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Heavy Hadron Weak Decay Form Factors to Next-to-Leading Order in $1/m_Q$

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Abstract

Based on the short-distance expansion of currents in the heavy quark effective theory, we derive the exact expressions for the heavy-to-heavy meson and baryon weak decay form factors to order $1/m_Q$ in the heavy quark expansion, and to all orders in perturbation theory. We emphasize that the Wilson coefficients in this expansion depend on a kinematic variable $\bar{w}$ that is different from the velocity transfer $w = v\cdot v'$ of the hadrons. Our results generalize existing ones obtained in the leading-logarithmic approximation. Some phenomenological applications are briefly discussed.

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1 Introduction

The heavy quark effective theory (HQET) is by now a well-established tool to investigate the properties of hadrons containing a single heavy quark [1, 2, 3, 4, 5, 6]. Most important are its applications to the weak decay form factors describing the semileptonic transitions of the type $H_b \to H_c\,\ell\,\bar\nu$, from a study of which one can extract the element V_{cb} of the Cabibbo-Kobayashi-Maskawa matrix [7, 8]. Here H_Q denotes a hadron containing a single heavy quark Q. Based on a short-distance expansion of the flavor-changing weak currents, HQET provides a systematic expansion of the decay amplitudes in powers of $1/m_c$ and $1/m_b$. At leading order the effective current operators are of the form $\bar h_{v'}^c\,\Gamma\,h_v^b$, where Γ denotes some Dirac matrix with the same quantum numbers as the current in the full theory, and h_v^Q are velocity-dependent fields that represent the heavy quarks in the effective theory. Isgur and Wise have shown that hadronic matrix elements of such operators can be parameterized by universal form factors that are independent of the flavor and spin of the heavy quarks and of the Dirac structure of Γ [9]. These so-called Isgur-Wise functions only depend on the quantum numbers of the cloud of light degrees of freedom surrounding the heavy quarks. In particular, transitions between two ground-state heavy mesons (pseudoscalar or vector) or baryons are each described by a single Isgur-Wise function, which is usually denoted by $\xi(v\cdot v')$ is the case of mesons, and $\zeta(v\cdot v')$ in the case of baryons. Here v and v' are the hadron velocities, which in the $m_Q \to \infty$ limit coincide with the velocities of the heavy quarks. The physical origin of this remarkable reduction of form factors is a spin-flavor symmetry of the leading-order effective Lagrangian of HQET.

At order $1/m_Q$ (we use m_Q as a generic notation for m_c or m_b), a complete set of dimension-four operators appears in the short-distance expansion of the weak currents and of the effective Lagrangian of HQET, which induce symmetry-breaking corrections to the heavy quark limit. Additional form factors are required to parameterize the matrix elements of these operators. Luke has shown that for meson decays one needs to introduce four additional functions as well as a mass parameter $\bar\Lambda$ [10], which can be interpreted as the effective mass of the light degrees of freedom [11]. In the case of baryon transitions, the introduction of an analogous parameter and a single new function are sufficient [12]. However, although the results of these papers are general and do in principle allow one to obtain the exact expressions for weak decay form factors to order $1/m_Q$, so far these expressions have not yet been derived. The reason is that at order $1/m_Q$ the short-distance expansion of the currents was not known beyond the leading-logarithmic approximation, which when applied to the scaling between m_b and m_c is known to be a very crude approximation of at best pedagogical relevance [13]. To get reliable numerical results, a full next-to-leading order calculation is unavoidable. Such a calculation is already very tedious at leading order in the heavy quark expansion, however, and to go to order $1/m_Q$ seemed very complicated.

In a recent paper we have shown that a "hidden" symmetry of HQET, namely its invariance under reparameterizations of the heavy quark momentum operator [14], leads to the surprising result that all the Wilson coefficients appearing at order $1/m_Q$ in the short-distance expansion of the weak currents can be related to the coefficients appearing at leading order [15]. By virtue of this result one can construct the expansion of any current to next-to-leading order in $1/m_Q$, without even knowing the explicit structure of the Wilson coefficients. For instance, the most general form of the vector current is

$$\begin{aligned}\bar c\,\gamma^\alpha b \to\ & C_1(\bar w,\mu)\,\bar h^c_{v'}\left[\gamma^\alpha + \gamma^\alpha\,\frac{i\not{D}}{2m_b} - \frac{i\overleftarrow{\not{D}}}{2m_c}\,\gamma^\alpha\right] h^b_v \\ & +\ C_2(\bar w,\mu)\,\bar h^c_{v'}\left[v^\alpha + \frac{iD^\alpha}{m_b} + v^\alpha\,\frac{i\not{D}}{2m_b} - \frac{i\overleftarrow{\not{D}}}{2m_c}\,v^\alpha\right] h^b_v \\ & +\ C_3(\bar w,\mu)\,\bar h^c_{v'}\left[v'^\alpha - \frac{i\overleftarrow{D}^\alpha}{m_c} + v'^\alpha\,\frac{i\not{D}}{2m_b} - \frac{i\overleftarrow{\not{D}}}{2m_c}\,v'^\alpha\right] h^b_v + \mathcal{O}(1/m_Q^2)\,.\end{aligned} \tag{1}$$

A similar expansion with coefficients $C_i^5(\bar w,\mu)$, and with γ_5 inserted after whatever object carries the Lorentz index α, obtains for the axial vector current. In these expressions, the kinematic variable $\bar w$ to be used in the Wilson coefficients is different from the velocity transfer $w = v\cdot v'$ of the hadrons. The relation is [15]

$$\bar w = w + \left(\frac{\bar\Lambda}{m_b} + \frac{\bar\Lambda}{m_c}\right)(w-1) + \mathcal{O}(1/m_Q^2)\,, \tag{2}$$

where $\bar\Lambda$ is defined as the asymptotic mass difference between a hadron H_Q and the heavy quark that it contains. In the $m_Q\to\infty$ limit, this difference approaches a finite value, $m_{H_Q} - m_Q \to \bar{\Lambda}$, which can be identified with effective mass of the light degrees of freedom in the hadronic bound state.[1]

