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Parity-Conserving Light-Cone Quantization of Quantum Field Theories*

OVID C. JACOB

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

ABSTRACT

Parity violation is a long standing problem in light-cone quantization.[1] We propose a new quantization on the light-cone which treats both the x^+ and the x^- coordinates as light-cone 'times.'This quantization respects both parity and time-reversal. We find that now both P^- and P^+ become dynamical.

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1. Introduction

Quantizing field theories on the light-cone[2,3,4,5] has been done for over twenty years. Usually, one picks the x^+ as the light-cone time, and most of the work is in constructing the 'correct' Hamiltonian P^- which gives the evolution in the light-cone variable x^+. There has been a question which always bothered us, and that is, why is x^+ 'special' ? Why not pick x^- as the light-cone time ? Very simple considerations show that this is not such a silly question. If we just use the simple definition of these variables

$$x^+ = x + t, x^- = x - t$$

then we find that under time reversal, $(t \to -t)$, $x^+ \to x^-$ and $x^- \to x^+$. Parity likewise mixes these two variables : under parity $(x \to -x)$, $x^+ \to -x^-$ and $x^- \to -x^+$. This suggests that we should treat these variables on equal footing.

There are stronger reasons though, coming from the study of the equations of motion which the solution needs to satisfy. The outline of this paper is as follow : first we'll compare the equations of motion we get in a Lagrangian formulation with those in Hamiltonian formulation; then we will the show how to quantize on the light-cone using <u>both</u> x^+ and x^- as light-cone 'times ' (A similar situation occurs in the quantization of $N = 2$ strings[6,7]).

2. Simple Considerations on Differential Equations

Let us study, for concreteness, ϕ^4 in 1+1-dimensions. We will first study this in equal-time quantization.

The Lagrangian is

$$\mathcal{L} = \partial_t\phi\partial_t\phi - \partial_x\phi\partial_x\phi - \frac{\lambda}{4!}\phi^4$$

This leads to the following hyperbolic[8] second order differential equations of motion :

$$(\partial_t^2 - \partial_x^2)\phi = \frac{\lambda}{3!}\phi^3$$

Integrating out this equation generates two constants which can be determined by two appropriate boundary conditions.

The usual path from the classical theory to the quantum theory is to construct the canonical pair of field ϕ and its sister momentum and to impose equal-time commutation relation on the Poisson bracket. Note that the Lagrangian formulation gives us a second order hyperbolic differential equation with two integration constants after integration over time. This integration over time gives us the time evolution of the system.

Let us see what happens when we go to the Hamiltonian picture. Here, by defining the momentum $\pi = d\phi/dt$, and by defining a Hamiltonian we transform the above second order differential equation into set(s) of coupled linear differential equations. In this case,

$$\frac{d\phi}{dt} = \{\mathcal{H}, \pi\}$$

$$\frac{d\pi}{dt} = -\{\mathcal{H}, \phi\}$$

where $\mathcal{H}$ is the Hamiltonian. Note that there are still two integration constants, one from each linear differential equation when we integrate over time. This means that there are still two boundary conditions to be specified. That's reassuring, since this means that we have the same physics as in the Lagrangian case since the two boundary conditions from the second order equation can describe the same physics as two boundary conditions coming from two first order differential equations. This is all probably well-known to the reader. We just repeat it here because this is not always taken into account when we come to light-cone quantization.

3. Equal-Time Quantization and Parity

Let us now define the usual energy-momentum tensor $T^{\mu\nu}$ in the equal-time quantization case :

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\mathcal{L}$$

and

$$P^\mu = \int dx^0 T^{\mu 0}$$

In our case this means

$$\mathcal{H} = P^0 = \int dx^0((\partial_x\phi)^2 + \frac{\lambda}{4!}\phi^4)$$

which is dynamic, and

$$P^1 = \int dx^0(\partial_x\phi\partial_t\phi)$$

which is kinematic i.e., free of interactions. It is easy to see that in this case, under time-reversal,

$$T[P^0] = -P^0, T[P^1] = P^1$$

while under parity

$$P[P^0] = P^0, P[P^1] = -P^1$$

This flows from the transformational properties of $x^0 = t$ and x under time-reversal and parity, respectively. We also have that

$$[P^0, P^1] = 0$$

Everything is as expected.

4. Differential Equations on the Light-Cone

The Lagrangian in this case is

$$\mathcal{L} = \partial_+\phi\partial_-\phi - \frac{\lambda}{4!}\phi^4$$

Following Chang, Root and Yan[9] we define the canonical conjugate variable π to ϕ thus

$$\pi = \frac{\partial\mathcal{L}}{\partial_+\phi}$$

if we define x^+ as the light-cone time. This leads to the following modified hyperbolic differential equations of motion :

$$\partial_+\partial_-\phi = \frac{\lambda}{3!}\phi^3$$

Note that now this equation is linear in light-cone time. This means that we get only one time integration constant, hence only one boundary condition, which is different from the equal-time quantization case . It is not clear how to interpret the x^- variable.

Another problem is that we don't recover the Hamilton's equations of motion: we should have two coupled equations

$$\partial_+ \pi = \{\mathcal{H}, \phi\}$$

and

$$\partial_+ \phi = \{\mathcal{H}, \pi\}$$

which upon integration over time should give us two integration constants or two boundary conditions. Instead , we have only

$$\partial_- \pi = \{\mathcal{H}, \phi\}$$

which gives us only one boundary condition. Probably this is different physics.

