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The Reggeon Trajectory in Exclusive and Inclusive Large Momentum Transfer Reactions*

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ABSTRACT

A fundamental prediction of perturbative QCD is that the Reggeon trajectories $\alpha_\rho(t)$ and $\alpha_{A_2}(t)$ governing charge exchange reactions at high energies $s \gg -t$ must monotonically approach zero at large spacelike momentum transfers. The asymptotic prediction $\lim_{-t\to\infty} \alpha_R(t) = 0$ reflects the fact that a weakly interacting quark-antiquark pair is exchanged in the $t-$channel. However, measurements of the inclusive processes $\pi^- p \to \pi^0 X$ at $s \simeq 300$ GeV^2 and 8 GeV$^2 > -t > 2$ GeV^2 indicate that the effective ρ trajectory becomes negative at large $-t$. We resolve the apparent contradiction between the perturbative QCD predictions and the experimental data by showing that the hard QCD part of the trajectory is weakly coupled and that its contribution will be hidden until much higher energy. We also show that Reggeon contributions to exclusive and inclusive mesonic exchange hadron reactions can be systematically studied in perturbative QCD. In particular, the Reggeon contributions to the reaction $\pi^- p \to \pi^0 X$ and $\pi^- p \to \pi^0 n$ is discussed in detail.

1. Introduction

According to Regge theory, a scattering process at high energies and fixed momentum transfer ($s \gg -t$) is controlled by the singularities in the partial wave amplitude continued to complex angular momentum. For an exclusive reaction $AB \to CD$ involving charge or other quantum number exchange, a fairly satisfactory description can be obtained by assuming that these singularities are simple poles ("Reggeons"), the residues of which, by virtue of unitarity, are factorizable:

$$M_{AB\to CD}(s,t) = \sum_R \beta_R(t) \left(\frac{s}{s_0}\right)^{\alpha_R(t)} \xi_R(t) \; . \tag{1}$$

Here $\alpha_R(t)$ is one of the Regge trajectories corresponding to the exchanged quantum number, $\xi_R(t) = \frac{1}{2}[e^{-i\pi\alpha_R(t)} \pm 1]$ is the associated signature factor determined from $s \leftrightarrow u$ crossing symmetry, and the residue function $\beta_R(t)$ factorizes as the product of Reggeon form factors

$$\beta_R(t) = F^R_{A\to C}(t) F^R_{B\to D}(t) \; . \tag{2}$$

The form factor $F^R_{A\to C}(t)$ can be thought of as a vertex function $\langle C|\, j_R(0)\, |A\rangle$ of "local" currents of effective spin $\alpha_R(t)$ described by the Reggeon.

Equation (1) neglects contributions from Regge cuts (unitarity corrections from multiple Reggeon exchange). Such contributions appear to be empirically small. The $1/N_c$ expansion of QCD provides a systematic procedure in which unitarity corrections are incorporated order by order in $1/N_c$, and we can expect that Regge cuts do not appear at leading order in this expansion, and that the only *moving* singularities are Regge poles. There may of course be other *fixed* singularities, *e.g.* fixed poles, in leading order.

The leading Regge trajectories with quark-antiquark quantum numbers, ρ, f, ω, and A_2, are approximately linear for $t > 0$ where they interpolate a sequence of meson states of progressively higher spin $j = \alpha_R(t = M_H^2)$. For $t < 0$ these Regge trajectories, extracted from fits to the high energy data, apparently continue this linearity for moderate t, suggesting the approximately linear form

$$\alpha_R(t) \approx \alpha_R(0) + \alpha'_R(0)t \tag{3}$$

with $\alpha_R(0) \approx 0.4$ and $\alpha'_R(0) \approx 0.8$ GeV^{-2}. This approximate linearity encouraged the dual model approach to strong interactions which in tree approximation assumed exactly linear trajectories. However, this assumption is not consistent with the expectations of perturbative QCD at large spacelike momentum transfer $-t \gg \Lambda^2_{QCD}$. For example, a mesonic charge-exchange Reggeon at large spacelike t can be simply identified with $q_a\overline{q}_b$ exchange in the t channel. Thus to lowest order one expects in QCD

$$\lim_{-t\to\infty} \alpha_R(t) = \frac{1}{2} + \frac{1}{2} - 1 = 0\ ; \tag{4}$$

i.e., the Regge trajectory asymptotically decreases to $\alpha_R(t) = 0$ at large $-t$. Similarly, in the case of a baryon exchange trajectory, we expect

$$\lim_{-t\to\infty} \alpha_R(t) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} - 1 - 1 = -\frac{1}{2}\ . \tag{5}$$

