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Multibunch Beam Break-up in Detuned Structures*

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Abstract

A key problem in next-generation linear collider designs utilizing multibunching is the control of multibunch beam break-up. One method of controlling the break-up is detuning, i.e., varying the frequency of the transverse deflecting modes by varying the cell dimensions within the accelerating structures. In this case, the beam break-up is sensitive to the resonances between the bunch frequency and some of the deflecting mode frequencies. It is also sensitive to errors in the fabrication and alignment of the accelerating structures. We examine these effects in the context of the SLAC NLC design.

Introduction

In the present SLAC design for a Next-generation Linear Collider (NLC), it is planned that a 125 ns train of bunches will be accelerated on each RF pulse. In order to control multibunch beam break-up, we must ensure that the transverse wake is kept sufficiently small over the length of the bunch train. We plan to vary the structure dimensions so that the frequencies of the lowest-passband synchronous modes approximately follow a truncated Gaussian distribution in each structure. This "detuning" of the frequencies provides a strong initial roll-off of the wake before the first bunch spacing is reached. In addition, to maintain sufficient suppression of the longer-range wake, we plan to interleave these detuned frequencies over about four different structure types. This provides a smoother and denser distribution of frequencies than is obtained with just one structure type. However, the success of the method requires that dipole-mode frequency errors and misalignments of the structures be kept sufficiently small; this is the focus of the present paper.

Detuning strategy and parameters

Intuitive understanding of the effects of detuning is most easily obtained by viewing the structure as consisting of a collection of uncoupled oscillators. A more correct treatment includes the effects of the small couplings between the oscillators; the simplest way to do this is via equivalent-circuit models. In this paper we use a "double-band" equivalent-circuit model [1], that takes account of the mixing of the TM_{110} and TE_{111} modes to produce a TM_{11}-like dipole mode, which is the most important mode for multibunch beam break-up.

In the NLC structure design [2] we have the freedom to shape the distribution of the dominant TM_{11}-like dipole mode frequency f_0 between its two end-cell values, while keeping the frequency f_{rf} of the accelerating mode fixed. Let the full-spread be Δf_{tot}, centered on frequency $\bar{f}_0$, and neglect the cell-to-cell coupling for the moment. For a truncated Gaussian distribution with standard deviation σ_f, the spacing between adjacent components is:

$$\delta f_i = \frac{\sqrt{2\pi}}{N-1}\sigma_f \exp\left[\frac{(f_{0,i}-\bar{f}_0)^2}{2\sigma_f^2}\right] \mathrm{erf}\left(\frac{n_\sigma}{2\sqrt{2}}\right) \quad . \qquad (1)$$

Here $n_\sigma \equiv \frac{\Delta f_{tot}}{\sigma_f}$ is the full width of the truncated distribution in units of σ_f, N is the number of cells in a structure, and $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$ is the usual error function. The fractional spacing in the central core of the distribution is approximately

$$\frac{\delta f}{\bar{f}_0} \approx \frac{\sqrt{2\pi}}{N-1}\frac{\sigma_f}{\bar{f}_0}\mathrm{erf}\left(\frac{n_\sigma}{2\sqrt{2}}\right) \quad . \qquad (2)$$

Including coupling via the equivalent circuit model modifies the distribution, in particular, the core spacing is increased somewhat and the tails of the distribution extend further out. For our parameters, the core frequency spacing in the uncoupled model is $\delta f/f \approx 3\times10^{-4}$; for the coupled double-band model, $\delta f/f$ is increased by a factor of about 1.4.

To obtain the overall frequency distribution for n structures with interleaving, we use the distribution as given by Eqn. (1), but we increase N by a factor n. For $n = 4$, the lowest frequency would be assigned to structure type 1, next lowest to type 2, next to type 3, next to type 4, next to type 1, and so on, cycling repeatedly through the structure types. The linac is built by cycling through the n structure types (we have generally used the pattern 1 3 2 4 1 3 2 4...). We have done simulations using "smooth focusing" in which there is a focusing element between each 1.8 meter structure. We have also used a more realistic FODO-type lattice, in which the number of structures between quads increases with energy, while maintaining an approximate $E^{1/2}$ dependence of the average beta function and reasonable magnetic field strengths. In either case, the initial value of the average betatron function β in the NLC main linac will be around 6 meters, and will increase approximately as the square root of the energy going along the linac. Thus, the betatron wavelength $2\pi\beta$ is much greater than the structure length everywhere in the linac. For $n = 4$ interleaved structure types, the effect is much the same as if the wake function $W(z)$ were an average of the wake function over the n structure types.

