Simulation and Compensation of Multibunch Energy Variation in NLC*

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Abstract

The SLAC NLC design for a next-generation linear collider utilizes multibunching (acceleration of a train of bunches on each RF fill) to increase the luminosity and energy efficiency. It is necessary to control the energy spread of the beam, in order to minimize chromatic emittance dilution and be within the energy acceptance of the final focus. It is anticipated that the NLC may run with bunch trains having length equal to a substantial fraction of the filling time. Multibunch energy simulation methods and compensation schemes appropriate to this regime are presented.

Introduction

Utilizing multibunching in a next-generation linear collider (NLC) requires that the energies of the bunches be tightly controlled. To be within the acceptance of the final focus system and to control chromatic emittance dilution in the linac, $\delta E/E$ needs to be less than about 0.15%.

By adjusting the timing of injection of the bunch train with respect to the RF pulse, and choosing the bunch spacing appropriately, one may cancel most of the energy variation between bunches in the train. The basic idea is to have the RF structure fill with sufficient extra energy between bunch passages to make up for the energy lost in accelerating the preceding bunches in the train.

However, with the simplest form of this "matched-filling" scheme [1], there is a "sag" in energy at the middle of the bunch train, and the longer the bunch train the greater the sag. In this paper we shall focus on compensation that permits running longer bunch trains ($\sim$ a filling time) while maintaining an acceptable energy spread. Long trains are under consideration as a way of obtaining the maximum possible luminosity and energy efficiency.

We begin by discussing the factors that affect the energy spread of the beam. A detailed simulation program has been written, the elements of which are outlined here. In this simulation, one may take account of input RF pulse shaping and timing, the dispersion of the RF pulse as it transits the structure, the longitudinal distribution of charge within the bunches, the long range wake (LRW) including both the fundamental (accelerating) mode and higher order modes (HOM's), the short range wake (SRW), and phasing of the bunches with respect to the crests of the RF. We shall focus only on the inter-bunch energy spread in the present paper.

Basis of Simulation

We begin by considering a single accelerating section, fed at its upstream end by an input RF waveform that travels to the other end and is absorbed in a load. At some specified time with respect to the entry of the RF pulse, a train of relativistic ($v = c$) bunches enters the structure, and the electrons in each bunch are accelerated by the fields (sum of RF pulse and wake fields) they encounter in the structure. The total charge in each bunch is divided into a finite number of longitudinal slices, small enough that the longitudinal position and the energy gain of the electrons in a given slice may be taken to be equal.

The total voltage gained in the section by slice r of bunch n may be broken down into

$$\Delta V_{n,r} = \Delta V_{n,r}^{rf} + \Delta V_{n,r}^{lrw} + \Delta V_{n,r}^{srw} \quad . \tag{1}$$

We denote the time of entry of this slice into the section by $t_{n,r}$, and longitudinal position in the section by s, where s runs from 0 to the structure length L, and consider each of these three contributions to the total section voltage gain.

The voltage $\Delta V_{n,r}^{rf}$ due to the RF pulse is

$$\Delta V_{n,r}^{rf} = \int_0^L E^{rf}(s, t_{n,r} + s/c)ds \quad , \tag{2}$$

where $E^{rf}(s,t)$ is the field of the RF waveform at location s and time t, obtained by propagating the input RF pulse $E^{rf}(0,t)$ down the structure. Let us specify the input RF pulse as

$$E^{rf}(0,t) = \hat{E}_0(t)\cos(\omega_{rf}t + \varphi_0) \quad . \tag{3}$$

The accelerating frequency ω_{rf} is assumed to have phase velocity in the structure equal to c and thus is synchronous with the charges to be accelerated.

If we neglect dispersion, i.e., assume the group velocity v_g is the same for all frequency components in the pulse (though it may in general still depend on s), we have

$$E^{rf}(s,t) = \hat{E}_0\left(t - \int_0^s \frac{ds'}{v_g(s')}\right)$$
$$\cdot \exp\left(-\int_0^s \alpha(s')ds'\right)\cos[\omega_{rf}(t - s/c) + \varphi_0] \quad . \tag{4}$$

In reality, dispersion makes the shape of the RF pulse change as it propagates through the structure, particularly for a sharply varying envelope $\hat{E}_0(t)$. Thus, one should Fourier analyze the input pulse,

$$E(\omega) = \int_{-\infty}^{\infty} E^{rf}(0,t)e^{-i\omega t}dt \quad , \qquad (5)$$

and propagate each of its frequency components according to the dispersion relation $\Gamma(\omega,s) = \alpha(\omega,s) + i\beta(\omega,s)$ for the structure:

$$E^{rf}(s,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(\omega)\exp[i\omega t - \int_0^s \Gamma(\omega,s')ds']\,d\omega \quad . \qquad (6)$$

Here we assume that the structure can be modelled locally as a band-pass filter, though its properties may change gradually with s.

