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A Fast Model-Calibration Procedure for Storage Rings*

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Abstract

The ever-increasing demand for better performance from circular accelerators requires improved methods to calibrate the optics model. We present a linear perturbation approach to the calibration problem in which the modeled BPM-to-corrector response matrix is expanded to first order in quadrupole strengths. The result is numerically fit to the measured response matrix yielding quadrupole strength errors, corrector strength errors, and BPM linearity factors. The large number of degrees of freedom in the fit allows a comprehensive error analysis, including the determination of BPM resolutions. In this way, a self-consistent first order optics model of SPEAR was generated which reproduces the measured tunes.

I. INTRODUCTION

In the course of developing an optics model for storage rings, a series of corrector kicks is typically applied to the beam and the resulting orbit shift is measured. Then, by simultaneously analyzing the horizontal and vertical orbit perturbations (and perhaps a measurement of dispersion), the on-line model is numerically verified, or updated if necessary. In the analysis procedure, the fitting parameters can include quadrupole strengths, corrector strengths, or beam energy errors, for instance. Although this multi-track analysis method improves the agreement between model and measurement, it is a manual process restricted to a limited set of measurements and fitting variables.

Recently however, a method for fast calibration of the optics model (CALIF) has been developed which automates the fitting procedure to include the full set of horizontal and vertical response matrix measurements. This method was originally based on a linear perturbation approach used for phased-array antenna design [1], but with re-interpretation for the application to accelerators. The matrix formalism allows us to expand the set of variable quadrupole strengths, solve for corrector strength and BPM linearity calibration factors, and estimate the BPM resolutions for the measured data set. The updated optics model, including statistically correlated error bars for all fitted quantities, can then be used to predict Twiss parameters at every element in the storage ring.

II. THE CALIF ALGORITHM

The objective of the CALIF algorithm is to obtain a consistent computer model of the as-built machine based on a set of difference orbit measurements. Using a first-order perturbation approach, we seek modeling errors in the following parameters

- Quadrupole gradients
- Corrector scale factors
- BPM scale factors
- BPM resolution errors

including a comprehensive error analysis of the results. From the difference orbit measurements, we first determine the BPM-to-corrector response matrix coefficients

$$\overline{C}^{ij} = \frac{\Delta x \text{ at BPM i}}{\Delta x' \text{ at corrector j}} \qquad (1)$$

which are then compared to the perturbed expression for the computer model prediction, namely,

$$\overline{C}^{ij} = C^{ij} + \sum_q \frac{\partial C^{ij}}{\partial k_q} \delta k_q \qquad (2)$$

where C^{ij} and $\partial C^{ij}/\partial k_q$ are the computer-model response-matrix coefficients and their derivatives with respect to the gradient of a particular quadrupole or quadrupole family, respectively. The C^{ij} and $\partial C^{ij}/\partial k_q$ are easily calculated with accelerator modeling codes such as COMFORT [2]. The δk_q are the sought-after gradient errors needed to explain the measured response-matrix coefficients $\overline{C}^{ij}$.

The solution of Equation 2 is strongly affected by errors in the linear-scale factors for both the correctors and BPMs. To take these affects into account, we augment the left-hand side of Equation 2 by variable corrector-scale factors, x^j, and BPM scale factors, y^i, to arrive at a relation among the unknown quadrupole-gradient errors, corrector scales, and BPM scales,

$$C^{ij} = y^i \overline{C}^{ij} x^j - \sum_q \frac{\partial C^{ij}}{\partial k_q} \delta k_q \qquad (3)$$

Furthermore, each $\overline{C}^{ij}$ has an intrinsic measurement error due to the limited resolution of the BPMs which is given by

$$\sigma\left(\overline{C}^{ij}\right) = \frac{\sigma(\text{BPM i})}{\Delta x' \text{ at correcor j}}. \qquad (4)$$

We initially assume the same value of σ for all BPMs.

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Now we are in a position to use Equation 3 in a linear least-squares fit, either for the corrector scales x^j and the gradient errors δk_q while keeping the BPM scales fixed, or for the BPM scales and the gradient errors while keeping the corrector scales fixed. In SPEAR eight quadrupole families, thirty correctors (x and y, respectively) and twenty-six BPMs yield up to a maximum of 1560 measured $\overline{C}^{ij}$. Since only about sixty parameters are fitted, we have a huge number of degrees of freedom which allows a careful error analysis.

The organization inside the CALIF code is as follows: first a setup file is read that contains status bits of the variable parameters to be included in the fit, the filenames of the measured difference orbits, and the computer model C^{ij} files.

Then, according to Equation 3, a matrix A is constructed in which the columns are related to the fitting parameters x^j and δk_q, and the rows are related to the $\overline{C}^{ij}$. Each row is weighted according to its associated measurement error, given by Equation. 4. In the next step, this over-determined set of linear equations is inverted. Using informal, but obvious notation, we get

$$(x^j : \delta k_q)^T = \left(\frac{A^T}{\sigma}\frac{A}{\sigma}\right)^{-1}\left(\frac{A^T}{\sigma}\right)\left(\frac{C^{ij}}{\sigma}\right), \qquad (5)$$

where $(A^T/\sigma\ A/\sigma)^{-1}$ is the covariance matrix from which the fit errors on $(x^j\!: \delta k_q)^T$ are deduced. The colon in Equation 5 indicates partitioning of the corrector scale and quadrupole-strength error vectors.