The NLC structures will have an accelerating frequency of 11.4 GHz (X-band), with irises and inner cell radii tapered to produce a truncated Gaussian before coupling is included. Parameters of this Gaussian are $\sigma_f = 2.5\%$ and $n_\sigma = 4$. The Q's of all the coupled modes were taken to be 6500 (the variation in Q calculated for the coupled modes was small, and for simplicity we neglected it). The envelope of the single-particle wake function averaged over a single structure is shown in Fig. 1(a). The envelope averaged over four structure types with interleaved frequency distributions is shown in Fig. 1(b); one sees that there is substantial additional suppression of the longer range wake. This is necessary for the 125 ns bunch train design now being proposed for NLC (note that the range shown for z is approximately

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the length of the train). Other parameters are: bunch charge $N = 0.65 \times 10^{10}$, bunch spacing $= 16\lambda_{rf} \approx 42$ cm, initial linac energy 15 GeV, final linac energy = 250 GeV, linac length = 6 km, initial $\bar{\beta} \approx 6$ m (and scaling as $\sqrt{E}$). The energy spread is assumed to be zero in these simulations.

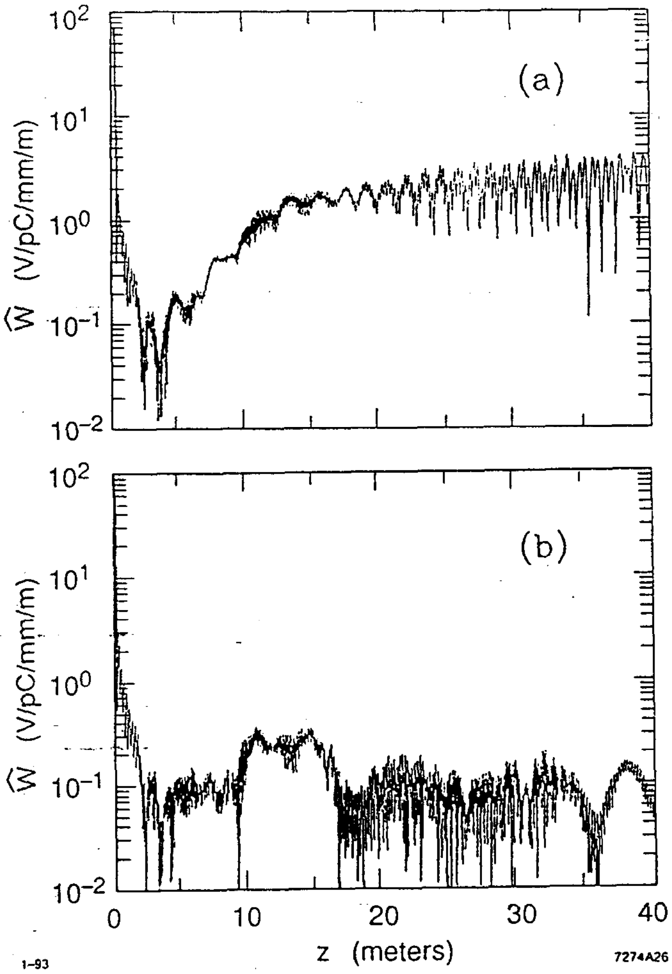

Figure 1. Envelope of the single-particle wake function $W(z)$ (a) for single structure type, (b) for four structure types with interleaved frequency distributions.

INJECTION JITTER

We begin by examining the case of a "perfect" linac (no frequency errors or misalignments), with a uniform initial offset of the beam. The transverse offset of the bunches at the end of the linac (in units of the initial offset, and normalized by factoring out the adiabatic damping due to acceleration) is shown for a single section type in Fig. 2(a) and for four section types with interleaved frequency distributions in Fig. 2(b). The advantage of using the four interleaved section types is mainly the strong suppression of the blow-up from injection jitter, as seen in this figure. For the case of a single section type, keeping the projected emittance growth of the multibunch beam below, say, 25% would require keeping the injection offset to less than 30% of the beam size. Note that it is only in the later part of the train that additional suppression is needed (cf. Fig. 1). For the case of four section types, the emittance growth will be only a few percent even with an injection offset comparable to the beam size.