For a narrow-band structure with small attenuation [2], α may be assumed independent of ω, and a good approximation to $\beta(\omega,s)$ is

$$\beta(\omega,s) = \frac{1}{d}\cos^{-1}\left[-\frac{\omega-\omega_0(s)}{\omega_{hw}(s)}\right] \quad , \qquad (7)$$

where d is the structure period (cell length), $\omega_0(s)$ is the mid-band frequency, and $\omega_{hw}(s)$ the half-width of the passband. In the usual situation, the accelerating mode lies in the range of β between 0 and π/d; it is common to choose $\beta(\omega_{rf},s)d = \frac{2\pi}{3}$.

The long range wake voltage $\Delta V_{n,r}^{lrw}$ felt by the slice is

$$\Delta V_{n,r}^{lrw} = \sum_{j=1}^{n-1}\int_0^L E^{lrw}(s,t_{n,r}+s/c,j)ds \quad , \qquad (8)$$

where $E^{lrw}(s,t,j)$ is the field at location s and time t, due to the wake left by bunch j.

There are two simpler special cases of particular interest. One is the constant-impedance (CZ) structure, made of identical cells so that its properties such as v_g, α, etc., are independent of s. For the CZ structure, we have

$$E^{rf}(s,t) = \frac{1}{\pi}\int_{\omega_{lo}}^{\omega_{up}}\left[\mathrm{Re}E(\omega)\cos[\omega t - \beta(\omega)s]\right.$$
$$\left. - \mathrm{Im}E(\omega)\sin[\omega t - \beta(\omega)s]\right]e^{-\alpha(\omega)s}d\omega \quad . \qquad (9)$$

The other special case is the constant-gradient (CG) structure, which when unloaded has constant electric field along the structure when fed with a constant amplitude input pulse. Thus the attenuation coefficient α for the electric field is zero. For such a structure

$$v_g(s) = \frac{\omega_{rf}}{Q}[L'-s] \quad , \qquad (10)$$

where $L' \equiv \frac{L}{1-e^{-2\tau}}$. We assume r (shunt impedance per unit length) and Q are approximately independent of s. For the CG structure,

$$E^{rf}(s,t) = \frac{1}{\pi}\int_{\omega_{lo}(s)}^{\omega_{up}(s)}\left[\mathrm{Re}E(\omega)\cos[\omega t - B(\omega,s)]\right.$$
$$\left. - \mathrm{Im}E(\omega)\sin[\omega t - B(\omega,s)]\right]e^{-\alpha(\omega)s}d\omega \quad , \qquad (11)$$

where $B(\omega,s)$ may be derived analytically [3]. The limits ω_{lo} and ω_{up} are the lower and upper passband boundaries of the fundamental mode. The CG structure is a fairly good representation of the detuned structures being contemplated for the SLAC NLC design.

Denoting the time when bunch j enters the structure by t_j^{ent}, and neglecting dispersion, we have for the LRW voltage in the CZ case:

$$E^{lrw}(s,t,j) = \sum_m -2\kappa_m q_j F_{m,j}\exp[-(t-t_j^{ent}-\frac{s}{c})\frac{\omega_m}{2Q_m}]$$
$$\cdot\cos[\omega_m(t-t_j^{ent}-\frac{s}{c})]$$
$$\cdot H(t-t_j^{ent}-\frac{s}{c})H(t_j^{ent}+\frac{s}{v_{g,m}}-t) \quad . \qquad (12)$$

Here the sum m runs over the modes in the wake (fundamental and HOM's); κ_m is the loss factor, Q_m the quality factor, and $v_{g,m}$ the group velocity of mode m. $F_{m,j}$ is a form factor that depends on the charge distribution of bunch j; for a Gaussian distribution, $F_{m,j} = \exp(-\omega_m^2\sigma_{t,j}^2/2)$. In future linear collider designs, the bunch length is very short, and we may take $F_{m,j} = 1$. $H(t)$ is the unit step function.

The contribution of the fundamental mode to the LRW in the CG case is

$$E_1^{lrw}(s,t,j) = -2\kappa_1 q_j F_{1,j}\cos[\omega_1(t-t_j^{ent}-\frac{s}{c})]$$
$$\cdot H(t-t_j^{ent}-\frac{s}{c})H(t_j^{ent}+t_s-t) \quad . \qquad (13)$$

where

$$t_s = -\frac{Q_{rf}}{\omega_{rf}}\ln(1-\frac{s}{L'}) \quad , \qquad (14)$$

and κ_1 is assumed to be independent of s. The HOM's in non-CZ structures may be treated by equivalent circuit models [4], but this is beyond the scope of the present paper.

Table 1: Parameters

RF frequency, $f_{rf} = 11.424$ GHz
Section length $= 1.8$ m
Attenuation $\tau = 0.505$
Fundamental mode $Q = 7107$
Fund. mode loss factor, $\kappa_1 = 203.75$ V/pC
Filling time, $T_f = 100$ ns
Bunch spacing $= 16\lambda_{rf} \approx 42$ cm
Bunch charge $= 1 \times 10^{10}$

COMPENSATION SCHEMES AND EXAMPLES

We model the linac as made up of CG sections (as defined above), with $2\pi/3$ phase advance per cell. bunch charge $= 1 \times 10^{10}$. Parameters are as shown in Table 1.

