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Proposal for a Feasibility Study of Very Large Aperture Quadrupole Magnets[1]

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Abstract

We seek expert assistance in performing a feasibility study for quadrupole magnets with inner radius at least 80 cm, field at that radius of at least 1.5 Tesla, and field integral at that radius of at least 3 Tesla-m. These magnets would be used as the spectrometer magnets in hadron-collider experiments that emphasize relatively low transverse momentum and large angular coverage, such as studies of B physics and of a variety of physics topics in the far-forward direction. The magnets would preferably be superconducting, but conventional construction may be considered as well. Should the magnets be deemed feasible, we then seek to initiate an R&D program to construct a prototype.

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Contents

List of Figures

List of Tables

1 Introduction

Quadrupole magnets have seldom, if ever, been used as spectrometer magnets in open-geometry high-energy-physics experiments. However, they have features that are appealing when good momentum analysis is required for particles both at large and small angles in a colliding-beam experiment. Most importantly, the analysis performed by a higher-multipole-magnet spectrometer in one region of phase space does not compromise the quality of analysis in other regions.

The excellent capability of solenoid-magnet spectrometers does not readily extend to particles with angles of less than $\sim 30°$ to the beams, as the cost of a solenoid magnet varies as $1/\tan\theta_{\rm min}$. Small-angle particles are better analyzed with a transverse magnetic field, commonly that of a dipole magnet. However, the interaction of particles with the beampipe in a colliding-beam experiment is aggravated by any transverse magnetic field on the beampipe, as discussed in more detail in Sec. 2. Because the transverse field of an nth-order-pole magnet varies as r^{n-1}, higher-multipole magnets such as quadrupoles are favored. If a relatively large-diameter beampipe is required, a sextupole magnet may be preferred.

Higher-multipole magnets have an additional benefit in experiments that utilize a secondary-vertex detector close to the beam. If the magnetic field on the vertex detector is weak the tracks will be nearly straight, and an on-line vertex trigger will be much easier to implement.

In Section 4 we sketch a detector for low-P_t hadronic physics at the SSC that uses quadrupole spectrometer magnets. A guide to the relevant scale of magnet parameters is summarized in Table 1.

Table 1: Parameters of quadrupole spectrometer magnets of a low-P_t-physics experiment for the SSC. R is the inner radius of the coils, L is the length of the magnet, B_0 is the magnetic field strength at radius R, and $P_0 = eB_0L/c$ is the momentum kick at radius R (except for the central quadrupole where the half length of the magnet is used in calculating the kick).

Type	R (m)	L (m)	B_0 (Tesla)	P_0 (GeV/c)	Stored Energy (10^6 Joule)	Iron (tons)
Forward	0.8	2	1.5	0.9	3.6	350
Forward	1.0	2	1.5	0.9	5.6	240
Central	1.6	6.8	3	3.1	180	1300

If a quadrupole-magnet spectrometer is to play a role at the SSC it is timely that its feasibility be evaluated, and a prototype R&D program be undertaken on the basis of a favorable evaluation. This effort requires participation of experienced magnet designers, whom we hope to interest with this proposal.

2 The Need for Quadrupole Spectrometer Magnets

The case for quadrupole-magnet spectrometers becomes most compelling when considering low-transverse-momentum hadronic physics at high center-of-mass energies [1]. The analysis of small-angle tracks after they have passed through the collider beampipe is compromised in detector configurations based on dipole spectrometer magnets. We discuss the interplay between beampipe design and optimum magnetic-field configuration in this Section.

2.1 The Beampipe Problem

Particles are produced in p-p collisions with typical transverse momentum $P_t \approx 400$ MeV/c and angular distribution

$$dN \propto \frac{d\theta}{\theta}.$$

The smallest angle to the beam of interest at the SSC is (400 MeV)/(20 TeV) $= 2 \times 10^{-5}$. To discuss the wide range of angles in a succinct manner it is customary to introduce the logarithmic angular variable called the pseudorapidity:

$$\eta = -\ln\tan\theta/2 \qquad \text{for which} \qquad d\eta = \frac{d\theta}{\sin\theta} \approx \frac{d\theta}{\theta}.$$

