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All-Order Quantum Gravity in Two Dimensions*

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ABSTRACT

We derive curvature counterterms in two-dimensional gravity coupled to conformal matter up to infinite order. By construction the higher-order action is equivalent to a finite first-order theory with an auxiliary scalar. Due to this equivalence it shares the following remarkable properties: There is no need for gravitational dressing of the cosmological constant, quantization is consistent for any conformal anomaly c of the coupled matter system, and if the coupled matter system is a $c = d$ -dimensional string theory in a Euclidean background then the effective string theory is $D = d + 2$ -dimensional with Minkowski signature $(1, D-1)$. The resulting quantum theory favours flat geometries and suppresses both parabolic and hyperbolic singularities.

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1. Introduction

The quantum theory of gravity remains one of the deep mysteries of fundamental physics. Recent years have seen enormous effort to understand at least the case of a two-dimensional universe [1]. There, at the classical level the Einstein-Hilbert action is trivial, but at the quantum level the Polyakov action shows up and causes the familiar difficulties if the system is coupled to matter [2]. In the conformal gauge the Polyakov action turns into the Liouville action. The quantum theory of this was first considered in [3,4] and a conformal field theory (CFT) treatment was given by David, Distler and Kawai [5]. The occurrence of the Polyakov/Liouville action is most easily understood in the framework of path integrals by using the conformal gauge and going to a translation invariant measure [3,5-7].

The central difficulty in quantizing two-dimensional gravity coupled to matter is the 'Liouville problem'. It arises with the only non-trivial term of the two-dimensional Einstein-Hilbert action: the cosmological constant. Coupling two-dimensional gravity to a conformal matter system of anomaly c, the matter system and gauge fixing contributions will induce the effective gravity action [3,2]

$$S(g) = \frac{1}{\pi} \int d^2x \sqrt{g} \;\; (Q_0 \; R\Delta^{-1}R + \lambda + \eta R) \; + \; S_C(g) \tag{1.1}$$

where $Q_0 = (26-c)/24$, R is the curvature in terms of the metric $g_{\alpha\beta}$, $\Delta = \frac{-1}{\sqrt{g}}\partial_\alpha \sqrt{g} g^{\alpha\beta}\partial_\beta$ and λ, η are arbitary constants. The first term is the Polyakov action (the Q_0 will be shifted after quantizing the metric), the next two terms give the Einstein-Hilbert action. If the system is defined on a closed manifold of genus h then $\frac{1}{2\pi}\int d^2x\sqrt{g}R = 2(1-h)$, the Euler number. (We use the conventions of [7].) The S_C is supposed to collect possible counterterms. Using the conformal gauge $g_{\alpha\beta} = e^{2\sigma}\hat{g}_{\alpha\beta}$ where $\hat{g}_{\alpha\beta}$ is a background-metric, quantizing the Liouville-mode σ and applying CFT methods to (1.1) without counterterms, the cosmological constant $\sqrt{g}\lambda = \sqrt{\hat{g}}e^{2\sigma}\lambda$ is seen to be not of weight (1,1) [5]. Therefore the action (1.1) is not a scalar, and general covariance is broken at the quantum level unless appropriate counterterms S_C are found. This is the 'Liouville problem'.

The usual consequence is to replace the cosmological constant by a 'dressed' version which is of weight (1,1) [5]

$$e^{2\sigma} \quad \rightarrow \quad e^{2\kappa\sigma} \qquad , \qquad \kappa = \frac{1}{12}\left(25 \; - \; c \; - \; \sqrt{(1-c)(25-c)}\right) . \tag{1.2}$$

This approach, however, has major drawbacks: the theory loses its geometrical character, i.e., the renormalized action can no longer be formulated in terms of the metric $g_{\alpha\beta}$ if $\kappa \neq 1$ and the dressed cosmological constant has to be complex if $1 < c < 25$ so that the renormalization (1.2) becomes senseless in this region.

Given these difficulties it is rather natural to search for an alternative renormalization program: the inclusion of higher-order counterterms. Such a renormalization procedure will be the content of this paper. Instead of (1.2) we will arrive at counterterms of the form

$$S_C(g) = \frac{Q}{2\pi\alpha} \int d^2x\sqrt{g}\ \sum_{m=2}^{\infty} q_m\ (\alpha R)^m \tag{1.3}$$

where $Q = (24-c)/24, \alpha \sim (length)^2$ is some renormalization scale and q_m are the coefficients that will be determined in what follows. We will find the renormalization $Q_0 \to Q$ in (1.1) after quantizing the gravity sector. Notice that in (1.3) no terms show up with derivatives acting on the curvature. This will be a feature of our result but may have been expected from the beginning by carefully reexamining the principle of ultralocality stated by Polchinski [8,7]. (We come back to this point in the summary.)

Many attempts have been made to avoid the problems associated with (1.2): W-gravity, quantum groups and others. It is fair to say that none of them succeeded so far. Recently, however, it was examined in greater detail that another problematic feature of two-dimensional gravity, the instabilities of surfaces related to covering with spikes and branched polymers is cured at scales $\ll 1/\mu,\ \mu \sim (length)^{-2}$ if a $(1/\mu)R^2$-term is added to (1.1) [9]. This is easily understood by realizing that the quadratic term suppresses arbitrary high curvature. In [9] the R^2-theory was studied by introducing an auxiliary field ϕ and analyzing instead the action $\int d^2x\sqrt{g}(\phi R + \mu\phi^2)$. Combining this with the results of [10] it can be seen immediately that the R^2-term is even enough to assure that the cosmological constant is of weight (1,1) without the shift (1.2) (see (2.7) and (2.8) below). However, new problems arise with the ϕ^2-term. In the conformal gauge $\sqrt{\hat{g}}e^{2\sigma}\phi^2$ will not be of weight (1,1). This is why the success of [9] would be complete only if $\mu \to 0$. This limit, however, establishes the constraint of vanishing curvature which may conflict with the Gauss-Bonnet theorem and therefore requires additional renormalization [10,11]. The potential ϕ^2 is ill-defined. Instead we have to work with another potential. This will be our starting point in the following. In summary, we take the point of view that the partial success of the R^2-theory is due to including a quadratic curvature term that is already enough to avoid (1.2) and suppresses arbitrary high curvature, the failure however is due to neglecting terms of even higher order.

Apparently a quantum analysis with (1.3) is a tough problem. Fortunately, there is a short cut. Let us write the counterterms in (1.1) as (we work with

Euclidean signature)

$$e^{-S_C(g)} = \int D_g \phi \; e^{-\widetilde{S}_C(g,\phi)} \tag{1.4}$$

This will allow us to use recent progress made in understanding two-dimensional gravity in the presence of an additional scalar ϕ, a 'dilaton'. In particular, we may use that if the metric-dilaton action is taken to be

$$\widetilde{S}_C(g,\phi) \;=\; \frac{1}{2\pi} \int d^2x \sqrt{g} \; (\; Q\phi R + \mu e^{\phi} \;) \tag{1.5}$$

then the Liouville problem is absent, (1.1) with (1.4) describes a well-defined quantum theory, and quantization is possible for any conformal anomaly c of the coupled matter system [10]. Contrary to the R^2-theory that corresponds to a ϕ^2-potential [9], the exponential potential in (1.5) describes a consistent quantum theory. Many aspects of the remarkable success related to (1.5) have reappeared in other frameworks including black hole physics [12,13]. (Using conformal gauge the action (1.1) turns into a non-linear sigma model, see (2.10) below. The metric-dilaton theories of [13] can be written in the same form. However, contrary to the discussion here, the relation between $X^{\pm}$, the Liouville mode σ, and the dilaton in [13] is such that $X^{\pm}$ are bounded and therefore the quantum theories in [13] are incomplete.) A recent study in terms of non-linear sigma models may be found in [14].