The variable $\bar{w}$ can be interpreted as the velocity transfer of free quarks. Consider the weak transition $H_b \to H_c + W^-$. The bottom quark in the initial state H_b moves on average (up to fluctuations of order $1/m_b$) with the hadron's velocity. When the W-boson is emitted, the outgoing charm quark has in general some different velocity. Let us denote the product of these velocities by $\bar{w}$. Since over short time scales the quark velocities remain unchanged, this is what is "seen" by hard gluons. After the W-emission, the light degrees of freedom still have the initial hadron's velocity. But they have to recombine with the outgoing heavy quark to form the final state H_c. Thus, a rearrangement is necessary, which happens over much larger, hadronic time scales by the exchange of soft gluons. In this process the velocity of the charm quark is changed by an amount of order $1/m_c$ (its momentum is changed by an amount of order Λ_{QCD}). Hence the final hadron velocity transfer w will differ from the "short-distance" quark velocity transfer $\bar{w}$ by an amount of order $1/m_Q$. The precise relation between w and $\bar{w}$ is determined by momentum conservation and is given in (2). In fact, this relation is nothing but the condition $(p_{H_b} - p_{H_c})^2 = (p_b - p_c)^2$, i.e., that the momentum transfer to the hadrons equals the momentum transfer to the heavy quarks. At zero recoil, no rearrangement is needed, and indeed $w = \bar{w} = 1$ in this limit.

In the case of flavor-changing currents, the Wilson coefficients in (1) are very complicated functions of $\bar{w}$, the heavy quark masses, and the renormalization scale μ. For the moment, the precise structure of these functions is not important. What will be relevant is that the μ-dependence can be factorized into a universal function $K_{\text{hh}}(\bar{w},\mu)$, which is independent of the heavy quark masses and is the same for any bilinear heavy quark current in HQET:

$$C_i^{(5)}(\bar{w},\mu) = \widehat{C}_i^{(5)}(m_b,m_c,\bar{w})\, K_{\text{hh}}(\bar{w},\mu)\,. \tag{3}$$

At zero recoil this function is independent of μ, and it is convenient to use a normalization such that $K_{\text{hh}}(1,\mu) = 1$. From now on we shall for simplicity write $\widehat{C}_i^{(5)}(\bar{w})$ and not display the dependence on the heavy quark masses.

In addition to the currents, one needs the effective Lagrangian of HQET

[1] This definition of $\bar{\Lambda}$ implicitly relies on a particular choice of the heavy quark mass, as discussed in detail in Ref. [11].

to order $1/m_Q$. For a heavy quark Q with velocity v, it reads [1, 2, 3, 4]

$$\mathcal{L}_{\rm eff} = \bar h_v^Q\, iv\cdot D\, h_v^Q + \frac{1}{2m_Q}\,\mathcal{L}_1^Q + \mathcal{O}(1/m_Q^2)\,, \tag{4}$$

where

$$\mathcal{L}_1^Q = \bar h_v^Q\,(iD)^2\,h_v^Q + C_{\rm mag}(\mu)\,\frac{g_s}{2}\,\bar h_v^Q\,\sigma_{\alpha\beta}\,G^{\alpha\beta}\,h_v^Q\,. \tag{5}$$

Similar to (3), the Wilson coefficient of the chromo-magnetic operator can be written in the factorized form

$$C_{\rm mag}(\mu) = \widehat{C}_{\rm mag}(m_Q)\,K_{\rm mag}(\mu)\,, \tag{6}$$

where $K_{\rm mag}(\mu)$ is independent of m_Q. The first operator in $\mathcal{L}_1^Q$ is not renormalized [14].

Explicit expressions for the short-distance coefficients can be found in the literature (see, e.g., Refs. [1, 4, 13]). With the exception of $C_{\rm mag}$, they are known to next-to-leading order in renormalization-group improved perturbation theory. However, as written above the short-distance expansions (1) and (4) are completely general and true to all orders in perturbation theory. Given these results, one can derive the exact expressions for the weak decay form factors of heavy mesons or baryons to order $1/m_Q$. This is the purpose of this paper. The resulting expressions are presented in an explicitly renormalization-group invariant form by introducing μ-independent Isgur-Wise functions and Wilson coefficients. They generalize approximate results obtained in leading-logarithmic approximation by Luke [10] and Georgi et al. [12]. In Sec. 2 we discuss the form factors for the decay $\Lambda_b\to\Lambda_c\,\ell\,\bar\nu$. The more complicated case of meson weak decays is considered in Sec. 3.

2 Baryon Form Factors

Consisting of a heavy quark and light degrees of freedom with quantum numbers of a spin-0 diquark, the ground-state Λ_Q baryons are particularly simple hadrons. In HQET they are represented by a spinor $u_\Lambda(v,s)$ that can be identified with the spinor of the heavy quark. For simplicity, we shall from now on omit the spin labels and write $u(v)\equiv u_\Lambda(v,s)$. The baryon matrix elements of the weak currents $V^\alpha=\bar c\,\gamma^\alpha b$ and $A^\alpha=\bar c\,\gamma^\alpha\gamma_5\,b$ can be parameterized by six hadronic form factors, which we write as functions of the baryon velocity transfer $w = v\cdot v'$:

$$\begin{aligned}
\langle\Lambda_c(v')|\, V^\alpha\,|\Lambda_b(v)\rangle &= \bar u(v')\Big[F_1(w)\,\gamma^\alpha + F_2(w)\,v^\alpha + F_3(w)\,v'^\alpha\Big]\,u(v)\,,\\
\langle\Lambda_c(v')|\, A^\alpha\,|\Lambda_b(v)\rangle &= \bar u(v')\Big[G_1(w)\,\gamma^\alpha + G_2(w)\,v^\alpha + G_3(w)\,v'^\alpha\Big]\gamma_5\,u(v)\,.
\end{aligned} \tag{7}$$