At the SSC, then, particles are produced nearly uniformly in the variable η for

$$|\eta| \lesssim 12 \qquad \text{(all hadronic interactions)}.$$

A variety of processes of interest at the SSC, such as cosmic-ray exotica, hard diffraction, and other large-s, small-t hard processes such as dilepton, dijet and direct-photon production populate the region

$$6 \lesssim |\eta| \lesssim 9 \qquad \text{(diffraction and other small-}t\text{ processes)}$$

Decays of b quarks at the SSC lead to final-state particles with

$$|\eta| \lesssim 7 \qquad (b\text{-quark decays}).$$

Processes that occur with higher transverse momentum lead to particles at larger angles to the beam, and hence distributed over a narrower range in η. Thus W and Z production extends out to

$$|\eta| \lesssim 5 \qquad (W \text{ and } Z \text{ production}).$$

while t-quark and Higgs-boson decays lead to relatively narrow angular distributions centered at 90° to the beams of

$$|\eta| \lesssim 3 \qquad (t\text{-quark and Higgs-boson decays}).$$

In a colliding-beam experiment the particles are generally analyzed only after they have passed through the beampipe. To minimize the interactions in the beampipe we first consider a thin Be pipe, say 400 μm thick. This is approximately 0.001 of a radiation length and

0.0007 of a pion interaction length. But for tracks at angles of less than 0.001 radian the pipe appears to be more than one radiation and interaction length thick. If we accept no more than a 10% interaction probability then the thin, straight pipe could not be used for angles less than 0.01, corresponding to good particle analysis only for

$$|\eta| < 5 \qquad (< 10\% \text{ interaction in thin, straight pipe}).$$

This 'beampipe problem' does not exist for t-quark and Higgs-boson physics, but is encountered near the edge of useful phase space for b-quark physics, and is very prominent in low-P_t hadronic physics.

2.2 Effect of a Magnetic Field on a Flared Beampipe

Flared beampipes are often proposed as a solution to the beampipe problem. But parallax effects due a finite-length luminous region at a collider render flares of little use for larger angles, *i.e.*, $|\eta| < 5$ [2]. For $|\eta| > 5$ flares will be quite effective at minimizing interactions in the beampipe – if spectrometer magnetic fields do not bend particles into the flare.

Suppose particles encounter a transverse magnetic field with 'kick' ΔP_t prior to a flare. The particle trajectories are deflected by angle

$$\Delta\theta = \frac{\Delta P_t}{P} \approx \frac{\Delta P_t}{P_t}\theta,$$

where θ is the production angle of the track. Because of the deflection, charged tracks produced within $\pm\Delta\theta$ of the cone angle θ_0 of the flare can intersect the flare as shown in Fig. 1, even if the luminous region had zero length.

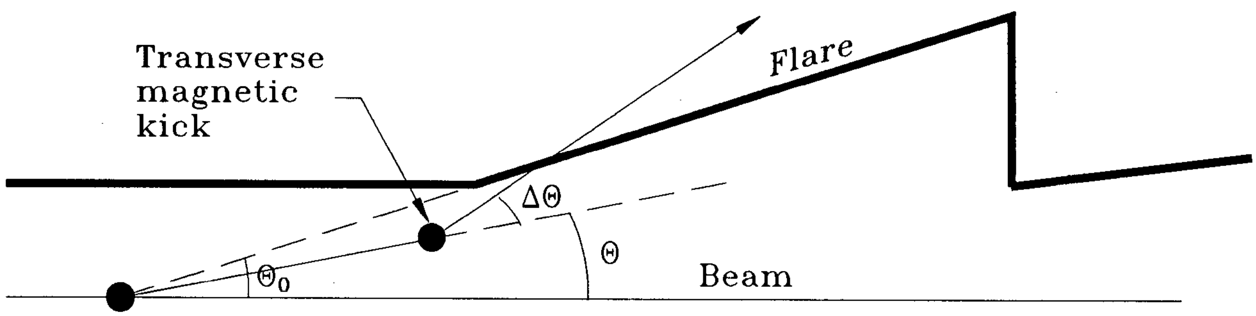

Figure 1: Illustration of how a transverse magnetic field prior to a flare in the beampipe can kick particles into the wall of the flare.

