Zero Modes in a $c = 2$ Matrix Model[*]

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ABSTRACT

Recently [1] Dalley and Klebanov proposed a light-cone quantized study of the $c = 2$ matrix model, but which ignores $k^+ = 0$ contributions. Since the non-critical string limit of the matrix model involves taking the parameters λ and μ of the matrix model to a critical point, zero modes of the field might be important in this study. The constrained light-cone quantization (CLCQ) approach of Heinzl, Krusche and Werner is applied . It is found that there is coupling between the zero mode sector and the rest of the theory, hence CLCQ should be implemented.

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Recently[1] Dalley and Klebanov proposed a light-cone quantized study of the $c = 2$ matrix model, but which ignores $k^+ = 0$ contributions. Since the non-critical string limit of the matrix model involves taking the parameters λ and μ of the matrix model to a critical point, zero modes of the field might be important in this study. The constrained light-cone quantization (CLCQ) approach of Heinzl, Krusche and Werner is applied . It is found that there is coupling between the zero mode sector and the rest of the theory, hence CLCQ should be implemented.

1. Introduction

Light-cone quantization has proven to be a very fruitful approach to study quantum field theories in 1+1-dimensions[2] , (as well as for studying non-critical strings[1],) since it is possible to define a sequence of compact operators of increased 'exactness' by increasing the a 'resolving' parameter K which governs the total momentum of the system.

It is known that the light- cone vacuum is trivial, i.e. it is identical to the perturbative vacuum , and it was not understood how one would get non-trivial topological behaviour like symmetry breaking. Only recently have we started to understand how to do this[3] .

This can be seen by noting that light-cone quantization is quantization of a system with constraints[4][5] , as the Regensburg group has noted.

Essentially a classical system can have constraints if there are some q's which don't have conjugate p's. If we want to pass to the quantum system, we cannot apply the canonical quantization procedure. In such cases we need to examine the nature of these constraints. Dirac calls these constraints first class if they commute among themselves, and second class if they do not.

For the first class case we can use the canonical quantization procedure, but need to restrict the Hilbert space of allowed states upon which the quantum operators are allowed to operate by imposing these first class constraints on them. Theories with a gauge symmetry are such an example.

For the second class, Dirac found that by modifying the Poisson braket he was able to define a consistent set of rules for quantization. In the case of light-cone quantization, this is the case we have to deal with.

Lack of implementation of a consistent quantization shows itself in unphysical aspects in the theory such as breakdown of Lorentz invariance, lack of gauge invariance or presence of ghosts . This has also shown up in light-cone quantization. Recently, Burkardt and Langnau showed that naive light-cone quantization of a Yukawa model leads to problems with rotational invariance[6] . Their solution is to add appropriate counterterms in the Hamiltonian and which restores the symmetry.[7] .

Recently, Dalley and Klebanov[1]applied DLCQ to the study of $c = 2$ matrix model. In this approach, one takes a double scaling limit[8] of a matrix model[9] as a way to study $c = 2$ non-critical strings[10] . In this limit, one lets the coupling g go to 1 and N go to ∞ but one keeps fixed some product of N and $(g - 1)$. In their work, Dalley and Klebanov took the N going to ∞ limit first and then looked for critical coupling. They interpreted the appearance of continuous states in the mass spectrum as the appearance of the Liouville mode[11] . They also found the presence of tachyonic states, which they identified with those of bosonic strings. In this paper I will to study the effect zero modes of the field might have on this $c = 2$ matrix model. The outline of the paper is as follows: first I will study a similar model , the ϕ^3 model , in CLCQ. Then I will apply the results obtained to the model of Dalley and Klebanov.

2. Constrained Light-Cone Quantization of the ϕ^3 Model

It is assumed that the reader is familiar with the way one implements CLCQ from previous work[4,3], so it In this study I will follow the work of Werner and collaborators[3].