Kirschner and Lipatov [1] and McGuigan and Thorn [2] have shown that the approach to the asymptotic form of the Reggeon trajectory from the sum of $t-$channel ladder diagrams in perturbative QCD is very slow: $\alpha_R(t) \sim O(1/\sqrt{\log|t|})$, and this approach is from above [3]. To leading logarithmic order, the QCD radiative corrections due to multiple gluon exchange between the exchanged quarks give the leading trajectory in the form shown in Fig. 1. For practical purposes it will be sufficient to use the numerical interpolating formula for the positive signature Regge trajectory given in Ref. [4]:

$$\alpha_R^{PQCD}(t) = \frac{\omega_{\max}}{\rho_1 + \rho_2\alpha_s^{-\frac{1}{2}}(-t)} \tag{6}$$

where $\omega_{\max} = \frac{C_F}{8\pi^2 b} \simeq 0.32$, for $b = \frac{11-\frac{2}{3}n_F}{16\pi^2} = 0.053$ (four flavors), and the fitted coefficients are: $\rho_1 = 1.14 \times \frac{3}{4} = 0.855$, $\rho_2 = 0.90 \times \frac{2}{\pi}(\frac{\omega_{\max}}{4\pi b})^{1/2} = 0.398$, Thus in practice, $\alpha_R^{\rm PQCD}(t)$ stays above 0.2 until very large $-t$ ($\alpha_R^{\rm PQCD}(t) = 0.160$ for $-t = M_Z^2$). Although data [5] at large $-t$ with $s \approx$ 300 GeV^2 do indeed show the effective ρ trajectory flattening off for $-t > 2$ GeV^2, it approaches a negative value (≈ -0.7) and not zero from above. Since the idea that the leading Regge trajectory in any channel is not monotonic is probably physical nonsense, we have a potential clash here between QCD and experiment.

The simplest resolution of this contradiction is that the "hard QCD" (*i.e.* flat) part of the ρ trajectory is so weakly coupled that its contribution is hidden by subleading contributions which are numerically dominant at currently available energies [6]. If this is indeed the case, then the effective trajectory extracted by Kennett *et al.* is not the true ρ trajectory but some averaged subleading set of trajectories that are still contributing because s is not large enough.[1] The estimates of the normalization of the hard QCD contribution that we will give below will indicate where the true asymptotic trajectory sets in.

2. Born term estimation

The pair of inclusive processes, $\pi^- p \to \pi^0(\eta)X$, singles out the $\rho(A_2)$ Regge trajectories respectively. According to perturbative QCD both the ρ and A_2 trajectories approach zero from above as $t \to -\infty$. We would like to be able to test these predictions by experiment. Existing experiments effectively probe s up to 200 to 300 GeV^2 and $-t$ up to 6 to 8 GeV^2. The most extensive studies have been done by Kennett *et al.* for π^0 production. In perturbative QCD, the hard part of the trajectories is generated by a sum of ladder gluon exchanges between the

1 For example, in the constituent interchange model discussed in Ref. [7], it is assumed that the dominant contribution to large momentum transfer fixed CM angle hadron scattering amplitudes is due to the interchange of the bound valence quarks. In this model the dominant interactions occur within the bound state wavefunctions, and the resulting meson exchange Reggeon trajectories have an effective behavior corresponding to $\alpha_R = -1$. Recent measurements of a number of two body scattering processes at BNL have shown that quark interchange processes strongly dominate gluon exchange contributions at fixed angles. See A. Carroll, *et al.* [8]

quark and antiquark exchanged in the charge changing process. The first diagram [Fig. 2a] in this sum that provides an s^0 behavior is a single rung of the ladder describing the annihilation of a u antiquark from the projectile with a quark from the target Bjorken [9]. has suggested a simple estimate of the strength of this lowest order contribution in terms of form factors and structure functions: since the gluon carries large momentum, we may approximate its contribution as $1/x_1x_2s$ times a contact interaction which through a Fierz transformation can be expressed as a vector exchange in the t channel. (The axial coupling does not contribute to the process $\pi^- p \rightarrow \pi^0(\eta)X$.) The resulting hadron matrix elements can then be identified as a form factor for the projectile times a form factor or structure function of the target, for exclusive and inclusive reactions, respectively. We can make this identification precise by incorporating into the definition of the Reggeon form factors and structure functions the reciprocal of the momentum fractions x_1 and x_2 of the annihilating quarks. Since these fractions are less than one, they will enhance an estimate based purely on ordinary electroweak form factors. In fact we shall show that one can reliably estimate the size of these enhancements in the Reggeon form factors using the various popular models for the wavefunctions of the mesons and nucleons [10,11], in terms of quark degrees of freedom. For the inclusive case, the kinematics actually fixes the momentum fraction of the quark in the target to be $x_2 = x_{Bj}$, so that factor can be incorporated without further approximation.

