OPTICAL STOCHASTIC COOLING*

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ABSTRACT

In this paper, we consider the utilization of optical amplifiers with bandwidth $\Delta f \cong 10^{14} Hz$ for use in stochastic cooling. It is shown that quadrupole and dipole wigglers can be used as pickups and kickers respectively. The proposed method increases the application of the stochastic cooling method beyond the traditional area of proton-antiproton cooling. For example, the method can also be applied for electron-positron cooling as well as for potential use in μ-meson cooling. The proposed method makes possible the independent choice of damping time and source of excitation of emittance for electron-positron damping rings, thereby providing a new direction for the design of low-emittance damping rings.

I. INTRODUCTION

Synchrotron radiation, electron cooling, and stochastic cooling are methods used to damp the emittance of charged particles beams. Heretofore, synchrotron radiation has been the only method used for damping emittances of electron and positron beams. The damping time τ for stochastic cooling of N particles is determined by the bandwidth Δf (see, for example, Ref. [1]). The damping time τ can be approximated by $\tau/T \cong N/\Delta f(c/l_b) \cong N_u$, where N_u is the number of the particles in the bandwidth, T is the period of revolution, and l_b is the bunch length. For a given number of particles, the limitations are determined by the bandwidth Δf of the amplifiers, pickups, and kickers. While successful for protons, if applied for electrons with the typical bandwidths used ($\cong 10^9 Hz$), this method gives longer damping times than those associated with synchrotron radiation. Therefore, this method has not been used for cooling electron and positron beams. However, if the bandwidth is extended to the optical range, stochastic cooling can be successfully applied to cool electrons and positrons as well. For practical utilization, of course, all hardware, such as pickups, amplifiers, and kickers, must function at typical optical bandwidths ($10^{14} Hz$).

Optical amplifiers exist with relative bandwidths $\Delta f/f$ in the range of 10 to 20% [2]. These amplifiers normally operate at a central wavelength λ of $\lambda \cong 0.3$ to $1\mu m$. The pickup must be sensitive to radiation in this bandwidth, which reflects the transverse fluctuations of the centroid of the beam charge. For example, a sequence of the quadrupole lenses with changing polarities, with the necessary period and number, a quadrupole wiggler (QW), can be used for this purpose. For example, a transverse kick can be fed back through an energy jump at a place with nonzero dispersion function in the lattice. A dipole wiggler, with corresponding period, can be used as the longitudinal kicker, if it is installed at a point of nonzero dispersion with the correct betatron phase shift relative to the quadrupole wiggler. The synchrotron radiation from the QW is due to fluctuations of the charge centroid.

This function of the QW is similar to that of pick-up electrodes in conventional schemes of stochastic cooling. The number of periods of the QW (the number of the

periods of FODO structure), defines the bandwidth of the radiation corresponding to the bandwidth of the optical amplifier. The betatron phase shift must be small to prevent enlarging the bandwidth by betatron oscillations. Polarization of the radiation makes it possible to distinguish fluctuations in both transverse directions independently.

After amplification, the optical signal from the QW goes to the longitudinal kicker, which is placed downstream in the beam trajectory, where, if the phase, amplitude, and dispersion function are chosen correctly, the fluctuation is damped to the necessary level. The general layout of the proposed method is represented in Fig. 1. The bend between the quadrupole and dipole wiggler is made in such a manner so that fluctuations which produced the radiation in the QW are not eliminated.

For different cooling schemes both transverse and longitudinal kickers can be used, as described in [3]. In optical cooling, transverse kickers are not effective, due to cancellation of the transverse force from the electric and magnetic field $\cong 1/2\gamma^2$ in the electromagnetic wave, which propagate in the same direction as the particle. So we consider only schemes with longitudinal kickers. In this way, the transverse emittance can be damped efficiently.

II. RADIATION FROM THE QUADRUPOLE WIGGLER

The radiation from the particles in the beam passing through the wiggler is related to the current density distribution. For a random distribution of particles in the beam, the field amplitude is known to depend on $\cong \sqrt{N}$, and the radiation intensity is known to depend on N, where N is the total number of the particles.