The dilaton ϕ appearing in (1.5) is an auxiliary field. It is therefore natural to eliminate it and ask for the form of the counterterms $S_C(g)$ that have to be included in (1.1). Doing so will be the content of this paper. Indeed, we find that the integration (1.4) can be performed explicitly. The metric-scalar theory (1.5) may then be understood as a first-order formulation of the higher-order theory with $S_C(g)$ and (1.1) may be seen as the effective action obtained from integrating out the auxiliary field.

In section 2 we review the cornerstones of the CFT analysis that reveal why (1.4) with (1.5) leads to a consistent quantum theory such that there is no need for the gravitational dressing (1.2). We also argue for the finiteness of (1.1) with (1.4), (1.5) from a diagrammatic point of view. In section 3 we compute the higher-order counterterms $S_C(g)$ by explicitly integrating out the auxiliary field in (1.4). Performing a path-integration requires some regularization of the surface. We work with standard triangulations. In section 4 we comment on the surprisingly (and convincingly!) sensible geometrical content of the resulting higher-order theory. Section 5 contains our summary.

2. The Gravity Action with Auxiliary Field

Let us use (1.4) to express $S_C(g)$ in terms of the first-order action (1.5). We will now shortly review that (1.4) with (1.5) indeed provides a solution of the Liouville problem. A detailed discussion can be found in [10]. We will also give a simple diagrammatic argument for the finiteness of the quantum theory. Notice that although the scalar in (1.5) is an auxiliary field it will obtain a mixed kinetic term with the Liouville mode in the conformal gauge. This gauge will be used in this section.

Upon quantizing the metric, the Liouville measure together with the dilaton-measure in (1.4) induces a renormalization of the coefficient in (1.1):

$$Q_0 \quad \rightarrow \quad Q \;=\; \frac{24-c}{24} \tag{2.1}$$

In the conformal gauge $g_{\alpha\beta} = e^{2\sigma}\hat{g}_{\alpha\beta}$ the curvature is $R = e^{-2\sigma}(\widehat{R} + \widehat{\Delta}\sigma)$ and so the effective gravity action (1.1) with (2.1), (1.4) and (1.5) reads

$$\begin{aligned} S(\sigma,\phi) = \frac{Q}{\pi}\int d^2x\sqrt{\hat{g}}\;(\;\hat{g}^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma + 2\sigma\widehat{R} \\ + \frac{1}{2}\hat{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\sigma + \frac{1}{2}\phi\widehat{R} + V(\sigma,\phi)) \end{aligned} \tag{2.2}$$

with potential

$$V(\sigma,\phi) = \lambda e^{2\sigma} \;+\; \mu e^{2\sigma+\phi}\;. \tag{2.3}$$

In (2.2) we rescaled λ, μ and did not write pure background terms. Using the local gauge $\hat{g}_{\alpha\beta} = \delta_{\alpha\beta}$ the stress-energy tensor $\widehat{T}_{\alpha\beta} = \frac{-4\pi}{\sqrt{\hat{g}}}\frac{\delta S}{\delta\hat{g}^{\alpha\beta}}$ is

$$\begin{aligned} \widehat{T}_{\alpha\beta} \;&= 2Q(\delta_{\alpha\beta}(\hat{\partial}^\gamma\sigma\partial_\gamma\sigma + \frac{1}{2}\hat{\partial}^\gamma\phi\partial_\gamma\sigma + V(\sigma,\phi)) \\ &\quad - 2\partial_\alpha\sigma\partial_\beta\sigma + 2\partial_\alpha\partial_\beta\sigma - 2\delta_{\alpha\beta}\hat{\partial}^2\sigma \\ &\quad + \frac{1}{2}(\partial_\alpha\partial_\beta\phi - \delta_{\alpha\beta}\hat{\partial}^2\phi - \partial_\alpha\phi\partial_\beta\sigma - \partial_\beta\phi\partial_\alpha\sigma)) \end{aligned} \tag{2.4}$$

The reason for writing $S_C(g)$ in its first-order form (1.4) with (1.5) is that the effective action (1.1) turns into (2.2) which can be analyzed by using simple CFT methods (for an easy review of these see [15]). Applying the standard David, Distler and Kawai procedure [5], we treat the potential terms (2.3) as a perturbation. Then

(2.2) without (2.3) is a CFT and, using complex coordinates, the σ and ϕ-fields obtain mixed propagators

$$\begin{aligned} < \phi(z,\bar{z})\phi(w,\bar{w}) > &= \frac{2}{Q} \ln |z-w|^2 \\ < \phi(z,\bar{z})\sigma(w,\bar{w}) > &= \frac{-1}{2Q} \ln |z-w|^2 \\ < \sigma(z,\bar{z})\sigma(w,\bar{w}) > &= 0 \end{aligned} \tag{2.5}$$

For the analytic part of (2.4) the operator product expansion (OPE) gives

$$: \widehat{T}(z) : : \widehat{T}(w) : = \frac{\frac{1}{2}(26-c)}{(z-w)^4} + \left[\frac{2}{(z-w)^2} + \frac{\partial_w}{z-w}\right] : \widehat{T}(w) : + ... \tag{2.6}$$

The complete system is free of the conformal anomaly since the anomaly $(26-c)$ arising in (2.6) just cancels the anomaly coming from the matter and gauge fixing sector.

The crucial test for consistency is whether the terms (2.3) can be included without breaking general coordinate invariance. This is where (1.1) without counterterms fails and eventually leads to (1.2). However, in the presence of (1.4) with (1.5), the propagators (2.5) imply that

$$: \widehat{T}(z) : : e^{2\alpha\sigma(w)+\beta\phi(w)} := \left[\frac{1}{(z-w)^2} + \frac{\partial_w}{z-w}\right] : e^{2\alpha\sigma(w)+\beta\phi(w)} : \tag{2.7}$$

has indeed solutions with $\alpha = 1$:

$$(\alpha, \beta) \quad = \quad (1,0) \quad \text{or} \quad (1,1)\ . \tag{2.8}$$

The first solution corresponds to the cosmological constant in (1.1), the second to the scalar self-coupling in (1.5). Thus the terms in (2.3) are of weight (1,1), they do *not* violate general covariance at the quantum level, there is *no* need for gravitational dressing and *no* restriction on the matter anomaly c appears!

In this paper, we are approaching a higher-order theory of gravity. As a consequence the metric propagator may include unphysical modes due to higher-order derivatives:

$$\frac{1}{\partial^4} \quad = \quad \frac{1}{2i\epsilon}\left(\frac{1}{\partial^2 - i\epsilon} - \frac{1}{\partial^2 + i\epsilon}\right) \quad , \quad \epsilon \to 0 \tag{2.9}$$

A natural question to worry about is whether these will induce a ghost problem. Again this question is most easily studied in the first-order formulation using (2.2).

With $X^+ = \sqrt{Q}\ (2\sigma + \phi),\ X^- = \sqrt{Q}\ 2\sigma$ the action (2.2) can be written

$$\begin{aligned} S(\sigma,\phi) \ = \ \frac{1}{2\pi}\int d^2x\sqrt{\hat g}\Big(\frac{1}{2}\hat\partial^\alpha X^+\partial_\alpha X^- + \sqrt{Q}(X^+ + X^-)\widehat{R} \\ + 2\lambda Q e^{X^-/\sqrt{Q}} + 2\mu Q e^{X^+/\sqrt{Q}}\Big)\ . \end{aligned} \tag{2.10}$$

If the coupled matter system is a c-dimensional string theory in a Euclidean background the gravitational part (2.10) with $X^\pm = X^0 \pm X^{c+1}$ turns this into a D=c+2-dimensional string theory in a background with Minkowski signature $(1, D-1)$. The negative contributions related to (2.9) cause no problems. The propagators of (2.2), (2.10) are given by (2.5) and are well-defined without violating causality.

