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Flavor Symmetries and The Problem of Squark Degeneracy

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Abstract

If supersymmetry exists at low energies, it is necessary to understand why the squark spectrum exhibits sufficient degeneracy to suppress flavor changing neutral currents. In this note, we point out that gauged horizontal symmetries can yield realistic quark mass matrices, while at the same time giving just barely enough squark degeneracy to account for neutral K-meson phenomenology. This approach suggests likely patterns for squark masses, and indicates that there could be significant supersymmetric contributions to $B-\bar{B}$ and $D-\bar{D}$ mixing and CP-violation in the K and B systems.

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Two solutions of the hierarchy problem have been suggested over the years: technicolor and supersymmetry. Perhaps the biggest problem for technicolor theories is that they tend to suffer from unacceptable flavor-changing neutral currents. Partial solutions to this problem have been offered, but the resulting models are extremely elaborate.[1] Supersymmetry, it is often argued, does not suffer from this problem. However, this is not so clear. At one loop, it is well known that there are diagrams contributing to $K^o - \bar{K}^o$ mixing which, for supersymmetry breaking masses below a TeV, are too large unless there is a high degree of degeneracy among squarks. The real part of this mixing, for example, leads to the requirement that[2,3,6]

$$\frac{\delta \tilde{m}_q^2}{m_{susy}^2} \frac{\delta \tilde{m}_{\bar{q}}^2}{m_{susy}^2} \frac{1}{m_{susy}^2} \lesssim 10^{-10} \text{ GeV}^{-2} \tag{1}$$

while for the imaginary part, the limit is about two orders of magnitude stronger. There are also limits on degeneracy from other processes: $B - \bar{B}$ mixing, $b \to s\gamma$, $\mu \to e\gamma$, *etc.* There are additional constraints on the size of certain CP-violating angles coming from $K - \bar{K}$, and the neutron and electron electric dipole moments. The question is, can one naturally satisfy all of these constraints? In some early models of supersymmetry breaking, these conditions were automatically satisfied because the breaking of supersymmetry was fed to squarks by gauge interactions.[4] In hidden sector supergravity theories, however, which provide the basis for much of our thinking about low energy supersymmetry, the situation is far less clear. It is often said that this degeneracy is perhaps reasonable, since, after all, gravity is “flavor blind.” On closer examination, however, this argument is seen to be without substance. In most models of the type which have been considered to date, there are operators which one can add to the theory, not suppressed by any (even approximate) symmetry, which give rise to an $\mathcal{O}(1)$ breaking of the degeneracy. This problem has been discussed in numerous places. In the context of supergravity theories, for example, it is considered in ref. 5. In ref. 7, this situation was anticipated for string theory and strategies for naturally raising the supersymmetry breaking scale into the multi-TeV region to alleviate this problem were proposed. Explicit departures from universality in simple orbifold models have been computed in ref. 8. Kaplunovsky and Louis[9] recently have reviewed this problem in the framework of string theory. They note that if supersymmetry breaking is associated principally with the dilaton, one will obtain some degree of degeneracy. However, they have also pointed out serious difficulties with such a scenario.

In early work on hidden sector supergravity models, it was suggested that one should simply postulate a large, approximate, flavor symmetry among squarks.[10] Indeed, while various other solutions to this problem might be contemplated, flavor symmetries seem a most natural framework. There are two immediate issues which one must face. First, whatever horizontal symmetry there may be is clearly very badly broken by the ordinary quark mass matrices. Second, we would prefer not to impose continuous global symmetries on the underlying theory. Such symmetries are almost certain to be broken by gravitational interactions, and are known not to arise in string theories.[11]

In this paper we will study models with non-abelian, gauged flavor symmetries, to determine whether these can assure an adequate degree of squark degeneracy while simultaneously allowing realistic quark mass matrices. We will describe simple models containing an $SU(2)_H$ horizontal symmetry in which there is adequate degeneracy to satisfy the limits coming from the real part of K-$\bar{K}$ mixing. To be more precise, a naive estimate, assuming all susy breaking parameters of order 300 GeV, gives a result about an order of magnitude larger than the experimental upper bound. This order of magnitude discrepancy is not disturbing. First, in the framework we consider, it is not unnatural to suppose that squarks of the first generation have TeV masses, while those of the third have smaller masses (so fine tuning of Higgs masses is not required). Alternatively, some of the parameters of order one in the model may be of order $1/3 - 1/10$. The limit on the imaginary part, two orders of magnitude stronger, is more problematic. To satisfy this constraint, it is necessary to make some further assumptions. Again, there are plausible regions of parameter space for which the imaginary part is sufficiently small. A different approach is to impose additional symmetries, such as discrete symmetries, to provide further suppression. This seems a reasonable thing to do, since such symmetries might be necessary to understand the fermion mass matrices.⋆ One also may want to consider additional assumptions about the nature of CP-violation. Note that bounds on gaugino mass phases from d_n and d_e also suggest additional assump-