5. Naive Light-Cone Quantization and Parity

Let us now define the energy-momentum tensor $T^{\mu\nu}$ in the naive light-cone quantization case :

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

and

$$P^\mu = \int dx^- T^{\mu +}$$

In this case, this means that

$$P^+ = \int dx^- (\partial^+ \phi)^2$$

while

$$P^- = \int dx^- \frac{\lambda}{4!}\phi^4$$

It is easy to see that in this case, under time-reversal,

$$T[P^+] \neq -P^-, T[P^-] \neq P^+$$

while under parity

$$P[P^+] \neq P^-, P[P^-] \neq -P^+$$

This flows from the transformational properties of x^+ and x^- under time-reversal and parity, respectively. In this case, the Hamiltonian P^- is not an eigenstate of either parity or time-reversal. Our proposal to fix this problem is to consider a new way to quantize on the light-cone.

6. New Light-Cone Quantization

The Lagrangian is

$$\mathcal{L} = \partial_+\phi\partial_-\phi - \frac{\lambda}{4!}\phi^4$$

This leads to the following modified hyperbolic second order differential equations of motion :

$$\partial_+\partial_-\phi = \frac{\lambda}{3!}\phi^3$$

Let us now treat the $^+$ and $^-$ variables on equal footing and let us define now two types of momenta:

$$p = \frac{\partial \mathcal{L}}{\partial(\partial_- \phi)} = \partial_+ \phi$$

$$r = \frac{\partial \mathcal{L}}{\partial(\partial_+ \phi)} = \partial_- \phi$$

Then there will be two types of Hamiltonians

$$\mathcal{H} = 2p\partial_- \phi - \mathcal{L} = pr + \frac{\lambda}{4!}\phi^4$$

and

$$\mathcal{K} = 2r\partial_+ \phi - \mathcal{L} = rp + \frac{\lambda}{4!}\phi^4$$

one, $\mathcal{H}$ for evolutions along x^+ and another, $\mathcal{K}$ for evolutions along x^-. We get now coupled linear differential equations :

$$p = \partial_+ \phi = \{\mathcal{H}, r\}$$

and

$$r = \partial_- \phi = \{\mathcal{K}, p\}$$

(these just reproduce the definitions above) as well as

$$\partial_- p = \partial_- \partial_+ \phi = \{\mathcal{K}, \phi\} = \frac{\lambda}{3!}\phi^3$$

$$\partial_+ r = \partial_+ \partial_- \phi = \{\mathcal{H}, \phi\} = \frac{\lambda}{3!}\phi^3$$

which give the equation of motion. The doubling of time variables would seem to imply the doubling of boundary conditions, yet we want the same physics as in the equal-time case. We have to show that some of the equations are redundant. Note that these last two equations are identical, so should have the same integration constants, or same boundary conditions. This means that we'll have two boundary conditions, like in the equal-time case.

7. Parity Conserving Light-Cone Quantization and Parity

Define now the energy-momentum tensor $T^{\mu\nu}$ thus

$$T^{\mu\nu} = \partial^{\mu}\phi\partial^{\nu}\phi - g^{\mu\nu}\mathcal{L}$$

We have now the following definition for P^{μ}

$$P^{\mu} = \int d\sigma_{\nu} T^{\mu\nu}$$

For our case where we treat x^{+} and x^{-} on equal footing, this means that $d\sigma^{\nu}$ has $^{+}$and^{-} components :

$$P^{\mu} = \int dx^{-} T^{\mu -} + \int dx^{+} T^{\mu +}$$

Just like in McCartor's case, the generators contain both x^{+} and x^{-} evolutions[10]. In this case, we get the following :

$$P^{+} = \int dx^{+} (\frac{\lambda}{4!}\phi^{4}) + \int dx^{-} (\partial^{+}\phi\partial^{-}\phi)$$

while

$$P^{-} = \int dx^{-} (\frac{\lambda}{4!}\phi^{4}) + \int dx^{+} (\partial^{-}\phi\partial^{+}\phi)$$

It is easy to see now that in this case, under time-reversal,

$$T[P^{+}] = P^{-}, T[P^{-}] = P^{+}$$

while under parity

$$P[P^{+}] = P^{-}, P[P^{-}] = P^{+}$$

This flows from the transformational properties of x^{+} and x^{-} under time-reversal and parity, respectively. In this case, the new definitions preserve both parity and time-reversal. We also have the usual property that

$$[P^{+}, P^{-}] = 0$$

like in the equal time quantization case.[11,12]

8. Constrained Light-Cone Quantization and Parity

It might be interesting to check parity invariance in the context of constrained light-cone quantization[5,12,11,13]. We can say even without doing any explicit calculations that both parity and time-reversal will be broken since in that construction we are really concerned with getting the correct $\mathcal{H} = P^{-}$, and we do not worry about P^{+}, which is supposed to be a kinematic generator anyway. The situation is just like in the naive light-cone

9. Conclusions

It is found that introducing two light-cone times, we recover the appropiate number of boundary conditions to have the same type of theory in both equal-time and light-cone time. This approach has the extra merit that it does not spoil parity of time reversal upon quantization, as does the naive light-cone quantization approach.

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