Notice that supersymmetrization is straightforward. To eliminate tachyons appearing in a string theory like (2.10) the action (1.5) may be replaced by its supersymmetric version using the results of [16].

The finiteness of (2.2), (2.10) may also be understood in a diagrammatic way. Whenever we work with a two-dimensional field-theory with second-derivative kinetic term and polynomial potential, a diagram with P internal lines and V vertices will have the superficial degree of divergency

$$\omega \ = \ 2\ (P - [V-1]) \ - \ 2\ P \ = \ 2\ (1-V)\ . \tag{2.11}$$

Only diagrams with $V = 1$ (or with $V = 1$ subdiagrams) can be divergent (see fig. 1). Such diagrams will appear if no $S_C(g)$ is included in (1.1). Then in the conformal gauge the cosmological constant together with the $<\sigma\sigma>$-propagator will induce such divergent diagrams. It is therefore not surprising that a renormalization (1.2) may occur. What happens if an R^2-term is included? Such a term will have an enormous impact due to higher derivatives like (2.9) contributing to the metric propagator. Again the analysis may be simplified by using an auxiliary field ϕ. In the conformal gauge the R^2-term will then result in an action like (2.10) with the replacement

$$e^{X^+/\sqrt{Q}} \qquad \rightarrow \qquad Q^{-1}\ e^{X^-/\sqrt{Q}}\ (X^+ - X^-)^2 \tag{2.12}$$

The propagators are given by (2.5). Now the Liouville propagator vanishes and in terms of $X^\pm$ the only non-zero propagator is $<X^+X^->$. It is then immediately obvious that the cosmological constant no longer causes a problem: using the vertices $\lambda e^{X^-/\sqrt{Q}}$ no divergent diagram can be drawn. This is the reason for the first solution in (2.8). On the other hand there is a new problem arising with (2.12).

The quadratic ϕ-potential introduces vertices with both X^+ and X^- legs. Closing these with $< X^+X^- >$ gives loop-diagrams of the divergent type. Therefore additional potential terms have to cancel these diagrams:

$$\sum_{n=0}^{\infty} \frac{Q^{-n/2}}{n!} e^{X^-/\sqrt{Q}} (X^+ - X^-)^n = e^{X^+/\sqrt{Q}} . \tag{2.13}$$

With (2.13) no divergent loop-diagram can be constructed and no additional counterterms are needed! This is again the potential of (2.10) and the reason why the action has to be of the form (1.5). The potential (2.13) corresponds to the second solution in (2.8). For $S_C(g)$ this implies that terms of order higher than R^2 have to be included. These terms will be derived in the next section.

Before calculating the integral (1.4) it may be worthwhile to remember the equivalence between higher-order gravity and first-order formulations with an additional scalar ϕ at the classical level. A higher-order gravity theory, given by some arbitrary function of the curvature

$$\frac{1}{2\pi\alpha} \int d^2x \sqrt{g}\, f(R) \tag{2.14}$$

where $\alpha \sim (length)^2$, can be turned into a first-order theory provided that the definition

$$\phi = \frac{1}{\alpha}\, f'(R) \tag{2.15}$$

with $' = \frac{d}{dR}$ allows to solve for R in terms of ϕ. Then a potential can be defined by

$$V(\phi) = \frac{1}{\alpha}\big(\, f(R) - Rf'(R)\, \big) \tag{2.16}$$

and (2.14) turns out to be equivalent to the first-order system

$$\frac{1}{2\pi} \int d^2x \sqrt{g}\, (\phi R + V(\phi)) \tag{2.17}$$

Therefore a higher-order gravity theory can be formulated as a first-order system by adding a scalar. This was first observed by Higgs [17], later rediscovered by Whitt [18] for $f(R) = R + \alpha R^2$ in $D = 4$ (of course the above procedure is possible for any space-time dimension) and has been extended to higher powers of Ricci and Riemann tensors in [19]. A recent application is [20] where (2.17) has been used to study four-dimensional black hole solutions in higher-derivative gravity.

Our first-order action (1.5) is indeed of the form (2.17). Assuming $\mu \sim 1/\alpha$ and neglecting topological terms, its classical analog (2.14) is given by

$$f(R) \;=\; Q\;\alpha R\;\ln\,(\alpha R)\;. \tag{2.18}$$

This action does not seem to make sense, in particular around zero curvature. The equivalence between (2.14) and (2.17) is however only a classical one. At the quantum level the corresponding equivalence must be established by integrating out the dilaton. Only if $V(\phi) \sim \phi^2$ will the result of this integration agree with the classical procedure. We have seen that in two dimensions $V(\phi)$ has to be exponential and so there is no alternative to performing the integration. This integration will be subject of the next section and will result in a higher-order quantum action clearly more acceptable than (2.18).

3. The Higher-Order Gravity Action

We now return to (1.4) and integrate out the auxiliary field. This will lead to the explicit form of the counterterms $S_C(g)$. In the last section we saw that using the action (1.5) in (1.4) ensures that we arrive at a consistent quantum theory (1.1) with no need for gravitational dressing (1.2). The absence of a kinetic term for ϕ in (1.5) is linked to the propagators (2.5) and thus essential for the quantum consistency reflected in the OPEs (2.6), (2.7). As a consequence the dilaton ϕ is an auxiliary field, there is no damping of arbitrary high frequencies and it is natural to eliminate it. Any path integration requires some regularization. We will work with triangulations (for an introduction to these see [21]).

We regularize the surface by a triangulation with V vertices, E edges and F faces (fig.2). The triangles are assumed to be of area $\pi\alpha$, where $\alpha \sim (length)^2$ is the regulator that will enter in (1.3). The total area A of the surface is

$$A = \int d^2x\sqrt{g} \simeq F\cdot\pi\alpha \tag{3.1}$$

The α is a kind of UV-cutoff. The continuum limit is obtained from $\alpha \to 0, F \to \infty$. The basic identities relating V, E and F are $V - E + F = 2(1-h)$ and $2E = 3F$. The regularization is then established by the replacements

$$\begin{gathered}\int d^2x\sqrt{g(x)}\;(...) \simeq \sum_{i=1}^{V}\; s_i\;(...)\quad,\quad s_i \;=\; \frac{N_i}{3}\;\pi\alpha \\ R(x)\;\simeq\;R_i\;=\;\frac{2\pi}{s_i}\left(1-\frac{s_i}{2\pi\alpha}\right)\end{gathered} \tag{3.2}$$

where N_i counts the nearest neighbours of vertex i. Moreover, we have to regulate the measure in the path-integral. With (3.2) we get for scalar fields X with value

X_i at vertex i

$$\parallel \delta X \parallel^2 \simeq \sum_{i=1}^{V} s_i \; (\delta X_i)^2 \; . \tag{3.3}$$

Using (3.2), (3.3) we may write (1.4) as

$$e^{-S_C(g)} \simeq \prod_{i=1}^{V} \sqrt{s_i} \int_C d\phi_i \quad e^{-\widetilde{S}_C(s_i,\phi_i)} \;\; = \;\; \prod_{i=1}^{V} \sqrt{s_i}\; e^{-S_C(i)} \tag{3.4}$$

where

$$\widetilde{S}_C(s_i,\phi_i) = \frac{s_i}{2\pi}\Big(Q\phi_i R_i + \mu e^{\phi_i}\Big) \; . \tag{3.5}$$

The path-integral factorizes which will be essential for performing the integration. We see immediately that no terms like $\alpha^3 R\Delta R, ...$ with Δ acting on R will appear and the counterterms will indeed be of the form (1.3).