The extent of the background-prone region, $[\theta_0 - \Delta\theta, \theta_0 + \Delta\theta]$, depends on the length of the flare and the location of the magnetic kick. Clearly longer flares are more prone to background. If the transverse magnetic kick is located near the production point or at the downstream end of the flare there is little ill effect. The worst case is when the kick is located midway between the production point and the far end of the flare.

In many detector designs the magnet kick at least 400 MeV/c, the average transverse momentum of the particles. In this case,

$$\Delta\theta \approx \theta,$$

and so the rapidity interval over which tracks can be deflected into the flare is

$$\Delta\eta = \ln(\theta_0 + \Delta\theta) - \ln(\theta_0 - \Delta\theta) \gg 1.$$

In this case a flared pipe causes backgrounds over a larger range of rapidity and is little better than a straight pipe.

This observation leads to the dictum that a collider detector for forward physics have magnetic fields on the beampipe only if their transverse kick (before particles pass through the beampipe) is much less than 400 MeV/c.

In particular, this excludes use of dipole magnets upstream of flared beampipes. The remaining options are solenoids, toroids, and higher-multipole magnets. As noted in the introduction, at small angles solenoids are not cost effective for momentum analysis. Toroids require a large mass of conductor and mechanical structure at their inner radius, near the beam axis, and therefore appear to us to be an awkward solution. Hence quadrupoles and higher-multipole magnets seem to be the best spectrometer magnets in an experiment whose coverage extends to $|\eta| > 5$.

3 Momentum Analysis with Quadrupole Magnets

In this Section we present some rules of thumb for the accuracy of momentum analysis in a quadrupole magnet spectrometer, based on calculation of the sagitta of particle trajectories. This will guide us in our discussion of useful configurations of spectrometer magnets in the following Section.

First we remark on the task of pattern recognition and tracking in a quadrupole spectrometer. In dipoles and solenoids a particle trajectory is bent in only one projection, and appears straight in the other. The 'nonbend' plane is often used to simplify track finding, and linking of track segments before and after the magnetic deflection. In a quadrupole a particle trajectory is deflected in both projections, with the magnitude of the deflection independent of azimuth but with the direction of the deflection a known function of the azimuth. Thus there remains a consistency check on the tracking in two projections, albeit of a slightly different form than in dipole/solenoid fields.

In addition, at small angles there is the simplifying feature that the motion in the x-projection decouples from that in the y-projection. However, at large angles ($|\eta| \lesssim$ 1-2) the particle motion is more complicated.

3.1 Forward Quadrupole

We first consider the case of a quadrupole magnet at some distance from the interaction point, so that tracks, while passing through the magnet, have nearly constant distance r from the beam. For simplicity we suppose that the tracking detectors related to this magnet extend to distance $2z$ from the interaction point at $z = 0$, where z is the distance to the magnet. Then if the track is deflected by angle $\Delta\theta$ in the magnet, the sagitta S of the trajectory as observed over distance $2z$ is