⋆ For a recent effort along these lines, see ref. 12.

tions such as spontaneous CP-violation. One can view these results in a positive light: generic models do not (quite) satisfy all constraints, so additional features must be considered – and perhaps additional predictions made.

The models we consider will be predictive: they will imply definite relations among squark masses. For example, models with $SU(2)_H$ symmetry predict that up or down squarks of the first two families are approximately degenerate, while third family squarks may have quite different masses. Similar degeneracies among sleptons are also expected. These models will also have interesting implications for $B - \bar{B}$ and $D - \bar{D}$ mixing and, possibly, $b \rightarrow s\gamma$.

One might hope that models of this kind would explain the many puzzling features of the fermion mass spectrum. We will not attempt this here. In particular, our models will require a rather large range of quark Yukawa couplings (though perhaps not quite as large a range as in the minimal standard model).

The first question one must address is the scale of breaking of the horizontal symmetry. We will distinguish two possibilities: breaking near M_p, and breaking much below M_p. A simple model with large-scale breaking is the following. Take the gauge group to be that of the standard model times an additional $SU(2)_H$. For purposes of enumerating the different particles and couplings, we will label the states by the quantum numbers they might have in an $SU(5) \times SU(2)_H$ unification. Note that we are not assuming an underlying $SU(5)$ symmetry, but simply using $SU(5)$ to classify the states. The three generations are then assumed to form doublets and singlets of the $SU(2)_H$. The states are

$$\bar{5}_a = (\bar{5}, 2) \quad 10_a = (10, 2) \quad \bar{5}_s = (\bar{5}, 1) \quad 10_s = (10, 1).$$

The Higgs particles are taken to be two singlets of $SU(2)_H$; this will avoid the problem of flavor changing currents mediated by Higgs particles. We will denote these by H_1 and H_2. To break $SU(2)_H$, one adds three fields transforming as $(1, 2)$: $\Phi_i^a, i = 1, 2, 3$. The model is then free of both perturbative and non-perturbative anomalies. Alternatively, one can add an $SU(2)_H$ doublet of right-handed neutrinos, in which case only two Φ_i singlets are added.

We will assume that supersymmetry is broken in a hidden sector, whose dynamics do not by themselves break any of these gauge symmetries. We will also assume that, after supersymmetry breaking, the potential for the fields Φ_i is such that these fields obtain large vev's. This assumption may seem unnatural, but it is often satisfied in string theories. First, there are "D-flat" directions (*i.e.*, directions in which the auxiliary D-fields in the $SU(2)_H$ gauge supermultiplet vanish) where some of these fields have vev's. F-flatness is known to arise in string theory in at least two ways. Moduli of string compactifications with $(2,2)$ world sheet supersymmetry are F-flat.[13] At points of enhanced symmetry, the moduli are typically charged (*e.g.*, at orbifold points); the enhanced symmetry could be our horizontal symmetry. Generically, however, the moduli *do* appear in the superpotential of the matter fields (some of these couplings may be exponentially suppressed at large radius).[13] F-flatness is also known to arise in the presence of discrete R-symmetries.[14] In either case, if some of the Φ fields acquire negative masses upon supersymmetry breaking, they can acquire large vev's. We will require that these be smaller than M_p by a factor of order 10. We will not attempt here to explain how this factor might arise, but simply argue that in a theory with small couplings it is not unnatural.†

To keep the discussion simple, we will assume that two singlets, Φ_1 and Φ_2, obtain vev's: $|\langle\Phi_1\rangle| = (0,\phi)^T$ and $|\langle\Phi_2\rangle| = (\phi,0)^T$.‡ In order to understand how this breaking of the $SU(2)_H$ symmetry feeds down to other fields, we need to examine the lagrangian more carefully. Let us focus first on the quark fields. Denoting quark doublets by Q and singlets by $\bar{d}$ and $\bar{u}$, the superpotential just below M_p contains dimension-four terms:

$$W_q = \lambda_1 \epsilon_{ab} Q_a \bar{d}_b H_1 + \lambda_2 \epsilon_{ab} Q_a \bar{u}_b H_2 + \lambda_3 Q_s \bar{d}_s H_1 + \lambda_4 Q_s \bar{u}_s H_2 \qquad (2)$$

These give rise to $SU(2)_H$ symmetric terms in the mass matrix. Clearly we need to assume that λ_1 and λ_2 are small (this might be arranged by means of a discrete symmetry).

