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GIBBS PHENOMENON SUPPRESSION AND OPTIMAL WINDOWING FOR ATTENUATION AND Q MEASUREMENTS*

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INTRODUCTION

There are basically four known techniques, spectral ratios (Hauge 1981, Johnston and Toksoz 1981, Moos 1984, Goldberg et al 1985, Patten 1988, Sams and Goldberg 1990), forward modeling (Chuen and Toksoz 1981), inversion (Cheng et al 1986), and first pulse rise time (Gladwin and Stacey 1974, Moos 1984), that have been used in the past to measure the attenuation coefficient or quality factor from seismic and acoustic data. Problems have been encountered in using the spectral techniques, which included: (1) the correction for geometrical divergence of the acoustic wave front; (2) the suppression of the Gibbs phenomenon or the ringing effect in the spectra; and (3) the elimination of contamination from interfering wave modes. Geometrical corrections have been presented by Patten (1988) and Sams and Goldberg (1990) for the borehole acoustics case. The remaining difficulties in the application of the spectral ratios technique come mainly from the suppression of the Gibbs phenomenon and optimal windowing of wave modes. This report deals with these two problems.

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FUNDAMENTALS

The amplitudes $R(x,f)$ of a seismic signal at frequency f recorded by a receiver at a distance or offset x from the source can be represented as,

$$R(x,f) = A(f)G(x)e^{-\alpha x}, \tag{1}$$

where $A(f)$ is the source term, $G(x)$ is the geometrical divergence which is assumed to be independent of frequency, and α is the attenuation coefficient.

For the purpose of demonstrating suppression of the Gibbs phenomenon and optimal windowing of first arrivals, we choose the simplest technique for geometrical correction; i.e., let $G(x) = 1/x$. Then, from equation (1) we obtain the attenuation between a receiver with an offset x_1 from the source and a second receiver with an offset x_2 from the source $(x_1 < x_2)$,

$$\alpha = \frac{1}{x_2 - x_1} \ln\left[\frac{R(x_1,f)x_1}{R(x_2,f)x_2}\right]. \tag{2}$$

The quality factor Q can then be calculated from the following relation,

$$Q = \frac{\pi f}{\alpha v}, \tag{3}$$

where v is the average formation velocity, ignoring velocity dispersion.

SUPPRESSION OF THE GIBBS PHENOMENON

The Gibbs phenomenon is caused by the truncation of Fourier series or the discrete Fourier transform (DFT) of data of finite length with discontinuities at or a difference between the end points (Antoniou 1979, Bloomfield 1976, Oppenheim and Schafer 1989, Papoulis 1962). The conventional way to treat this problem is to use windows such as Hanning or

Hamming to force the end points to become zero or near zero. However, the weighting the data points in between the end points in the conventional windows distorts the spectra and thus causes erroneous estimation. An ideal way to suppress the Gibbs phenomenon is to force the end points to become equal or zero without weighting of the data points in between. This can be achieved by: (1) Moving the zero line to the first point of the data—shifting the zero line will only introduce a dc component into the spectra which will not affect the spectral content of interest and can simply be ignored in measurements of the spectral ratios; (2) flipping the data over to create a new data set twice the original length with the original last data point as the center of mirror images—this flipping creates an even function (Bracewell 1965, Bloomfield 1976) which, as we shall see, will not affect the spectral ratios measurements. However, it has recently been observed that, since in a DFT the input data are implicitly treated as a periodic waveform with period equal to the data length, the Gibbs phenomenon actually arises from the incomplete periodicity of the waveform; i. e., its period ends before the waveform has completed a full cycle (Pan 1993). Zero-line shifting is thus not really needed for the suppression of the Gibbs phenomenon, if the end data points are equal, to make the waveform to become a complete cycle. Therefore, in the practical application of FFT, if the length of the even function is not a power of two, instead of shifting the zero-line and filling in with zeros, the blank points in between the length of the even function and the length of FFT can be filled in with the first data point or the last point of the even function. The results will be the same.

Data flipping is not really "windowing" in the conventional sense. Instead, it creates a new data set having the spectral content of the original data, but without forced weighting to distort the spectra, or difference between the end points to produce the Gibbs phenomenon. The technique doubles the data length before transforming into the frequency domain. The spectral resolution is thus accordingly doubled. The spectral resolution may be further refined by flipping the data more than once.

