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APPLICATIONS AND COMPARISONS OF METHODS OF COMPUTINGTHE S MATRIX OF 2-PORTS†

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Abstract

We report on the application of three different methods of computing the S Matrix for 2-port microwave circuits. The four methods are modal expansions with field matching across boundaries, time domain integration of Maxwell's equations as implemented in MAFIA, HFSS (high frequency structure simulator), and the KKY frequency domain method. Among the applications to be described are steps in rectangular waveguides and irises in waveguides.

I. INTRODUCTION

The Kroll-Kim-Yu (KKY) [1] method of determining the S matrix of 2-port microwave circuits is an elementary algebraic procedure which can be used in conjunction with any computer program which determines the resonant frequency and electromagnetic fields of closed cavities. As such it may be thought of as supplementary to program packages which accomplish the same objective, but which may not be available to a particular user. It may also be used to provide mutual validation of alternate procedures. While the basic theory of the method is given in [1], no actual examples were presented. The purpose of this paper is to remedy this deficiency, and thereby to demonstrate the practicality of the method. As an example of the symmetric case we discuss reflecting iris design in circular waveguide for application to SLED II [2]. As an example of the unsymmetric case we chose a symmetric H-plane step in rectangular waveguide (i.e. discontinuous increase of waveguide width). As will be discussed below, this simple geometry allow us to compare the KKY results with those obtained from highly accurate mode matching calculations. Comparison with other lattice based program packages will also be given.

II. THE KKY METHOD

A. Description Of The Method

Following [1], we consider a lossless 2-port and parameterize its S-matrix as follows:

$$S_{11} = -\cos(\theta)\exp[j(\phi + d\phi)] \quad (1)$$

$$S_{12} = S_{21} = -j\sin(\theta)\exp(j\phi) \quad (2)$$

$$S_{22} = -\cos(\theta)\exp[j(\phi - d\phi)] \quad (3)$$

Unique values for the parameters θ, ϕ, and $d\phi$ as functions of frequency are defined by restricting their ranges as follows : $-\pi/2 < \theta \le \pi/2$, $-\pi/2 < d\phi \le \pi/2$, and $-\pi < \phi \le \pi$. From the definition of the S matrix:

$$b_i = S_{ij}a_j \quad (4)$$

where a_i and b_i are incoming and outgoing wave amplitudes respectively.

Now let us suppose that we have available the field configuration and frequency of some mode of the 2-port transformed into a closed cavity by shorting the waveguides associated with the ports at distance L_i from the reference planes. We consider here only the case in which one has single mode propagation in each waveguide. Then we have:

$$\begin{aligned} b_i / a_i &= -\exp(2j\psi_i) \\ a_2 / a_1 &= r\exp[j(\psi_1 - \psi_2)] \end{aligned} \quad (5)$$

where ψ_i is k_iL_i and r is the ratio of the incoming wave amplitudes evaluated at the shorts. The wave amplitude ratio r is readily computed from appropriately chosen field amplitudes as will become clear from the examples. Substituting Eqs. (3) and Eqs. (1) into Eqs. (2) provides us with two equations for the three unknown S matrix parameters. Through algebraic and trigonometric manipulation we obtain explicit expressions for θ and ϕ in terms of the known quantities ψ_i and r and the unknown $d\phi$:

$$\tan(\theta) = 2\sin(d\psi)/(r - 1/r) \quad (6)$$

where $d\psi = \psi_1 - \psi_2 - d\phi$, and $\bar{\phi}$ is defined in [1].

B. The Symmetric Case

A symmetric structure with symmetrically selected reference planes has $S_{11} = S_{22}$, and hence $d\phi = 0$. Thus for such circuits (provided we have chosen $L_1 \ne L_2$) these formulas determine the complete S matrix for each frequency which appears in the mode spectrum of a computer run.

We have applied the method to the design of a number of circular iris's in circular TE_{01} waveguide. For application to SLED II, design to a specified value of reflection coefficient was required. The iris thickness was specified for mechanical reasons to be .080 inches, the waveguide diameter was 1.75 inches, and the problem was to

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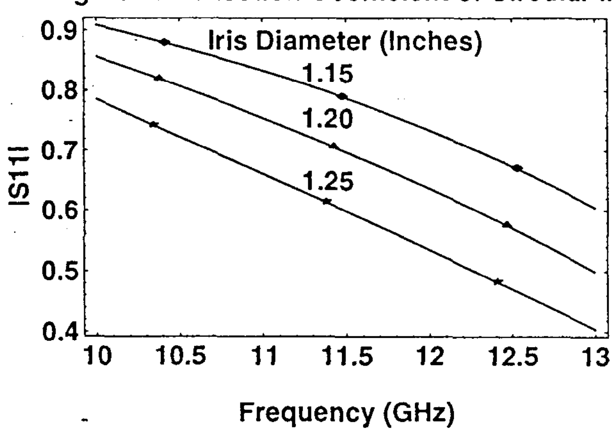
determine the correct iris diameter. Three URMEL runs were carried out, one for each of three choices of iris diameter, and each with the waveguides shorted at 2.15 inches and 1.80 inches respectively from the center of the iris.