In addition to the annihilation diagram just described there is an exchange diagram [Fig. 2b] in which a d quark from the meson is exchanged with a u quark from the proton with a gluon exchanged between the two exchanged quarks. This diagram gives a u^0 behavior which at fixed t is also an s^0 behavior. Iterations of the gluon exchange in this t, u diagram again build up the ρ or A_2 Regge trajectory. This t, u ladder exchange combines with the first mentioned ladder exchange to supply the familiar signature factor $(e^{-i\pi\alpha} \pm 1)/2$. The A_2 trajectory has even signature (+) and the ρ trajectory odd signature (−). In the Born term estimate the trajectories are effectively at zero so strictly speaking one predicts a non-zero coupling only for the even signature exchange (A_2 trajectory, η production). However, once Reggeization has been incorporated, both the even and odd signature trajectories lie above zero and both signature factors are nonvanishing. For example, in the large N_c limit the ρ and A_2 trajectories stay degenerate and at large $-t$ are $O(\sqrt{N_c\alpha_s(-t)})$. In fact, in this limit the only difference between the even and odd signature contributions is the signature factor. Both amplitudes are nonzero since $\alpha_\rho, \alpha_{A_2} > 0$. Multiplying the Born term estimate for η production by the ratio $\xi_-(\alpha_R)/\xi_+(\alpha_R)$ evaluated at nonzero α_R provides a large N_c estimate for π_0 production. For small α_R this suggests a suppressed amplitude for π_0 compared to η production[2]. However, for finite N_c the odd signature trajectory is slightly higher by $O(1/N_c^2)$ than the even signature trajectory in perturbative QCD. The steeper energy dependence of the odd signature contribution will eventually compensate for the small signature factor so that at extremely high energies π_0 production will dominate over η production. As shown in Ref. [4], the difference between the magnitude of even and odd signature amplitudes can be neglected in practice up to SSC energies.

The inclusive cross section for the process $\pi^- p \to \pi^0 X$ has the general form:

$$\frac{d^2\sigma}{dt d\omega'} = \frac{1}{32\pi^2\omega^2} \sum_X \int dX (2\pi)^4 \delta(P_X - k - p) \, \frac{1}{2m_p} \, |\mathcal{M}_X|^2 \tag{7}$$

with momenta of π^-, π^0 defined as $k = (\omega, \vec{k})$ and $k' = (\omega', \vec{k}')$ respectively. We can compare this with the cross section for lepto-production:

$$\frac{d^2\sigma}{dt d\omega'} = \frac{\pi}{\omega^2} \left(\frac{\alpha}{Q^2}\right)^2 W_{\mu\nu}[(k+k')^\mu (k+k')^\nu - (q^\mu q^\nu + Q^2 g^{\mu\nu})] \tag{8}$$

where $q = k' - k$ is the momentum transfer and $Q^2 = -q^2 = -t$. The Born term estimate for the leading Reggeon behavior is obtained by substituting $(2\alpha/3Q^2)$

2 Note that since the physical η contains a significant strange quark component (transforming approximately as an $SU(3)_{\text{flavor}}$ octet), η production is further suppressed by a factor of roughly 3 compared to π_0 production. (A fictitious "η" made up only of up and down quarks is 1/3 octet and 2/3 singlet.)

in the electroproduction formula by $\alpha_s(1-1/N_c^2)/4x_2 s$, and replacing the factor $[(k+k')^\mu(k+k')^\nu-(q^\mu q^\nu+Q^2 g^{\mu\nu})]$ for the electron by $|\sqrt{2}F^\pi_{1/x}|^2(k+k')^\mu(k+k')^\nu$ for the pion where $F^\pi_{1/x}$ is the pion form factor with an extra $1/x$ in the wave function convolution. The $\sqrt{2}$ converts the pion form factor to the charge-raising isovector form factor. In all models of the pion form factor which are based on a quark wave function symmetric under $x \to 1-x$ the effect of the $1/x$ enhancement is to multiply the form factor by a factor of two.[3]

These substitutions can be understood as follows:

1. The up quarks in the proton are the ones that participate in the charge exchange so we want the up quark contribution to the electroproduction amplitude which is proportional to $2\alpha/3$. The electroproduction measurement includes a piece from scattering from the d quark but this is small (there are twice as many up quarks and each contributes 4 times as much) and we don't try to remove it.

2. The quark annihilation gluon is in the crossed channel w.r.t. the photon in electroproduction, so a Fierz transform routes the spinor lines as in electroproduction. The Fierz factor is $-1/2$ to each of axial and vector couplings, but the axial doesn't contribute here. Also the T coupling is subleading by a power of s.

3. The color factors are $\alpha_s \sum_a \lambda_1^a \lambda_2^a/4$. But since the quark antiquark must be in a singlet state, $\sum_a(\lambda_1^a+\lambda_2^a)^2=0$ which implies that the color factors are $-\alpha_s P_0(N_c^2-1)/2N_c$ where P_0 is the singlet projector. It is just $\delta_\alpha^\beta \delta_\gamma^\delta/N_c$. The Kronecker delta's just tie together the quark lines of the initial and final pion and those of p and X, leaving one more factor of $1/N_c$.

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3 The form factor is given as $\int_0^1 dx f(x)/(1-x)$ where x is the momentum fraction of the quark coupling to the current. The enhancement supplies another factor of $1/x$. The identity $1/x(1-x)=1/x+1/(1-x)$ together with $f(x)=f(1-x)$ gives the result.