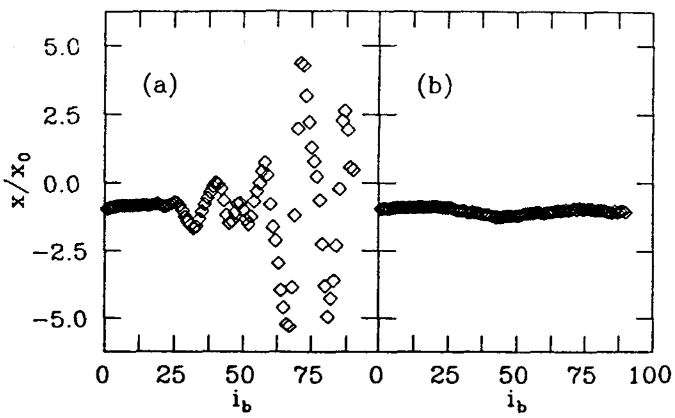

Figure 2. Transverse offset of bunches at end of linac (no errors), in units of initial uniform offset x_0 of train, as function of bunch number i_b, (a) for single section type, (b) for four section types.

FREQUENCY ERRORS

We examined the effects of small random variations in the frequencies, due, for example, to fabrication errors. There are two extreme cases. The first is that in which the error on each frequency in the design distribution is the same in all sections of a given type (but random from cell to cell). This case, which we shall refer to as "systematic" errors generally leads to an increase of the longer range wake field. The second case, which we denote "random" is that in which the errors are independent for each section and each frequency; in this case there is some averaging of the errors over many sections.

The expected rms size of the random frequency errors due to machining precision is $\sigma_{e,ran} = 1 \times 10^{-4}$ (fractional error, i.e. $\delta f / f_0$) [3]. This is comparable to the spacing in the core of the four-interleaved-section frequency distribution, which is one reason why it is not advantageous to have more than about four section types. We hope to keep the systematic component $\sigma_{e,sys}$ smaller than this; for illustration, we look at the cases $\sigma_{e,sys} = 3 \times 10^{-5}$ and $\sigma_{e,sys} = 1 \times 10^{-4}$. Fig. 3 shows histograms of the fractional emittance increase of the multibunch beam compared to that of a single bunch (using four structure types), assuming an initial offset of the beam equal to the bunch size, for fifteen different distributions of systematic errors generated at each value of $\sigma_{e,sys}$. If we also include random errors uncorrelated from section to section, having $\sigma_{e,ran} = 1 \times 10^{-4}$, there is little additional effect on the projected emittance.

RESONANCES

Fig. 4 shows the maximum transverse offset of the bunches, for the point having the largest emittance growth in Fig. 3. The sharp onset of transverse growth at some point in the train is typical of cases with large growth and is due to the fact that the wake function at a number of successive bunches has the same sign. This can happen when frequencies in the detuned distribution are close to a "resonance" with the bunch frequency.

The 21st and 22nd harmonic of the bunch frequency lie within the range of detuned frequencies and have significant kick factors, according to the double-band model. The resonances themselves are not so much a problem as are nonuniformities in the frequency distribution of

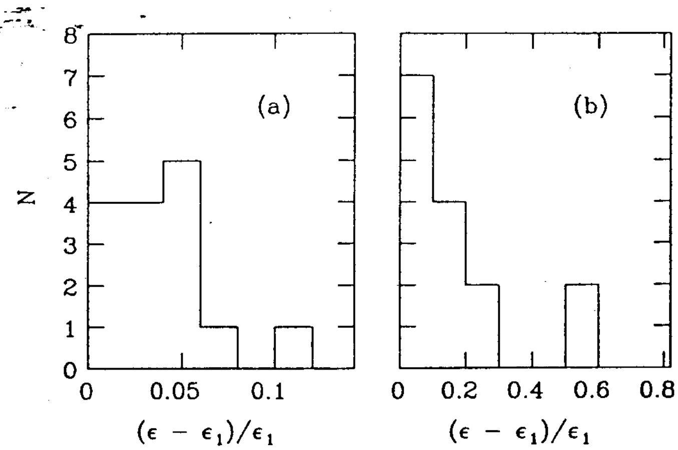

Figure 3. Histogram of fractional emittance increase (w.r.t. accelerator axis) of multibunch beam compared to emittance ϵ_1 of a single bunch, for fifteen different systematic error distributions, (a) at $\sigma_{\epsilon,sys} = 3 \times 10^{-5}$, and (b) at $\sigma_{\epsilon,sys} = 1 \times 10^{-4}$.