$$S \approx \frac{z\Delta\theta}{2},$$

as shown in Fig. 2

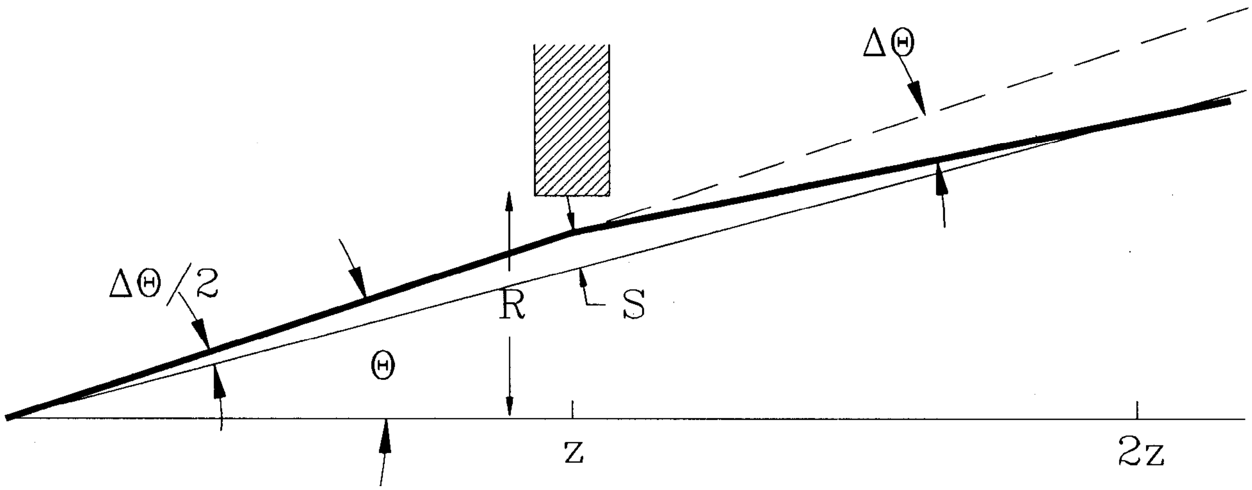

Figure 2: Illustration of the sagitta of the trajectory of a small-angle particle that passes through a forward quadrupole magnet as distance z for the interaction region. The tracking detectors associated with the quadrupole spectrometer extend out to $2z$.

A track at distance r from the beams passing through a quadrupole with inner radius R of the coils experiences a 'kick' of

$$\Delta P = \frac{r}{R} P_0, \qquad \text{defining} \qquad P_0 \equiv \frac{eB_0L}{c},$$

where B_0 is the field at radius R, and L is the length of the quadrupole. The deflection angle due to the kick is

$$\Delta\theta \approx \frac{\Delta P}{P} = \frac{\Delta P}{P_t}\theta,$$

where θ is the production angle of the particle with respect to the beam. Then the sagitta is

$$S \approx \frac{1}{2}\frac{P_0}{P_t}\left(\frac{\theta}{\theta_0}\right)^2 R,$$

where $\theta_0 \approx R/z$ is the angle to the coil of the quadrupole.

Of course, the accuracy of the momentum analysis depends on the accuracy of the measurement of the sagitta:

$$\frac{\sigma_P}{P} = \frac{\sigma_{P_t}}{P_t} = \frac{\sigma_S}{S} = 2\frac{\sigma_S}{R}\frac{P_t}{P_0}\left(\frac{\theta_0}{\theta}\right)^2.$$

A tracking system based on gaseous drift chambers might provide $\sigma_S \approx 200\ \mu$m. A reasonable kick in the quadrupole is $P_0 = 1$ GeV/c (3.3 Tesla-m). Thus if we desire momentum resolution of, say, 0.1% for a track with $P_t = 1$ GeV/c just grazing the inner edge of the coil ($\theta = \theta_0$), we need a magnet of radius $R = 40$ cm.

The momentum resolution varies as $1/\theta^2$, so a single quadrupole usefully covers only a limited range of angles, from some $\theta_{\min}$ up to θ_0. For a given resolution at $\theta_{\min}$ the ratio $\theta_0/\theta_{\min}$ does not depend on θ_0, *i.e.*, on the location of the magnet, supposing the radius R and kick P_0 of the magnet are held fixed. Thus the magnet provides useful momentum analysis over a fixed rapidity interval

$$\Delta\eta = \ln(\theta_0/\theta_{\min}),$$

independent of the location of the magnet.

The coverage $\Delta\eta$ per magnet should not be too small or a large number of magnets will be required. We feel that $\Delta\eta$ should be at least 1, corresponding to $\theta_{\min} = \theta_0/e$, and a momentum resolution at $\theta_{\min}$ that is $e^2 = 7.4$ times worse than that at θ_0. The worst momentum resolution is then

$$\left.\frac{\sigma_P}{P}\right|_{\max} = 15\frac{\sigma_S}{R}\frac{P_t}{P_0}.$$

To fix the design parameters, some criterion is needed as to the worst acceptable momentum resolution. Here we take that criterion from B-physics, for which it is highly desirable that the B-meson mass resolution be small compared to the mass of a pion. This would provide rejection against decays in which one pion goes undetected. It also would permit clear separation of the B_s meson from the B_u and B_d mesons, whose mass difference is roughly one pion mass. Thus we seek $\sigma_M \approx 25$ MeV/c^2 at $M \approx 5$ GeV/c^2, or $\sigma_M/M \approx 0.5\%$. But $\sigma_M/M \approx \sqrt{2}\sigma_P/P$ for a two-body decay, so we desire $\sigma_P/P \lesssim 0.35\%$. Furthermore, this resolution is desired for tracks of $P_t \approx 1$ GeV/c, as is typical of secondary particles from B-meson decay.