Making all these substitutions leads to

$$\begin{aligned}
\frac{d^2\sigma}{dt\,d\omega'} &= \frac{9}{4}\,\frac{\pi}{\omega^2}\left(\frac{\alpha_s}{4sx_2}\right)^2\left(\frac{8}{9}\right)^2 2|F^{\pi^-}_{1/x}(Q^2)|^2 W_{\mu\nu}[(k+k')^\mu(k+k')^\nu] \\
&= \frac{9}{4}\,\frac{\pi}{\omega^2}\left(\frac{\alpha_s}{4sx_2}\right)^2\left(\frac{8}{9}\right)^2 8F^2_{\pi^-}(Q^2)W_{\mu\nu}[(k+k')^\mu(k+k')^\nu] \\
&= \frac{8\pi}{9\omega^2}\left(\frac{\alpha_s}{sx_2}\right)^2 F^2_{\pi^-}(Q^2)\,\frac{W_2}{m_p^2}\,[(s-u)/2]^2 \\
&\approx \frac{8\pi\alpha_s^2}{9\omega^2 x_2^2}\,F^2_{\pi^-}(Q^2)\,\frac{\nu W_2}{m_p^2\nu}\,.
\end{aligned} \tag{9}$$

Now use $m_p \nu x_2 = Q^2/2$ and multiply by $(\hbar c)^2 \approx 389\mathrm{GeV}^2\mu\mathrm{bn}$ to convert to $\mu\mathrm{bn/GeV}^3$:

$$\begin{aligned}
\frac{d^2\sigma}{dt\,d\omega'}\,(\mu\mathrm{bn/GeV}^3) &\approx \frac{389\cdot 16\pi\alpha_s^2}{9m_pQ^2\omega^2x_2}F^2_{\pi^-}(Q^2)\nu W_2(x_2) \\
&\approx 2170\,\frac{\alpha_s^2}{m_pQ^2\omega^2x_2}\,F^2_{\pi^-}(Q^2)\nu W_2(x_2)\,.
\end{aligned} \tag{10}$$

We also present this estimate in terms of Feynman $x_F \approx \omega'/\omega$. Note that x_2 in the above formula is Bjorken x: $x_2 = x_{Bj} = Q^2/2m_p\nu = Q^2/2m_p\omega(1-x_F)$. Then we have

$$\frac{d^2\sigma}{dt\,dx_F}(\mu\mathrm{bn/GeV}^2) \approx 4340\,\frac{\alpha_s(\widehat{s})^2(1-x_F)}{Q^4}\,F^2_{\pi^-}(Q^2)\nu W_2(x_{Bj})\,. \tag{11}$$

The strong coupling constant in this formula should be evaluated at the scale

$$\widehat{s} = x_1x_2s = 2m_px_1x_{Bj}q = \frac{x_1Q^2}{(1-x_F)} \approx \frac{Q^2}{2(1-x_F)}\,. \tag{12}$$

In the triple Regge limit[4] (Pomeron, 2 Reggeons), $x_{Bj} \sim 0$, $1-x_F \sim 0$, assuming $\alpha_P(0) = 1$ and a Reggeon trajectory $\alpha_R(Q^2)$, the cross section should have the

4 This limit of an inclusive process requires $s \gg M^2 \gg -t = Q^2$ where M is the invariant mass of the unobserved particles. The first inequality (fulfilled if $x_F \sim 1$ or $x_{Bj} \sim 1$) means that the process $\pi^- + p \to \pi^0 + X$ can be approximated by the exchange of a Reggeon $\alpha_R(t)$. The second inequality (fulfilled if $x_{Bj} \sim 0$) means that the sum over X, related by the optical theorem to a forward elastic amplitude, can be approximated by the exchange of the Pomeron $\alpha_P(0)$.

asymptotic form

$$\frac{d^2\sigma}{dt\,dx_F}\,(\mu\text{bn}/\text{GeV}^2) \approx G(Q^2)\,(1-x_F)^{1-2\alpha_R}\ . \tag{13}$$

Thus we see that the Born estimate corresponds to putting $\alpha_R \approx 0$.

The experiment of Kennett *et al.* for π^0 production uses an incident beam of 200 GeV pions so $\omega = 200$ GeV. The experimental range of Q^2 is from 1 to 8 GeV^2. Picking 4 as typical and consulting the literature for F_π at that value gives $F_\pi = .075$ so $F_\pi^2/m_p Q^2 \omega^2 \approx 3.74 \cdot 10^{-8}$ so for these values our estimate is

$$\frac{d^2\sigma}{dt\,d\omega'} \approx 0.81 \cdot 10^{-4}\ \frac{\alpha_s^2}{x_2}\ \nu W_2(x_2)(\mu\text{bn}/\text{GeV}^3)\ . \tag{14}$$

The Kennett *et al.* data give roughly $2 \cdot 10^{-4}(\mu\text{bn}/\text{GeV}^3)$ at $x_2 \approx .2$ and $3.5 \cdot 10^{-3}$ at $x_2 \approx 1/15$. This is to be compared to $4 \cdot 10^{-4}\alpha_s^2 \nu W_2(x_2)(\mu\text{bn}/\text{GeV}^3)$ and $1.2 \cdot 10^{-3}\alpha_s^2 \nu W_2(x_2)(\mu\text{bn}/\text{GeV}^3)$ respectively. Since νW_2 is roughly 0.3 for $x_2 < .2$ and α_s should surely be less than 1 at these scales, we see that we estimate the hard QCD part to be hidden in the noise of the Kennett *et al.* data.