A quadrupole wiggler consists of a sequence of focusing and defocusing lenses, i.e., a FODO structure. For steady current of particles passing through a quadrupole field wiggler, for each particle at a given transverse position x, there exists another particle at $-x$ position, which is accelerated in the opposite direction. This process yields destructive radiation interference in the forward direction. Hence, radiation in a quadrupole field is defined by the fluctuation of current density. The period of the FODO structure is chosen to obtain the desired radiation wavelength. If the

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particle causes only a small part of the betatron oscillation in the whole QW structure, then the central wavelength is defined by the period $2 \cdot L$ of the FODO structure, $\lambda_u \cong L/\gamma^2$. This wavelength must fall within the frequency range of the optical amplifier, i.e., $\lambda_u \cong 0.3$ to $1 \mu m$ [2]. If we take $\gamma \cong 10^3$, then $L \cong 100$ cm, or a period of about 200 cm.

The undulatory factor K is defined:

$$K = \frac{e \cdot H_\perp \cdot 2 \cdot L}{2 \cdot \pi \cdot m \cdot c^2} \; . \tag{1}$$

For an optimal undulatory factor K, $K \cong 1/\sqrt{2}$ [4], we need only the field strength $H_\perp \cong G \cdot A_0 \cong 40$ G for electrons and $8T$ for protons, where G is the gradient of the lens and A_0 is the amplitude of the betatron oscillation.

The number of photons radiated by one particle on the first harmonic (calculated in the energy interval from E_{max} to $E_{max}(1 - \frac{1}{M})$) can be estimated for $K \leq 1$ according to [4]

$$\Delta N_{\gamma 1} \cong 4 \cdot \alpha \cdot \frac{K^2}{1 + K^2} \; , \tag{2}$$

where $\alpha = e^2/\hbar c$, M is the number of periods and the energy of the quanta E_γ defined by the formula

$$E_\gamma = \frac{2 \cdot \pi \cdot \hbar \cdot c \cdot \gamma^2}{L \cdot (1 + K^2 + \gamma^2 \theta^2)} \; , \tag{3}$$

where θ is the angle of observation. We will be interested in the range $K \leq 0.7$. The number of photons radiated by the particles with amplitude A in the relative bandwidth $\Delta f/f \cong 1/M$ becomes

$$\Delta N_{\gamma 1} \cong \alpha \cdot (A/A_0)^2 \cong \alpha \cdot \frac{\epsilon}{\epsilon_0} \; , \tag{4}$$

where ϵ_0 is the initial emittance at the beginning of the cooling, when $K \cong 0.7$ and ϵ is the emittance after cooling. For a bunch with N particles and length l_b, the

number of particles in the bandwidth is equal to

$$N_u \cong M \cdot N \cdot (\lambda_u/l_b) \cong M \cdot N \cdot (L/l_b)/\gamma^2. \tag{5}$$

The number of photons ΔN_r in the bandwidth is defined by fluctuations of the centroid of charge from these N_u particles and is given by

$$\Delta N_{\gamma 1} \cong \alpha \cdot N_u \cdot (A/A_0)^2 \cong \frac{1}{3} \cdot \alpha \cdot N_u \cdot \frac{\epsilon}{\epsilon_0} \ , \tag{6}$$

and the total number of radiated photons in the bandwidth in each pass is

$$\Delta N^t \cong \frac{1}{3} \cdot \alpha \cdot N \cdot \frac{\epsilon}{\epsilon_0} \ . \tag{7}$$

During damping, the factor K also decreases, which introduces a wavelength shift $\Delta\lambda \simeq 1/(1 + K^2)$. There are techniques available for overcoming this problem. For example, the beam envelope in the wiggler can be dynamically changed to restore the K factor.