In a DFT, the discontinuities at or difference between the end data points are equivalent to the multiplication of the data by a boxcar function of unit height, which, after being transformed into the frequency domain, becomes the convolution of the spectra with a sinc function (Antoniou 1979, Bracewell 1986). Data flipping eliminates the difference between the end data points, and thus clears the discontinuous truncation of the periodic waveform by the boxcar function. This means that in the time domain, the data are no longer multiplied by a boxcar function, but become an impulse train of complete periodicity. Once transformed into the frequency domain, the spectra are, therefore, no longer convoluted with a sinc function but are still a periodic impulse train (Pan 1993, Oppenheim and Schafer 1989, Papoulis 1962). The Gibbs phenomenon thus disappears, and the spectra become truly those of the data . Data flipping has created an even function $r'(x,t)$ by doubling the data length against the last data point. That is,

$$r'(x,t) = \begin{cases} r(x,t) & t = 0,1,2,\ldots,N-1 \\ r(x,2N-1-t) & t = N,N+1,\ldots,2N-1 \end{cases}, \qquad (4)$$

where $r(x,t)$, t = 0, 1, 2, ..., N-1, is the original data set, and the equal data points $r'(x,\mathrm{N-1})$ and $r'(x,\mathrm{N})$ are the center of mirror images of the even function.

Assuming that x is independent of t, then the spectra of $r'(x,t)$ are,

$$R'(x,f') = \frac{1}{2}\left[R(x,f') + e^{j\frac{2\pi f'}{N}} R^{*}(x,f')\right] \qquad f' = 0,\frac{1}{2},1,\frac{3}{2},\ldots,\frac{(N-1)}{2}, \qquad (5)$$

where $j = \sqrt{-1}$ and $R(x,f')$ are the spectra of $r(x,t)$, which are identical to $R(x,f)$ in equation (1) except that $f' = f/2$, and $R^{*}(x,f')$ are the conjugate of $R(x,f')$. The even symmetry of $r'(x,t)$ gives $R'(x,f') = e^{j\frac{2\pi f'}{N}} R'^{*}(x,f')$, which implies that the transform is not real. This is because the even function is of even number and its symmetry is not about the origin (Bloomfield, 1976). The average $R(x,f')$ and its phase-shifting conjugate will induce a phase interference in $R'(x,f')$, but the spectral content of the original data is not altered. It will therefore not affect the spectral ratios measurements. Data flipping doubles the spectral resolution; i. e., $f' = f/2 = 0, 1/2, 1, \ldots, (N-1)/2$. This means in $R'(x,f')$ the discrete frequency has been cut in half while the spectral length is twice that of $R(x,f)$, from N/2 to N. The derivation of equation (5) and related equations can be found in the Appendix.

The data flipping technique has been used to design a digital filter via the FFT algorithm (Pan 1993). The filter is able to suppress the Gibbs phenomenon in the time domain with no transition between passbands and stopbands, no waveform distortion or spectral leakage, and the end-effects are minimum. The filter can provide a lowpass, bandpass, highpass, bandstop, notch, or single-frequency-pass filtering by simple manipulation of the band limits. The technique is also successful for suppression of the ringing effect in the frequency domain.

Simplistic synthetic wavelets shown in Figure 1 are used for the preliminary test of the technique. The wavelets are generated using a 20-point sine function centered at 1000 hertz with a spectral band of 500–1500 hertz. The attenuation is calculated using $Q = 50$, $v = 17187.5$ ft/s, $x_1 = 660$ ft, $x_2 = 990$ ft (Kjartasson 1979), and velocity dispersion is ignored. The wavelets are purposely generated with discontinuities at the starting points.

The attenuation coefficient and quality factor Q measured using the data flipping technique are respectively plotted against frequency in Figures 2 and 3 together with those using Hanning and Hamming windows and those with no windows. Figure 2 shows that data flipping gives an almost linear dependence of attenuation on frequency within the spectral band of the wavelets, while the other techniques give either ringing or scattered values. Figure 3 shows that, within the spectral band of the wavelets, data flipping is able to recover a constant Q value of 50 used in the synthesis of the wavelets, while the other techniques either overestimate or underestimate Q in the same spectral band. The technique deteriorates slightly near the edges of the wavelet spectral band. The large anomalies at the starting points in Figures 2 and 3 are the result of measurements at frequencies below the 500-hertz spectral band. The attenuation and Q measured using the Hamming window and using no windows are cut short at about 1400 hertz because they ring wildly above that frequency.