The quantity r for this configuration is given by the negative of the ratio of the maximum magnetic fields at each end. The sign is to some extent a convention, but with this choice, S_{12} is unity when the iris diameter coincides with the waveguide diameter. This configuration yields eight modes in the frequency range lying between the cutoff frequencies of the TE_{01} and TE_{02} modes, and S matrix parameters could have been determined for all of them. This was actually carried out, however, only for the three frequencies closest to the design frequency, 11.424 GHz, of SLED II. The parameters at the design frequency were obtained by interpolation from these data at each of the three diameters, and the diameter required to provided the specified reflection coefficient was again found by three point interpolation from these numerical values. Experimental values were obtained in the course of the SLED-II measurements reported in [2] and are in excellent agreement with the theoretical values.

Since the completion of the above, the mode matching program to be discussed below has been extended to allow very accurate evaluation of the S matrix for these iris's. The curves obtained with the mode matching method are shown in figure 1 (the points are obtained with the KKY method).

Figure 1: Reflection Coefficient of Circular Iris



C. Application To An Asymmetic Case

We have seen in the previous section that from a mode at a specific frequency with a specific choice of shorting lengths we get two equations for determining the three S matrix parameters. To obtain additional equations it is merely necessary to find a different pair of shorting lengths which produces an independent solution at the same frequency. There is, in fact, a continuum of (L_1, L_2) pairs which satisfy this condition, but one such is actually more than enough. That is, from a second set with, say L_1', L_2', and an associated r', we have four equations for the three parameters. We can obviously write down a set of expressions analogous to Eqs. (5) to (10) in terms of the primed quantities and temporarily designate the associated S matrix parameters as primed quantities. Then setting $d\phi = d\phi'$ and setting $\tan(\theta)$ equal to $\tan(\theta')$ in the unprimed and primed versions of Eq. (5), we obtain:

$$\tan(d\phi) = \frac{[(r'-1/r')\sin(D\psi) - (r-1/r)\sin(D\psi')]}{[(r'-1/r')\cos(D\psi) - (r-1/r)\cos(D\psi')]} \quad (7)$$

where $D\psi = \psi_1 - \psi_2$. We may then obtain ϕ and ϕ' from Eq. (6) and its primed counterpart using $d\phi$ as obtained from Eq. (11). They should, of course, be equal to one another and the extent to which this will be found to be the case depends upon the accuracy of the computer programs which produce the input data and the accuracy with which the frequencies of the primed and unprimed cases have been matched.

As a test of the practicality of the KKY method in the asymmetric case we considered a rectangular waveguide in which the width increased from 0.4 inches to 0.6 inches. With L_1 and L_2 measured from the junction equal to 1.1 inches and 1.3 inches respectively, a MAFIA computation produced eight modes in the frequency range between the TE_{10} and TE_{30} cutoffs. (Because the junction is transversely symmetric there is no coupling between the TE_{10} and TE_{20} modes.) MAFIA runs with equal lengths of 1.2, 1.25, and 1.3 inches produced corresponding sets of eight modes whose frequencies bracketed each of the modes of the unequal length set. The $L_1 = L_2$ values needed to match each of the eight frequencies of the unequal length run were determined by interpolation from the three equal length runs. The associated r values were determined by interpolation from the r values from the three computed lengths. To determine the r values we used $r = -K(H_2/H_1)$ where the H_i are the maximum magnetic fields at the ends of the waveguides and K is given by: $K = [(f^2 - f_{c1}^2)/(f^2 - f_{c2}^2)]^{1/4}$, where f and f_{ci} are the mode frequency and waveguide cutoff frequencies respectively. Thus four MAFIA runs yielded the data to compute the S matrix parameters at eight frequencies, providing thereby a comprehensive description of the behavior of the S matrix over a broad range of frequencies. A subset of results obtained will be shown in connection with those obtained by other methods in the next section. The ϕ values to be shown were obtained from the unequal length set. The discrepancies between them and those obtained from the interpolated equal length set were very small, a result which provides us with a consistancy check.

III. COMPARISON WITH OTHER METHODS

The S matrix of both the circular iris and the H-plane step can be very reliably and accurately computed by the mode matching technique. In order to ascertain the veracity of the KKY method we conducted a detailed comparison between the KKY method, a computer code written utilizing the mode matching method, MAFIA in the time domain and HFSS.