The quadrupole wiggler is not the only choice to act as a pick-up electrode in optical cooling. The dipole wiggler also can be used as a broadband beam position monitor, but it requires additional optical devices. Sextupole and octupole wigglers can also be used to generate the radiation from the beam which can then be correlated with the transverse density distribution of the beam.

III. NECESSARY ENERGY CHANGE AND AMPLIFICATION

The energy $\Delta\epsilon_y$ radiated by N^t photons, produced by a beam whose center is off in position by $\Delta x \cong A/(N_u)^{1/2}$, is given approximately by

$$\Delta\mathcal{E}_\gamma \cong 2 \cdot \pi \cdot \hbar \cdot c \cdot \gamma^2/L \cdot N^t \cong 2 \cdot (\frac{e^2}{L}) \cdot N \cdot \gamma^2 \cdot (A/A_0)^2. \tag{8}$$

Following the general idea of stochastic cooling, we must damp this amplitude Δx with a kicker. The well-known longitudinal kicker method can be used for optical cooling; a transverse displacement is provided by an energy change $\Delta E/E$ at a lattice position with nonzero dispersion function (see, e.g., [3]).

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To damp the amplitude $\Delta x \cong A/(N_u)^{1/2}$, we must provide an energy change $\Delta E/E$ which corresponds to

$$\Delta x \cong A/(N_u)^{1/2} \cong \eta \cdot (\Delta E/E), \tag{9}$$

where η is the dispersion function of the lattice of the dipole wiggler. If P is the undulatory factor of the dipole wiggler and the number of periods M is the same as in the quadrupole wiggler, then

$$\eta \cdot \frac{e \cdot E_\perp \cdot P \cdot L}{E \cdot \gamma} \cdot M = A/(N_u)^{1/2} \ . \tag{10}$$

The energy ϵ that must be contained in the photon radiation is given by

$$\mathcal{E} = \frac{c}{4 \cdot \pi} \cdot E_\perp^2 \cdot \mathcal{S} \cdot \frac{l_b}{c} \cong \frac{c}{4 \cdot \pi} \cdot \frac{A^2}{N_u} \cdot \left(\frac{E \cdot \gamma}{e \cdot P \cdot L \cdot \eta \cdot M} \right)^2 \cdot \mathcal{S} \cdot \frac{l_b}{c} \ , \tag{11}$$

where $\mathcal{S}$ is the cross section of the emitted radiation light spot at the interaction point. If we compare with the energy radiated by the particles in the quadrupole wiggler, the coefficient amplification κ is obtained

$$\kappa \cong (\mathcal{E}/\Delta \mathcal{E}_\gamma)^{1/2} \cong \frac{1}{4} \cdot \frac{l_b}{r_0} \cdot \frac{\gamma}{N} \cdot \frac{\Delta E}{E} \cdot \frac{\Delta f}{f} \ , \tag{12}$$

where $r_0 = e^2/mc^2$ and $\mathcal{S}$ is estimated to be $\mathcal{S} \cong \pi \cdot A_0^2 \cong 2 \cdot \pi \cdot \lambda \cdot L \cdot M$. In expression 12 we used the optimal beam size as the transverse size of coherence and assumed $P \cong 1$. The dispersion function was also estimated to be $\eta \cong A_0/(\Delta E/E)$, where $(\Delta E/E)$ is the energy spread in the beam and $M \cong f/\Delta f$.

If we substitute $N \cong 1 \cdot 10^{10}, l_b \approx 15 \ cm, M = 5, \Delta E/E \cong 10^{-3}$, and $\gamma \cong 10^3$ (500 MeV) in expression 12, we obtain $\kappa \cong 3 \cdot 10^2$ for an electron damping ring which is well below the typical $\kappa \cong 10^5$, the typical amplification factor [2]. For the above parameters, the required pulsed power is $\cong 5$ kW at an average power of $\cong 25$ W, assuming a repetition rate of $\cong 10$ MHz. Thus, there is no apparent limitation for the

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design of such an amplifier to effectively cool positron/electron beams by this method. The amplification coefficient κ increases for protons by the γ factor as $r_\gamma = r_p/r_0$, where r_p is the classical radius of a proton. The power approaches the technical limits of present-day amplifiers. If, however this relatively fast damping time is not necessary, the amplification coefficient can be decreased to an acceptable level.