Another interesting characteristic of the attenuation and Q measured using the data flipping technique is that their frequency dependence is invariant with velocity change, while those measured using the other techniques are not. Figure 4 plots Q measurements corresponding to those in Figure 3 except that the formation velocity has been changed from 17187.5 ft/s to 7000 ft/s in the synthesis of the wavelets. The plot shows that the Q measured using this technique keep the same form as those in Figure 3, while those measured using other techniques diverge considerably more from $Q = 50$ than those in Figure 3.

OPTIMAL WINDOWING

The data flipping technique can also be applied within windows. One hundred-point synthetic sine-function wavelets with a spectral band of 700-1300 hertz have been generated for this purpose. Measurements of Q with 20-point, 40-point, 60-point, and 80-point windows are shown in Figure 5. The measurements are all close to 50 except some occasional scatters that may be caused by a slight mismatch between corresponding spectra arising due to the conjugate phase shift. Sams and Goldberg (1990) reported that windowing degrades the Q estimates in the borehole acoustic data, and that the estimates are dependent on window length. Data flipping doubles the window length, which improves windowing by suppressing the Gibbs phenomenon, and also makes the Q measurement less sensitive to window length for longer windows.

Fixed-length windows are not optimal for the isolation of first arrivals. A window with too short a length may leave out the lower frequency content of the first arrivals (due to the progressive delay of those lower frequencies by dispersion), and will thus underestimate the attenuation and overestimate Q. Whereas, a window with too long a length may jeopardize the effort to eliminate contamination from the lower velocity arrivals. A floating-length window, designed to pick up the first arrivals on a cycle-by-cycle basis, improves the results. The first cycles of the arrivals can be picked up in two ways. Examples of selecting the first one and one-half (1 1/2) cycles to the zero line (point 0') to include the peak of the second cycle, as well as selecting one and three-fourth (1 3/4) cycles to the trough of the second cycle, are shown in Figure 6. The difference between the two selections is small; however, the results in attenuation and Q measurements are surprisingly different. As shown in Figure 7, the arrivals chosen up to 1 1/2 cycles underestimate Q (or overestimate the attenuation) at lower frequencies and overestimate Q (or underestimate the attenuation) at higher frequencies, while those chosen up to 1 3/4 cycles give a Q close to 50 and an attenuation nearly linearly dependent on frequency. For the general case, the implication of this observation of these data might be that the derivative or slope at the truncation points should be kept as close to zero as possible.

Finally, we should also note that, as shown in Figures 2, 3, and 4, the attenuation and Q are measured within the spectral bands of the wavelets excluding the sidelobes. Ringing or scatter shown by the non-data-flipping measurements in these figures is therefore not related to the sidelobes but is an effect of the Gibbs phenomenon within the spectral band of the wavelets.

CONCLUSIONS

A preliminary test of the data flipping technique using a sine-function wavelet reveals: (1) the technique is effective for the suppression of the Gibbs phenomenon or the ringing effect in the spectral ratios measurement and is invariant to velocity change; (2) after the Gibbs phenomenon has been suppressed, windowing will not degrade attenuation and Q measurements, but the measurements do show a dependence on window length; (3) the optimal way to pick up the first arrivals for attenuation and Q measurements through data flipping is to pick up to a point at which the derivative or slope is zero or nearly zero, such as at the trough of the second cycle of the first arrivals; and (4) attenuation and Q measurements are least scattered within the spectral band of the first arrivals.