The aim is to construct an expansion of $F_i(w)$ and $G_i(w)$ in powers of $1/m_Q$, and to relate the coefficients in this expansion to universal, mass-independent functions of the velocity transfer. Given the operator product expansion of the weak currents as in (1), this is achieved by evaluating the matrix elements of the effective current operators. The baryon matrix elements of the dimension-three operators can be parameterized in terms of a single Isgur-Wise function $\zeta(w,\mu)$ defined by [16, 17, 18, 19]

$$\langle\Lambda_c(v')|\,\bar h_{v'}^c\,\Gamma\,h_v^b\,|\Lambda_b(v)\rangle = \zeta(w,\mu)\,\bar u(v')\,\Gamma\,u(v)\,. \tag{8}$$

As discussed by Georgi, Grinstein, and Wise [12], the power corrections of order $1/m_Q$ involve contributions of two types. The first come from the dimension-four operators in the expansion of the currents. Their matrix elements can be related to the generic matrix element

$$\langle\Lambda_c(v')|\,\bar h_{v'}^c\,\Gamma\,iD_\beta\,h_v^b\,|\Lambda_b(v)\rangle = \zeta_\beta(v,v',\mu)\,\bar u(v')\,\Gamma\,u(v)\,. \tag{9}$$

The equation of motion $iv\cdot D\,h_v^Q = 0$ allows one to relate $\zeta_\beta(v,v',\mu)$ to the leading-order Isgur-Wise function:

$$\zeta_\beta(v,v',\mu) = \frac{w\,v_\beta - v'_\beta}{w+1}\,\bar\Lambda\,\zeta(w,\mu)\,. \tag{10}$$

The form factors also receive corrections from insertions of the higher-dimension operators of the effective Lagrangian into matrix elements of the leading-order currents. However, baryon matrix elements with an insertion of the chromo-magnetic operator vanish, since the total spin of the light degrees of freedom is zero. Insertions of the kinetic operator preserve the Dirac structure of the currents, hence effectively correcting the Isgur-Wise function $\zeta(w,\mu)$. The total effect is

$$\langle\Lambda_c(v')|\,i\!\int\!\mathrm{d}x\,T\,\{\,\bar h_{v'}^c\,\Gamma\,h_v^b(0),\mathcal{L}_1^b(x)\,\}\,|\Lambda_b(v)\rangle = 2\bar\Lambda\,\chi(w,\mu)\,\bar u(v')\,\Gamma\,u(v)\,. \tag{11}$$

We have factored out $\bar{\Lambda}$ to obtain a dimensionless form factor $\chi(w,\mu)$. Insertions of $\mathcal{L}_1^c$ can be parameterized by the same function.

Given these definitions, one can readily work out the explicit expressions for the hadronic form factors $F_i(w)$ and $G_i(w)$ at next-to-leading order in the $1/m_Q$ expansion. Of course, the physical form factors should be written in terms of renormalized universal functions rather than the μ-dependent functions that parameterize the matrix elements in the effective theory. According to (3), the μ-dependence of the Wilson coefficients of any bilinear heavy quark current can be factorized into a universal function $K_{\rm hh}(\bar{w},\mu)$, which is normalized at zero recoil. It is now important to recall that the variable $\bar{w}$ differs from the hadron velocity transfer by terms of order $1/m_Q$. Using (2) we find

$$K_{\rm hh}(\bar{w},\mu) = K_{\rm hh}(w,\mu) + \left(\frac{\bar{\Lambda}}{m_b} + \frac{\bar{\Lambda}}{m_c}\right)(w-1)\,\frac{\partial}{\partial w}\,K_{\rm hh}(w,\mu) + \ldots\,. \qquad (12)$$

The second term gives an extra contribution to the renormalization of $\chi(w,\mu)$. We define

$$K_{\rm hh}(\bar{w},\mu)\left[\zeta(w,\mu) + \left(\frac{\bar{\Lambda}}{m_b} + \frac{\bar{\Lambda}}{m_c}\right)\chi(w,\mu)\right] \equiv \zeta_{\rm ren}(w) + \left(\frac{\bar{\Lambda}}{m_b} + \frac{\bar{\Lambda}}{m_c}\right)\chi_{\rm ren}(w)\,, \qquad (13)$$

so that

$$\begin{aligned} \zeta_{\rm ren}(w) &= \zeta(w,\mu)\,K_{\rm hh}(w,\mu)\,,\\ \chi_{\rm ren}(w) &= K_{\rm hh}(w,\mu)\,\chi(w,\mu) + \zeta_{\rm ren}(w)\,(w-1)\,\frac{\partial}{\partial w}\ln K_{\rm hh}(w,\mu)\,. \end{aligned} \qquad (14)$$

Note that, because of $K_{\rm hh}(1,\mu)=1$, the renormalized universal functions agree with the original functions at zero recoil.

For the presentation of our results we find it convenient to introduce the dimensionless ratios $\varepsilon_Q = \bar{\Lambda}/2m_Q$, and to collect certain $1/m_Q$ corrections that always appear in combination with the Isgur-Wise function into a new function

$$\widehat{\zeta}_{bc}(w) \equiv \zeta_{\rm ren}(w) + (\varepsilon_c + \varepsilon_b)\left[2\chi_{\rm ren}(w) + \frac{w-1}{w+1}\,\zeta_{\rm ren}(w)\right]. \qquad (15)$$

Because of the dependence on the heavy quark masses this is no longer a universal form factor. However, the flavor dependence is irrelevant as long as one considers $\Lambda_b \to \Lambda_c$ transitions only. Furthermore, we shall see below that $\widehat{\zeta}_{bc}(w)$ and $\zeta_{\rm ren}(w)$ obey the same normalization at zero recoil. Let us factorize the hadronic form factors according to

$$F_i(w) = N_i(w)\,\widehat{\zeta}_{bc}(w)\,, \qquad G_i(w) = N_i^5(w)\,\widehat{\zeta}_{bc}(w)\,. \tag{16}$$