The most promising strategy useful for bunch trains of length a filling time or longer, is to pre-fill the structure in such a way that the energy gain of each bunch during the transient period approximates the energy gain of each bunch in the steady state [5]. In the simplest form of this scheme, the amplitude of the input RF field is linearly ramped during the first filling time, then the bunch train is injected at the beginning of the second filling time. The input RF pulse used in our example is shown in Fig. 1. However, since dispersion creates large amplitude variations on the front of the RF pulse, it is desirable to wait an additional 10 nsec before injecting the bunch train, to allow the worst of these dispersion "wiggles" to propagate out of the structure. The resulting fractional energy deviation, for a 90-bunch train is shown in Fig. 2. Were it not for the effects of dispersion, the steady state would be reached at about bunch 70, as can be seen in Fig. 2(a), where dispersion is not included in the calculation. Fig. 2(b) shows the result with dispersion included, but with its effects diminished by allowing the front 10 nsec of the pulse to propagate out of the structure before injecting the bunch train.

Figure 2. Fractional energy deviations of bunches in long-pulse pre-filling compensation scheme, (a) ignoring dispersion, (b) including dispersion.

Finally, we mention a related strategy [7], in which one approximately equalizes the bunch energies via the matched-filling method, and then modulates the RF input during the time when the train is passing through the structure, to compensate the "sag" one would otherwise get in the middle of the bunch train. Such a scheme may be the best for trains which are of order a half of the filling time in length.

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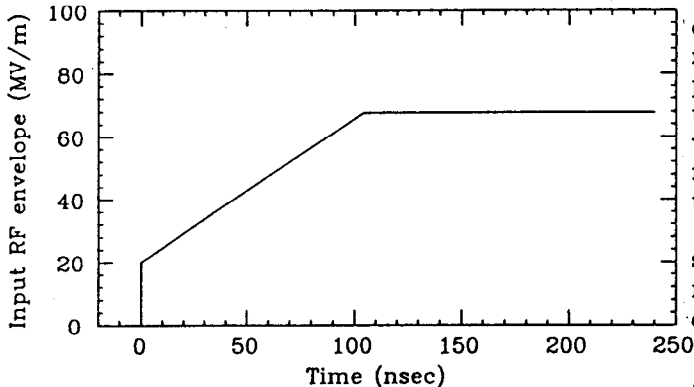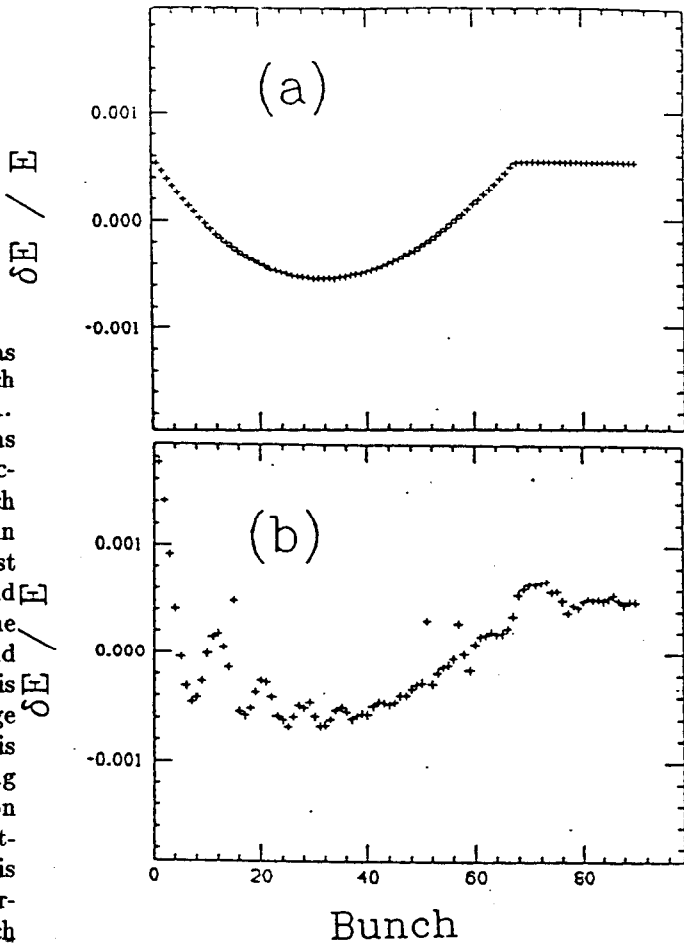

Figure 1. Envelope of input RF pulse, used in long-pulse pre-filling compensation scheme.

Another possible scheme for long bunch trains involves staggering the timing of the RF pulses in different sections [6]. Space prevents a detailed discussion of this scheme here; the overall energy compensation (net effect over a sum of sections) obtained with this scheme is similar to that of the preceding scheme. It does not, however, keep the energy as well compensated locally.

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