# Experimental Study of Crossing Angle Collision ${ }^{\dagger}$ 

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#### Abstract

The non-linear coupling due to the beam-beam interaction with crossing angle has been studied. The major effect of a small ( $\sim 12 \mathrm{mrad}$ ) crossing angle is to excite $5 \mathrm{Q}_{\mathrm{x}} \pm \mathrm{Q}_{\mathrm{s}}=$ integer coupling resonance family on large amplitude particles, which results in bad lifetime. On the CESR, a small crossing angle $(\sim 2.4 \mathrm{mr})$ was created at the IP and a reasonable beam-beam tune-shift was achieved. The decay rate of the beam is measured as a function of horizontal tune with and without crossing angle. The theoretical analysis, simulation and experimental measurements have a good agreement. The resonance strength as a function of crossing angle is also measured.


## I. INTRODUCTION

A primary objective of modern $e^{+} e^{-}$collider development is to achieve very high luminosity to meet the requirements of high energy physics. The design luminosity of these colliders, so called B-Factories, $\Phi$-Factories, etc., is about 50 times as highras that achieved in current colliders. To obtain this luminosity, the new designs employ two rings, with each ring being filled with large number of bunches to make the collision rate at the single interaction point large. The natural way to brïng the two beams into collision and separating them thereafter is to have a small crossing angle. However, studies ${ }^{[1]}$ show that synchrobetatron resonances are excited by crossing angle beam-beam interaction. This paper analyzes this problem, and provides the results of an experimental measurement. The conclusion of analysis, simulation, and experiment agrees reasonably well.

## II. ANALYTICAL ANALYSIS AND SIMULATION

The reason why synchrobetatron coupling is introduced by a crossing angle is that, due to the angle, the distance between a particle and the center of the counter bunch is modulated by the particle's longitudinal position. As a result, the beam-beam kick, which is a function of the distance, is modulated by the longitudinal motion too.

A resonance analysis method can be developed based on difference equations ${ }^{[2]}$. Considering horizontal and longitudinal planes, particle motion in a linear ring with a thin, nonlinear kick can be described by
$x_{t+1}-2 \cos \mu_{x} x_{t}+x_{t-1}=-\beta_{x} \sin \mu_{x} \mathrm{~F}\left(x_{t}+\tan \Phi \cdot s_{t}\right) \cos ^{2} \Phi$, $s_{t+1}-2 \cos \mu_{s} s_{t}+s_{t-1} \approx 0$.
Where $\mathrm{F}(r)$ is the beam-beam kick, $\Phi$ is the crossing angle, and $t$ stands for turn number. For small crossing angle, the longitudinal component of the kick is neglected. It is easy to see that the linear solutions of (1) are:
$x_{t}=A_{x} \cos \left(\mu_{x} t\right), \quad s_{t}=A_{s} \cos \left(\mu_{s} t\right)$.
As the first step approximation, insert (2) into the right-

[^0]hand side of (1), and do a Fourier expansion. Then, the right-hand-side of first equation of ( 1 ) is written as:
$\frac{1}{2} \sum_{\mathrm{m}, \mathrm{n}} c_{m, n} \cos \left[\left(m \mu_{x}+n \mu_{s}\right) t\right]+d_{m, n} \cos \left[\left(m \mu_{x}-n \mu_{s}\right) t\right]$
Naturally, a similar form of solution of (1) is expected:
$x_{t}=\frac{1}{2} \sum_{\mathrm{m}, \mathrm{n}} a_{m, n} \cos \left[\left(m \mu_{x}+n \mu_{s}\right) t\right]+b_{m, n} \cos \left[\left(m \mu_{x}-n \mu_{s}\right) t\right]$
Substitute (3), (4) into (1), the relation between driving terms and response terms is found:
\[

$$
\begin{equation*}
(a, b)_{m, n}=\frac{(c, d)_{m, n}}{2 \sin \frac{1}{2}\left[(m+1) \mu_{x} \pm n \mu_{s}\right] \sin \frac{1}{2}\left[(m-1) \mu_{x} \pm n \mu_{s}\right]} \tag{5}
\end{equation*}
$$

\]

Near resonances $(m \pm 1) \mathrm{Qx} \pm n \mathrm{Qs}=$ integer, the denominator is small. Then, $(a, b)_{m, n}$ has strong response to $(c, d)_{m, n}$. Therefore, we can say that $c_{m, n}$ and $d_{m, n}$ drive these resonances.

Spectrum of crossing angle beam-beam kick


Figure (1). The power spectrum of the crossing angle beambeam kick.

Figure (1) shows the two-dimensional FFT power spectrum of the beam-beam kick. From the picture, one can easily see that the strongest driving terms are at $m=4, n=1$ and $m=6, n=1$. According to the previous analysis, both these two terms will drive $5 \mathrm{Q}_{\mathrm{x}} \pm \mathrm{Q}_{s}=$ integer resonances. It is natural to conclude that the $5 \mathrm{Q}_{x} \pm \mathrm{Q}_{s}=$ integer resonances are the strongest coupling resonances.

In order to analysze the effect of a crossing angle, a computer simulation similar to Piwinski's work ${ }^{[3]}$ was made. The storage ring was model by a linear ring and a thin-kick beam-beam interaction with crossing angle. Three dimensional motion is simulated. Particles are launched in 6 dimensional phase space with $6 \sigma$ amplitudes. The program scans the horizontal fractional tune from 0 to 1 . The maximum amplitude of all particles ever reached during the 1000 -turn tracking is recorded as a function of horizontal tune. Figure (2) plots the maximum horizontal amplitude versus fractional tune. It shows that, besides the one-dimensional
fractional tune. It shows that, besides the one-dimensional resonances which exist also in head-on collision, the strongest coupling resonances are $5 \mathrm{Q}_{\mathrm{x}} \pm \mathrm{Q}_{s}=$ integer family.