$SU(2)_H$-violating terms arise from higher dimension couplings, of which there are a wide variety. For example, in the d-quark sector, we have:

$$\frac{1}{M_p}(\lambda_5^i \epsilon_{ab} \Phi_a^i Q_b \bar{d}_s H_1 + \lambda_6^i \epsilon_{ab} \Phi_a^i Q_s \bar{d}_b H_1) + \frac{1}{M_p^2}(\lambda_7^{ij} \epsilon_{ab}\epsilon_{cd} \Phi_a^i \Phi_c^j Q_b \bar{d}_d H_1 + ...) \quad (3)$$

† One possible origin of this scale is through the appearance of a Fayet-Iliopoulos D-term.[15]
‡ The precise alignment of Φ_1 and Φ_2 will not be important to us, except when we consider additional discrete symmetries. In such cases, the alignment considered here is natural.

Similar terms are present in the up-quark sector. Some points should be noted immediately. First, Φ/M_p (times coupling constants) cannot be too small; in the limit that $\Phi \to 0$, there is no mixing of the third generation with the first two. As we remarked above, the $SU(2)$ symmetric terms in the light quark matrices must be small, so m_c and m_s must go as Φ^2/M_p^2; this quantity thus cannot be much smaller than 10^{-2}. With this restriction on Φ, there is no difficulty in obtaining reasonable fermion masses and KM angles, provided one is willing to take several Yukawa couplings to be small and comparable, as in the standard model. Hopefully, other horizontal schemes could be more predictive. We leave the exploration of this question to future work.

For simplicity,[§] we choose the various Yukawa couplings so that the down quark mass matrix entries $m^d_{ij} d_i \bar{d}_j$ satisfy

$$m^d = \begin{pmatrix} \sim m_d & \sim m_d & \sim m_d \\ \sim m_d & \sim m_s & \sim m_s \\ \sim m_d & \sim m_s & \sim m_b \end{pmatrix}, \tag{4}$$

with similar assumptions for the up-quark matrix. The quark masses and eigenstates then take on a particularly simple form. For example, the down masses are given by

$$m_d = m_{11} - \frac{m_{12} m_{21}}{m_s}, \quad m_s = m_{22} - \frac{m_{23} m_{32}}{m_b}, \quad m_b = m_{33}. \tag{5}$$

The down mass eigenstates are given by

$$|d_i\rangle = x^d_{ij} |j\rangle, \qquad |\bar{d}_i\rangle = \bar{x}^d_{ij} |\bar{j}\rangle, \tag{6}$$

where $i = 1, 2, 3$ correspond to d, s, b, respectively, and $|1\rangle$ corresponds to the vector $(1, 0, 0)^T$, *etc.* With a mass matrix of the form of eq. (4), the x^d_{ij} are given

§ If, for example, $m^d_{31} \sim m^d_{32} \sim m_b$, then the mixings of the first and second generation $\bar{d}$ quarks with the third generation quarks are large, and the $SU(2)$ symmetry will not lead to sufficient degeneracy for the $K - \bar{K}$ system.

by

$$x_{ii}^d \sim 1, \quad x_{21}^d \sim \frac{m_{12}^d}{m_s} \sim V_{us}, \quad x_{32}^d \sim \frac{m_{23}^d}{m_b} \sim V_{cb}, \quad x_{31}^d \sim \frac{m_{13}^d}{m_b} \sim V_{ub}. \tag{7}$$

The remaining x_{ij} follow from orthonormality of the eigenstates. The $\bar{x}_{ij}$ are obtained by complex conjugating the above and interchanging indices on the m_{ij}. Expressions for the up masses and eigenstates are completely analogous. Knowledge of the x_{ij} and $\bar{x}_{ij}$ will be required to estimate the various off-diagonal squark mass matrix entries of relevance to FCNC's.