IV. DAMPING TIME AND TEMPERATURE

At each pass the individual particle gets a kick which is correlated with its coordinate in the pickup $\Delta x \cong A/N_u$, and an uncorrelated kick $\Delta x \cong A/(N_u)^{1/2}$ which heats the particle. The resulting action under optimal conditions gives a change of emittance per turn as

$$\frac{d\epsilon}{dn} + \frac{\epsilon}{N_u} = i,$$

where i is the effective source of excitation. So, the number of revolutions is approximately N_u. The process of cooling will stop when the number of radiated quanta is of the order unity. This gives a decrease in emittance equal to

$$\alpha \cdot N_u \cdot \epsilon_f/\epsilon_0 \cong 1, \text{ or } \epsilon_f/\epsilon_0 \cong \frac{1}{\alpha \cdot N_u} . \tag{14}$$

For our example, the reduction factor is 10^3. Of course, the noise of the amplifier must be of the same order, i.e., one photon in the coherence volume.

The damping time τ in the exponential $\epsilon \cong \epsilon_0 \cdot \exp(-t/\tau)$ is approximately equal to

$$\tau \cong T \cdot \frac{N}{(\Delta f/f)} \cdot (\lambda_u/l_b) = N_u/f_0, \tag{15}$$

where f_0 is the frequency of revolution for $f_0 = 10$ MHz, this implies that $\tau \cong 10^5/10^7 = 10$ ms for a single cooling system installed in a damping ring.

V. TRANSVERSE DYNAMICS

The equations which define the equilibrium emittance in the damping ring are as follows [5]:

$$\frac{d\epsilon_{x,y}}{dt} \cong < \left(H_{x,y} + \frac{\beta_{x,y}}{\gamma^2} \right) \frac{d}{dt}(\Delta E/E)_{\iota}^2 > -\epsilon_{x,y} \cdot \left(\frac{1}{\tau_s} + \frac{1}{\tau} \right) \, , \tag{16}$$

where τ_s, τ is the cooling time by synchrotron radiation and stochastic cooling correspondingly,

$$H_{x,y} = \frac{1}{\beta_{x,y}} \left(\eta_{x,y}^2 + \left(\beta_{x,y}\eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\eta_{x,y} \right)^2 \right) \, , \tag{17}$$

where $\beta_{x,y}$ is the envelope function, $\eta_{x,y}$ is the dispersion function, and $\eta'_{x,y}$ its derivative. The averaging $(< \ldots >)$ is made over the longitudinal distance; x and y denote the transverse coordinates.

The energy spread is defined by the relation

$$\frac{d}{dt}(\Delta E/E)_{\iota}^2 = \frac{d}{dt}(\Delta E/E)_{\text{IBS}}^2 + \frac{d}{dt}(\Delta E/E)_{\text{QE}}^2. \tag{18}$$

It should be noted that there are two distinct components, denoted by the subindices QE and IBS, which refer to quantum excitations and intra-beam scattering, respectively. The reduction in energy spread due to cooling is given by the expression

$$\frac{d}{dt}(\Delta E/E)^2 = \frac{d}{dt}(\Delta E/E)_{\iota}^2 - (\Delta E/E)^2 \cdot (\frac{1}{\tau_s} + \frac{1}{\tau}) \, . \tag{19}$$

The increase in the energy spread due to IBS can be expressed as

$$\frac{d}{dt}(\Delta E/E)_{\text{IBS}}^2 \cong \frac{N \cdot r_0^2 \cdot (\epsilon_x\beta_x)^{1/2}Ln_c \cdot c}{\gamma^3 \cdot \epsilon_x \cdot (\epsilon_y \cdot \beta_y)^{1/2}\sigma_1 \cdot \sigma_{\text{sx}}} \, , \tag{20}$$

where $\sigma_{sx} = (\epsilon_x\beta_x + \eta_x^2(\Delta E/E)^2)^{1/2}, Ln_C$ is the Coulomb's logarithm, $\sigma_1 \simeq l_b/2$, while the increase in the energy spread attributable to QE can be approximated by

$$\frac{d}{dt}(\Delta E/E)_Q^2 \cong \frac{55}{48 \cdot (3)^{1/2}} \cdot \frac{r_0^2 \cdot c \cdot \gamma^5}{\alpha \cdot |\rho|^3} \, , \tag{21}$$

where ρ is the bending radius.

