}, \tag{20}$$

obtained by integrating over $k_\perp$ in eq. (19), is subject to errors from the region in which $k_\perp$ is small.

III. CONCLUSIONS

We have introduced and defined the instantaneous wavefunctions, eq. (7), by which mesons partake in interactions involving the exchange of instantaneous fermions. They take a remarkably simple form, requiring no consideration of the higher Fock states involved in the meson wavefunction.

Using these results, we have written rotationally invariant LCPTh amplitudes corresponding to a single Feynman diagram. In this manner, we have been able to relate the newly introduced instantaneous wavefunctions to the well-known propagating wavefunctions by the relation in eq. (18); thus the consideration of instantaneous interactions introduces no new degrees of freedom into the calculation of hadronic amplitudes.

Finally, we have used the form thus derived for the instantaneous wavefunction to express rotationally invariant quantities in terms of the conventional wavefunction and fragmentation amplitude alone; from this, we have derived a constraint, eq. (19), on the behavior of these functions.

ACKNOWLEDGMENTS

We would like to thank S. Brodsky for many helpful conversations.

REFERENCES

[1] G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).

[2] These are distinct from 'simultaneous' interactions, which arise from the fact that the hadronic time-evolution operator does not commute with the hard-scattering Hamiltonian. The latter do not contribute to hadronic processes at leading twist, while instantaneous terms do.

[3] S. J. Brodsky, R. J. Roskies, and R. Suaya, *Phys. Rev.* **D8**, 4574 (1973).

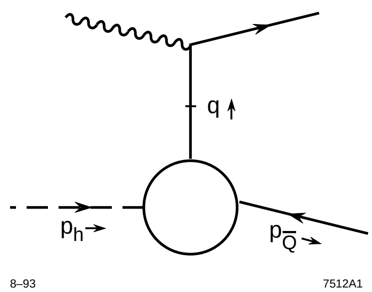
[4] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 675 (1972).

[5] See *e.g.* S. J. Brodsky and T. Hyer, SLAC-PUB-6219 (in preparation).

[6] This can be proved by identifying the square of the term in brackets as $-u/s$.

[7] The s helicity in the final state is now negative because the interaction with a scalar flips fermion spins. Thus this process again probes the wavefunction $\psi_{K^*_\uparrow \to \bar{d}_+ s_+}$.

[8] Since the left-hand side of the equation does not depend on $\hat{u}$, F must be a function of $k_\perp$ only. Thus we obtain the additional result $\mathcal{M} = (-t/s)F(k_\perp) = f(t)/s$ at large momentum transfer t, which is the expected Regge behavior for an amplitude resulting from the exchange of a single spinor particle.



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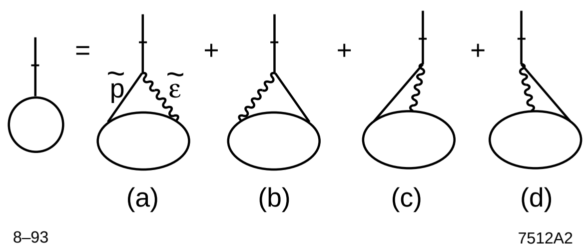

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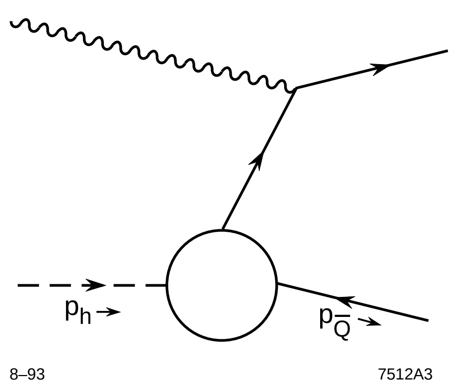

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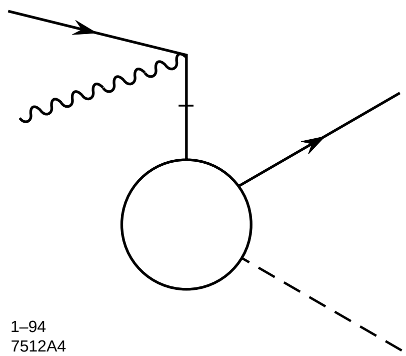

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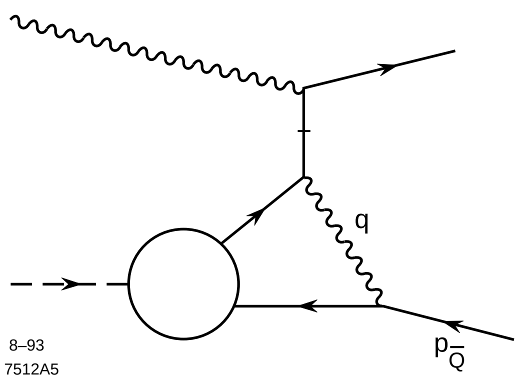

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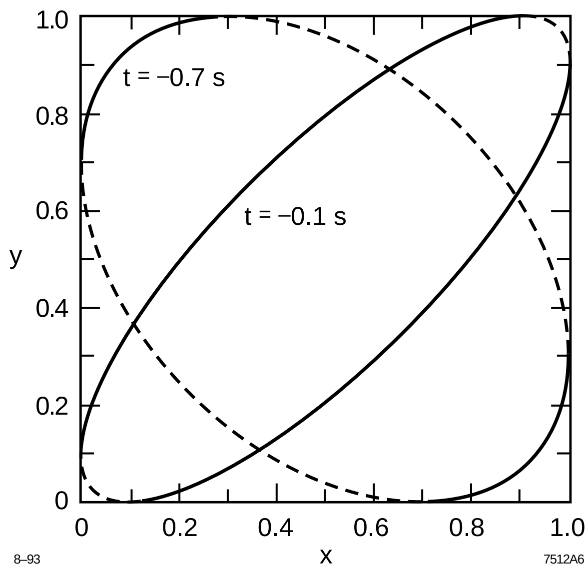

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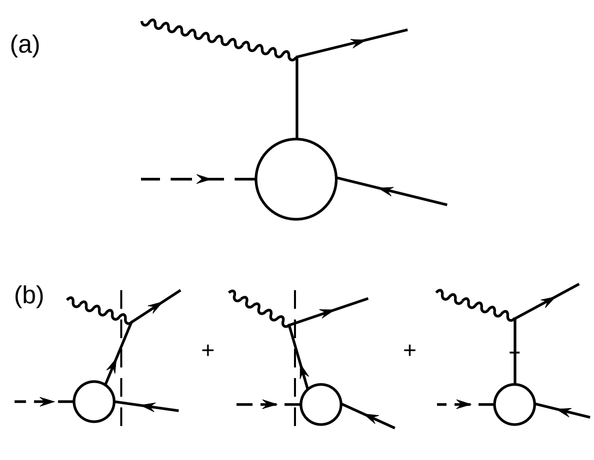

Figure 7: