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Parke-Taylor amplitudes in the multi-Regge kinematics*

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ABSTRACT

The Parke-Taylor multigluon amplitudes are examined in the multi-Regge kinematics, which assumes strong rapidity ordering of the produced gluons, and are used to compute the n-gluon production cross section and the gluon-gluon total cross section.

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In hadron scattering processes with large momentum transfer Q^2, of the order of the center of mass energy s, we evaluate the physical quantities of interest by performing perturbative QCD calculations of parton cross sections as series expansions in the strong coupling constant α_s. Since the calculation of the coefficients in the series becomes quickly complicated as one goes to higher orders in the expansion, only very few coefficients are usually computed, that is calculations are performed at a fixed (and small) order in α_s.

At the Tevatron, LHC and SSC hadron colliders a new kinematical region, the semihard region, characterized by scattering processes with $s >> Q^2 >> \Lambda^2_{QCD}$, becomes important. In this region the momentum transfer is large enough to allow perturbative QCD calculations, but so much smaller than the center of mass energy that processes with production of a large number of partons become relevant. In the series expansion of the parton cross section each coefficient contains the logarithm of a large ratio of kinematical invariants, of the order of $\ln(s/Q^2)$, and the effective expansion parameter becomes the product $\alpha_s \ln(s/Q^2)$, which may be $O(1)$. Thus in the series expansion we have to retain many higher orders, which open up several real channels and are the cause of the abundant production of partons. To keep all this into account properly it is useful to have techniques that resum all the orders in the effective expansion parameter. To this end it is necessary to have analytical, albeit approximate, expressions for multiparton amplitudes.

In the semihard region, the leading contribution to scattering processes always comes from the exchange of a particle of highest spin, in our case a gluon, in the crossed channel[1] . In the case of multiple gluon emission the rapidity interval between the scattered partons is filled with gluons. In the multi-Regge kinematics, which yields the leading logarithmic contribution to the cross section, the rapidities of the emitted gluons are strongly ordered. Fadin, Kuraev and Lipatov[2] computed long ago the multigluon amplitude in the multi-Regge kinematics. This amplitude contains all the virtual radiative corrections, whose effect is to reggeize the gluons exchanged in the crossed channel[2][3] . Balitsky, Fadin, Kuraev and Lipatov (BFKL)[4] computed then the total parton-parton cross section, by putting the multi-gluon amplitude in the multi-Regge phase space, integrating out the rapidities of the produced gluons, and reducing the dependence of the cross section on the gluon transverse momenta to the resolution of an integral equation. In this equation the infrared real and virtual divergences exactly cancel and thus the eigenvalues do not depend on the infrared cutoff. The total parton-parton cross section is found then to have a power growth with s, with the power depending on the eigenvalue of the integral equation. Via the optical theorem the total parton-parton cross section is related to the forward elastic parton-parton scattering with color-singlet exchange in the crossed channel, the perturbative QCD pomeron.

Another multigluon amplitude, the Parke-Taylor amplitude[5] , which is a tree-level multigluon amplitude in a helicity basis, with a particular choice for the gluon helicities, is available in the literature. It is not specific to a particular kinematical region. It has been used to make approximate calculations of the four- and five-jet production rates[6] , which have been found to be in good agreement with the data[7] .

In this paper we want to consider the Parke-Taylor multigluon amplitude, for the production of an arbitrary number of gluons, in the multi-Regge kinematics, study the color flows of the produced gluons on the Lego plot in azimuthal angle and rapidity, and compute the total gluon-gluon cross section. Since the virtual radiative corrections are missing in the Parke-Taylor amplitudes, we will have to cut off the infrared real divergences and we expect the slope of the pomeron trajectory to depend on the infrared cutoff.

We will find that introducing an infrared cutoff is not enough, and we also have to regulate the behavior of the amplitudes in the ultraviolet, to avoid the rise of an unphysical singularity in the total cross section. The same happens also in the BFKL multigluon amplitude if we neglect the contribution of the virtual radiative corrections. Thus a multigluon amplitude without virtual radiative corrections seems inherently ill-suited for the calculation of a fully inclusive quantity, like the total cross section.

Parke-Taylor amplitudes in the multi-Regge kinematics

A tree-level multigluon amplitude can be written in an SU(N_c) Yang-Mills theory as

$$\mathcal{M}_n = \sum_{[1,2,...,n]'} tr(\lambda^{a_1}\lambda^{a_2}\cdots\lambda^{a_n})\, m(p_1,\epsilon_1;p_2,\epsilon_2;...;p_n,\epsilon_n), \tag{1}$$

where $a_1, a_2, ..., a_n$, $p_1, p_2, ..., p_n$, and $\epsilon_1, \epsilon_2, ..., \epsilon_n$ are respectively the colors, momenta and helicities of the gluons, λ's are the color matrices in the fundamental representation of SU(N_c) and the sum is over the noncyclic permutations of the set $[1, 2, ..., n]$. The gauge-invariant subamplitudes $m(p_1,\epsilon_1;p_2,\epsilon_2;...;p_n,\epsilon_n)$ enjoy several properties[8] , like incoherence to leading order in $1/N_c$

$$\sum_{colors} |\mathcal{M}_n|^2 = N_c^{n-2}(N_c^2-1) \sum_{[1,2,...,n]'} \left[|m(p_1,\epsilon_1;p_2,\epsilon_2;...;p_n,\epsilon_n)|^2 + O(N_c^{-2})\right], \tag{2}$$

and cyclical and reversal symmetry

$$\begin{aligned} m(p_1,\epsilon_1;p_2,\epsilon_2;...;p_n,\epsilon_n) &= m(p_2,\epsilon_2;...;p_n,\epsilon_n;p_1,\epsilon_1) \\ m(p_n,\epsilon_n;p_{n-1},\epsilon_{n-1};...;p_2,\epsilon_2;p_1,\epsilon_1) &= (-1)^n m(p_1,\epsilon_1;p_2,\epsilon_2;...;p_n,\epsilon_n). \end{aligned} \tag{3}$$

If we assume that all the gluons are outgoing, the subamplitude for the maximally helicity violating configuration $(-,-,+,\cdots,+)$ is given by[5]

$$m(p_1^-,p_2^-,p_3^+,...,p_n^+) = i g_s^{n-2} \frac{<12>^4}{<12><23>\cdots<n1>}. \tag{4}$$

where the spinor product is defined as $<pq> = \overline{\psi_-}(p)\psi_+(q)$, with $\psi_\pm(p) = \frac{1}{2}(\mathrm{I}\pm\gamma_5)\psi(p)$. $\psi(p)$ is a massless Dirac spinor, normalized in such a way that $|<pq>|^2 = 2(p\cdot q)$.

By replacing the Parke-Taylor subamplitudes (4) into (1) and using the incoherence to leading order in the number of colors (2), we obtain the square of the multigluon Parke-Taylor amplitudes, summed over colors and the maximally helicity violating configurations, to leading order in $1/N_c$. From this we straightforwardly derive the n-gluon production squared Parke-Taylor amplitude (fig. 1), averaged over colors and helicities of the incoming gluons

$$|\mathcal{M}(p_A,p_0,p_1,...,p_{n+1},p_B)|^2 = 2\frac{1}{4(N_c^2-1)}(g_s^2)^{n+2}N_c^{n+2}$$
$$\sum_{i>j}s_{ij}^4 \sum_{[A,0,1,...,n+1,B]'}\frac{1}{s_{A0}s_{01}\cdots s_{n,n+1}s_{n+1,B}s_{AB}}, \tag{5}$$

where we label the incoming gluon momenta as p_A and p_B, and $i,j = A,0,...,n+1,B$. The overall factor 2 at the beginning of the right hand side of (5) is present only in the inelastic case $n \neq 0$, and keeps into account the different maximally helicity violating configurations (-,-,+,$\cdots$,+) and (+,+,-,$\cdots$,-).

We parametrize the momenta of the produced gluons in terms of the rapidity η, and the momentum $p_\perp$ and the azimuthal angle ϕ in the plane transverse to the beam axis. Then the kinematical invariants are given by

$$\begin{aligned} s_{Ai} &= \sqrt{s}\,|\vec{p}_{i,\perp}|\,e^{-\eta_i} \\ s_{Bi} &= \sqrt{s}\,|\vec{p}_{i,\perp}|\,e^{\eta_i} \\ s_{ij} &= 2\,|\vec{p}_{i,\perp}|\,|\vec{p}_{j,\perp}|\,[cosh(\eta_i-\eta_j)-cos(\phi_i-\phi_j)], \end{aligned} \tag{6}$$

where $s = s_{AB}$ is the center of mass energy of the scattering process and $i,j = 0,...,n+1$. Now we want to specify the n-gluon production squared Parke-Taylor amplitude to the semihard regime, where $s >> m^2 >> \Lambda_{QCD}^2$, and $\vec{p}_\perp^{\,2} \simeq m^2$ is the characteristic value of the transverse momentum of the produced gluons. To pick up the leading contribution in $ln(s/m^2)$ we consider the multi-Regge kinematics, where the gluon rapidities are strongly ordered

$$\eta_A \simeq \eta_0 >> \eta_1 >> \cdots >> \eta_{n+1} \simeq \eta_B. \tag{7}$$

In this kinematics the sum over helicities becomes

$$\sum_{i>j} s_{ij}^4 = 4\, s^4 \left(1 + O(s^{-1})\right), \tag{8}$$

with $i, j = A, 0, ..., n+1, B$. If we assume that s-channel helicity conservation for the incoming gluons holds, which is the case in multi-Regge kinematics[2,3], then we have 2^{n+2} possible helicity configurations in the n-gluon production amplitude. Indeed, fixed the helicity configuration for the incoming gluons, s-channel helicity conservation allows for 2^n different helicity configurations for the $n+2$ outgoing gluons. Then the sum over the four helicity configurations for the incoming gluons gives the figure quoted above. Thus from (8) together with the extra factor 2 in (5), for $n \neq 0$, due to the different maximally helicity violating configurations (-,-,+,$\cdots$,+) and (+,+,-,$\cdots$,-), we see that the Parke-Taylor amplitudes count correctly the number of helicity configurations for the elastic case $n = 0$ and for the 1-gluon production case.

To study the color ordering of the gluons we introduce the reduced squared amplitude

$$|M(p_A, p_0, p_1, ..., p_{n+1}, p_B)|^2 = s^4 \sum_{[A,0,1,...,n+1,B]'} \frac{1}{s_{A0}\, s_{01} \cdots s_{n,n+1}\, s_{n+1,B}\, s_{AB}}, \tag{9}$$

and the function

$$F_{ij} = \frac{e^{\eta_i - \eta_j}}{2[cosh(\eta_i - \eta_j) - cos(\phi_i - \phi_j)]}. \tag{10}$$

In the multi-Regge kinematics F_{ij} becomes

$$F_{ij} = \begin{cases} 1 + O\left(e^{-(\eta_i - \eta_j)}\right), & \text{if } \eta_i > \eta_j; \\ e^{2(\eta_i - \eta_j)}, & \text{if } \eta_i < \eta_j. \end{cases} \tag{11}$$

where we assumed that the rapidity interval between any two gluons is large enough that we can neglect the azimuthal-correlation term in (10). Thus in the

Parke-Taylor amplitudes in multi-Regge kinematics (7) the azimuthal correlation between the produced gluons is not a leading order effect. We notice, though, that in the BFKL multigluon amplitudes[2] there is azimuthal correlation between the produced gluons, due to the propagators of the gluons exchanged in the t channel. Only at the end of the day, in the solution of the BFKL integral equation, we do realize that the azimuthal correlation is a subleading effect.

In considering the sum over colors, we have to sum over all the non-cyclic permutations of the set [A, 0, 1,..., n+1, B] in (9). To do so, let us fix the position of gluon A in the set and move gluon B one step at a time to the left, and for each position of gluon B we consider all the permutations of the $n+2$ outgoing gluons. This will exhaust all the non-cyclic permutations of the set above. They are not all different, though, since for each color ordering in (9) there is the reversed ordering which, because of (3), yields the same contribution

$$\begin{aligned}[A,0,...,m,B,m+1,...,n+1] &= [n+1,...,m+1,B,m,...,0,A] \\ &= [A,n+1,...,m+1,B,m,...,0].\end{aligned} \tag{12}$$

To begin with, let us consider the color ordering [A, 0, 1,..., n+1, B], plus all the permutations of the outgoing gluons. By using the kinematical invariants of (6) and the function F_{ij}, the reduced squared amplitude (9) becomes

$$|M(p_A,p_0,p_1,...,p_{n+1},p_B)|^2 = \frac{s^2}{\prod_{i=0}^{n+1}\vec{p}_{i,\perp}^{\,2}} \sum_\sigma \prod_{i=0}^{n} F_{i,i+1}, \tag{13}$$

where $\sum_\sigma$ represents the permutations of the $n+2$ outgoing gluons in the color configuration $[A,0,1,...,n+1,B]$, while keeping fixed the incoming gluons A and B. In fig.2 we represent the color configuration [A, 0, 1,..., n+1, B] in terms of color lines in the fundamental representation of SU(N_c). Permuting the outgoing gluons in the color ordering [A, 0, 1,..., n+1, B], we see that the only permutation which respects the strong rapidity ordering (7) is the identity $\sigma(i) = i$, all the others giving a contribution that, because of (11), is $O(e^{-|\Delta\eta_{ij}|})$. Thus, to leading order in rapidity, the reduced squared amplitude (13) becomes

$$|M(p_A, p_0, p_1, ..., p_{n+1}, p_B)|^2 = \frac{s^2}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^2} [1 + O(e^{-|\Delta\eta_{ij}|})]. \tag{14}$$

Now let us take the color ordering [A, 0,..., j-1, j+1,..., n+1, B, j], where $j = 0, ..., n+1$ and we consider all the permutations of the $n+1$ gluons between gluons A and B. The squared amplitude is

$$|M(p_A, p_0, ..., p_{j-1}, p_{j+1}, ..., p_{n+1}, p_B, p_j)|^2 = \\ \frac{s^2}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^2} \sum_\sigma \sum_{j=0}^{n+1} F_{0,1} \cdots F_{j-2,j-1} F_{j-1,j+1} F_{j+1,j+2} \cdots F_{n,n+1} . \tag{15}$$

This corresponds to the configuration of fig.3(a), which we untwist in fig.3(b). The untwisted diagram can be conventionally thought of as a double-sided Lego plot in rapidity and azimuthal angle[9] . In this picture $\sum_\sigma$ in (15) represents the permutations of the $n+1$ gluons on the "front" of the Lego plot. We notice that the parameters of the gluon on the "back" of the Lego plot do not appear in (15). For each gluon that we bring to the back of the Lego plot, there is one permutation, the identity $\sigma(i) = i$, that gives a leading contribution to (15) and yields a strong rapidity ordering of the gluons on the front of the Lego plot. Any other permutation violates this rapidity ordering and is $O(e^{-|\Delta\eta_{ij}|})$. Then the leading contribution to (15) is

$$|M(p_A, p_0, ..., p_{j-1}, p_{j+1}, ..., p_{n+1}, p_B, p_j)|^2 = (n+2) \frac{s^2}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^2} [1 + O(e^{-|\Delta\eta_{ij}|})]. \tag{16}$$

If we move gluon B one more place to the left we have the color ordering [A, 0,..., j-1, j+1,..., k-1, k+1,..., n+1, B, k, j], where $j, k = 0, ..., n+1$, and we consider all the permutations of the gluons to the left of gluon B, independently from the ones to the right. The corresponding squared amplitude is

$$|M(p_A, p_0, \ldots, p_{j-1}, p_{j+1}, \ldots, p_{k-1}, p_{k+1}, \ldots, p_{n+1}, p_B, p_k, p_j)|^2 = \frac{s^2}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^{\,2}} \sum_{\sigma_F \sigma_B} \sum_{j<k} F_{0,1} \cdots F_{j-1,j+1} \cdots F_{k-1,k+1} \cdots F_{n,n+1}\, F_{j,k}, \tag{17}$$

with the configuration of fig.4(a), and its untwisted version fig.4(b), with two gluons on the back of the Lego plot. $\sum_{\sigma_F \sigma_B}$ represents the permutations of the gluons on the front and on the back of the Lego plot. In fig.4(b), for each two gluons that we bring to the back of the Lego plot there is one permutation of the gluons on the front, the identity $\sigma_F(i) = i$, that gives a leading contribution to the squared amplitude, and conversely for each set of n gluons on the front there is the identical permutation of the two gluons on the back $\sigma_B(i) = i$, that gives a leading contribution. So, in order to have the leading order in rapidity, we must take the identical permutation both on the front and the back of the Lego plot, which yields a strong rapidity ordering of the gluons on the two sides of the Lego plot. Since there are $\binom{n+2}{2}$ such configurations which respect the strong rapidity ordering of the gluons on the front and the back of the Lego plot, the leading contribution to (17) is

$$|M(p_A, p_0, \ldots, p_{j-1}, p_{j+1}, \ldots, p_{k-1}, p_{k+1}, \ldots, p_{n+1}, p_B, p_k, p_j)|^2 = \binom{n+2}{2} \frac{s^2}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^{\,2}} \left[1 + O(e^{-|\Delta\eta_{ij}|})\right]. \tag{18}$$

Then in general, given a color configuration to which corresponds an untwisted diagram that has m gluons on the back of the Lego plot, there are $\binom{n+2}{m}$ color configurations which respect the strong rapidity orderings of the gluons on the front and the back of the Lego plot and give a leading contribution to the squared amplitude. Then, to leading order in rapidity, the reduced squared amplitude (9) becomes

$$|M(p_A,p_0,p_1,...,p_{n+1},p_B)|^2 = 2^{n+2}\frac{s^2}{\prod_{i=0}^{n+1}\vec{p}_{i,\perp}^{\,2}}[1+O(e^{-|\Delta\eta_{ij}|})]. \tag{19}$$

From (5), (8) and (19), we can now write the n-gluon production squared Parke-Taylor amplitude, averaged over colors and helicities of the incoming gluons, to leading order in rapidity in the multi-Regge kinematics, as

$$|\mathcal{M}(p_A,p_0,...,p_{n+1},p_B)|^2 = 2\frac{(2g_s^2N_c)^{n+2}}{N_c^2-1}\frac{s^2}{\prod_{i=0}^{n+1}\vec{p}_{i,\perp}^{\,2}}[1+O(e^{-|\Delta\eta_{ij}|})]. \tag{20}$$

As mentioned after (5), the overall factor 2 at the beginning of the right hand side of (20) is missing in the elastic case $n=0$.

We finally notice two features of the multigluon amplitudes which do not depend on the particular kinematics chosen: *i*) in the Parke-Taylor amplitudes there is interaction only between gluons on the same side of the Lego plot; *ii*) the two sides of the Lego plot are indistinguishable. Property *i*) is hinted in (17); property *ii*) holds because of the reversal symmetry (3) and (12), which is not peculiar of the multigluon amplitudes (1) at the tree-level[10] and implies that for each color ordering with n_F gluons on the front and n_B gluons on the back of the Lego plot there is a color ordering with n_B gluons on the front and n_F gluons on the back which yields the same contribution to (9).

It is clear that also the BFKL amplitude must admit a distribution of the produced gluons on a double-sided Lego plot. We have not been able, though, to make such an identification.

Parke-Taylor gluon-gluon total cross section

We are now in the position to compute the n-gluon production and the gluon-gluon total cross sections from the Parke-Taylor amplitudes in the multi-Regge kinematics. In order to dispose of the infrared divergences we will cutoff the gluon transverse momenta $p_\perp$ at a characteristic scale m. This corresponds to the following experimental setting: in hadron-hadron scattering, we tag two jets at a large rapidity interval on the Lego plot and count all the accompanying jets produced in between, and we require that all the jets have transverse momentum larger than a cutoff m[11,12]. The phase space for the production of $n+2$ gluons in multi-Regge kinematics, where $s >> m^2$ and $\vec{p}_\perp^{\,2} \simeq m^2$ and the gluon rapidities are strongly ordered (7), can be written as

$$d\Pi_{n+2} = \frac{1}{2s}\left(\prod_{i=1}^{n}\frac{d\eta_i}{4\pi}\right)\left(\prod_{i=0}^{n+1}\frac{d^2p_{i,\perp}}{(2\pi)^2}\right)(2\pi)^2\,\delta^2(\sum_{i=0}^{n+1}p_{i,\perp}), \tag{21}$$

where we have used the conservation of energy and longitudinal momentum to fix the rapidities of the gluons at the extremes of the Lego plot.

The case where two gluons become collinear, and the related function F_{ij} (10) blows up, is not included in the multi-Regge kinematics. It may appear, though, as a contribution at the boundary of the integration in the phase space (21). To rule this out, we strictly enforce the strong rapidity ordering (7), i.e. we assume that any two gluons cannot get closer in rapidity than a fixed cutoff $\bar{\eta}$, defined in such a way that (11) is valid over the whole phase space and the azimuthal correlation is negligible everywhere. The exact value of $\bar{\eta}$ is actually unimportant, since in the leading logarithmic approximation the whole rapidity interval $\eta_A - \eta_B$ is defined up to an additive constant. In the following we will assume that a cutoff $\bar{\eta}$ has been introduced and we will neglect it. It is worth recalling though that the BFKL multigluon amplitudes do not have such a problem at the boundary of the phase space, since there is an explicit cancellation of the real and virtual infrared contributions in the BFKL integral equation.

Using (20) and (21), we can compute the n-gluon production cross section. For $n = 0$ we obtain the tree-level elastic cross section, i.e. the Born term for gluon-gluon scattering

$$\sigma_{elas} = \frac{8}{N_c^2 - 1} \frac{\pi \alpha_s^2 N_c^2}{2m^2}, \tag{22}$$

in agreement with ref.11. As we see from (21), in the n-gluon production cross section

$$\sigma_n = \frac{1}{2s} \int d\Pi_{n+2} |\mathcal{M}(p_A, p_0, \dots, p_{n+1}, p_B)|^2, \tag{23}$$

the integrals over transverse momentum are linked by the δ-function. We disentangle them using the integral representation in impact parameter b space of the δ-function, and obtain

$$\int \left(\prod_{i=0}^{n+1} \frac{d^2 p_{i,\perp}}{(2\pi)^2} \right) \frac{1}{\prod_{i=0}^{n+1} \vec{p}_{i,\perp}^{\,2}} (2\pi)^2 \, \delta^2 \Big(\sum_{i=0}^{n+1} p_{i,\perp} \Big) = 2\pi \int_0^\infty db \, b \left[\frac{K_0(bm)}{2\pi} \right]^{n+2}, \tag{24}$$

where we have introduced the modified Bessel function K_0, and the cutoff m to regulate the infrared behavior of the transverse momentum. Since $K_0(x)$ is exponentially decreasing for $x >> 1$ and increases only logarithmically for $x << 1$,

$$K_0(x) \simeq - \left(\gamma + \ln \frac{x}{2} \right), \tag{25}$$

with γ the Euler-Mascheroni constant, the integral over the impact parameter b is well defined. The transverse-momentum-conserving δ-function has provided two more powers of the momentum in the denominator of the left hand side of (24), and thus has suppressed the ultraviolet growth of the transverse momentum, as expected, since the ultraviolet divergences are an artifact of the loop corrections and do not appear at tree level.

We perform the integrals over the gluon rapidities, bound by the rapidity interval $\eta_0 - \eta_{n+1} \simeq \eta_A - \eta_B = \ln(s/m^2)$ between the gluons at the extremes of the Lego plot, using the strong ordering (7). Then, fixing $x = bm$ and $z = \frac{\alpha_s N_c}{\pi} \ln(s/m^2)$ and using (24), the n-gluon production cross section (23) becomes

$$\sigma_n = \frac{8}{N_c^2 - 1} \frac{2\pi\alpha_s^2 N_c^2}{m^2} \frac{z^n}{n!} \int_0^\infty dx\, x\, K_0^{n+2}(x). \tag{26}$$

It shows the growth in rapidity characteristic of the multi-Regge kinematics. Since the main contribution to the integral over the impact parameter comes from its lower end, we may use the approximation (25) for K_0. Then the integral over the impact parameter in (26) has a factorial growth which approximately compensates the factorial in the denominator, due to the integration in rapidity. Thus the series (26) becomes geometrical. Using (22) and (26), we can write the total cross section for gluon-gluon scattering as

$$\sigma_{tot} = \frac{8}{N_c^2 - 1} \frac{2\pi\alpha_s^2 N_c^2}{m^2} \left(\int_0^\infty dx\, x\, K_0^2(x)\, e^{zK_0(x)} - 1/4 \right). \tag{27}$$

The integral shows an exponential growth in a double logarithm, which for a large enough z leads to a singularity, i.e. the series constructed from (26) is not integrable, even though the single terms (26) in the series are. We remark that the BFKL total cross section[4] does not share such a behavior, since the virtual radiative corrections precisely cancel the doubly logarithmic growth of the real ones, and one is left over with an exponential growth in rapidity.

Performing the integral over the impact parameter in (27), we obtain

$$\sigma_{tot} = \frac{8}{N_c^2 - 1} \frac{\pi\alpha_s^2 N_c^2}{m^2} \left[e^{-\gamma z} \frac{\Gamma(1 - z/2)^4}{\Gamma(2 - z)} - \frac{1}{2} \right]. \tag{28}$$

where Γ is the Euler gamma function. When $z \geq 2$, the total cross section becomes singular.* The singularity comes from the lower end in the integral over the impact parameter, i.e. from the ultraviolet behavior of the transverse momenta. That is because the reasoning which follows (24) applies only to finite n, but not to the infinite resummation (27).

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* Conversely, we may integrate each term in the series before resumming it, using the approximation (25), and we obtain $\sigma_{tot} \sim (1 - z/2)^{-3}$ for $z \sim 2$, in agreement with (28).

We notice, though, that the transverse momentum of each of the produced gluons cannot grow beyond a value $p_{max} < \sqrt{s}$, because of energy conservation. Thus we regulate the integral over the impact parameter in (26) requiring that $b > 1/p_{max}$, and we evaluate it using (25) and the saddle-point approximation. As long as $i)$ $n+2 < 2\log\frac{p_{max}}{m}$, the integral is well approximated by

$$\int_{m/p_{max}} dx\, x\, K_0^{n+2}(x) \sim (n+2)!\,, \tag{29}$$

and the n-gluon cross section σ_n has a geometrical growth;
when $ii)$ $n+2 > 2\log\frac{p_{max}}{m}$, the integral is better approximated by

$$\int_{m/p_{max}} dx\, x\, K_0^{n+2}(x) \sim \left(\frac{m}{p_{max}}\right)^2 \log^{n+2}\frac{p_{max}}{m}, \tag{30}$$

and the n-gluon cross section behaves like

$$\sigma_n \sim \frac{1}{p_{max}^2}\frac{1}{n!} z^n \log^{n+2}\frac{p_{max}}{m}. \tag{31}$$

If $p_{max} = O(m)$, then condition $ii)$ is easily fulfilled and the n-gluon cross section shows a $1/m^2$ behavior, typical of very high energy cross sections, times an exponential growth in rapidity, in agreement with the BFKL theory; if $p_{max} = O(\sqrt{s})$, then cross sections with a small number of gluons exhibit a geometrical growth and cross sections with a large number of gluons exhibit a $1/s$ behavior, times an exponential growth in a double logarithm. It looks like the latter cross sections might give a small contribution to the total cross section, but, as long as $z > 2$, the exponential growth wins over the $1/s$ behavior. Indeed the contribution of the n-gluon cross sections, with large n, to the total cross section σ_{tot} is

$$\sum_n \sigma_n \sim \frac{1}{m^2}\log^2\frac{p_{max}}{m} e^{(z-2)\ln\frac{p_{max}}{m}}, \qquad \text{with} \quad \mathrm{n}+2 > 2\log\frac{\mathrm{p_{max}}}{\mathrm{m}} \tag{32}$$

which dominates over the contribution of σ_n with small n to σ_{tot}, as long as $z > 2$. In the BFKL theory, the virtual corrections suppress the exponential growth in the double logarithm by not allowing the gluon transverse momenta to become much larger than m.

(32) shows the doubly logarithmic growth, typical of a kinematical regime where there is a strong ordering both in rapidity and transverse momentum. So even if we have explicitly suppressed this regime, as said in the discussion which follows (21) since it does not belong to the multi-Regge kinematics, it does reappear in the total cross section. It would happen the same in the BFKL total cross section if we neglected the virtual radiative corrections. Indeed if in the BFKL integral evolution equation we suppressed the term that describes the reggeization of the gluon exchanged in the t channel, i.e. we discarded the virtual radiative corrections, there would be no cancellation of the divergences in the eigenvalue of the integral equation, and in the BFKL total cross section we would have to regulate by hand the ultraviolet growth of the transverse momentum, obtaining the same behavior as in (32).

This shows an inherent difference between the Parke-Taylor and the BFKL multigluon amplitudes. Because of the cancellation of the virtual and real infrared contributions in the BFKL integral equation, the BFKL amplitudes are particularly suited for the calculation of a fully inclusive quantity, like the total cross section. For such a quantity the Parke-Taylor amplitudes do not fare well. They may be better suited for the calculation of exclusive quantities, like the n-gluon production cross section (26), where the excessive growth due to the lack of virtual radiative corrections may be taken care of by using appropriate kinematical cuts, as we have seen above.

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FIGURE CAPTIONS

1) The n-gluon production amplitude.

2) Multigluon amplitude in the color configuration [A, 0, 1,..., n+1, B].

3) (a) Multigluon amplitude in the color configuration [A, 0,..., j-1, j+1,..., n+1, B,j], and (b) its untwisted version.

4) (a) Multigluon amplitude in the color configuration [A, 0,..., j-1, j+1,..., k-1,k+1,..., n+1, B, k, j], and (b) its untwisted version.

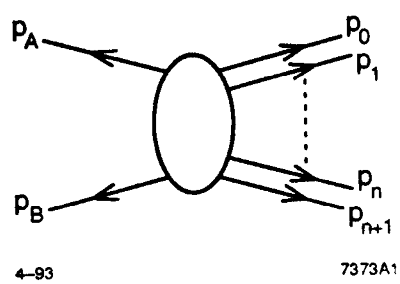


Fig. 1

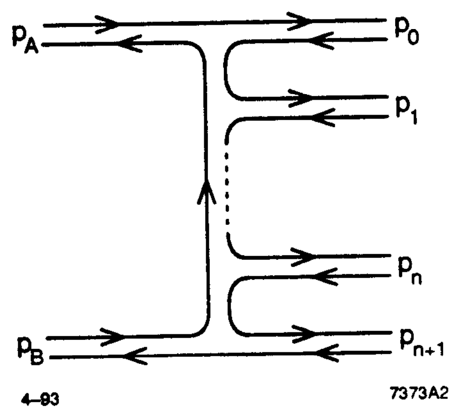


Fig. 2

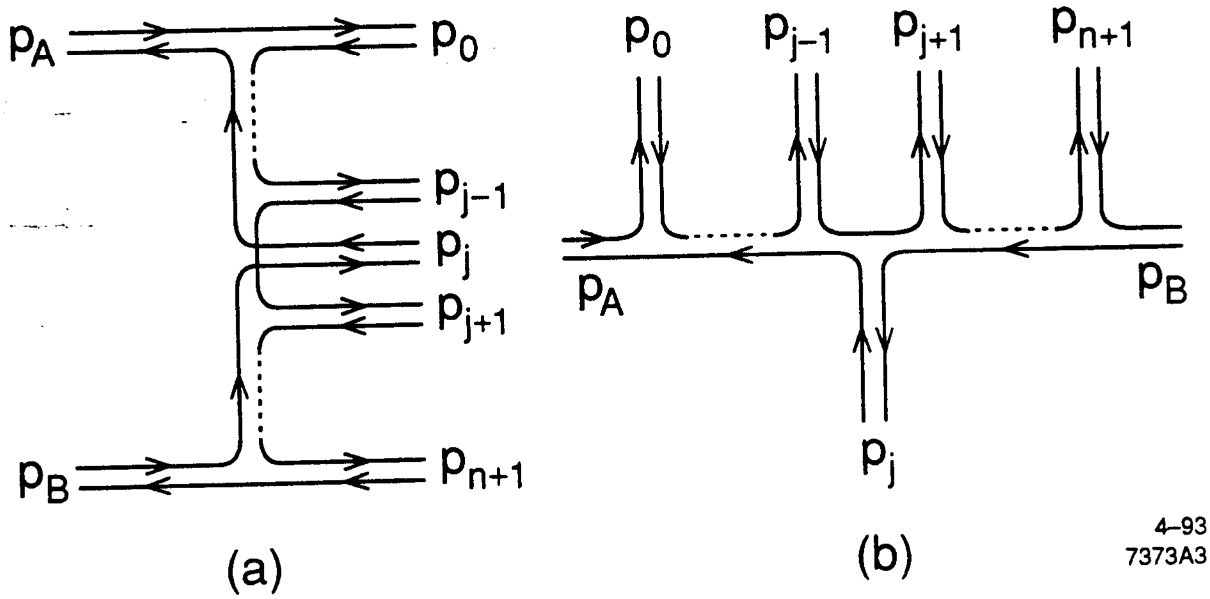
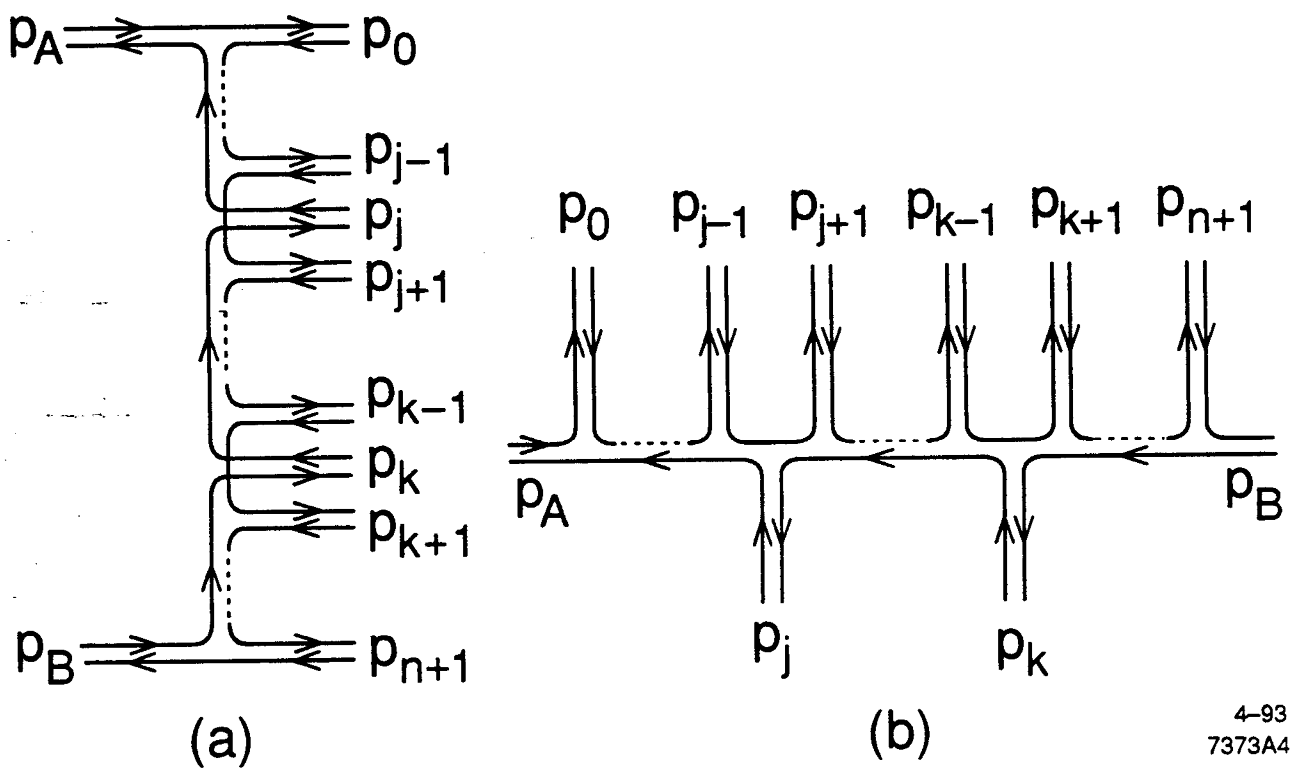


Fig. 3



p_A
p_0
p_{j-1}
p_j
p_{j+1}
p_{k-1}
p_k
p_{k+1}
p_B
p_{n+1}
(a)
p_0 p_{j-1} p_{j+1} p_{k-1} p_{k+1} p_{n+1}
p_A
p_B
p_j
p_k
(b)
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Fig. 4