Note that, given eq. (4) and it's analogue for the up sector, the KM angles are essentially generated in the down sector. The (32) and (31) entries in eq. (4) are unrelated to the KM angles and, in general, can be as large as a few GeV. We will see that in this limit gluino graphs can give Br($b \to s\gamma$) at the level of the latest CLEO[16] bound, 5.4×10^{-4}. However, in this case we will have difficulty with the $K - \bar{K}$ constraints.

What about the squark mass matrices? We are assuming that the underlying supergravity theory is the most general one consistent with its symmetries. Such a theory is described, in general, by three functions, the Kähler potential, K, the superpotential W (which we have already discussed), and a function f which describes the gauge couplings. With our assumptions, the Kähler potential is not of the so-called "minimal" type, and will give rise to violations of universality. If we denote "hidden sector fields" generically by z and visible sector fields by y, we can characterize the violations of degeneracy and proportionality quite precisely. For small y, we can expand K in powers of y. Rescaling the fields, we can write

$$K = k(z, z^*) + y_i^* y_i + \ell_{ij}(z, z^*)\, y_i^* y_j + ... \tag{8}$$

Examining the form of the potential in such a theory, it is easy to see that proportionality and degeneracy occur if ℓ is proportional to the unit matrix. There is no reason for this to occur in general. However, the $SU(2)_H$ symmetry significantly restricts the form of ℓ. Expanding ℓ in powers of ϕ, the leading terms for the squark fields lead to $SU(2)_H$-symmetric squark mass terms of the form (using the same symbol for the scalar field as for the superfield)

$$V_{soft} = \tilde{m}_1^2 |Q_a|^2 + \tilde{m}_2^2 |Q_s|^2 + \tilde{m}_3^2 |\bar{u}_a|^2 + \tilde{m}_4^2 |\bar{u}_s|^2 + ...$$

$$+A_1\lambda_1 Q\bar{d}H_1 + A_2\lambda_2 Q\bar{u}H_2 + ... + \text{h.c.}, \tag{9}$$

Here, $\tilde{m}_i$ and A_i are of order m_{susy}. Terms linear and quadratic in Φ give rise to symmetry-breaking terms of the type:

$$\delta V^2_{soft} = \frac{m^2_{susy}}{M_p}(\gamma_1 \Phi_1 Q Q^*_s + ...) + \frac{m^2_{susy}}{M^2_p}(\gamma'_1 \Phi_1 Q \Phi_2 Q^* + ...) \tag{10}$$

and

$$\delta V^3_{soft} = \frac{m_{susy}}{M_p}\lambda^1_5 Q\bar{d}_s H_1(\eta_1\Phi_1 + \eta_2\Phi_2 + \eta_3\Phi^*_2)$$

$$+\frac{m_{susy}}{M^2_p}\lambda^{11}_7 Q\bar{d}H_1(\eta'_1\Phi_1\Phi_1 + \eta'_2\Phi_1\Phi_2 + \eta'_3\Phi_1\Phi^*_2) + ... \tag{11}$$

We have omitted $SU(2)_H$ indices on Q, $\bar{u}$, $\bar{d}$ but terms with all possible contractions should be understood. Here γ, γ', η and η' are dimensionless numbers. The couplings η_3 and η'_3 may seem surprising, since they are not among the usual allowed soft breaking terms. These terms, however, are *supersymmetric* terms, arising because the superpotential of the effective theory will in general contain terms like $m_{susy}\Phi_i\Phi_j$. By 't Hooft's naturalness criterion,[17] many of the couplings in eqs. (9)-(11) should not be much less than one; the theory does not become any more symmetric if these quantities vanish. Some, however, can naturally be small; later, we will consider discrete symmetries which might suppress certain dangerous ones.

The resulting down squark mass matrices are of three types; $\tilde{m}^2_{LL}$, $\tilde{m}^2_{LR}$, and $\tilde{m}^2_{RR}$, where L and R refer to left-handed and right-handed squarks, respectively. For example, for $\tilde{m}^2_{LL}$ one obtains

$$\tilde{m}^2_{LL} = diag(\tilde{m}^2_1, \tilde{m}^2_1, \tilde{m}^2_2) + \delta\tilde{m}^2. \tag{12}$$