For coverage of $\Delta\eta = 1$ per quadrupole, this criterion leads to a desired resolution of about $0.35/7.4 = 0.05\%$ for tracks grazing the inner edge of the magnet coil. Thus in our example of a magnet with kick of 1 GeV/c we would need a coil radius of 80 cm. This is the value used in Sec. 4 where we sketch a detector based on quadrupole magnets.

We have not considered how close to the beams the tracking detectors of the spectrometer can safely be. The issue is partly the higher occupancy closer to the beams, and partly the higher radiation damage there. Gaseous tracking detectors may be of limited use at luminosity 10^{32} cm^{-2}sec^{-1} for radii less than 10 cm. Then if 80 cm is the inner radius of a forward quadrupole magnet the maximum angular coverage in a single magnet is a factor of 8, corresponding to $\Delta\eta = 2$. Of course, the kick of such a magnet would have to be e times as large as one designed to cover only $\Delta\eta = 1$ if the same momentum resolution is to be maintained at the smallest angles within their respective coverages.

3.2 Central Quadrupole

Momentum analysis is also important at large angles both for low-P_t hadron physics and for b-quark physics, so we consider the capability of a quadrupole magnet in the central region as well. Typically, 'central' means $|\eta| \lesssim 1.5$, corresponding to angles larger than 25-30°.

In the central region the momentum analysis must be accomplished by tracking chambers that lie entirely within the magnetic volume (since the magnet coil and return yoke surround this volume). The trajectories inside a quadrupole field have displacements that are cubic in path length (compared with quadratic dependence of helical trajectories in a dipole), which leads to the following expression for the sagitta [3]:

$$S = \frac{\sqrt{3}}{27}\frac{P_0}{P_t}R\sqrt{\cos^2\theta + \sin^2\theta\cos^2 2\phi}\;\frac{\tan\theta_0}{\sin\theta} \times \begin{cases} 1 & \text{if track reaches radius} R, \\ \frac{\tan^3\theta}{\tan^3\theta_0} & \text{if track exits at } z = \pm L, \end{cases}$$

In this, the central quadrupole covers the interval $(-L, L)$ along the beam, and $\tan\theta_0 = R/L$. We define $P_0 = eB_0L/c$ as before.[2] The azimuthal factor $\sqrt{\cos^2\theta + \sin^2\theta\cos^2 2\phi}$ arises because there are four directions ϕ in a central quadrupole for which tracks emanating at $\theta = 90°$ from the origin experience no transverse magnetic field.[3]

Comparing with the expression for the sagitta in a forward quadrupole spectrometer, we see that a central quadrupole yields a sagitta only about 1/8 as large for a magnet with the same radius R and kick P_0. To obtain the same momentum accuracy in a central quadrupole spectrometer as in a forward one will require, say, a magnet of twice the radius, and twice the field at the coil. The stored energy in the central quadrupole would then be 32 times larger, noting that its length is $2L$. Of course, any central magnet that delivers good momentum resolution is a sizable object.

4 Example Detectors Based on Quadrupole Magnets

Using the results of Sec. 3 we now consider the design of a low-P_t spectrometer based on a series of quadrupole magnets. At large η (small angles) the magnets are far apart in space and therefore relatively simple to configure. In the central region the magnets must be close to one another but not, of course, overlapping. Hence the design naturally proceeds from the central region outwards to insure mechanical compatibility of the various elements.

Another consideration is that the central magnet should be larger than the forward magnets, as discussed in Sec. 3. The central magnet may well be so costly that it cannot be afforded in the first phase of the experiment. So the design should still be compatible with good physics capability without the central magnet, while reserving space for it in a second phase of the experiment.