Then the exact next-to-leading order expressions for the correction factors are:

$$\begin{aligned} N_1(w) &= \widehat{C}_1(\bar w)\left[1 + \frac{2}{w+1}\,(\varepsilon_c + \varepsilon_b)\right],\\ N_2(w) &= \widehat{C}_2(\bar w)\left(1 + \frac{2w\,\varepsilon_b}{w+1}\right) - \Big[\widehat{C}_1(\bar w) + \widehat{C}_3(\bar w)\Big]\,\frac{2\varepsilon_c}{w+1}\,,\\ N_3(w) &= \widehat{C}_3(\bar w)\left(1 + \frac{2w\,\varepsilon_c}{w+1}\right) - \Big[\widehat{C}_1(\bar w) + \widehat{C}_2(\bar w)\Big]\,\frac{2\varepsilon_b}{w+1}\,,\\ N_1^5(w) &= \widehat{C}_1^5(\bar w)\,,\\ N_2^5(w) &= \widehat{C}_2^5(\bar w)\left(1 + \frac{2\varepsilon_c}{w+1} + 2\varepsilon_b\right) - \Big[\widehat{C}_1^5(\bar w) + \widehat{C}_3^5(\bar w)\Big]\,\frac{2\varepsilon_c}{w+1}\,,\\ N_3(w) &= \widehat{C}_3^5(\bar w)\left(1 + 2\varepsilon_c + \frac{2\varepsilon_b}{w+1}\right) + \Big[\widehat{C}_1^5(\bar w) - \widehat{C}_2^5(\bar w)\Big]\,\frac{2\varepsilon_b}{w+1}\,. \end{aligned} \tag{17}$$

It is remarkable that, up to an overall unknown function $\widehat{\zeta}_{bc}(w)$, the baryon form factors at order $1/m_Q$ are completely determined in terms of ε_i and the short-distance coefficient functions. Notice that the Wilson coefficients are functions of the "short-distance" quark velocity transfer $\bar w$, whereas the remaining kinematic expressions and the universal form factors depend on the hadron velocity transfer w. In leading-logarithmic approximation, where $\widehat{C}_1 = \widehat{C}_1^5$ and all other coefficients are set to zero, our exact expressions reduce to the approximate results obtained in Ref. [12].

For the numerical evaluation of the correction factors $N_i^{(5)}(w)$ we use the next-to-leading order expressions for the Wilson coefficients from Ref. [13]. As input parameters we take $m_b = 4.80$ GeV and $m_c = 1.45$ GeV for the heavy quark masses, and $\Lambda_{\overline{\rm MS}} = 0.25$ GeV (for $n_f = 4$) in the two-loop expressions for the running coupling constant. The resulting values of the short-distance coefficients for different values of the quark velocity transfer $\bar w$ are given in Table 1. For the physical processes of interest, we also show the

$\bar{w}$	$w_{\Lambda_b\to\Lambda_c}$	$w_{\bar{B}\to D^*}$	$w_{\bar{B}\to D}$	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_1^5$	$\widehat{C}_2^5$	$\widehat{C}_3^5$
1.0	1.00	1.00	1.00	1.14	−0.08	−0.02	0.99	−0.12	0.04
1.1	1.05	1.06	1.07	1.11	−0.08	−0.02	0.97	−0.11	0.04
1.2	1.11	1.13	1.15	1.08	−0.08	−0.02	0.95	−0.11	0.04
1.3	1.16	1.19	1.22	1.06	−0.07	−0.02	0.93	−0.10	0.04
1.4	1.22	1.25	1.29	1.03	−0.07	−0.02	0.91	−0.10	0.03
1.5	1.27	1.31	1.37	1.01	−0.07	−0.02	0.89	−0.09	0.03
1.6	1.33	1.38	1.44	0.99	−0.06	−0.02	0.88	−0.09	0.03
1.7	1.38	1.44	1.51	0.97	−0.06	−0.02	0.86	−0.09	0.03
1.8	1.43	1.50	1.59	0.95	−0.06	−0.02	0.85	−0.08	0.03

Table 1: Short-distance coefficients for $b \to c$ transitions.

corresponding values of the hadron velocity transfer w. They are obtained from (2) by using $\bar{\Lambda}_{\rm baryon} = 0.84$ GeV and $\bar{\Lambda}_{\rm meson} = 0.51$ GeV, which give the correct hadron masses. The corresponding values of the parameters ε_Q for Λ_Q baryons are $\varepsilon_c \approx 0.29$ and $\varepsilon_b \approx 0.09$. From (17) we then obtain the results shown in Table 2. The correction factors $N_i^{(5)}$ are given in dependence of the baryon velocity transfer w over the kinematic region accessible in $\Lambda_b \to \Lambda_c\,\ell\,\bar{\nu}$ decays. We find that symmetry-breaking corrections can be quite sizable in heavy baryon decays. This is not too surprising, since $\varepsilon_c \approx 0.3$ sets the natural scale of power corrections, and the QCD corrections are typically of order $\alpha_s(m_c)/\pi \approx 0.1$. We note, however, that upon contraction with the lepton current the form factors $F_2(w)$ and $G_2(w)$ become suppressed, relative to $F_3(w)$ and $G_3(w)$, by a factor $m_{\Lambda_c}/m_{\Lambda_b} \approx 0.4$. The apparently large corrections to these form factors thus become less important when one computes physical decay amplitudes.