The equilibrium emittance is defined by the condition

$$\frac{d\epsilon_{x,y}}{dt} = 0 \ . \tag{22}$$

Thus, as the energy is lowered, the source of excitation of the transverse emittance from synchrotron radiation is also reduced by $1/\gamma^5$ for fixed radius, and the effective time

$$\frac{1}{\tau_s} + \frac{1}{\tau} \cong \frac{1}{\tau} \tag{23}$$

is approximately the same as the time for stochastic cooling.

The primary source of heat for the beam is associated with intra-beam scattering (IBS). Present designs for low-emittance damping rings are optimized under the condition that the synchrotron radiation defines the damping time. Because the damping times for optical cooling are practically independent of the beam envelope functions in the damping ring, optimization must be done at relatively lower energy in order to minimize IBS. IBS can be decreased by enlarging the vertical and horizontal beam envelope functions, which were reinforced to keep the invariant $H_{x,y}$ as small as possible. By using this optimization procedure the invariant emittance can be decreased to sufficiently low values.

VI. CONCLUSION

Stochastic cooling offers a powerful technique for reducing energy spreads in electron and positron beams when extended to optical bandwidths. All the hardware such as pickups, amplifiers and kickers have analogs in the optical region, such as quadrupole wigglers, optical amplifiers and dipole wigglers. The estimated optical amplification of signals, energetics, damping times, and final beam temperatures, are all within the realm of practical implementation for obtaining low-emittance (low-energy spread) electron and positron beams. This technique may also be useful for developing low-emittance damping rings for future linear colliders and free electron lasers. By considering the use of damping rings for linear collider applications, the

emittance can be lowered while maintaining the same luminosity which makes it possible to decrease the number of particles, thereby lowering the damping time. By reducing the number of particles, problems associated with space-charge effects can also be reduced for this application. Proton-antiproton rings have distinctly different dimensional and temporal scales. Therefore, it is possible to install a few optical cooling devices and obtain significant damping by using the techniques proposed in this paper. The proposed method also offers the possibility for consideration in cooling μ-meson beams.

We would like to thank S. Heifets and A. Zholents for useful discussions.

VII. REFERENCES

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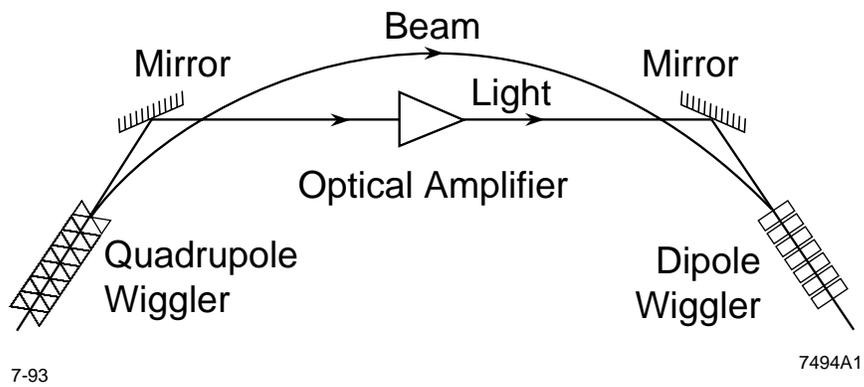5. *H. Bruck, Accèlèrateures Circulaires de Particules*, Press Universitaures de France, 1966.

Fig. 1. General layout of the system.