We have to specify the integration contour C. To integrate along the real axis seems to be senseless. This is obvious after recognizing the structure of a Γ-function:

$$\int_{-\infty}^{+\infty} d\phi \; e^{a\phi \; - \; be^{\phi}} \quad = \quad e^{-a\ln b}\; \Gamma(a) \quad , \quad Re\; a > 0 \quad (b > 0) \; . \tag{3.6}$$

This integral is divergent for Re $a \leq 0$. Applied to (3.4) this would imply that whenever a given geometry has one vertex with $QR_i \geq 0$ the ϕ-integration would be divergent.

One should not try to establish convergence by changing the integrand in (3.6), simply because (1.5), (3.5) is the only action we know to solve the problems of quantization. This will exclude subtracting the singularities from the integrand leaving as finite pieces the Γ-function between its poles. Apparently, the only way we can make sense out of (3.4) is to choose the integration contour to be different from (3.6). This, however has to be subject to another requirement: we may change the contour only such that perturbation theory, in particular the OPEs (2.6), (2.7) and the arguments leading to (2.11) and below are not affected. Perturbation theory is obtained by coupling the fields to sources and then performing a (real) shift in the path-integration. Therefore, we should not cut the ϕ-integration in (3.7), for example to $[0, \infty)$ or $(-\infty, 0]$. Cutting the contour would introduce boundaries in field space which are subtle issues to deal with.

Fortunately, there is a way to make sense out of (3.4) such that all these requirements are satisfied. Instead of cutting the contour we may extend it. Let us consider the complex e^{ϕ}-plane. The integration in (3.6) is along the positive real axis. For non-integer a (or $Q\frac{s_i}{2\pi}R_i$ in (3.4)) there is a cut in this plane starting from the origin. Instead of using the ϕ-integration of (3.6) we may go to the integration contour shown in fig. 3a, b:

$$\int_{-\infty}^{+\infty} d\phi \quad \rightarrow \quad \frac{1}{2\pi i}\int_{C} d\phi \tag{3.7}$$

In the $-e^{\phi}$-plane the contour comes in along the positive real axis, encircles the origin counterclockwise and goes back to infinity. Using this replacement in (3.6) we obtain [23]

$$\frac{1}{2\pi i}\int_{C} d\phi \; e^{a\phi - be^{\phi}} = e^{-a\ln(-b)} \; \frac{1}{\Gamma(1-a)} \quad , \quad |a| \; < \infty \quad (b<0) \tag{3.8}$$

Here, we have chosen the phase such that the integral is real. In (3.8) we arrived at Hankel's representation of the Γ-function which is well-defined everywhere on the complex plane. (A replacement similar to (3.7) has been used in Liouville theory in the context of regularizing the area-integration to obtain finite expression for correlation functions [22]). The finiteness of (3.8) may be understood by realizing that for Re $a > 0$ the vertical part of C gives vanishing integration. For Re $a \leq 0$ this integration diverges, thereby cancelling corresponding divergencies arising in the integration along the horizontal parts of C. Obviously perturbation theory is not affected if ϕ takes values along this contour.

Using the integration contour C in (3.4) we may apply (3.8) and obtain

$$S_C(i) \;=\; -Q\frac{s_i}{2\pi}R_i \ln\Big(-\mu\frac{s_i}{2\pi}\Big) + \ln\Gamma(1+Q\frac{s_i}{2\pi}R_i) \tag{3.9}$$

In order to write this as an action, we should remember the heat-kernel expansion for the δ-function which is at the origin of the Liouville action appearing in the conformal gauge [7,6]. This reads

$$\frac{\Delta^2 x\sqrt{g}}{2\pi}\Big(\; R(x) + \frac{1}{\Lambda^2} \; \Big) = 6\pi\Delta^2 x \; \delta^{(2)}(0) \tag{3.10}$$

where $\Lambda \sim (\mathit{length}) \rightarrow 0$ is some UV-cutoff. Using our regularization the analog relation is

$$\frac{s_i}{2\pi}\left(R_i + \frac{1}{\alpha} \right) = 1 \tag{3.11}$$

which is easily derived from (3.2). Including $\sum_i$ in (3.9) we may use (3.11) to write S_C as an action. According to (3.2) this action is equivalent to

$$\begin{aligned} S_C(g) \;=\; & \frac{1}{2\pi\alpha}\int d^2x\sqrt{g}\Bigg(Q\alpha R\ln(1+\alpha R) \\ & \qquad\qquad + (1+\alpha R)\ln\Gamma\big(1+Q\frac{\alpha R}{1+\alpha R}\big)\Bigg) \;. \end{aligned} \tag{3.12}$$

This is the action we have been looking for. Although it may look rather complicated we will see below that it is just of a form that is geometrically meaningful. Notice that instead of writing the term $\frac{-1}{24\pi}\int d^2x\sqrt{g}\; R\Delta^{-1}R$ in (3.12) which is induced from the dilaton measure in (1.4), we keep track of it by renormalizing the coefficient Q_0 in (1.1). Quantizing the Liouville mode also, this will imply

$$Q_0 \quad \rightarrow \quad Q \;=\; \frac{24-c}{24}$$

as mentioned before in (2.1). In the discrete case this metric dependence of the measure shows up in the triangulation itself and the factor $\sqrt{s_i}$ appearing in (3.4). Such a factor comes with any matter field measure. In (3.14) we separated it from (3.9). In (3.12) we also did not write the topological term $-2Q(1-h)\ln(-\mu\alpha)$. The coefficient μ does affect only this topological term. A natural choice would be $-\mu \sim 1/\alpha$. (Notice that the ϕ along the horizontal part of D has imaginary part $\pm i\pi$ so that with $\mu < 0$ in (1.5) and (3.5) $\mu e^{\phi} > 0$.)

Contrary to (2.18) the terms (3.12) are well-behaved around zero curvature, they just vanish if $R = 0$. The action (3.12) is formulated at some scale α. As a consequence possible fluctuations of the geometry are restricted. Indeed, with (3.2) we always have

$$(\;1+\alpha R_i\;) \;=\; \frac{6}{N_i} \;>\; 0\;. \tag{3.13}$$

Therefore no singularity problems arise in (3.12).

In (1.3) we expected an infinite series of counterterms:

$$S_C(g) \;= \frac{Q}{2\pi\alpha}\int d^2x\sqrt{g}\;\sum_{m=2}^{\infty}\; q_m\;(\alpha R)^m\;. \tag{3.14}$$

Such a form may be more familiar from perturbation theory. So let us deduce the coefficients q_m from (3.12). We have to use

$$\begin{aligned}\ln(1+z) &= -\sum_{n=1}^{\infty} \frac{1}{n} (-z)^n \\ \left(\frac{1}{1+z}\right)^k &= k \sum_{n=0}^{\infty} \frac{1}{k+n} \binom{k+n}{k} (-z)^n\end{aligned} \qquad (3.15)$$

if $|z| < 1$, and [23]

$$\ln\Gamma(1+w) = -\gamma w + \sum_{n=2}^{\infty} \frac{\zeta(n)}{n} (-w)^n \qquad (3.16)$$

if $|w| < 1$, where $\gamma = 0.57721...$ is Euler's constant and ζ is Riemann's Zeta-function. With $z = \alpha R, w = \alpha R/(1+\alpha R)$ this can be applied to (3.12) and the coefficients in (3.14) turn out to be

$$q_m = \frac{(-1)^m}{m-1}\left[1 + \sum_{k=1}^{m-1} kQ^k \binom{m-1}{k} \frac{\zeta(k+1)}{k+1}\right]. \qquad (3.17)$$

Upon expanding (3.12) there is also a topological term $-2\gamma Q(1-h)$ arising. We did not write this in (3.14). Since the radius of convergence for this series is at $\alpha R_i = (1+Q)^{-1}$ one may prefer to work with the closed expression (3.12) instead of expanding it.