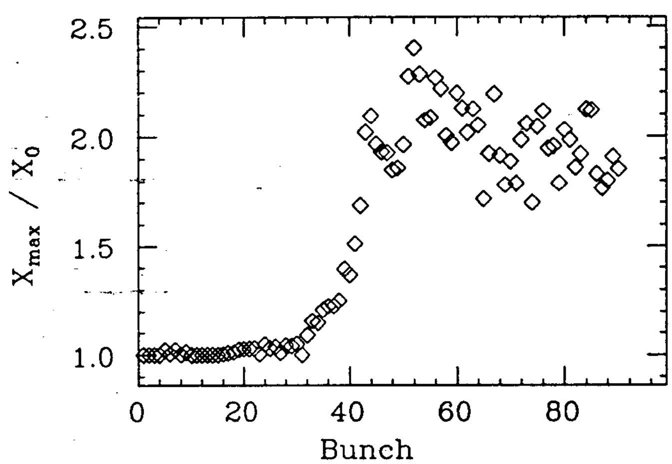

Figure 4. Maximum transverse offset x/x_0 as a function of bunch number i_b, for the $\sigma_{\epsilon,sys} = 1 \times 10^{-4}$ distribution with largest emittance growth in Fig. 3.

modes in their vicinity. Note that the resonant wakes are 90° out of phase with the bunches. Even with no frequency errors, a non-symmetric location of the resonant peak relative to neighboring modes can lead to a net wake function sum at a given bunch. This is especially true near the end of the train, where the resonant width starts to become comparable to the mode spacing. Nonuniformities, such as those for frequency errors, can lead to a net resonant wake sum, which is potentially much larger than that for a uniform, symmetric distribution.

MISALIGNMENTS

We have also examined the effects of misalignments of the acceleration structures. As a simple model, we assume each misaligned piece of structure contains i_m cells, (where i_m may vary from 1 to the number of cells in a whole structure), the misalignment within each such piece is uniform, and the misalignments are random with rms size σ_m. Fig. 5 shows the tolerance $t_{25\%}$ on σ_m to produce a 25% emittance blow-up as a function of i_m, assuming four interleaved structure types. Here we have assumed a FODO-type lattice, with beam size in the initial focusing quad equal to 3.6 μm (comparable results are obtained using smooth focusing). The wake kicks are calculated assuming that the modes themselves are not significantly distorted by the presence of the misalignments.

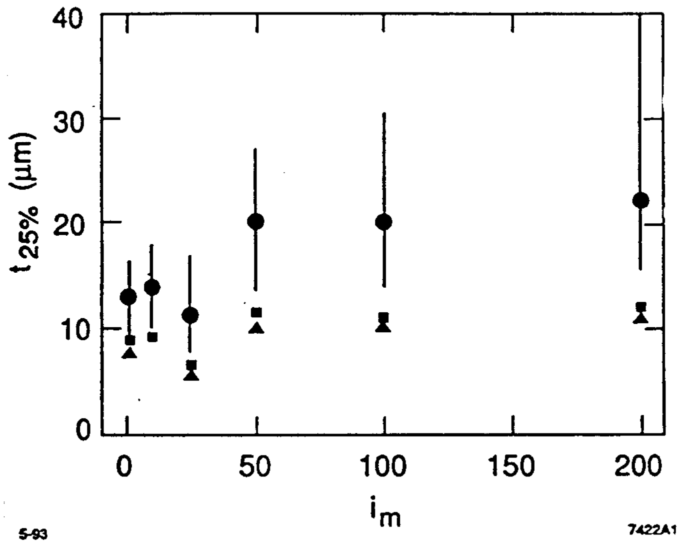

Figure 5. Tolerance (in μm) of rms misalignments for 25% emittance growth (w.r.t. beam centroid) of the multibunch beam as a function of number of cells i_m per uniformly misaligned piece of structure. 174 different random distributions were calculated at each value of i_m. The error bars show the rms on each side of the mean (●). The (■) and (▲) show the 10%, and 5% points respectively.

We see that the tolerance is fairly insensitive to i_m, although it is tightest when i_m is around 20. The loosest tolerance is for misalignment of entire structures, since the coherence of the detuned frequency distribution within a section is preserved. The tolerance for smoother misalignment distributions is less tight than the above case of random uniformly-offset pieces of structures.

CONCLUSIONS

Using four structure types with interleaved frequencies, the jitter tolerance is greater than the beam size, to keep the emittance growth to a few percent. The tolerance on frequency errors that are the same for all sections is a few parts in 10^5 to keep the emittance growth to a few percent. For frequency errors that are uncorrelated in different sections, the tolerance is looser than the expected machining precision of a part in 10^4. For misalignments, the tolerance to keep the emittance growth below 25% with 95% confidence ranges from 5 to 10 μm depending upon the correlation length. It is probably possible to loosen the alignment tolerances using appropriate trajectory correction algorithms; this will be a subject of future work.

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