Our design uses a central quadrupole that covers $|\eta| < 2.5$, followed by several forward quadrupoles each covering $\Delta\eta = 1$. In the central quadrupole, particles with $|\eta| < 1.5$ will reach the coil radius R, but tracks with $1.5. < |\eta| < 2.5$ leave through the end surface at $z = L$. If the central magnet has coil radius $R = 1.6$ m, as suggested in Sec. 3.2, then it half length L is 3.4 m, as shown in Fig. 3.

[2] Note that $P_0 R\tan\theta_0 = eB_0R^2/c$, which may be a more familiar form for central magnets.

[3] We believe the above expression for the sagitta holds for an nth-order-pole magnet of circular cross section with 2ϕ replaced by $n\phi$, and the coefficient $\sqrt{3}/27$ replaced by $1/(n+1)^{2+1/n}$. The total solid angle over which the sagitta is reduced by any given amount due to the azimuthal factor is independent of the order of the multipole.

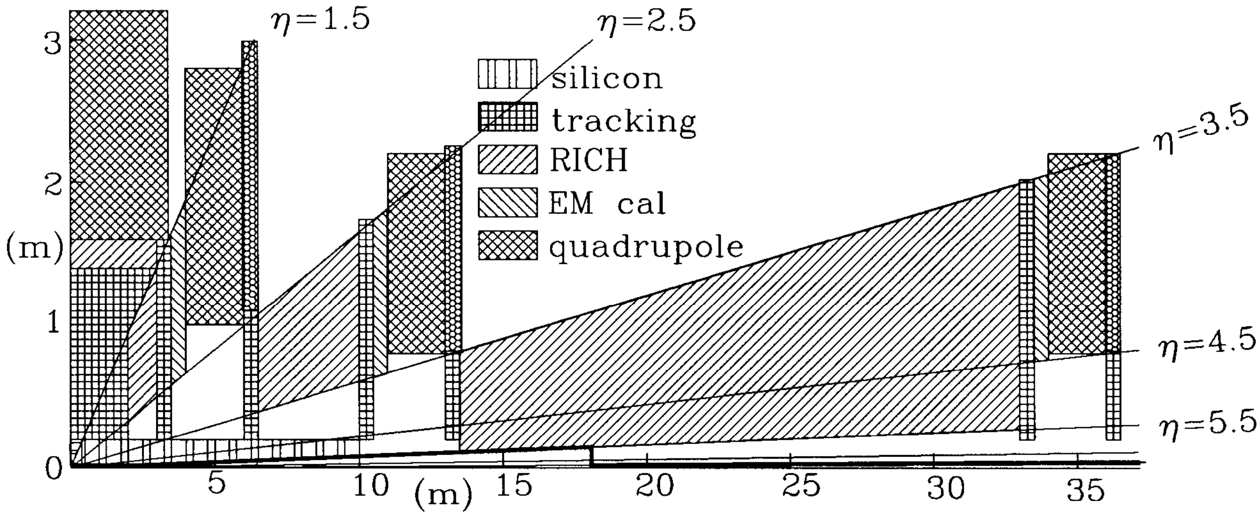

Figure 3: Quarter section of an SSC spectrometer based on quadrupole magnets. The vertical scale is five times the horizontal. Only the inner 35 meters of the spectrometer are shown here.

Guided by Sec. 3.1 the forward quadrupole magnets are expected to be about 2 m long with 0.8 m inner radius, and 1.5 Tesla field at that radius. However, if such a magnet is used to cover the pseudorapidity interval $2.5 < \eta < 3.5$ it would extend from $z = 2.8$ to 4.8 m, and be partly inside the central quadrupole. It appears more reasonable that this magnet have an inner radius of 1 m, and be located between $z = 4$ and 6 m, as shown in Fig. 3.

The subsequent forward quadrupoles then fit well assuming their inner radii are 0.8m, as shown in Fig. 3.

At the SSC there is about ± 100 m free space available downstream from the collision point at an interaction region where the luminosity is restricted to 10^{32} cm^{-2}sec^{-1}. Beyond 30-40 m it is appropriate to place one more magnetic stage. This might again be a quadrupole, although a dipole (or one of each) is a possible option as well.