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APPENDIX

SPECTRA OF AN EVEN FUNCTION

The DFT ot the spectra of the even function $r'(x,f)$, t = 0,1,2,...,2N-1 in equation (4) are,

$$R'(x,f) = \frac{1}{2\mathbf{N}} \sum_{t=0}^{2\mathbf{N}-1} r'(x,t) e^{-j\frac{2\pi f t}{2\mathbf{N}}}$$

$$= \frac{1}{2\mathbf{N}} \sum_{t=0}^{\mathbf{N}-1} r(x,t) e^{-j\frac{2\pi f t}{2\mathbf{N}}} + \frac{1}{2\mathbf{N}} \sum_{t=\mathbf{N}}^{2\mathbf{N}-1} r(x,2\mathbf{N}-1-t) e^{-j\frac{2\pi f t}{2\mathbf{N}}}$$

$$f = 0,1,2,\ldots,N-1 \quad , \qquad \text{(A1)}$$

where j = √-1. Now let $f' = f/2$ and substitute $t' = 2N-1-t$ into the second term of equation (A1). Since summation is independent of direction and its index is dummy, and in a DFT the waveform is periodic with its period equal to the data length, then we have,

$$R'(x,f') = \frac{1}{2}\left[\frac{1}{\mathbf{N}} \sum_{t=0}^{\mathbf{N}-1} r(x,t) e^{-j\frac{2\pi f' t}{\mathbf{N}}} + \frac{1}{\mathbf{N}} \sum_{t'=\mathbf{N}-1}^{0} r(x,t') e^{-j\frac{2\pi f'(2\mathbf{N}-1-t')}{\mathbf{N}}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\mathbf{N}} \sum_{t=0}^{\mathbf{N}-1} r(x,t) e^{-j\frac{2\pi f t}{2\mathbf{N}}} + \frac{1}{\mathbf{N}} \sum_{t'=0}^{\mathbf{N}-1} r(x,t') e^{j\frac{2\pi f' t'}{2\mathbf{N}}} e^{j\frac{2\pi f'}{\mathbf{N}}}\right]$$

$$= \frac{1}{2}\left[R(x,f') + e^{j\frac{2\pi f'}{\mathbf{N}}} R^*(x,f')\right] \qquad f' = 0,\frac{1}{2},1,\frac{3}{2}\ldots,\frac{(N-1)}{2}. \qquad (5)$$

Here one should note that there is a slight difference between $R'(x,f)$ and $R'(x,f')$ in theoretical interpretation. $R'(x,f)$ is the direct DFT of $r'(x,t)$, which implies a frequency sampling interval of $1/2N\Delta t$, where Δt is the time sampling interval. Whereas, $R'(x,f')$ represents the average of $R(x,f')$ and its phase-shifting conjugate, which implies an original frequency sampling interval of $1/N\Delta t$. Physically $R'(x,f)$ and $R'(x,f')$ are identical, with exactly the same spectral resolution and spectral length. The discrete frequency of $R'(x,f')$ therefore has to split; i.e., $f' = f/2 = 0, 1/2, 1, 3/2, \ldots, (N-1)/2$.

On the other hand, from equation (4) we know that $r'(x,t) = r'(x, 2N-1-t)$ for $t = 0,1,2,\ldots,2N-1$. We then obtain,

$$
\begin{aligned}
R'(x,f') &= \frac{1}{2N}\sum_{t=0}^{2N-1} r'(x,t)e^{-j\frac{2\pi f t}{2N}} = \frac{1}{2N}\sum_{t=0}^{2N-1} r'(x,2N-1-t)e^{-j\frac{2\pi f t}{N}} \\
&= \frac{1}{2N}\sum_{t'=2N-1}^{0} r'(x,t')e^{-j\frac{2\pi f(2N-1-t')}{2N}} = \frac{1}{2N}\sum_{t'=0}^{2N-1} r'(x,t')e^{j\frac{2\pi f t'}{2N}}e^{j\frac{2\pi f}{2N}} \\
&= e^{j\frac{2\pi f}{2N}}\left[\frac{1}{2N}\sum_{t'=0}^{2N-1} r(x,t')e^{j\frac{2\pi f t'}{2N}}\right] \\
&= e^{j\frac{2\pi f}{2N}} R'^{*}(x,f) \qquad f = 0,1,2,\ldots,N-1 \quad .
\end{aligned}
\tag{A2}
$$

Now substituting $f' = f/2$, we have

$$
R'(x,f') = e^{j\frac{2\pi f'}{N}} R'^{*}(x,f') \qquad f' = 0,\frac{1}{2},1,\frac{3}{2},\ldots,\frac{(N-1)}{2}
\tag{A3}
$$

LIST OF FIGURES