Figure (2) Maximum horizontal amplitude vs. tune for crossing angle collision. $\left(\mathrm{Q}_{\mathrm{S}}=0.081\right)$

## III. EXPERIMENTAL MEASUREMENT

The experiment is designed to observe the $5 \mathrm{Q}_{x}+\mathrm{Q}_{s}$ resonance excited by the crossing angle collision, which is predicted by the theory in previous section. The experiment is based on the setup of the CESR crossing angle experiment ${ }^{[4]}$. CESR has been running with multi-bunch mode ( 7 bunches of $e^{-}$on 7 bunches of $e^{+}$). The key point of making multi-bunch mode possible is to separate bunches at crossing points around the ring except at the interaction point where the detector is located. In CESR, four electrostatic separators are used to separate electron and positron orbits at parasitic crossing points. As shown in figure (3), the orbits (thin lines) are separated at 13 would be collision points, but merged between the two south (lower) separators, including the IP where the collision takes place. The crossing angle lattice is essentially a modified version of the normal operation lattice with the bunches separated at the collision points except the IP. An anti-symmetric voltage applied to the south separators will create anti-symmetric orbits about the IP. This is displayed in figure (3) as the thick lines.

The experiment was performed in a way similar to the simulation. The strong-weak beam-beam interaction is achieved by colliding a 2 mA beam on 10 mA beam. The beam size and bcam current decay rate is measured while scanning the horizontal tune in the $5 \mathrm{Q}_{x}+\mathrm{Q}_{s}$ resonance region. A high decay rate peak was observed on the resonance when the crossing angle was turned on. However, the peak disappeared when the crossing angle was turned off. Figure (4) shows the tune scan data with and without crossing angle. For comparison, the simulation results are shown in figure (4) too. One can easily see the agreement between them. Note that the vertical axis represents different quantities in experimental data and simulation. The reason is that the calculation is only qualitative. Nevertheless, they both reflect
the same physical phenomena. Meanwhile, the vertical beam size is measured. No beam blow up is observed at the same resonance, with or without crossing angle. This implies that this effect applies only on beam tail, which is what the theoretical analysis and simulation predicted.


Figure(3). Diagram of the orbits for crossing angle experiment


Figure (4). (a)upper: Simulation result, maximum amplitude versus horizontal tune. (b)lower: Experimental data, decay rate as a function of horizontal tune.

A two dimensional tune scan was also performed to check the resonance. The result with crossing angle on is shown in figure (5). In this part of experiment, strong-strong beambeam interaction was employed, because, a weak beam cannot survive after crossing the resonance many times. The $5 \mathrm{Q}_{x}+\mathrm{Q}_{s}$ resonance corresponds to the light vertical line on the left. With this result, the resonance is better identified due to its consistent appearance and independent of vertical tune.

The resonance strength, in terms of peak decay rate, is


Figure (5) Electron decay rate measured in two dimensional tune scan with 2 mrad crossing angle. The lighter shade indicates higher decay rate. The horizontal and vertical tune frequencies, the product of the tune and the revolution frequency, are in kHz . (The revolution frequency is 390 kHz ). also measured as a function of crossing angle. Figure (6) plots the measured result. Each line in the picture is from a single tune scan with certain crossing angle. The crossing angle ranges from about $\pm 1.4 \mathrm{mrad}$ to $\pm 2.5 \mathrm{mrad}$. For crossing angle smaller than $\pm 1.4 \mathrm{mrad}$, there is no clear decay rate peak being measured.

Crossing Angle Experiment


Figure (6).Tune scans versus crossing angle.
The maximum decay rate from figure (6) is plotted as a function of the half crossing angle in figure (7), one can easily see the rise of the resonance strength as the crossing angle increases. The simulation result is also plotted for comparison. Again, the quantities in vertical axis are different, so that the comparison is only qualitative. However, from both plots, a saturation effect can be seen. Simulation shows that the saturation goes up to $\pm 12 \mathrm{mrad}$.

Unfortunately, the crossing angle in the experiment cannot go larger, because it is limited by machine aperture. The last data point raises again. The reason may be that the crossing angle has been pushed to the limit of the physical aperture at this angle. The tight physical aperture certainly enhances the decay rate. We also cannot exclude other driving sources.

(a)Maximum amplitude on the resonance vs. crossing angle.

Experimental data

(b). Peak decay rate on the resonance vs. crossing angle

Figure (7). Resonance strength as a function of crossing angle.

## IV. CONCLUSIONS

The study shows a good consistency among analytical analysis, computer simulation and experiment on the strongest coupling resonance family excited by the crossing angle beam-beam interaction. This resonance family, $5 \mathrm{QX}^{ \pm} \mathrm{Q}_{\mathrm{S}}=$ integer, will result in a bad lifetime in operation.

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## VI. REFERENCES

[1] A. Piwinski, IEEE Trans. NS-24, p1408 (1977).
[2] Peggs, S. and Talman, R., "Nonlinear Problems in Accelerator Physics," Ann. Rev. Nucl. Part. Sci. , vol. 36, 287, 1986.
[3] Piwinski, A., "Simulations of Crab Crossing in Storage Rings," SLAC-PUB-5430, Feb. 1991.
[4] Rubin, D. and et al, "Beam-beam Interaction with a Horizontal Crossing Angle," to be published in Nuclear Instruments and Methods A or CLNS 92/1183.


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