More recent experiments (see *e.g.* Ref. [12]) at the Fermilab Tevatron have studied these inclusive processes with a 500 GeV π^- beam. Unfortunately, only scattering angles $\theta \approx \sqrt{2m_p x_{Bj}(1-x_F)/\omega}$ larger than about 20 mrad are measured. To improve the extraction of the true Reggeon trajectory over the Kennett data we would want to probe $1 - x_F < .05$ and $x_{Bj} < .1$. At $\omega = 500$ GeV, we require scattering angles smaller than 3 to 4 mrad. We hope such experiments will be seriously considered. At the SSC, even higher energies would be available, but, of course, measurements must be made at correspondingly smaller angles (< 1 mrad)!

The above calculation offers an explanation of the puzzling apparent absence of the $q\bar{q}$ annihilation process in a variety of allegedly short distance experiments. This absence was noticed long ago in studies of fixed angle scattering of hadrons at high energies [3]. The explanation is that the process couples so weakly that it is swamped by sub-asymptotic longer distance effects.

We have also made a similar estimate for the hard QCD contribution to the exclusive $\pi^- p \rightarrow \pi^0 n$ charge exchange reaction, again confirming that existing experiments should not yet have detected it. This is fortunate since those same data extract an effective ρ trajectory that is *negative*, conflicting with expectations from QCD.

3. Reggeization

For a more sophisticated estimate we can incorporate the effects of Reggeization. In perturbative QCD there is actually an infinite accumulation of Regge pole trajectories approaching 0 as $-t \rightarrow \infty$. This bundle of trajectories simulates a square root branch cut in the angular momentum plane whose effect can be summarized as multiplying the Born approximation to the underlying quark scattering cross section by the factor

$$\frac{2}{\pi} \left|\xi^{\pm}\right|^2 \frac{\widehat{s}^{2\alpha_R}}{(\alpha_R \ell n \widehat{s})^3} \, . \tag{15}$$

In the process of expressing $\widehat{s}$ in terms of s we note that the enhancement factors $1/x$ in the wave function convolution become instead $x^{\alpha_R - 1}$. So one has a trade-off between this reduced enhancement and an extra enhancement from the $s^{2\alpha_R}/(\alpha_R \ell n \widehat{s})^3$.

In general, the quark-antiquark annihilation scattering amplitude in the color singlet channel can be written as

$$A^P(\widehat{s}, t) = \frac{\gamma_\mu \otimes \gamma^\mu}{\widehat{s}} \, \frac{\delta_{aa'} \delta_{bb'}}{N_c} \, M^P(\widehat{s}, t) \, , \tag{16}$$

where a, b and a', b' label the color states of the initial and final quarks and antiquarks. The signature corresponding to even (odd) parts of the amplitude under the transformation $\hat{s} \leftrightarrow \hat{u}$ is $p = +(-)$. The Born approximation corresponds to

$$
\begin{aligned}
M^+(\hat{s}, t) &= 2\pi\alpha_s \frac{N_c^2 - 1}{N_c} \\
M^-(\hat{s}, t) &= 0 \; .
\end{aligned}
\tag{17}
$$

In the Regge kinematic region where $\hat{s} \gg -t \gg \Lambda_{QCD}^2$, higher order corrections like $\sim \alpha_s[(\alpha_s/\pi)y^2]^n$ are important, when y, the rapidity, defined by $y = \ell n(\hat{s}/-t)$, is large. Summing over this double logarithmic series with running coupling leads to [4]

$$
M^+(\hat{s}, t) = 0.895 \times 64\pi^3 b\alpha_R^2(t) \left(\frac{\alpha_s(t)}{2\pi C_F}\right)^{1/2} \left(\frac{\hat{s}}{-t}\right)^{\alpha_R(t)} \; . \tag{18}
$$

Here, we just take the leading Reggeon trajectory which is given by Eq. (6). In the fixed coupling case, one finds to good numerical accuracy $M^-(\hat{s}, t) \approx iM^+(\hat{s}, t)$. We shall assume this also holds in the running coupling case. Thus, in order to incorporate the effects of Reggeization, one simply takes the first line of Eq. (9) and replaces $2\pi\alpha_s\frac{N_c^2-1}{N_c}$ by $M^p(\hat{s}, t)$ as given by Eq. (18). We would like to express the scattering cross section in terms of the parton distribution so $(3/2)^2\nu W_2$ is replaced by $x_2 G_{u/p}(x_2, Q^2)$. Here $G_{u/p}(x_2, Q^2)$ is the u quark probability distribution in the target p. As usual, one can relate this probability distribution to the electromagnetic structure functions $F_2(x, -t) = \sum_b e_b^2 x G_{q_b/B}(x, -t)$ and the light-cone Fock states of the target:

$$
G_{q_b/B}(x, -t) = \sum_n \int_0^1 \Pi dx \int \Pi \frac{d^2k_\perp}{16\pi^3} \left|\psi_{nB}^\dagger(x_i, k_{\perp_i}, \lambda_i)\right|^2 \sum_{b'=b} \delta(x - x'_b) \; . \tag{19}
$$