The first and second terms originate in V_{soft} and δV^2_{soft}, respectively. In the interaction basis, the (13), (23), (31) and (32) entries of $\delta\tilde{m}^2$ are proportional to $\frac{\phi}{M_p}$ and the remaining entries are proportional to $\frac{\phi^2}{M^2_p}$. In the quark mass eigenstate basis, the same is true (see eqs. (6)). For example, $\tilde{m}^2_{ds}$ is given by

$$\tilde{m}^2_{ds} = (\tilde{m}^2_2 - \tilde{m}^2_1)x^*_{13}x_{23} + \delta\tilde{m}^2_{12} + (\delta\tilde{m}^2_{11} - \delta\tilde{m}^2_{22})x_{21} + \delta\tilde{m}^2_{13}x_{23} + \delta\tilde{m}^2_{32}x^*_{13} + ... \quad (13)$$

Similar statements hold for the $\tilde{m}^2_{RR}$ and for the up-sector. The matrix elements satisfy the promising hierarchy $\tilde{m}^2_{ds} << \tilde{m}^2_{db}, \tilde{m}^2_{sb}$.

What about proportionality of the quark and squark mass matrices? The $\tilde{m}^2_{LR}$ matrix corresponding to a fermion matrix, m, satisfying the hierarchy in eq. (4), is of the form

$$\tilde{m}^2_{LR} \approx \begin{pmatrix} A_{11}m_{11} & A_{12}m_{22} & A_{13}m_{23} \\ A_{21}m_{22} & A_{22}m_{22} & A_{23}m_{23} \\ A_{31}m_{32} & A_{32}m_{32} & A_{33}m_{33} \end{pmatrix}. \quad (14)$$

For a general potential, the violations of proportionality for the $\tilde{m}^2_{LR}$ matrix for the first two generations are of order Φ^2/M_p^2. This is small enough for the $K - \bar{K}$ and $D - \bar{D}$ systems.

We now discuss implications for FCNC's. Bounds on off-diagonal down squark masses[3] from the $K - \bar{K}$ and $B - \bar{B}$ mass differences are summarized in Table 1. Estimates of these quantities in the $SU(2)_H$ model are collected in Table 2. Similar estimates are obtained in the up-sector for $\tilde{m}^2_{LL}$ and $\tilde{m}^2_{RR}$, but $\tilde{m}^2_{LR}$ entries will be related to the up quark mass matrix and will be somewhat larger. This will turn out to be of significance for $D - \bar{D}$ mixing. We see that for ~ 300 GeV squarks and gluinos, some of the dimensionless couplings in δV^2_{soft} will have to be $\sim \frac{1}{10} - \frac{1}{3}$ to obtain satisfactory Δm_K and Δm_B. If the squarks of the first two generations have masses of order 1 TeV then all dimensionless couplings in the scalar potential can be $\mathcal{O}(1)$. Provided the third generation squarks are comparatively light[18] (perfectly possible in this sort of model), this does not imply any fine tuning. Alternatively, as will be described below, discrete symmetries can further suppress the most dangerous couplings.

We have not included limits from Br$(b \to s\gamma)$ in Table 1. From gluino graphs with LR squark mass insertions[3] the new CLEO bound of 5.4×10^{-4} implies*

$$\left(\frac{\tilde{m}^2_{b\bar{s}}}{\tilde{m}^2}\right)^2 \frac{1}{\tilde{m}^2} \lesssim 8 \times 10^{-10}, \tag{15}$$

and the same for $\tilde{m}^2_{s\bar{b}}$. The $SU(2)_H$ model gives

$$\tilde{m}^2_{b\bar{s}} = A^d_{32} m^d_{32} + \bar{x}^d_{23} A^d_{33} m^d_{33} + ...$$

where the A^d's are of order m_{susy} as given in eq. (14). For $m_{32} \sim m_s$, as in eq. (4), this is too small to give interesting contributions to $b \to s\gamma$. However, if m^d_{32} is as large as a few GeV but less than m_b, which is allowed from the point of view of fermion masses and mixing angles, then, from eq. (15), we see that gluino graphs can contribute to Br($b \to s\gamma$) at the level of 5.4×10^{-4} for squarks as heavy as 300 GeV; as noted earlier, however, this may lead to difficulties for $K - \bar{K}$.