4. The Geometrical Implications

In the foregoing sections we determined $S_C(g)$ by insisting on quantum consistency in the continuum limit. Let us now study the geometrical implications of the resulting quantum action. We mentioned in the introduction that already the presence of a quadratic curvature term R^2 in (1.1) is enough to pacify the problem of spikes covering the surface and branched polymers at small scales [9]. At these scales the surfaces turn out to be smooth since high curvature is suppressed by the quadratic term. There is, however, no reason to expect that such a feature would survive the inclusion of even higher-order terms. In this section we will find that although our $S_C(g)$ contains curvature terms up to infinite order it is indeed of a form that favours flat geometries.

Since $S_C(g) \simeq \sum_i S_C(i)$ the effect of (3.12) is to introduce a weight

$$e^{-S_C(i)} \;=\; \Big(\frac{N_i}{6}\Big)^{Q\left[1-\frac{N_i}{6}\right]} \frac{1}{\Gamma\Big(1+Q\big[1-\frac{N_i}{6}\big]\Big)} \; e^{-2\eta'(1-\frac{N_i}{6})} \tag{4.1}$$

at each vertex i. The dependence on the conformal dimension c of the coupled matter system enters via $Q = (26-D)/24$ where $D = c+2$ is the dimension of the effective theory (see (2.10)). To arrive at (4.1) we used (3.9) without the topological term that was dropped in (3.12). The last factor in (4.1) corresponds to an additional $\eta' \frac{1}{\pi} \int d^2x \sqrt{g} R$. With $\eta' = Q\gamma/2$ this will cancel the topological term arising when (3.12) is expanded into (3.14). Notice that the first factor in (4.1) was studied some time ago [24]. Here the essential new feature is the appearence of the inverse Γ-factor.

Let us first discuss the region $D < 26$ ($Q > 0$). The weight (4.1) is zero for

$$N_i \;=\; 0 \quad \text{or} \quad 6(1+Q^{-1}),\; 6(1+2Q^{-1}),\; \ldots \tag{4.2}$$

The maxima between these zeros are obtained if N_i obeys

$$Q(\frac{6}{N_i}-1) - Q\ln\frac{N_i}{6} + Q\psi(1+Q(1-\frac{N_i}{6})) + 2\eta' \;=\; 0 \tag{4.3}$$

where $\psi(z) = \frac{d}{dz}\ln\Gamma(z)$. To begin we may concentrate on the region between the first two zeros (see fig. 4). We comment on the other regions when discussing singularities. Using $\eta' = Q\gamma/2$ the maximum of (4.1) is then obtained at

$$N_i \;=\; 6 \quad : \quad \frac{d}{dN_i}\, e^{-S_C(i)} \;=\; 0 \tag{4.4}$$

since $\psi(1) = -\gamma$. Therefore our $S_C(g)$ is such that it favours flat geometries.

Before discussing singularities we should recall some basics about Riemann surfaces. For any point on the surface the geometry of a local neighbourhood may be thought of as being induced from a three-dimensional Euclidean space into which this local region is embedded [25]. In order to regularize the quantum theory we represented the Riemann surfaces in terms of F equilateral triangles of area $\pi\alpha$. At a vertex i with N_i nearest neighbours (see fig. 2) the local geometry will be flat ($N_i = 6$), parabolic ($N_i < 6$), or hyperbolic ($N_i > 6$). It is easily visualized that in a three-dimensional embedding space a locally hyperbolic geometry ('saddle-surface') gets singular at $N_i = 12$. A global embedding may require a higher-dimensional space [25]. At finite scale α global embedding theorems have to be applied and vertices with $N_i \geq 12$ may occur.

What about singularities? Let us first study the case of an effective $D = 2$ dimensional string theory, i.e., the case of pure gravity. Then $Q = 1$ and the region between the first two zeros of (4.2) is $0 < N_i < 12$ (fig. 4). Obviously the weight (4.1) decreases for small N_i-values, thereby suppressing parabolic singularities. On the other hand we just mentioned that also $N_i = 12$ has geometrical significance. It corresponds to a hyperbolic singularity in the continuum limit. At finite scale α we have to consider also vertices with $N_i \geq 12$. It turns out that vertices with arbitrary large N_i are strongly suppressed (see fig. 5). This is a remarkable result. We find that our $S_C(g)$ not only favours flat geometries, but also suppresses both parabolic and hyperbolic singularities.

When matter is included, $D > 2$ ($Q < 1$), the geometric content gets less obvious. The maximum of (4.1) remains at flat geometries. With increasing D the suppression of non-flat geometries is weakened. In particular the second zero $6(1 + Q^{-1})$ in (4.2) (and all higher zeros) will increase (see fig. 4). Given the interpretation of D as target-space dimension this may reflect that with increasing embedding dimension there is 'more space' to include triangles. Notice that random triangulations will also include configurations that do not correspond to Riemann surfaces. Nevertheless, vertices with arbitrary large N_i are strongly suppressed also for $D > 2$ (see fig. 5) and Riemannian geometries may be expected to be dominant even if arbitrary triangulations are considered. Definite statements should be left to future studies, e.g., in the framework of numerical simulations.

Alternatively, the region beyond the second zero, $N_i \geq 6(1 + Q^{-1})$, can be eliminated completely if the weight (4.1) is defined in terms of the expansion (3.14), (3.17) instead of (3.12). Then whenever $N_i \geq 6(1 + Q^{-1})$ the series is divergent and the weight is zero.

To study the critical limit $D \to 26$ and $D > 26$ it is helpful to look back at the first-order fomulation (2.2), (2.10). Let us choose the coupled matter system to be a $c = d$-dimensional string theory in a flat background with Euclidean metric δ_{ij}. With $X^{\pm} = X^0 \pm X^{d+1}$ and not writing the potential terms (2.3) the gravitational action (2.2), (2.10) gives

$$\begin{aligned} S(\sigma, \phi) \ + \frac{1}{4\pi} \int d^2x \sqrt{\hat{g}} \ \ \hat{g}^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j \delta_{ij} \\ = \frac{1}{4\pi} \int d^2x \sqrt{\hat{g}} \ (\ \hat{g}^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \ + \ O(Q) \ ) \end{aligned} \tag{4.5}$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, ..., +1)$, $\mu, \nu = 0, 1, ..., D - 1 = d + 1$ is a Minkowski metric. As usual the target-space metric and world-sheet metric in (4.5) are related by

$$\hat{g}_{\alpha\beta} \;=\; \frac{\partial X^\mu}{\partial x^\alpha}\frac{\partial X^\nu}{\partial x^\beta}\,\eta_{\mu\nu} \;+\; O(Q)\;. \tag{4.6}$$

The background geometry on the world-sheet is induced from the target-space and corrected by quantum contributions for the non-critical case. With (4.5), (4.6) the relation to the critical theory is obvious since $O(Q)$ vanishes as $D \rightarrow 26$.