Making all these substitutions one has

$$
\frac{d^2\sigma}{dt d\omega'} (\pi^- p \to \pi^0(\eta) X) = \frac{1}{8\pi N_c^2} \frac{|M^p|^2}{m_p Q^2 \omega^2} \left|F^R_{d/\pi^- \to u/\pi^0(\eta)}\right|^2 G_{u/p}(x_2, Q^2) \; , \tag{20}
$$

or in terms of Bjorken variable x_2

$$
\frac{d^2\sigma}{dt dx_2} (\pi^- p \to \pi^0(\eta) X) = \left\langle \frac{d\sigma}{dt}(x_2 s, t) \right\rangle \left|F^R_{d/\pi^- \to u/\pi^0(\eta)}\right|^2 G_{u/p}(x_2, Q^2) \tag{21}
$$

where

$$\left\langle \frac{d\sigma}{dt}(s,t) \right\rangle = \frac{1}{4\pi s^2 N_C^2} \left| M^+(s,t) \right|^2 , \tag{22}$$

is the quark-quark backward scattering cross section (or quark-antiquark annihilation) averaging over the initial helicity states and summing over the final helicity states. The required Reggeon form factors $F^R_{d/\pi^- \to u/\pi^0(\eta)}$ [Fig. 3] can be most easily represented as convolutions of light-cone wavefunctions [11], where the frame is chosen such that $-t = Q^2 = q_\perp^2$:

$$F^R_{q_a/A \to q_c/C}(t) = \sum_n \sum_{\lambda_i} \int_0^1 \overline{\Pi} dx \int \overline{\Pi} \frac{d^2 k_\perp}{16\pi^3} \psi^\dagger_{nC}(x_i, k'_{\perp_i}, \lambda_i) x^{\alpha_R(t)-1} \psi_{nA}(x_i, k_{\perp_i}, \lambda_i) , \tag{23}$$

where $k'_\perp = k_\perp + (1-x) q_\perp$ for active quarks $q_a \to q_c$ which couples to the exchanged Reggeon and $k'_{\perp_i} = k_{\perp - i} - x_i q_\perp$ for the spectator quarks. Here the sum is over all contributing states of the hadrons A and C. The notation $\overline{\Pi}$ implies integration subject to $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n k_{\perp_i} = 0$. The QCD-Reggeon form factor is thus computable as an overlap of light-cone wavefunctions for a transition current that replaces quark q_a with q_c. All the rest of the constituents of each Fock state n of hadrons A and C must match in quantum number and helicity.

Exclusive hadron scattering reactions in which all hadrons are identified such as $\pi^- p \to \pi^0 n$ have the advantage that one can make a specific identification of the Reggeon contributions. The scattering amplitude involves quantum number exchange and the Reggeon contribution factorizes. The hadron amplitude is given by the hard scattering of the Reggeon exchange multiplied by the product of respective form factors. The exclusive scattering cross section is given by:

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) = \left| F^R_{d/\pi^- \to u/\pi^0}(t) \, F^R_{u/p \to d/n}(t) \right|^2 \left\langle \frac{d\sigma}{dt}(s,t) \right\rangle , \tag{24}$$

In general, if hadrons A and C are related with internal symmetry, one may be able to relate the Reggeon form factor $F^R_{A \to C}(t)$ to the electromagnetic transition form factor $F^{em}_{AA}(t)$ and $F^{em}_{CC}(t)$. Using the same notation, the helicity-conserving electromagnetic transition form factor $F^{em}_{AA}(t)$ is given by $F^{em}_{A\to A}(t) = \sum e_{q_a} F^R_{q_a/A\to q_a/A}(t)$ with $\alpha_R(t) = 1$. In case of proton and neutron, the wavefunction $\psi^\dagger_{d/n}$ can be replaced by $\psi^\dagger_{u/p}$ by isospin symmetry and so one can write

$$
\begin{aligned}
F^R_{u/p\to d/n}(t) &= \sum_n \sum_{\lambda_i \lambda_i'} \int_0^1 \Pi dx \int \Pi \frac{d^2 k_\perp}{16\pi^3}\, \psi^{n\,\dagger}_{u/p}\, x^{\alpha_R(t)-1}\, \psi^n_{u/p} \\
&= \left(\sum_n \sum_{\lambda_i \lambda_i'} \int_0^1 \Pi dx \int \Pi \frac{d^2 k_\perp}{16\pi^3}\, \psi^{n\,\dagger}_{u/p} \psi^n_{u/p} \right) \left\langle x^{\alpha_R(t)-1} \right\rangle \\
&= \left(F^{em}_{pp}(t) + \frac{1}{2} F^{em}_{nn}(t) \right) \left\langle x^{\alpha_R(t)-1} \right\rangle \,.
\end{aligned}
\tag{25}
$$

Here, the argument of the wavefunctions are not shown explicitly for the sake of simplicity. We use the simple mean value theorem to go from the first line to the second line and note that $\langle x^{\alpha_R(t)-1}\rangle$ is of order of 1. Thus, in principle, we can predict not only the effective Reggeon power dependence of exclusive quantum number exchange reactions, but also their normalization.