The mode matching method entails a decomposition of the tangential electromagnetic field in region 1 into the form:

$$\mathbf{E}_t = \sum_{n=1}^{\infty} (a_1^n \exp[-jk_1^n z] + b_1^n \exp[jk_1^n z])\sqrt{Z_1^n}\,\mathbf{e}_1^n \quad (8)$$

$$\mathbf{H}_t = \sum_{n=1}^{\infty} (a_1^n \exp[-jk_1^n z] - b_1^n \exp[jk_1^n z])\sqrt{Y_1^n}\,\mathbf{h}_1^n \quad (9)$$

where **e** and **h** are the characteristic mode functions of the structure, k_1^n is the wavenumber of mode n and, $Y_1^n = 1/Z_1^n$ is the characteristic admittance of mode n. A similar expansion of fields is made in region 2. The electromagnetic fields on both sides of the waveguide junction (z = 0) are equated to each other and this results, in principle, in an infinite set of coupled equation for the mode coefficients a_1^n, a_2^n, b_1^n and b_2^n, which in practice, are truncated to N in region 1 and M in region 2. The solution to these equations for a WN (wide to narrow) transition enables the normalized generalized scattering matrix to be obtained in the compact form:

$$S_{11} = Y^{1/2}(Y + Y_I)^{-1}(Y - Y_I)Y^{1/2} \quad (10)$$

$$S_{12} = Y^{1/2}[2a(\hat{Y} + Y_{II})^{-1}]\hat{Y}^{-1/2} \quad (11)$$

$$S_{21} = \hat{Y}^{1/2}[2a^{-1}(Y + Y_I)^{-1}]Y^{-1/2} \quad (12)$$

$$S_{22} = \hat{Y}^{1/2}(\hat{Y} + Y_{II})^{-1}(\hat{Y} - Y_{II})\hat{Y}^{-1/2} \quad (13)$$

where a is the matrix of scalar products of mode functions integrated over the cross-sectional area of the aperture region, Y and $\hat{Y}$ are diagonal matrices with elements Y_1^n and Y_2^n respectively, and the admittance of the smaller waveguide viewed from the larger and vice-versa, respectively, are given by:

$$Y_I = (a^{-1})^t \hat{Y} a^{-1}, \text{ and } Y_{II} = a^t Y a \quad (14)$$

The above relation reveals the interesting result that the impedance of the smaller guide, viewed from the larger waveguide, is given exactly (i.e. no matrix inversions are required), in terms of a summation of the product of matrix elements. Moreover, the above results for the scattering matrix are applicable to both an H-plane step and to a transition in the radius of circular waveguide. Furthermore, for a WNW (wide to narrow to wide) transition, as is apposite to the SLED-II iris, the overall S matrix may be obtained by using the inherent symmetry properties of the system [3], or by cascading [4] the WN matrix with a matrix corresponding to a shift in phase along the length of waveguide and, with a matrix corresponding to a NW transition; the latter matrix is, of course, readily obtained from the WN matrix. In our computations we have utilized both methods to provide a check as to the efficacy of the calculations.

A representative sample of S matrix parameters as a function of frequency (GHz) for an H-plane step in rectangular waveguide is shown in table 1. for four different methods, namely, MM (mode matching) using both the cascaded approach and the symmetrical approach (the difference between the two methods is in the eighth decimal place), KKY, MAFIA applied in the time domain, and HFSS.

Table 1: S Parameters For An H-Plane Step

f	θ	ϕ	dϕ	Method
15.1590	-1.00935	-1.35441	1.14863	MM
	-1.00987	-1.35374	1.13839	KKY (ω)
	-1.00858	-1.35159	1.13868	MAFIA (t)
	-1.00910	-1.34078	1.12459	HFSS
16.0159	-1.22911	-1.40363	1.03246	MM
	-1.22843	-1.40268	1.02089	KKY (ω)
	-1.22847	-1.40105	1.02126	MAFIA (t)
	-1.22632	-1.38731	.983884	HFSS
17.7145	-1.37354	-1.44954	.854241	MM
	-1.37297	-1.44981	.841649	KKY (ω)
	-1.37283	-1.44762	.840349	MAFIA (t)
	-1.37179	-1.45437	.811546	HFSS
19.3840	-1.43435	-1.47581	.694889	MM
	-1.43368	-1.47632	.678899	KKY (ω)
	-1.43364	-1.47447	.678542	MAFIA (t)
	-1.43185	-1.48811	.630695	HFSS

The agreement between MM, KKY, and MAFIA in the time domain, for ϕ and θ is encouraging. This indicates that the absolute value of either the reflection coefficient or the transmission coefficients and indeed, the phase of the transmission coefficients, may be quite reliably obtained by either method. However, the values for dϕ only agree in the first and second decimal place, indicating that the phase of the reflection coefficients of the S matrix cannot be reliably upon.

V. REFERENCES

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