For pure Liouville theory the region $D > 26$ is known to be troublesome. There $Q < 0$, and the kinetic Liouville term will cause the σ-integration to diverge. This is usually dealt with by rotating $\sigma \rightarrow i\sigma$, a transformation with unclear geometrical content. In our case we have another problem. With $Q < 0$ the weight (4.1) and in particular the zeros (4.2) do not seem to make sense. Both problems may be cured by rotating the two world-sheet coordinates $x^\alpha \rightarrow ix^\alpha$ (followed by $X^j \rightarrow iX^j$ to leave the matter sector invariant). As a result the gravity action will change sign which may be expressed by $Q \rightarrow -Q$. This is just the correction we need. There may indeed be some motivation for this transformation. If we move D from $D < 26$ to $D > 26$ the $X^\pm$-values are rotated from the real axis to the imaginary axis. This is due to the factor $\sqrt{(26-D)/24}$ included in their definition. Since target-space and world-sheet coordinates X^μ and x^α are related by (4.5), (4.6) it may then be natural to rotate also the x^α into the imaginary direction. Since we work with Euclidean signature on the world-sheet this procedure simply corresponds to applying a different Wick-rotation for $D > 26$. Notice that even with $Q \rightarrow -Q$ the second zero $6(1-Q^{-1})$ of (4.1) will be at $N_i < 12$ if $D > 50$. It is unclear whether this procedure is appropriate. Of course, one should not exclude that a sensible quantization can only be established for $D \leq 26$. However, since from the perturbative point of view the quantum theory is well-defined for any D (see section 2) there ought to be hope that also in the non-perturbative setting we could make sense out of the theory if $D > 26$. Again, we should leave definite statements to future investigations. However, one may speculate that the behaviour of (4.1) in the presence of matter ($Q < 1$) is related to geometrical phenomena that become dominant in the region $D > 26$. Thus it may be advisable to obtain a clear geometrical picture for the region $D < 26$ first.

Finally, we may consider the scaling behavior of the partition function. Let us rescale the area A by $A \;\rightarrow\; \Lambda A$. Using the first-order formulation (1.4), (1.5) where the conformal gauge turns (1.1) into (2.2), the rescaling of the area is expressed by $2\sigma \rightarrow 2\sigma + \ln\Lambda$. We may use the standard method to turn the sum over geometries into a sum over partition functions for fixed area $Z(A)$ [27,5]. The string susceptibility ξ is then defined by [26]

$$Z(A) \;\sim\; e^{-\frac{\lambda}{\pi}A}\;A^{\xi-3}\;. \tag{4.7}$$

Compensating in (2.2) the shift of σ by shifting the ϕ-integration $\phi \to \phi - \ln\Lambda$ one obtains $\xi = 2 + \frac{D-26}{12}(1-h)$. This value does not agree with the semi-classical value $\xi^0 \approx \frac{D}{6}(1-h)$, $D \to -\infty$ [27,28]. If on the other hand we set $\mu \sim 1/\alpha$ and accompany the rescaling of the area (3.1) by $\alpha \to \Lambda\alpha$ then the Lagrangian (1.5) is invariant, no shift in ϕ is necessary and one obtains $\xi = 2 + \frac{D-26}{6}(1-h)$. This result would agree with the semi-classical limit. To clarify the situation notice that the weight (4.1) is independent of the area α. In (3.9) we found that upon regularizing (1.4), (1.5) in terms of triangles and integrating out ϕ, the only α-dependence shows up in a topological term $-2Q(1-h)\ln(-\mu\alpha)$. This term was canceled in (4.1). With $A \to \Lambda A$, $\alpha \to \Lambda\alpha$ it would disturb the semi-classical limit unless μ scales as $1/\alpha$. Therefore, assuming that μ in (2.2) scales with $1/\alpha$ corresponds to cancelling the topological factor arising in the integration (1.4). It is another appealing feature of our results that, contrary to the string susceptibility resulting from (1.2) [2,5], our ξ is real for any D.

5. Summary

The (Liouville-)problem of quantizing two-dimensional gravity with conformal matter of anomaly c can be solved if the metric $g_{\alpha\beta}$ is accompanied by an auxiliary field ϕ. Choosing the appropriate action (1.5) for ϕ the cosmological constant turns out to be of conformal weight (1,1) without any need for gravitational dressing (1.2) and quantization becomes possible for any c [10]. (See also related work in [12-14].) This can be shown by using standard David, Distler and Kawai CFT-methods. In section 2 we also gave simple diagrammatic arguments. (It is convenient to analyze the quantum consistency in the conformal gauge $g_{\alpha\beta} = e^{2\sigma}\hat{g}_{\alpha\beta}$. In this gauge ϕ obtains a mixed propagator with the Liouville-mode σ and if the coupled matter system is a d-dimensional string theory in Euclidean background the effective theory is a $D = d + 2$ string theory in a background with Minkowski-signature $(1, D-1)$.)

The auxiliary character of ϕ is essential for quantum consistency. Any kinetic term for ϕ would destroy the finiteness of the theory. Given this auxiliary nature of ϕ we wanted to eliminate it and asked for the corresponding higher-order action $S_C(g)$. This was done in section 3. The ϕ-potential has to be exponential and so ϕ could be eliminated only by explicitly performing its path-integration. (Only with quadratic potential would the path-integration agree with simple elimination by equation of motion.) In order to regularize this integration we triangulated the two-dimensional universe and had to extend the ϕ-contour to the complex plane. Then ϕ could be integrated out explicitly and, using the triangulated version of the heat-kernel expansion, the result could be formulated as an action with higher-order curvature terms.

The weights (4.1) introduced by the higher-order terms are such that flat geometries are dominating. Discussing the case of pure gravity in section 4, we found both parabolic and hyperbolic singularities to be suppressed. In the presence of matter the geometrical content becomes less obvious. Still flat geometries are favoured but the suppression of non-flat geometries is weakened as the critical limit ($D = 26$) is approached. Without a clear geometric understanding of the region $D < 26$ it may be too early to make statements on $D > 26$. In particular we do not know whether our result should also be applied to the non-Riemannian regime, e.g., to triangulations that do not correspond to regular surfaces. We have to leave definite statements to further investigations, possibly in the framework of numerical simulations.

Essential features of the results presented in this paper may have been expected from the beginning by using Polchinski's principle of ultralocality [8,7]. In two dimensions the Einstein-Hilbert action is trivial. The only dynamics is induced from the measure in the path-integral. Any counterterms will have to make this measure well-defined and so the path-integration. Therefore, one may speculate that counterterms $S_C(g)$ correspond to ambiguities in the measure. The principle of ultralocality states that the measure and thus any ambiguity is a local product over space-time points. This is often used to argue that any ambiguity in the measure is restricted to the cosmological constant. However, the notion of 'locality' can only be defined together with a regularization of the path-integral. Here, we regularized the two-dimensional surface in terms of triangles and therefore 'locality' refers to the plaquette around each vertex i, see fig. 2. Local quantities are quantities defined on each plaquette seperately. If, following the principle of ultralocality, we assume the counterterms to reflect the local structure of the measure, then we may include curvature terms R_i but not differences $R_i - R_j, i \neq j$, or in terms of the scalar field no $\phi_i - \phi_j, i \neq j$. It is therefore not surprising that the counterterms factorize as it happens in (4.1). So we find the auxiliary nature of ϕ and in $S_C(g)$ the absence of terms with derivatives acting on R related to structures of the measure.

The theory presented certainly has a rich structure that is waiting to be discovered. It should open the avenue to study string theories with $c > 1$ (which actually is $c > 0$ in our framework). Future studies should illuminate the geometrical content in more detail. Correlation functions may be most conveniently discussed along the lines of [29,11] using the first-order formalism. This will also hold for analyzing the spectrum. Some matrix-model formulation would be highly wellcome. Here, we introduced triangulations only as a tool to regularize the two-dimensional surface and integrate out the auxiliary field to gain some insight into the higher-order theory. Using this regularization the next step would be to implement the summation over geometries as a summation over triangulations. Given the particularly simple form of the first-order formulation with auxiliary field one may speculate that a two-matrix model would be appropriate. (Notice in the conformal gauge the action (2.10) includes coupling to a background charge and some progress has been made to relate minimal CFTs to two-matrix models [30].) As long as matrix-model formulations are absent one may study non-perturbative effects using numerical simulations. These should be straightforward using the weight (4.1). Dynamical triangulations with higher-order curvature terms have recently been studied in [31] (with lattice sizes up to 400,000 triangles) and earlier in [24]. Contrary to the weights used there, the weight (4.1) originates from a consistent continuum theory.