Consider then the Langrangian

$$\mathcal{L}_C = \mathcal{L}_0 + \mathcal{L}_I$$

where

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu\phi^2$$

and

$$\mathcal{L}_I = -\frac{\lambda\phi^3}{3!}$$

In light-cone coordinates this is

$$\mathcal{L}_C = \partial_+\phi\partial_-\phi + \frac{1}{2}\mu\phi^2 - \frac{\lambda}{3!}\phi^3$$

To solve the theory we introduce a box of dimensions 2L in the x^- directions. Define now the zero mode background field ω

$$\omega(x^+) = P * \phi(x^+, x^-) = \frac{1}{2L}\int_{-L}^{L} dx^- \phi(x^+, x^-)$$

This is the P-space or zero mode projection of the scalar field ϕ. Its complement

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Q is defined thus

$$\varphi(x^+, x^-) = Q * \phi(x^+, x^-) = \phi(x^+, x^-) - P * \phi(x^+, x^-)$$

The free part of the Lagrangian becomes :

$$\mathcal{L}_0 = \partial_+\varphi\partial_-\varphi - \frac{1}{2}\mu(\varphi + \omega)^2$$

Then the interaction part of the Lagrangian becomes

$$\mathcal{L}_I = \frac{1}{2}\mu(\varphi + \omega)^2 - \frac{\lambda}{3!}(\varphi + \omega)^3$$

$$= \frac{1}{2}\mu\varphi^2 + \frac{1}{2}\omega^2 + \mu\varphi\omega - \frac{\lambda}{3!}(\varphi^3 + \omega^3 + 3\varphi^2\omega + 3\varphi\omega^2)$$

The canonical φ momentum is

$$\pi_\varphi = \frac{\partial\mathcal{L}_C}{\partial(\partial_+\varphi)} = \partial_-\varphi$$

leading to the constraint

$$\theta_1(x^+, x^-) = \pi_\varphi(x^+, x^-) - \partial_-\varphi(x^+, x^-)$$

The result for the background field ω is

$$\pi_\omega \approx 0$$

and the next constraint is

$$\theta_2(x^+) = \pi_\omega(x^+)$$

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These degrees of freedom have the following canonical commutation

$$\left\{\varphi(x^+, x^-), \pi_\varphi(x^+, y^-)\right\} = \delta(x^- - y^-)$$

and

$$\left\{\omega, \pi_\omega\right\} = 1$$

We obtain the following result for the canonical Hamiltonian $\mathcal{H}_C$

$$\mathcal{H}_C = \frac{1}{2}\mu(\varphi + \omega)^2 - \frac{\lambda}{3!}(\varphi + \omega)^3$$

We implement the constraints by adding extra terms to the canonical Hamiltonian H_C to obtain the 'proper' Hamiltonian H_p

$$H_P(x^+) = H_C(x^+) + \int\limits_{-L}^{L} dx^- u_1(x^+, x^-)\theta_1(x^+, x^-) + \theta_2(x^+)u_2(x^+)2L$$

The Lagrange multipliers u_1 and u_2 are determined by requiring that they have zero Poisson commutator with the modified Hamiltonian H_P . If the first commutator is not zero, we continue to take commutators until zero is obtained[5]. We use the cannonical commutation expressions for φ and ω introduced above. The following expression is obtained for u_1

$$u_1(x^+, x^-) = \int\limits_{-L}^{+L} dy^- G^Q(x^-, y^-)\frac{1}{4}\{\mu\varphi(x^+, y^-) + \mu\omega(x^+)+$$

$$+\frac{\lambda}{2!}(\varphi(x^+, y^-)^2 + 2\varphi(x^+, y^-)\omega(x^+) + \omega(x^+)^2)\}$$

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where $G^Q(x, y)$ is the Q-projected Green's function

$$G^Q(x, y) = \frac{1}{2}sgn(x - y) - \frac{x - y}{2L}$$

To determine u_2 we need to work a bit harder. The first calculation gives

$$\partial_+\theta_2 = \{\theta_2, \mathcal{H}_P\}$$

$$\partial_+\theta_2 = -\mu\omega(x^+) + \frac{\lambda}{2!}\omega(x^+)^2 +$$

$$+ \int_{-L}^{L} dx^- \frac{\lambda}{2!}\{\varphi(x^+, x^-)^2 + 2\omega(x^+)\varphi(x^+, x^-)\} = \theta_3(x^+) \approx 0$$