At large momentum transfer, the behavior of form factors can be obtained by iterating the Fock state equations of motion to isolate the perturbative calculable hard scattering amplitude [11]. The leading power behavior is determined by PQCD dimensional counting rules: $F^R_{A\to C}(t) \sim t^{-n}$ where n is the minimum number of spectator quarks in the overlap of LC valence Fock states of A and C: $F^R(t) \sim t^{-1}$ for meson transitions and $F^R(t) \sim t^{-2}$ for baryon transitions[5]. Thus PQCD predicts not only the energy dependence of exclusive reactions: $\frac{d\sigma}{dt}(AB \to CD) \propto s^{2\alpha_R(t)-2}$ at large fixed $-t$, but also the $t-$ dependence of the Reggeon coefficient functions at large $-t$.

Thus exclusive and inclusive hadron reactions in the region $s \gg -t \gg \Lambda^2_{QCD}$ allow a detailed look into a QCD at the intersection between non-perturbative Regge

5 In the case of baryon trajectories $MB \to BM$ the transition form factor has one spectator, so again $F^R(t) \sim t^{-1}$.

phenomena and perturbative QCD. At low momentum transfer, such reactions have been traditionally used to explore the systematics of Regge phenomenology, triple Reggeon couplings, etc. However, at large momentum transfer, the underlying quark and gluon structure of the Reggeon becomes apparent, and one can use the methods of perturbative QCD and the light-cone Fock structure of hadrons to make detailed and elegant predictions directly from the theory.

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FIGURE CAPTIONS

1) The Reggeon trajectory. Dotted line is the usual linear form with $\alpha_R(0) = 0.4$, $\alpha'_R(0) = 0.8$ GeV^{-2} while the solid line is the PQCD Eq. (6).
   a. $-t$ from 0 to 1 GeV^2.
   b. $-t$ from 0 to 5 GeV^2.

2) a. $\overline{u}u$ annihilation in $\pi^- p \to \pi^0 X$.
   b. du backward scattering in $\pi^- p \to \pi^0 X$.

3) The Reggeon form factor where $\otimes$ denotes the insertion of operator $x^{\alpha_R(t)-1}$