Since the bounds from Δm_B are just barely satisfied, the model could have very rich implications for CP-violation in the B system.[19] As remarked above, the $\tilde{m}^2_{LR}$ entries have interesting consequences for $D - \bar{D}$ mixing. Given the current experimental bound of $\frac{\Delta m_D}{m_D} < 6.97 \times 10^{-14}$ one obtains the following constraints, taking $F_D = 200\ MeV$ and $B_D = 1$:

$$\left(\frac{\tilde{m}^2_{u\bar{c}}}{\tilde{m}^2}\right)^2 \frac{1}{\tilde{m}^2} \lesssim 10^{-9}\ \text{GeV}^{-2} \tag{16}$$

for $\tilde{m}_g \sim \tilde{m}$; the bound is 7×10^{-10} for $\tilde{m}_g \sim 0.1\ \tilde{m}$. By way of comparison, in the $SU(2)_H$ model we expect for the above quantities $2 \times 10^{-10} \eta'^2$ for 300 GeV squark masses, and $2 \times 10^{-12} \eta'^2$ for ~ 1 TeV squark masses. Recent HQET calculations[20] for the standard model lead to $\frac{\Delta m_D}{m_D} \sim 10^{-17} - 10^{-16}$. So it is clear that one can readily obtain Δm_D one to two orders of magnitude larger in the $SU(2)_H$ model!

What about constraints from CP-violation? The bounds from ϵ_K on the imaginary parts of the various quantities constrained by Δm_K are about two orders of

$\star$ Graphs with LL and RR insertions give small contributions, except for very light $O(100\ GeV)$ squarks, which are disfavored by $K - \bar{K}$ bounds.

magnitude stronger than indicated in Table 1. Constraints on non-degeneracy will depend on the size of the phases entering these quantities. For example, if the phases are of order unity, as would be expected in models with explicit CP-violation, then for ~ 1 TeV first and second generation squarks, some of the γ' couplings in δV_{soft} would have to be of order $\frac{1}{10}$. Couplings of this size or smaller might arise as a consequence of discrete symmetries, as discussed below. Alternatively, these squarks could be even heavier. This will not lead to fine-tuning of Higgs parameters, since, as already mentioned, their Yukawa couplings are small. Note that with this choice of parameters, large CP-violating supersymmetric contributions in the B-system are possible, since the relevant phases can be of order unity.

The bound on the neutron electric dipole moment, d_n, most strongly constrains $Im(m_{\tilde{g}} A_{11} m^q_{11})$, and $Im(\tilde{m}^2_{u\bar{t}} \tilde{m}^2_{t\bar{u}} A^u_{33} m^u_{33})$. The former constraint has been widely considered[21] in the MSSM. The situation here is much the same: for ~ 300 GeV squark, gluino and A_{ij} trilinear scalar masses, one requires $Arg[m_{\tilde{g}} A_{11} m^q_{11}] \sim 10^{-2}$, while for ~ 1 TeV squarks, phases of order unity are permissible. The same turns out to be true for $Arg[\tilde{m}^2_{u\bar{t}} \tilde{m}^2_{t\bar{u}} A^u_{33} m^u_{33}]$, given that the $B - \bar{B}$ constraints on $\tilde{m}^2_{LR}$ are satisfied in our model.

In scenarios of spontaneous CP-violation, the relevant phases might naturally be of order 10^{-2}, in which case the ϵ_K, and d_n constraints are more comfortably accomodated[22]. One interesting possibility is that it is the Φ field vev's which spontaneously break CP. This is consistent, for example, with the idea described above that these fields could be moduli of a string compactification.[23] In such a scheme the phase of the gluino, in particular, will be at most of order Φ^2/M_p^2. However, it appears difficult to suppress $Arg\langle\Phi\rangle$, so that the other phases of relevance to d_n and ϵ_K are likely to be large. Perhaps if the scale of CP-violation is somewhat smaller than the scale of horizontal symmetry breaking one can naturally obtain smaller phases.

An alternative strategy for accomodating CP-violation bounds is to increase the amount of squark degeneracy by adding additional abelian discrete or continuous horizontal symmetries. One notices that all terms in eqs. (10) and (11) which contribute to off-diagonal entries (here we are refering to the interaction basis) in $\tilde{m}^2_{LL}$, $\tilde{m}^2_{RR}$ and $\tilde{m}^2_{LR}$ can, in principle, be eliminated by additional symmetries. It is not hard to construct models with such symmetries and realistic quark mass matrices. The smallness of off-diagonal squark mass matrix entries is limited by the x_{ij}, $\bar{x}_{ij}$, or KM angles. This can lead to further suppression of order θ_c^2 for $K - \bar{K}$ and of order $\left(\frac{V_{cb}}{\gamma}\right)^2$ for $B - \bar{B}$. Moreover, in many cases, the lowest dimension operators are CP-conserving, providing adequate suppression for Im $K^o - \bar{K}^o$.