Since this equation does not give us an expression for u_2, we take the commutator
of the new constraint, θ_3, with the full Hamiltonian. We get the following results
for the commutators with θ_1 and θ_2 respectively:

$$\{\theta_3, \theta_1\} = \frac{1}{2L}(\lambda\varphi(x^+, x^-) - \mu)$$

and

$$\{\theta_3, \theta_2\} = \frac{1}{2L}(2\lambda\omega(x^+) - \mu)$$

Putting this in the commutator with $\mathcal{H}_P$, we get

$$u_2(x^+) = \frac{-1}{2L} \int_{-L}^{L} dx^- \frac{\lambda\varphi(x^-, x^+) - \mu}{2\lambda\omega(x^+) - \mu} u_1(x^-, x^+)$$

To determine the new commutation relations, construct the Dirac braket $\{,\}^*$

$$\{A, B\}^* = \{A, B\} - \sum_{ij}\{A, \psi_i\}\{\psi_i, \psi_j\}^{-1}\{\psi_j, B\}$$

and here the ψ_i's are all second class constraints, so that the inverse is meaningful.

With this, the following Dirac brackets are obtained :

$$\{\varphi(x^+, x^-), \varphi(x^+, y^-)\}^* = -\frac{1}{4}G^Q(x^-, y^-)$$

and

$$\{\varphi(x^+, x^-), \pi_\varphi(x^+, y^-)\}^* = +\partial_-^{y^-}G^Q(x^-, y^-)$$

These are as expected. The interesting result is that we also get

$$\{\omega(x^+), \varphi(x^+, x^-)\}^* = -\frac{1}{4}\int_{-L}^{L} dy^- G^Q(x^-, y^-)\frac{\lambda\varphi(x^+, y^-) - \mu}{2\lambda\omega(x^+) - \mu}$$

This indicates that there is coupling between the non-zero modes of the field of the scalar field and the background field ω, the zero mode of the scalar field.

There is also coupling between the zero modes of the field and the momentum of the non-zero modes of the field :

$$\{\omega(x^+), \pi_\varphi(x^+, x^-)\}^* = \frac{1}{4}\int_{-L}^{L} dy^- G^Q(x^-, y^-)\frac{\lambda\varphi(x^+, y^-) - \mu}{(2\lambda\omega(x^+) - \mu)^2}2\lambda$$

On the other hand we get the following Dirac brackets for the zero mode of the field

$$\{\omega(x^+), \pi_\omega(x^+)\}^* = \{\omega(x^+), \omega(x^+)\}^* = \{\pi_\omega(x^+), \pi_\omega(x^+)\}^* = 0$$

This means that upon Dirac quantization this quantity is not dynamical. Nonetheless, since it depends on the quantized field ϕ, and since it couples to the non-zero part of the field, it is important in studying non-trivial topological properties of the theory.

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3. The c=2 Matrix Model

Let us study now this Dirac (constrained) quantization applied to the $c = 2$ matrix model introduced by Dalley and Klebanov[1]. The Lagrangian is

$$\mathcal{L} = Tr\left\{1/2(\partial_\alpha M)^2 + 1/2\mu M^2 - \frac{\lambda}{3\sqrt{N}}M^3\right\}$$

where $M(x^-, x^+)$ are $N \times N$ hermitian matrices. We apply now the method of constrained quantization described above and get the following for the full Hamiltonian

$$P^-(x^+) = \int\limits_{-L}^{L} dx^- Tr\left\{\frac{1}{2}\mu M^2 - \frac{\lambda}{3\sqrt{N}}M^3\right\} +$$

$$+ \int\limits_{-L}^{L} dx^- Tr\left\{U_1(x^+, x^-)\Theta_1(x^+, x^-)\right\} + Tr\left\{\Theta_2(x^+)U_2(x^+)2L\right\}$$

where the $U's$ are the new Lagrange multipliers and the $\Theta's$ the new constraints :