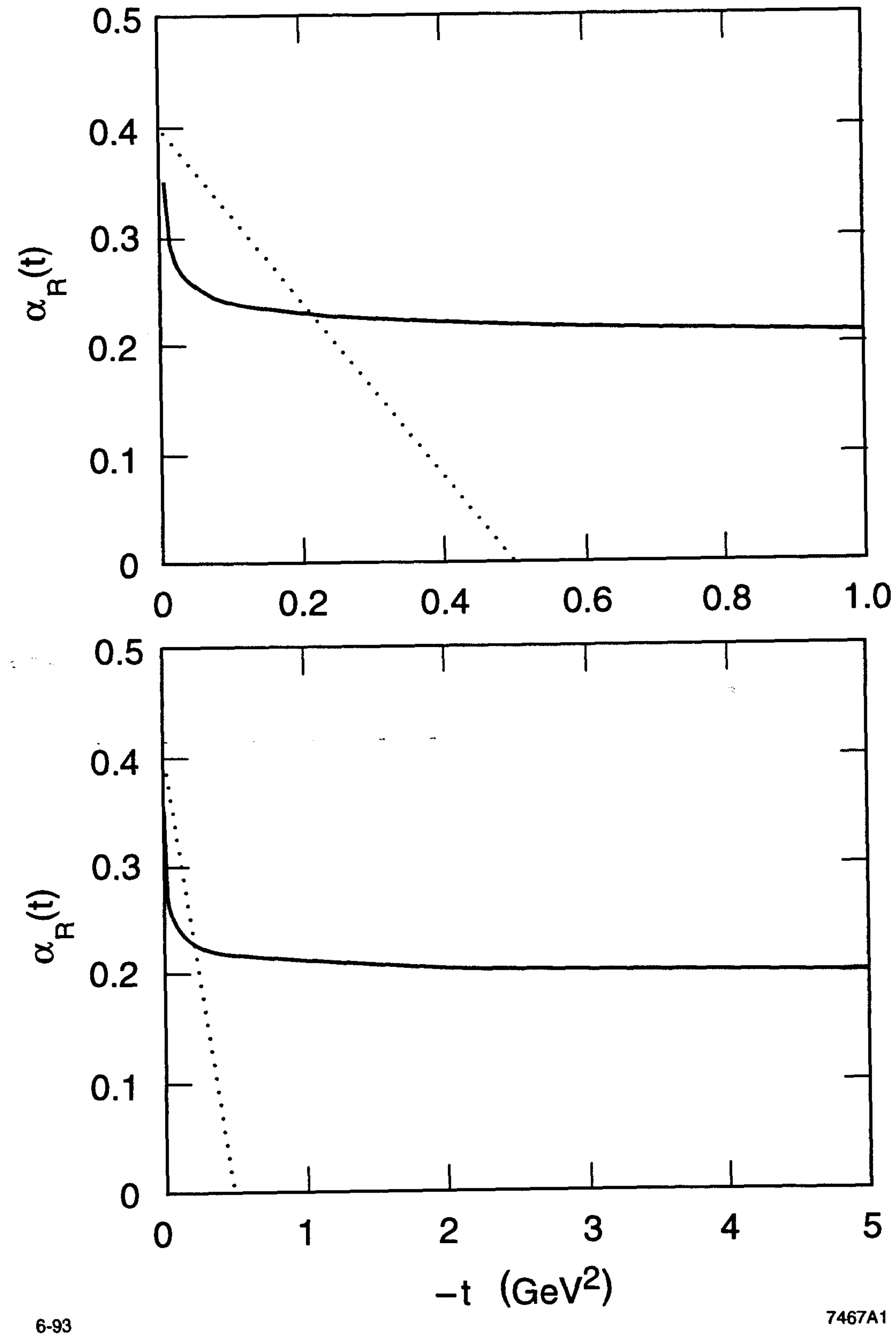
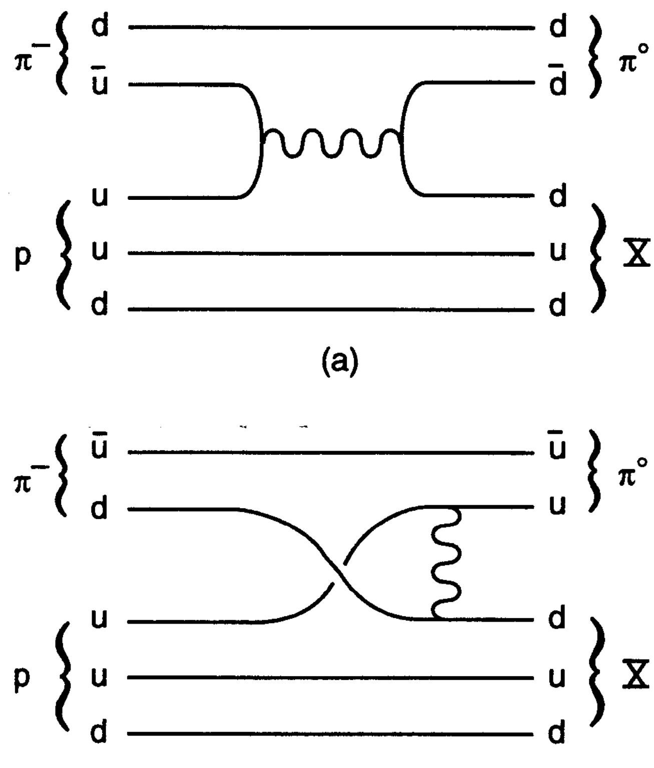
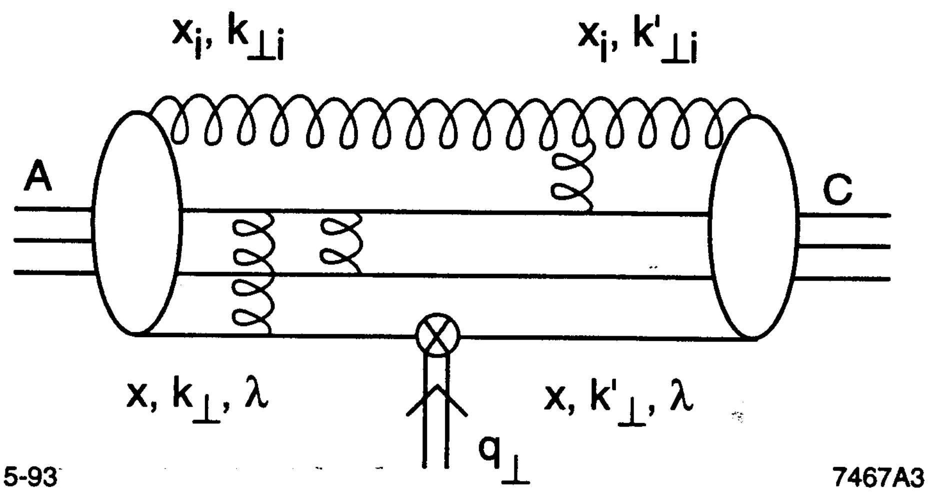

0.5
0.4
0.3
0.2
0.1
0
$\alpha_R(t)$
0
0.2
0.4
0.6
0.8
1.0
0.5
0.4
0.3
0.2
0.1
0
$\alpha_R(t)$
0
1
2
3
4
5
$-t$ (GeV^2)
6-93
7467A1

Fig. 1



9-93 7467A4

Fig. 2



x_i, $k_{\perp i}$
x_i, $k'_{\perp i}$
A
C
x, $k_\perp$, λ
x, $k'_\perp$, λ
$q_\perp$
5-93
7467A3

Fig. 3