One can also carry out the above program making use of other symmetry groups, such as $SU(3)$ or non-abelian discrete groups. The SU(2) models have the virtue of simplicity, which is in large part due to the gross features of the quark mass spectrum: large mass splitting and small mixing angles between the third family and the first two.

Let us turn now to the possibility of breaking at lower scales. We will not attempt here to construct explicit models, but confine ourselves to some general remarks. First, there are a number of approaches one might adopt. We have already remarked that if the Higgs carry $SU(2)_H$ quantum numbers, there are likely to be problems with flavor changing neutral currents from Higgs exchange. Still, such models are clearly worthy of exploration.

An alternative possibility, following ref. 12, is to suppose that at some new scale, not far from the flavor symmetry breaking scale, there are some $SU(2)_L$-singlet vector-like quarks, some of which are in doublets of $SU(2)_H$. There are also $SU(2)_H$-breaking doublets ϕ_i, like in the large-scale model, which couple the light and heavy quarks. Without a terribly complicated structure at this scale, integrating out the heavy fields produces couplings of light quarks analogous to those in eq. (3) with M_p replaced by the heavy quark scale.

We can summarize all of this by saying that it is easy to construct models in which horizontal symmetries adequately suppress flavor changing neutral currents. This view suggests patterns of masses which may differ from assumptions which are conventional in model building. For example, at very high energies, the third generation (left and right) squarks are not likely to be degenerate with those of the first two. Moreover, in the simplest models, the squarks of the first two generations should have masses of order a TeV, while the top squark (to avoid naturalness problems) should be comparatively light. The simplest models, which only make use of an $SU(2)$ horizontal symmetry, offer no understanding of the quark mass matrix. Such an understanding may require more intricate symmetry patterns, which, as we have illustrated, may lead to even tighter degeneracy. Perhaps, after all, supersymmetry may yield insights into the problems of flavor.

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REFERENCES

1. For recent efforts to solve these problems, see L. Randall, MIT preprint MIT-CTP-2112 (1992) and H. Georgi, Harvard preprint HUTP-92-1037 (1992).
2. H.P. Nilles, *Phys. Rep.* **110** (1984) 1.
3. F. Gabiani and A. Masiero, *Nucl. Phys.* **B322** (1989) 235
4. For a recent attempt along these lines, see M. Dine and A. Nelson, UCSD preprint UCSD/PTH 93-05 (1993).
5. H. Georgi, *Phys. Lett.* **169B** (1986) 231; L.J. Hall, V.A. Kostelecky and S. Raby, *Nucl. Phys.* **B267** (1986) 415.
6. J.S. Hagelin, S. Kelley, and T. Tanaka, MIU preprint MIU-THP-92/59.
7. M. Dine, A. Kagan and S. Samuel, *Phys. Lett.* **243B** (1990) 250.
8. L. Ibanez and D. Lust, *Nucl. Phys.* **B382** (1992) 305; B. de Carlos, J.A. Casas and C. Munoz, CERN-TH.6436/92 and *Phys. Lett.* **299B** (1993) 234
9. V.S. Kaplunovsky and J. Louis, preprint CERN-TH. 6809/93 UTTG-05-93.
10. L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359.
11. T. Banks and L. Dixon, *Nucl. Phys.* **B307** (1988) 93.
12. M. Leurer, Y. Nir and N. Seiberg, Rutgers preprint RU-92-59, (1992).
13. M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nucl. Phys.* **B278** (1986) 769; L. Dixon, in *Superstrings, Unified Theories, and Cosmology 1987*, G. Furlan, et al, eds., World Scientific (Singapore, 1988).
14. M. Dine and N. Seiberg, *Nucl. Phys.* **B306** (1988) 137.
15. M. Dine, N. Seiberg and E. Witten, *Nucl. Phys.* **B289** (1987) 589.