$$\Theta_1(x^+, x^-) = \Pi_{\mathcal{M}}(x^-, x^+) - \partial_-\mathcal{M}(x^-, x^+) \approx 0$$

and

$$\Theta_2(x^+) = \Pi_\Omega(x^+) \approx 0$$

The analysis goes through as in the previous chapter - except that now I have the extra indices, since TrM^2 means $\sum_{ij} M_{ij}M_{ji}$; I'll suppress these indices from

now on. As before, I split the field M into a zero mode part

$$\Omega(x^+) = P * M(x^+, x^-) = \frac{1}{2L} \int\limits_{-L}^{L} dx^- M(x^+, x^-)$$

and

$$\mathcal{M}(x^+, x^-) = Q*M(x^+, x^-) = \delta*M(x^+, x^-) - P*M(x^+, x^-) = M(x^+, x^-) - \Omega(x^+)$$

is the non-zero part. I get the following form for the new hamiltonian

$$\mathcal{H}_P = P^-(x^+) = \int\limits_{-L}^{L} dx^- Tr\Big\{\frac{1}{2}\mu M^2 - \frac{\lambda}{3\sqrt{N}} M^3\Big\} +$$

$$+ \int\limits_{-L}^{L} dx^- Tr\Big\{(\Pi_{\mathcal{M}}(x^-, x^+) - \partial_- \mathcal{M}(x^-, x^+))U_1(x^+, x^-)\Big\} + Tr\Big\{\Pi_\Omega(x^+)U_2(x^+)2L\Big\}$$

where the $\Pi_{\mathcal{M}}$ is the momentum canonical to $\mathcal{M}$ and Π_Ω the momentum canonical to Ω. The $U's$ are found to be

$$U_1(x^+, x^-) = \int\limits_{-L}^{+L} dy^- G^Q(x^-, y^-)\frac{1}{4}\Big\{\mu\mathcal{M}(x^+, y^-) + \mu\Omega(x^+) +$$

$$+ \frac{\lambda}{2!}(\mathcal{M}(x^+, y^-)^2 + 2\mathcal{M}(x^+, y^-)\Omega(x^+) + \Omega(x^+)^2)\Big\}$$

and

$$U_2(x^+) = \frac{-1}{2L} \int\limits_{-L}^{L} dx^- (2\lambda\Omega(x^+) - \mu I)^{-1}(\lambda\mathcal{M}(x^-, x^+) - \mu I)U_1(x^-, x^+)$$

As before, the interesting part is that there is coupling between the zero mode

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sector and the non-zero mode sector due to the following commutator

$$\{\Omega(x^+), \mathcal{M}(x^+, x^-)\}^* = -\frac{1}{4} \int\limits_{-L}^{L} dy^- (2\lambda\Omega(x^+) - \mu I)^{-1} G^Q(x^-, y^-)(\lambda\mathcal{M}(x^+, y^-) - \mu I)$$

In this case there is also coupling between the zero modes of the field and the momentum of the non-zero modes of the field :

$$\{\Omega(x^+), \Pi_{\mathcal{M}}(x^+, x^-)\}^* = \frac{1}{4} \int\limits_{-L}^{L} dy^- (2\lambda\Omega(x^+) - \mu I)^{-2} G^Q(x^-, y^-)(\lambda\mathcal{M}(x^+, y^-) - \mu I)2\lambda$$

4. Conclusions

The constrained light-cone quantization indicates that there is coupling between the zero mode sector and the nonzero mode sector. This means that the analysis of Dalley and Klebanov[1] needs to be redone in light of this result. It is unclear at this point if there is still the kind of excitations which Dalley and Klebanov associated with Liouville mode. This is because the type of critical behaviour studied might involve excitations of the zero modes of the field which were previously left out. A more careful study is necessary to discover what happens now in the double scaling limit of this matrix model.

There is also the possibility that the tachyonic mode which they discover might be due to instabilities in the ϕ^3 theory rather than due to the bosonic string. In a recent paper, Hiller and Swenson studied the Wick-Cutkosky model which is similar to the ϕ^3 model considered by Dalley and Klebanov, and found instabilities in vacuum , as expected for a cubic theory.

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