Vector current conservation implies that $\sum_i F_i(1) = 1$ in the limit of equal baryon masses. Taking into account that $\sum_i \widehat{C}_i(1) = 1$ in this limit [13], we conclude that $\zeta_{\rm ren}(1) + 4\varepsilon_Q \chi_{\rm ren}(1) = 1$, which must be satisfied for any value of m_Q. Hence, we recover the well-known normalization conditions $\zeta_{\rm ren}(1) = 1$ and $\chi_{\rm ren}(1) = 0$. It also follows that $\widehat{\zeta}_{bc}(1) = 1$, which justifies the definition of this function in the first place. At zero recoil, the normalization of the baryon form factors is thus completely determined by (17). The most important consequence of these relations is that the quantities $\sum_i F_i(w)$ and

w	$\sum_i N_i$	N_2	N_3	N_1^5	N_2^5	N_3^5
1.00	1.03	−0.42	−0.12	0.99	−0.48	0.17
1.11	0.98	−0.37	−0.11	0.94	−0.43	0.15
1.22	0.94	−0.34	−0.10	0.91	−0.39	0.14
1.33	0.90	−0.31	−0.09	0.88	−0.35	0.13
1.44	0.87	−0.29	−0.09	0.85	−0.32	0.12

Table 2: Correction factors for the $\Lambda_b \to \Lambda_c$ decay form factors.

$G_1(w)$ do not receive any $1/m_Q$ corrections at zero recoil. This is the analog of Luke's theorem for baryon decays [12]. Because of this result, it might be possible to extract an accurate value of V_{cb} from the measurement of semileptonic Λ_b decays near zero recoil, where the decay rate is governed by the form factor G_1 alone. The deviations from the prediction $G_1(1) = \widehat{C}_1^5(1) \approx 0.99$ are of order $1/m_Q^2$ and are expected to be small [20].

3 Meson Form Factors

The most important application of heavy quark symmetry is to derive relations between the form factors parameterizing the exclusive weak decays $\bar{B} \to D\,\ell\,\bar{\nu}$ and $\bar{B} \to D^*\ell\,\bar{\nu}$. A detailed theoretical understanding of these processes is a necessary prerequisite for a reliable determination of the element V_{cb} of the quark mixing matrix. We start by introducing a convenient set of six hadronic form factors $h_i(w)$, which parameterize the relevant meson matrix elements of the flavor-changing vector and axial vector currents $V^\alpha = \bar{c}\,\gamma^\alpha b$ and $A^\alpha = \bar{c}\,\gamma^\alpha\gamma_5\, b$:

$$\begin{aligned} \langle D(v')|\, V^\alpha\, |\bar{B}(v)\rangle &= h_+(w)\,(v+v')^\alpha + h_-(w)\,(v-v')^\alpha\,, \\ \langle D^*(v',\epsilon)|\, V^\alpha\, |\bar{B}(v)\rangle &= i\,h_V(w)\,\epsilon^{\alpha\beta\mu\nu}\,\epsilon^*_\beta\,v'_\mu\,v_\nu\,, \\ \langle D^*(v',\epsilon)|\, A^\alpha\, |\bar{B}(v)\rangle &= h_{A_1}(w)\,(w+1)\,\epsilon^{*\alpha} - \Big[h_{A_2}(w)\,v^\alpha + h_{A_3}(w)\,v'^\alpha\Big]\epsilon^*\!\cdot v\,. \end{aligned} \qquad (18)$$

Here $w = v\cdot v'$ is the velocity transfer of the mesons. For simplicity we work with a nonrelativistic normalization of states. To obtain the standard relativistic normalization one has to multiply the right-hand sides of (18) by $\sqrt{m_B\, m_{D^{(*)}}}$.

In HQET, the doublet of the ground-state pseudoscalar and vector mesons can be represented by a combined tensor wave function

$$\mathcal{M}(v) = \frac{1+\not v}{2} \begin{cases} -\gamma_5\,; & \text{pseudoscalar meson,} \\ \not\epsilon\,; & \text{vector meson.} \end{cases} \tag{19}$$

We shall use a notation where $M(v)$ represents $\bar B$ or $\bar B^*$, and $M'(v')$ stands for D or D^*. Meson matrix elements of the leading-order currents can then be written as [5]

$$\langle M'(v')|\,\bar h^c_{v'}\,\Gamma\,h^b_v\,|M(v)\rangle = -\xi(w,\mu)\,\mathrm{Tr}\Big\{\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\Big\}\,, \tag{20}$$

where $\xi(w,\mu)$ is *the* Isgur-Wise function. The $1/m_Q$ corrections have been analyzed by Luke [10]. Matrix elements of the dimension-four current operators in (1) can be related to

$$\langle M'(v')|\,\bar h^c_{v'}\,\Gamma\,iD_\beta\,h^b_v\,|M(v)\rangle = -\mathrm{Tr}\Big\{\,\xi_\beta(v,v',\mu)\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\Big\}\,. \tag{21}$$

The tensor form factor $\xi_\beta(v,v',\mu)$ has components proportional to v_β, v'_β, and γ_β. The equation of motion yields two relations among these three, and the final result can be written in the form

$$\xi_\beta(v,v',\mu) = \frac{\bar\Lambda}{w+1}\,\xi(w,\mu)\,\Big\{\Big[w-\eta(w)\Big]\,v_\beta - \Big[1+\eta(w)\Big]\,v'_\beta - (w+1)\,\eta(w)\,\gamma_\beta\Big\}\,, \tag{22}$$

where $\eta(w)$ is a renormalization-group invariant function [21].[2] A second class of $1/m_Q$ corrections comes from insertions of higher-dimension operators of the effective Lagrangian. The corresponding matrix elements have the structure

$$\begin{aligned} &\langle M'(v')|\,i\!\int\!\mathrm{d}x\,T\,\{\,\bar h^c_{v'}\,\Gamma\,h^b_v(0),\mathcal{L}^b_1(x)\,\}\,|M(v)\rangle \\ &\quad = -2\bar\Lambda\,\chi_1(w,\mu)\,\mathrm{T}\Big\{\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\,\Big\} \\ &\qquad - 2\bar\Lambda\,C_{\mathrm{mag}}(\mu)\,\mathrm{Tr}\Big\{\,\chi_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'(v')\,\Gamma\,P_+\,\sigma^{\alpha\beta}\mathcal{M}(v)\,\Big\}\,, \end{aligned} \tag{23}$$

and similar for an insertion of $\mathcal{L}^c_1$. Here $P_+ = \frac{1}{2}(1+\not v)$. Again we have factored out $\bar\Lambda$ in order for the form factors to be dimensionless. The kinetic operator