Figure 4 shows the sagitta of tracks with 1 GeV/c transverse momentum as would be observed as a function of η in the spectrometer of Fig. 3. In the central region ($\eta \lesssim 1$) there is a loss of resolution at some azimuths as discussed in sec. 3.2. In each forward quadrupole spectrometer the sagitta varies as $e^{-2(\eta-\eta_0)}$. If a dipole magnet were placed at 70 m from the collision point to cover the far-forward region, the sagitta would be as shown by the dashed curve, which varies as $e^{-(\eta-\eta_0)}$.

A detailed beampipe design involves issues beyond the scope of this note. For this example we consider a beampipe that is straight in the central region, with flares at angles corresponding to pseudorapidities of $\eta = 5.5$ and 7.5. These flares end at 18 and 100 m, respectively, from the interaction point. The maximum radius of the flares never exceeds 20 cm.

A scenario for a first phase of the experiment is shown in Fig. 5 in which the large central quadrupole is absent. However a new quadrupole, identical to that covering $2.5 < \eta < 3.5$, is used to cover $1.5 < \eta < 2.5$ to nearly nominal momentum resolution. This quadrupole

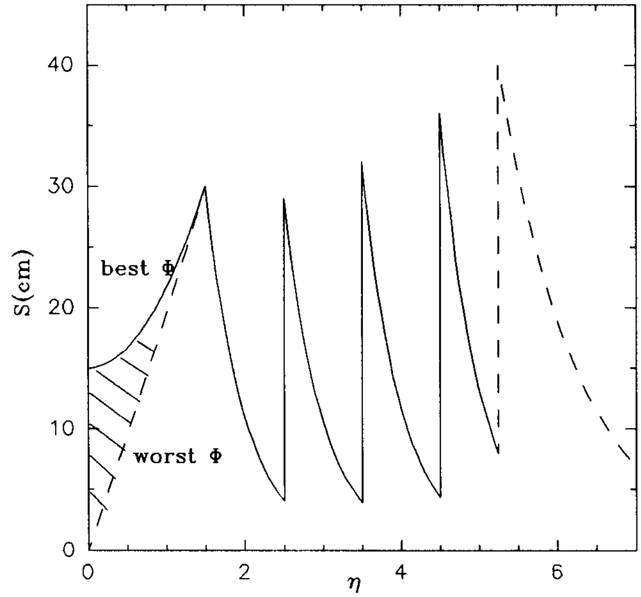

Figure 4: The sagitta as a function of η of charged tracks with $P_t = 1$ GeV/c that would be observed in the spectrometer of Fig. 3. A dipole magnet is assumed to cover the far-forward region.