As a conclusion we find that to cure the Liouville problem and quantize two-dimensional gravity with conformal matter of arbitrary anomaly c there is a choice of two prices that have to be paid: Either an auxiliary field ϕ is included or - equivalently and discussed in this paper - curvature terms up to infinite order will appear.

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FIGURE CAPTION

FIG.1 In two-dimensional field theories with second-derivative kinetic term and polynomial potential only one-vertex (sub-)diagrams can be divergent.

FIG.2 A piece of triangulation of a two-dimensional surface. The broken lines show the dual graph, describing plaquettes of area $s_i = N_i \pi \alpha/3$ around a vertex i with N_i nearest neighbours.

FIG.3 Integration contour for the auxiliary field ϕ.

FIG.4 The weight (4.1) between its first two zeros, introduced at each vertex i due to the higher-order terms.

FIG.5 The weight (4.1) suppresses arbitrary large numbers N_i of nearest neighbours. (The solid and broken curves correspond to fig. 4.)

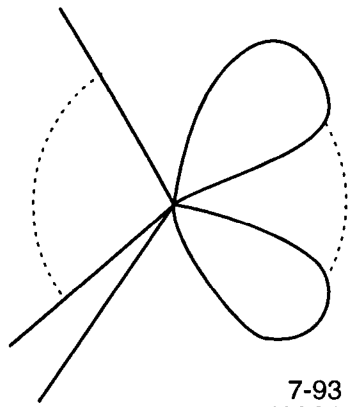

Fig. 1

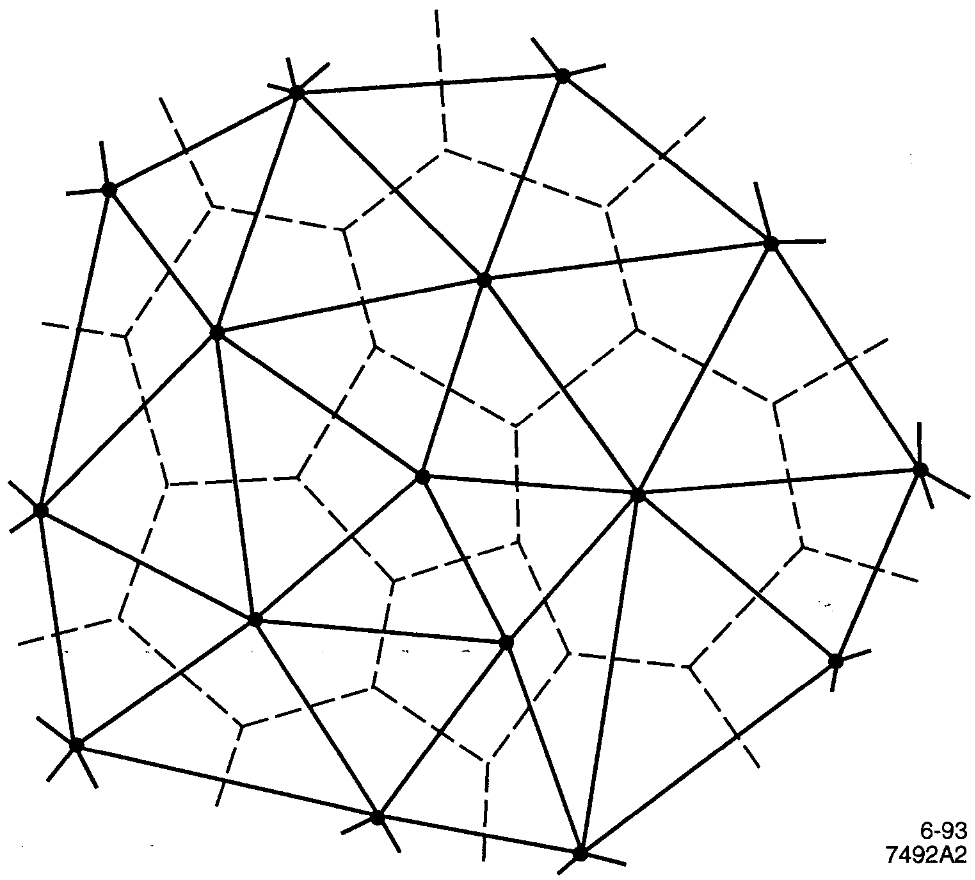

Fig. 2

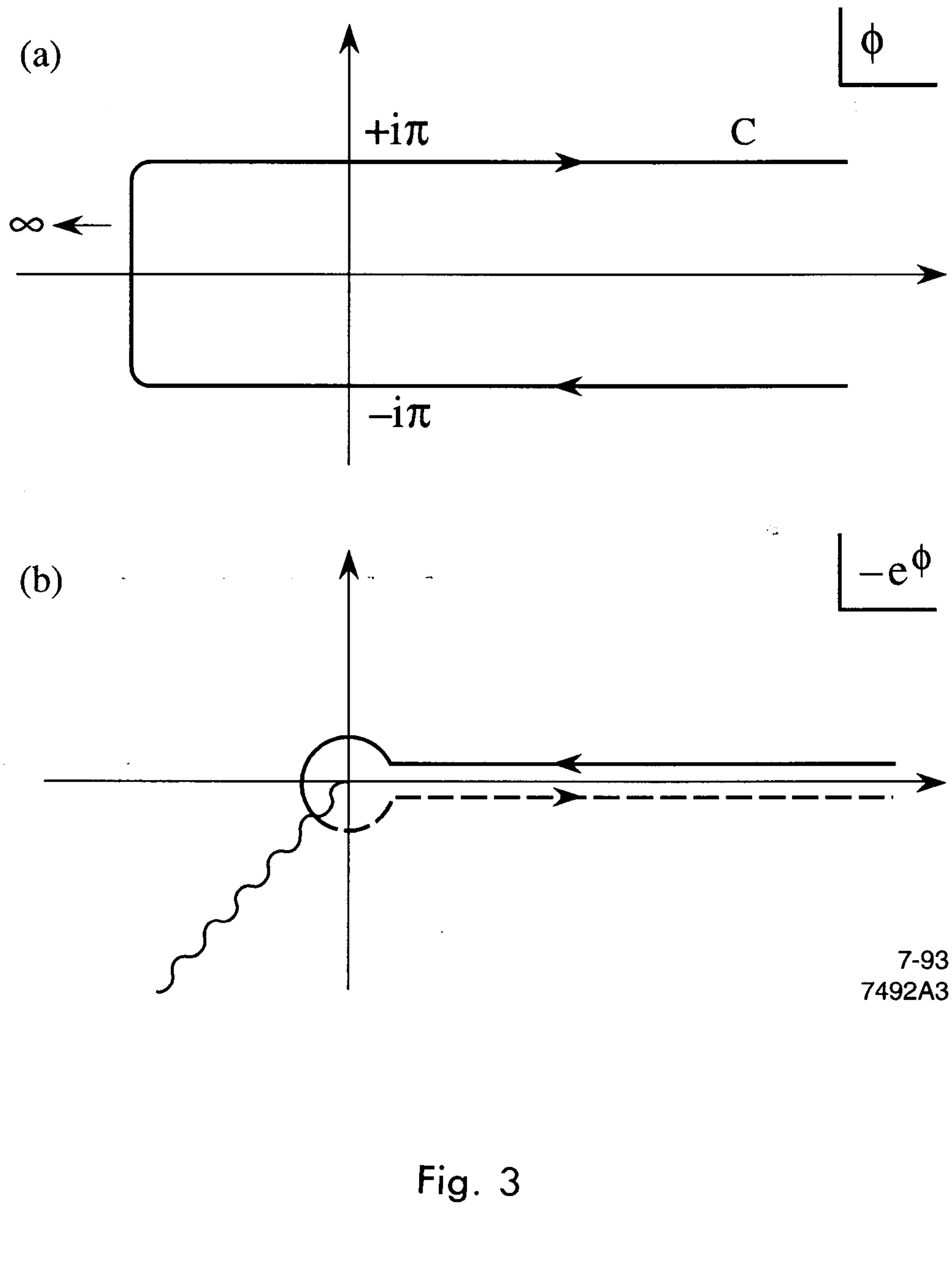

Fig. 3

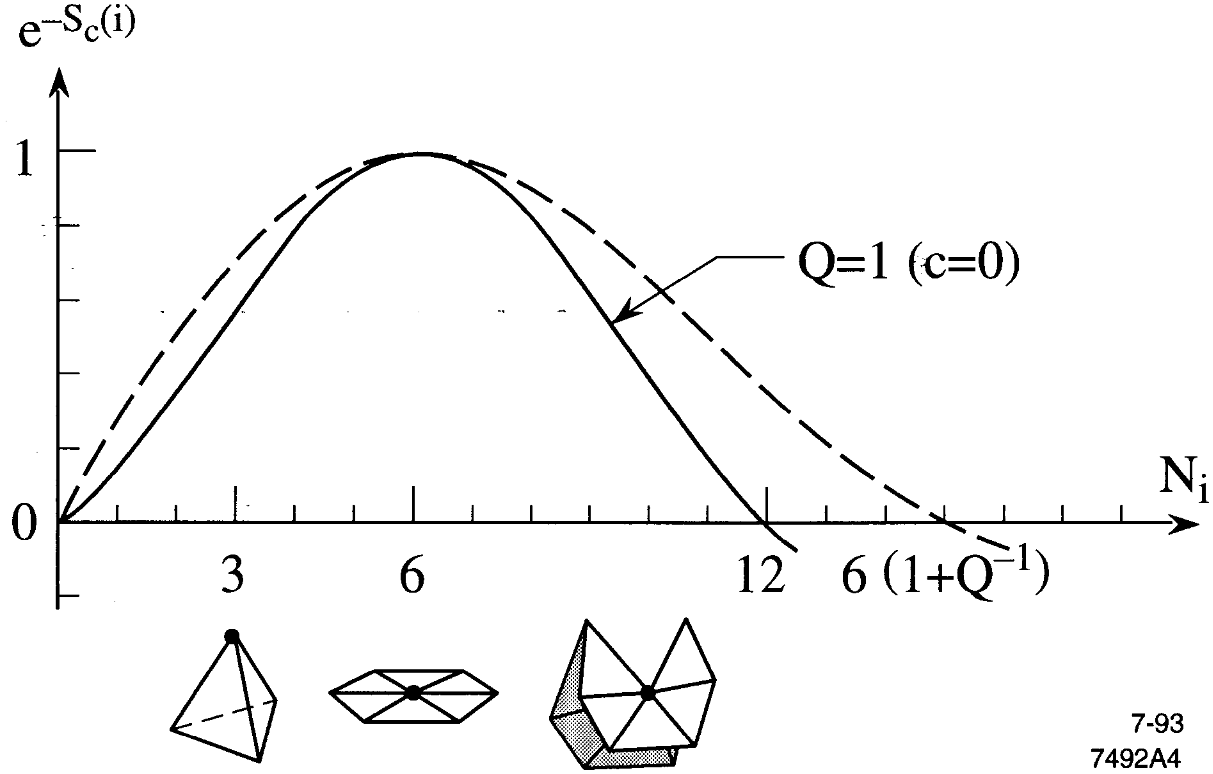

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Fig. 4

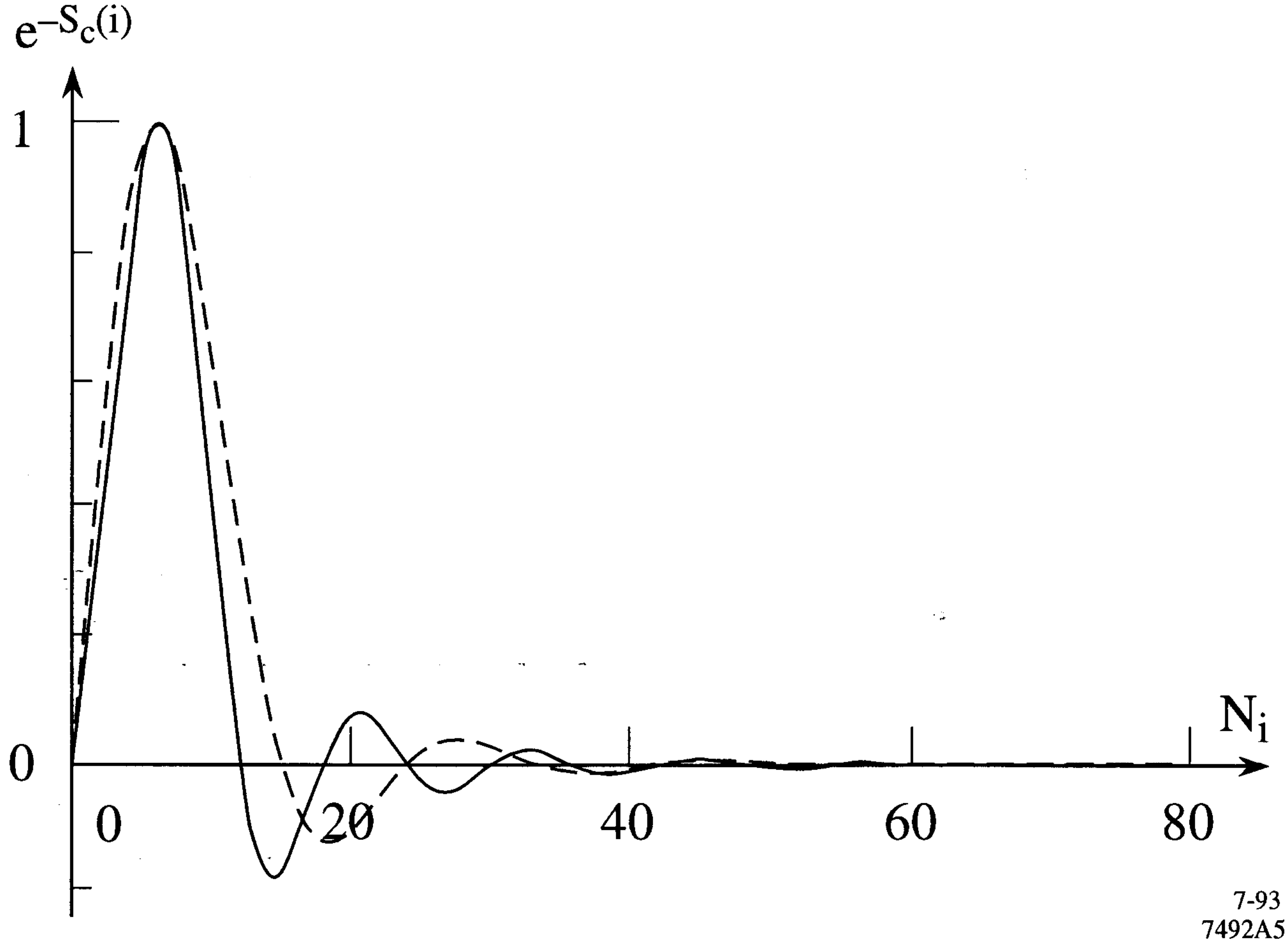

Fig. 5