[2] In the notation of Ref. [10], one has $\bar\Lambda\,\eta(w) = \xi_3(w,\mu)/\xi(w,\mu)$.

contained in $\mathcal{L}_1^b$ transforms as a Lorentz scalar. An insertion of it does not affect the Dirac structure of the matrix element. Hence, the corresponding function $\chi_1(w,\mu)$ effectively corrects the Isgur-Wise function. The chromomagnetic operator, on the other hand, carries a nontrivial Dirac structure. An insertion of it brings a matrix $\sigma^{\alpha\beta}$ next to the meson wave function $\mathcal{M}(v)$. In addition, a propagator separates this insertion from the heavy quark current, resulting in a projection operator P_+ to the right of Γ. This explains the structure of the second trace in (23). Because of $v_\alpha\, P_+\, \sigma^{\alpha\beta}\mathcal{M}(v) = 0$, the most general decomposition of the tensor form factor $\chi_{\alpha\beta}(v,v',\mu)$ is

$$\chi_{\alpha\beta}(v,v',\mu) = i\,\chi_2(w,\mu)\,v'_\alpha\gamma_\beta + \chi_3(w,\mu)\,\sigma_{\alpha\beta}\,. \tag{24}$$

The terms proportional to $\chi_3(w,\mu)$ in (23) can be simplified by means of the identity $P_+\,\sigma^{\alpha\beta}\mathcal{M}(v)\,\sigma_{\alpha\beta} = 2d_M\,\mathcal{M}(v)$, where $d_P = 3$ for pseudoscalar and $d_V = -1$ for vector mesons. It follows that, irrespective of the structure of the current, the function $\chi_3(w,\mu)$ always appears in combination with the Isgur-Wise function, but with a coefficient that is different for pseudoscalar and vector mesons. It thus represents a spin-symmetry violating correction to the meson wave function.

What remains to be done is to introduce renormalized form factors. We define $\xi_{\rm ren}(w)$ and $\chi_1^{\rm ren}(w)$ in analogy to $\zeta_{\rm ren}(w)$ and $\chi_{\rm ren}(w)$ in (14). For the remaining functions, we define

$$\chi_i^{\rm ren}(w) = K_{\rm mag}(\mu)\,K_{\rm hh}(w,\mu)\,\chi_i(w,\mu)\,;\qquad i=2,3. \tag{25}$$

It is again convenient to introduce quantities $N_i(w)$, which contain the symmetry-breaking corrections to the heavy quark limit, by

$$h_i(w) = N_i(w)\,\xi_{\rm ren}(w)\,. \tag{26}$$

As in the baryon case we denote $\varepsilon_Q = \bar\Lambda/2m_Q$. We stress, though, that the numerical values of these parameters are different in the two cases. It is useful to define three new functions $L_{P,V}(w)$ and $L_3(w)$ by

$$\begin{aligned}
\xi_{\rm ren}(w)\,L_P(w) &= 2\chi_1^{\rm ren}(w) - 4\,\widehat C_{\rm mag}(m_Q)\,\Big[(w-1)\,\chi_2^{\rm ren}(w) - 3\chi_3^{\rm ren}(w)\Big]\,,\\
\xi_{\rm ren}(w)\,L_V(w) &= 2\chi_1^{\rm ren}(w) - 4\,\widehat C_{\rm mag}(m_Q)\,\chi_3^{\rm ren}(w)\,,\\
\xi_{\rm ren}(w)\,L_3(w) &= 4\,\widehat C_{\rm mag}(m_c)\,\chi_2^{\rm ren}(w)\,.
\end{aligned} \tag{27}$$

L_P and L_V are corrections to the Isgur-Wise function which always appear for pseudoscalar and vector mesons, respectively, irrespective of the structure of the current [20]. We first present the results for N_+ and N_{A_1}, which will play a special role in the analysis below. They are:

$$\begin{aligned}
N_+(w) &= \Big[\widehat{C}_1(\bar{w}) + \frac{w+1}{2}\Big(\widehat{C}_2(\bar{w}) + \widehat{C}_3(\bar{w})\Big)\Big]\Big\{1 + \varepsilon_c L_D(w) + \varepsilon_b L_B(w)\Big\} \\
&\quad + \varepsilon_c\,\frac{w-1}{2}\Big\{[1-2\eta(w)]\,\widehat{C}_2(\bar{w}) + [3-2\eta(w)]\,\widehat{C}_3(\bar{w})\Big\} \\
&\quad + \varepsilon_b\,\frac{w-1}{2}\Big\{[3-2\eta(w)]\,\widehat{C}_2(\bar{w}) + [1-2\eta(w)]\,\widehat{C}_3(\bar{w})\Big\}\,, \\
N_{A_1}(w) &= \widehat{C}_1^5(\bar{w})\Big\{1 + \varepsilon_c L_{D^*}(w) + \varepsilon_b L_B(w)\Big\} \\
&\quad + \varepsilon_c\,\frac{w-1}{w+1}\Big\{\widehat{C}_1^5(\bar{w}) + 2\eta(w)\,\widehat{C}_3^5(\bar{w})\Big\} \\
&\quad + \varepsilon_b\,\frac{w-1}{w+1}\Big\{[1-2\eta(w)]\,\widehat{C}_1^5(\bar{w}) + 2\eta(w)\,\widehat{C}_2^5(\bar{w})\Big\}\,.
\end{aligned} \tag{28}$$