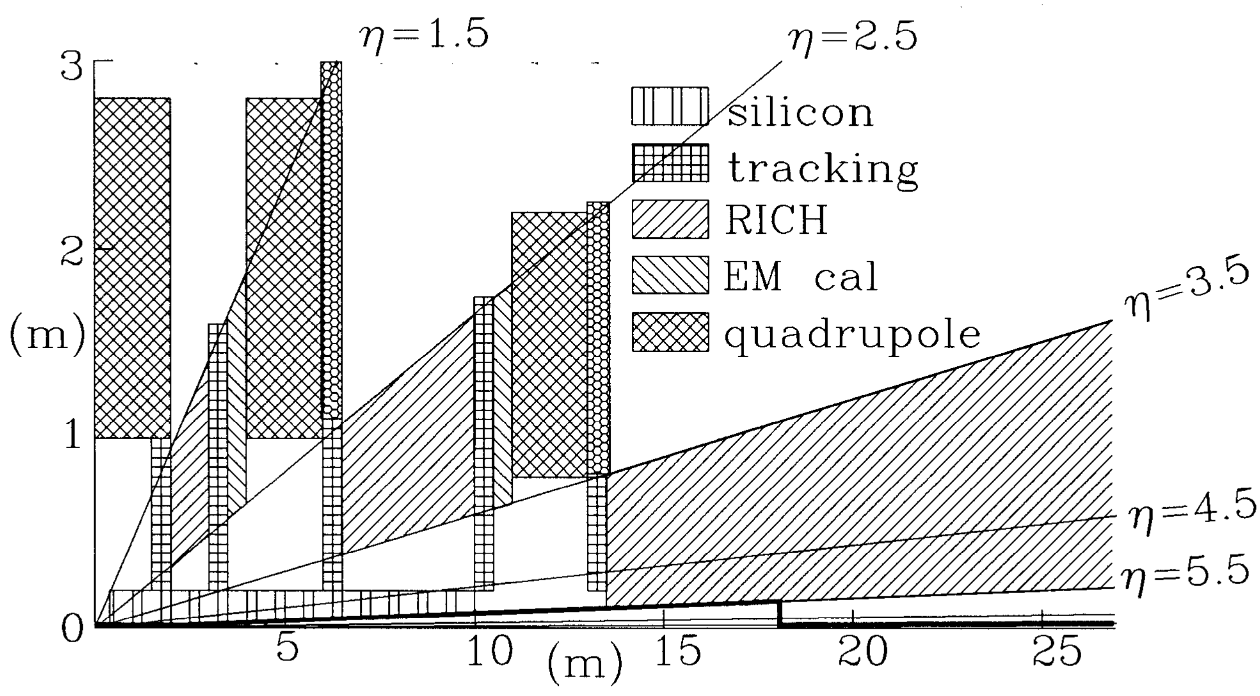

Figure 5: Quarter section of the inner 25 m of a possible first phase of an SSC spectrometer based on quadrupole magnets.

actually covers $|\eta| < 1.5$ as well, but offers momentum resolution about 5 times worse than before. The sagitta of 1-GeV/c-transverse-momentum tracks in the phase-1 scenario is sketched in Fig. 6.

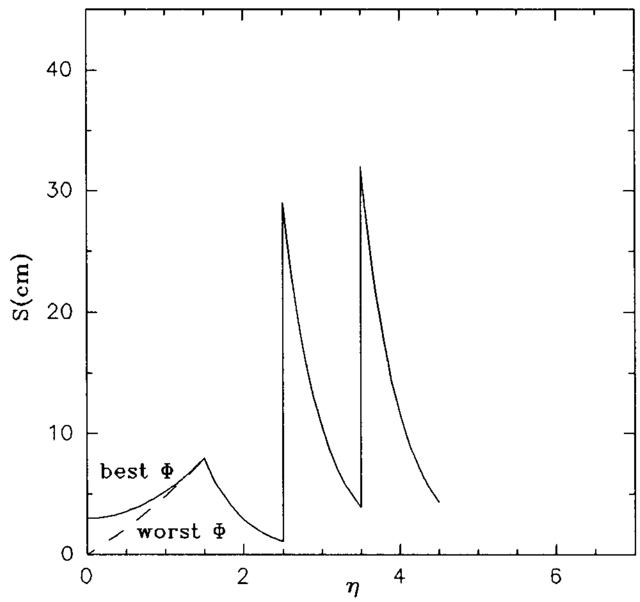

Figure 6: The sagitta of charged tracks with $P_t = 1$ GeV/c that would be observed in the spectrometer of Fig. 5.

Another option, intermediate with respect to these cases, would be to use a central quadrupole with the larger aperture and length (1.6 m and ± 3.4 m, respectively) but to reduce the peak field from 3 T to 1.5 T. The degradation of resolution in that case is only a factor of 2.

5 Open Questions

Beyond the pre-eminent issue of whether large-aperture quadrupoles can be built, the next question is what is the maximum reasonable peak field. In Secs. 3 and 4 we tried to consider only magnets with 1.5-Tesla peak field, but found that this is marginal at best for the central quadrupole.

An essential design question is the relative feasibility and cost of the various candidate designs of such magnets. The options include a scaled-up version of $\cos 2\phi$ superconducting accelerator quadrupoles, superferric quadrupoles with conventional geometry of poles and return-yoke, and large "conventional" iron and copper structures.

It might well be feasible to build the forward quadrupoles, but not the much larger central quadrupole. An alternative configuration of the spectrometer with a central solenoid has been discussed in ref. [3] and elsewhere.

The proposed feasibility study will provide crucial information needed to converge on the basic design of a low-P_t-physics experiment at the SSC.

6 References

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