16. E. Thorndike, CLEO Collaboration, talk given at the meeting of the APS, Washington, DC (1993).
17. G. 't Hooft, in *Recent Developments in Gauge Theories*, G. 't Hooft et. al, Eds., Plenum (New York) 1980.
18. R. Barbieri and G.F. Giudice, *Nucl. Phys.* **B306** (1988) 77; G.G. Ross and R.G. Roberts, *Nucl. Phys.* **B377** (1992) 571; B. de Carlos and J.A. Casas, CERN-TH. 6835/93.
19. L.I. Bigi and F. Gabbiani, *Nucl. Phys.* **B352** (1991) 309.
20. H. Georgi, Harvard preprint HUTP-92/A049; T. Ohl, G. Ricciardi and E.H. Simmons, Harvard preprint HUTP-92/A053.
21. J. Ellis, S. Ferrara and D.V. Nanopoulos, *Phys. Lett.* 114B (1982) 231; J. Polchinski and M. Wise, *Phys. Lett.* **125B** (1983) 393; W. Buchmuller and D. Wyler, *Phys. Lett.* **121B** (1983) 321.
22. A. Pomarol, *Phys.Rev.* **D47** (1992) 273 ; R. Garisto and G. Kane, TRIUMF preprint TRI-PP-93-1, (1993); L. Hall and S. Weinberg, Texas preprint UTTG-22-92.
23. A. Strominger and E. Witten, *Comm. Math. Phys.* **101** (1985) 341.

Table 1a. Bounds on various components of the squark mass matrix from $K^o - \bar{K}^o$ mixing. Here, $m_{\tilde{g}}$ is the gluino mass and $\tilde{m}$ is a typical first-second generation squark mass. All quantities in GeV^{-2}. We have taken $F_K = 170$ MeV and $B_K = 1$.

$\frac{m_{\tilde{g}}}{\tilde{m}}$	$\frac{\tilde{m}^4_{ds}}{\tilde{m}^6}$	$\frac{\tilde{m}^2_{ds}\tilde{m}^2_{\bar{d}\bar{s}}}{\tilde{m}^6}$	$\frac{\tilde{m}^4_{\bar{d}\bar{s}}}{\tilde{m}^6}$
1	2×10^{-9}	3×10^{-11}	10^{-10}
.1	4×10^{-10}	10^{-10}	2×10^{-11}

Table 1b. Bounds on various components of the squark mass matrix from $B^o - \bar{B}^o$ mixing. We have taken $F_B = 230$ MeV and $B_B = 1$.

$\frac{m_{\tilde{g}}}{\tilde{m}}$	$\frac{\tilde{m}^4_{db}}{\tilde{m}^6}$	$\frac{\tilde{m}^2_{db}\tilde{m}^2_{\bar{d}\bar{b}}}{\tilde{m}^6}$	$\frac{\tilde{m}^4_{\bar{d}\bar{b}}}{\tilde{m}^6}$
1	10^{-8}	10^{-9}	2×10^{-9}
.1	2×10^{-9}	2×10^{-9}	7×10^{-10}

Table 2a. Predictions of the $SU(2)_H$ model for $K^o - \bar{K}^o$ mixing for representative values of squark masses. The dimensionless parameters γ, η, *etc.*, in this and the following table are as defined in the text. All quantities in GeV^{-2}.

$\tilde{m}$	$\frac{\tilde{m}^4_{ds}}{\tilde{m}^6}$	$\frac{\tilde{m}^2_{ds}\tilde{m}^2_{\bar{d}\bar{s}}}{\tilde{m}^6}$	$\frac{\tilde{m}^4_{d\bar{s}}}{\tilde{m}^6}$
300 GeV	$10^{-9}\gamma'^2$	$10^{-9}\gamma'^2$	$3 \times 10^{-12}\eta'^2$
1 TeV	$10^{-10}\gamma'^2$	$10^{-10}\gamma'^2$	$3 \times 10^{-14}\eta'^2$

Table 2b. Predictions of the $SU(2)_H$ model for $B^o - \bar{B}^o$ mixing for representative values of squark masses.

$\tilde{m}$	$\frac{\tilde{m}^4_{db}}{\tilde{m}^6}$	$\frac{\tilde{m}^2_{db}\tilde{m}^2_{\bar{d}\bar{b}}}{\tilde{m}^6}$	$\frac{\tilde{m}^4_{d\bar{b}}}{\tilde{m}^6}$
300 GeV	$10^{-7}\gamma^2$	$10^{-7}\gamma^2$	$3 \times 10^{-12}\eta^2$
1 TeV	$10^{-8}\gamma^2$	$10^{-8}\gamma^2$	$3 \times 10^{-14}\eta^2$