Notice again that the Wilson coefficients are functions of the quark velocity transfer $\bar{w}$, whereas the universal form factors depend on the meson velocity transfer w. The expressions for the remaining four form factors are more lengthy. To display them we omit the dependence on $\bar{w}$ and w. We find:

$$\begin{aligned}
N_- &= \frac{w+1}{2}\,(\widehat{C}_2 - \widehat{C}_3)\Big\{1 + \varepsilon_c L_D + \varepsilon_b L_B\Big\} \\
&\quad - \varepsilon_c\Big\{(1-2\eta)\Big[\widehat{C}_1 - \frac{w-1}{2}\,(\widehat{C}_2 - \widehat{C}_3)\Big] + (w+1)\,\widehat{C}_3\Big\} \\
&\quad + \varepsilon_b\Big\{(1-2\eta)\Big[\widehat{C}_1 + \frac{w-1}{2}\,(\widehat{C}_2 - \widehat{C}_3)\Big] + (w+1)\,\widehat{C}_2\Big\}\,, \\
N_V &= \widehat{C}_1\Big\{1 + \varepsilon_c L_{D^*} + \varepsilon_b L_B\Big\} \\
&\quad + \varepsilon_c\Big\{\widehat{C}_1 - 2\eta\,\widehat{C}_3\Big\} + \varepsilon_b\Big\{[1-2\eta]\,\widehat{C}_1 - 2\eta\,\widehat{C}_2\Big\}\,, \\
N_{A_2} &= \widehat{C}_2^5\Big\{1 + \varepsilon_c L_{D^*} + \varepsilon_b L_B\Big\} + \Big[\widehat{C}_1^5 + (w-1)\,\widehat{C}_2^5\Big]\,\varepsilon_c L_3 \\
&\quad - \frac{2\varepsilon_c}{w+1}\Big\{(1+\eta)\,(\widehat{C}_1^5 + \widehat{C}_3^5) - \frac{w+1}{2}\,(1+2\eta)\,\widehat{C}_2^5\Big\}
\end{aligned} \tag{29}$$

$$
\begin{aligned}
&+\frac{2\varepsilon_b}{w+1}\,\widehat{C}_2^5\left\{\frac{3w+1}{2}-(w+2)\,\eta\right\},\\
N_{A_3} &= (\widehat{C}_1^5+\widehat{C}_3^5)\left\{1+\varepsilon_c L_{D^*}+\varepsilon_b L_B\right\}-\left[\widehat{C}_1^5-(w-1)\,\widehat{C}_3^5\right]\varepsilon_c L_3\\
&+\frac{\varepsilon_c}{w+1}\left\{(w-1-2\eta)\,(\widehat{C}_1^5-\widehat{C}_3^5)+4w\,(1+\eta)\,\widehat{C}_3^5\right\}\\
&+\varepsilon_b\left\{(1-2\eta)\,(\widehat{C}_1^5+\widehat{C}_3^5)+\frac{2}{w+1}\,(w\eta-1)\,\widehat{C}_2^5\right\}.
\end{aligned}
$$

These expressions are the main result of this paper. In leading-logarithmic approximation, where $\widehat{C}_1=\widehat{C}_1^5$ and all other coefficients are set to zero, they reduce to the approximate expressions obtained by Luke [10].

It is difficult to extract much information from the complicated expressions for N_i without a prediction for the subleading universal form factors $\eta(w)$ and $\chi_i^{\rm ren}(w)$. Recently, these functions have been investigated in great detail using the QCD sum rule approach. The interested reader is referred to Refs. [21, 22] for further discussion. Some important, general observations can be made without such an analysis, however. Vector current conservation implies that $L_P(1)=L_V(1)=0$, from which one can derive the well-known normalization conditions $\xi_{\rm ren}(1)=1$ and $\chi_1^{\rm ren}(1)=\chi_3^{\rm ren}(1)=0$. Inserting this into (28), one finds that the $1/m_Q$ corrections in N_+ and N_{A_1} vanish at zero recoil. This is Luke's theorem [10], which implies that the leading power corrections to the meson form factors $h_+(1)$ and $h_{A_1}(1)$ are of order $1/m_Q^2$. In particular, it follows that

$$
h_{A_1}(1)=\widehat{C}_1^5(1)+\mathcal{O}(1/m_Q^2)\,. \tag{30}
$$

This relation plays a central role in the model-independent extraction of V_{cb} from $\bar{B}\to D^*\ell\,\bar{\nu}$ decays [8]. In this case, even the second-order corrections have been analyzed in detail and are found to be suppressed [20].

It has been emphasized in Ref. [23] that Luke's theorem does not protect the remaining form factors in (29). For instance, the $\bar{B}\to D\,\ell\,\bar{\nu}$ decay amplitude at zero recoil is proportional to the combination

$$
h_+(1)-\sqrt{S}\,h_-(1)=\left[\widehat{C}_1(1)+\widehat{C}_2(1)+\widehat{C}_3(1)\right]\left\{1+S\cdot K\right\}, \tag{31}
$$

where $S=(\frac{m_B-m_D}{m_B+m_D})^2\approx 0.23$. The $1/m_Q$ corrections enter this expression through $h_-(1)$ and are contained in the quantity K. Using the analytic expressions for the Wilson coefficients given in Ref. [13], we find

$$K = \delta_1 + (\varepsilon_c + \varepsilon_b) \Big[(1 + \delta_1) - 2(1 + \delta_2)\, \eta(1)\Big] , \qquad (32)$$

with

$$\begin{aligned} \delta_1 &= \frac{1+z}{1-z} \left\{ \frac{4\alpha_s(m_c)}{3\pi} - \frac{2\alpha_s(m_b)}{3\pi} + \frac{4\alpha_s(\bar{m})}{3\pi}\, \frac{z}{1-z} \bigg(\frac{\ln z}{1-z} + 1 \bigg) \right\} , \\ \delta_2 &= \frac{4\alpha_s(m_c)}{3\pi} - \frac{2\alpha_s(m_b)}{3\pi} . \end{aligned} \qquad (33)$$

Here $z = m_c/m_b$, and $\bar{m}$ denotes the geometric average of the heavy quark masses ($\bar{m} \approx 2.23$ GeV). Note that δ_1 has a smooth limit as $z \to 1$. Numerically, we find $\delta_1 \approx 6\%$ and $\delta_2 \approx 9\%$, and thus

$$K \approx 0.3 - 0.5\, \eta(1) . \qquad (34)$$

Recently, the function $\eta(w)$ has been calculated using the QCD sum rule approach, with the result that $\eta(w) = 0.6 \pm 0.2$ essentially independent of w [21]. We observe that, by a fortunate accident, this leads to an almost perfect cancellation in (34), i.e., $K \approx 0.0 \pm 0.1$. This means that the $1/m_Q$ corrections to the $\bar{B} \to D\, \ell\, \bar{\nu}$ decay rate at zero recoil are highly suppressed.

Further predictions can be made for ratios of meson form factors, in which some of the universal functions drop out [24]. An important example is the ratio $R = h_V/h_{A_1}$, which can be extracted from a measurement of the forward-backward asymmetry in $\bar{B} \to D^*\ell\, \bar{\nu}$ decays. From (28) and (29) it is readily seen that R is independent of the functions $\chi_i^{\rm ren}(w)$. In fact, at order $1/m_Q$ the following simple expression can be derived:

$$R = F_1(w) \left\{ 1 + \frac{2\varepsilon_c}{w+1} + \frac{2\varepsilon_b}{w+1} \Big[1 - 2F_2(w)\, \eta(w) \Big] \right\} . \qquad (35)$$

Note that the form factor $\eta(w)$ enters in the $1/m_b$ corrections only. The functions $F_i(w)$ contain the short-distance corrections and are given by

$$\begin{aligned} F_1(w) &= 1 + \frac{4\alpha_s(m_c)}{3\pi}\, r(w) , \\ F_2(w) &= 1 + \frac{2\alpha_s(\bar{m})}{3\pi}\, \frac{(w^2-1)\, r(w) + (w-z)\, \ln z}{1 - 2wz + z^2} , \end{aligned} \qquad (36)$$

where $r(w) = \ln(w + \sqrt{w^2-1})/\sqrt{w^2-1}$. The second function is almost independent of w over the kinematic range accessible in semileptonic decays: $F_2(w) \approx 0.9$. Assuming that the sum rule estimate $\eta(w) \approx 0.6$ is at least approximately correct, we observe again a substantial cancellation: $1 - 2F_2(w)\,\eta(w) \approx -0.1$. This means that the $1/m_b$ corrections in R can be safely neglected, and up to terms of order $1/m_Q^2$ the form factor ratio can be predicted in an essentially model independent way. Note that both the QCD and the $1/m_c$ corrections are positive. As a consequence, the deviations from the symmetry limit $R = 1$ are rather substantial. For instance, using $\bar\Lambda = 0.5 \pm 0.2$ GeV and neglecting the $1/m_b$ corrections we obtain $R = 1.33 \pm 0.08$ at zero recoil.

4 Conclusions

Starting from the observation that the structure of the short-distance expansion of heavy quark currents is to a large extent determined by a reparameterization invariance of HQET, we have derived the exact expressions for the meson and baryon weak decay form factors to next-to-leading order in $1/m_Q$. The results are presented in an explicitly renormalization-group invariant form by introducing renormalized Isgur-Wise form factors. The final formulas do not rely on explicit expressions for the Wilson coefficient functions and are thus valid to all orders in perturbation theory.

We have emphasized that beyond the leading order in $1/m_Q$ it is necessary to distinguish between the hadron velocity transfer $w = v\cdot v'$ and a related variable $\bar w$, which can be interpreted as the short-distance velocity transfer of the heavy quarks. Whereas the universal form factors of HQET are functions of w, the variable $\bar w$ appears in the short-distance coefficient functions.

Our final expressions for the meson form factors in (28) and (29), and for the baryon form factors in (17), generalize approximate results derived in leading-logarithmic approximation by Luke [10] and Georgi et al. [12]. These new formulas should be used in further analyses of semileptonic decays of heavy mesons or baryons.

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References

[1] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990).

[2] H. Georgi, Phys. Lett. B **240**, 447 (1990).

[3] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B368**, 204 (1992).

[4] A.F. Falk, B. Grinstein, and M.E. Luke, Nucl. Phys. **B357**, 185 (1991).

[5] A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. **B343**, 1 (1990).

[6] For a comprehensive review, see: M. Neubert, SLAC preprint SLAC–PUB–6263 (1993), to appear in Physics Reports.

[7] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)].

[8] M. Neubert, Phys. Lett. B **264**, 455 (1991).

[9] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).

[10] M.E. Luke, Phys. Lett. B **252**, 447 (1990).

[11] A.F. Falk, M. Neubert, and M. Luke, Nucl. Phys. **B388**, 363 (1992).

[12] H. Georgi, B. Grinstein, and M.B. Wise, Phys. Lett. B **252**, 456 (1990).

[13] M. Neubert, Phys. Rev. D **46**, 2212 (1992); Nucl. Phys. **B371**, 149 (1992).

[14] M. Luke and A.V. Manohar, Phys. Lett. B **286**, 348 (1992).

[15] M. Neubert, Phys. Lett. B **306**, 357 (1993).

[16] N. Isgur and M.B. Wise, Nucl. Phys. **B348**, 276 (1991).

[17] H. Georgi, Nucl. Phys. **B348**, 293 (1991).

[18] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B355**, 38 (1991).

[19] F. Hussain, J.G. Körner, M. Krämer, and G. Thompson, Z. Phys. **C51**, 321 (1991); F. Hussain et al., Nucl. Phys. **B370**, 259 (1992).

[20] A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993); **47**, 2982 (1993).

[21] Z. Ligeti, M. Neubert, and Y. Nir, SLAC preprint SLAC–PUB–6146 (1993), submitted to Phys. Rev. D.

[22] M. Neubert, Z. Ligeti, and Y. Nir, Phys. Lett. B **301**, 101 (1993); Phys. Rev. D **47**, 5060 (1993).

[23] M. Neubert and V. Rieckert, Nucl. Phys. **B382**, 97 (1992).

[24] M. Neubert, Phys. Rev. D **46**, 3914 (1992).