Next, the BPM resolution errors are deduced by calculating the contribution of each BPM to the total χ^2 for the problem. The BPM resolutions are then rescaled so that each BPM contributes equally, and the χ^2 is forced to unity. Inconsistent (noisy) BPMs are rejected at this stage. This procedure is iterated until the χ^2 remains close to unity, which typically takes one to three iterations.

In a final step, the updated solution for the corrector scales remains constant, and an iterative procedure similar to the one just described is launched in order to fit the BPM scale factors y^i and the quadrupole gradient errors.

The procedure for alternately fitting the corrector scales and the BPM scales is iterated typically four times until a self-consistent set of gradient errors, corrector scales, BPM scales, and BPM resolution errors is found. A normal run for SPEAR usually involves a total of about fifteen fits, where each fit takes about one minute on a VAX8700. The bulk of this time is spent inverting the matrix needed for the calculation of the covariance matrix.

Recently, the CALIF program and associated drivers used to compute 4 × 4 response matrix elements have been updated to include arbitrary numbers of quadrupole, corrector, and BPM elements. These modifications make it possible to apply the CALIF program to most storage rings. For machines with strong focusing, the linearity of the partial derivatives may be valid only in a restricted range. In this case, the step size used for the quadrupole strengths in each iteration of the fitting procedure can be adjusted to achieve faster convergence.

III. EXPERIMENTAL RESULTS

The first application of the CALIF program was carried out using measurements of the bare SPEAR lattice, with all insertion devices and skew quadrupoles turned off. The 4 × 4 corrector response matrix $\overline{C}^{ij}$ was measured relative to a flat-orbit configuration where the beam was steered to the center of the BPMs. Due to the long time required to measure the response matrix, only one measurement was made for each horizontal and vertical corrector. The peak closed-orbit perturbations were about 2–3 mm in SPEAR, and the tune shift produced by the corrector kicks was within the frequency line width as measured by the spectrum analyzer.

Next, we extracted the on-line optics model for computation of the theoretical corrector-response matrix, and its derivatives with respect to the quadrupole family strengths. The derivatives were computed with COMFORT [2] by evaluating $\Delta C^{ij}/\Delta k_q$ for values Δk_q on the order of 1×10^{-3}. Finally, a set-up file was compiled directing CALIF to the measured data and computed response matrices.

The results of the CALIF computation are listed in the following table. Only a few of the horizontal correctors and BPMs are shown as examples:

Quadrupole	Initial Value	Final Value	Error (+/-)
Q3	-0.9316 (m^{-2})	-0.9293	0.188E-03
Q2	0.3700 (")	0.3713	0.135E-03
Q1	-0.2543	-0.2651	0.843E-03
QFA	0.7711	0.7701	0.102E-02
QDA	-0.7214	-0.7314	0.442E-03
QFB	0.4714	0.4730	0.538E-03
QF	0.4301	0.4266	0.218E-03
QD	-0.6651	-0.6685	0.157E-03

Corrector (x)	Initial Value	Final Value	Error (+/-)
HCORR1	1.0	0.839	0.023
1BB2T	1.0	1.051	0.025
2BB2T	1.0	1.156	0.029

BPM (x)	Initial Value	Final Value	Error (+/-)
WIS1	1.0	1.033	0.017
1S2	1.0	1.074	0.031
2S3	1.0	1.037	0.026

Tune	Initial Value	Final Value	Measurement
Q_x	6.864	6.834	6.838
Q_y	6.635	6.753	6.749

From the table, we find that the tunes of the calibrated model agree in both planes to within 0.004 with the measured tunes. Since the tunes were not part of the fitting procedure, this result gives us confidence in the fidelity of the calibrated model. For the quadrupole strengths, we found deviations of less than $0.01 m^{-2}$, with error bars of less than ±0.001. The corrector scale errors were in the range of <10 percent, with 3 percent accuracy. These results indicate that the matrix "A" discussed in Section II was well conditioned.

For the BPM resolutions, we found the average vertical value of σ was about 100 microns, which is a plausible result for SPEAR. In the horizontal plane, the resolutions were larger, about 200–250 microns, possibly due to the button geometry or longitudinal misalignment of components in SPEAR. Following the installation of new BPMs and re-alignment of SPEAR, we will repeat the process and compare results. The entire process, including measurement and data analysis, takes only about two hours.

IV. CONCLUSION

A conceptually simple and fast way to calibrate the linear optics model for storage rings was developed and tested on SPEAR with great success. One of the primary advantages of this technique is that the problem has a large number of degrees of freedom that allow a careful error analysis of the solution. When applied to SPEAR, for instance, the eight quadrupole magnet strength errors were found to generally be less than 0.01m^{-2}, with error bars less than ±0.001. With these errors corrected, the model tunes now agree to within 0.004 with the measured tunes in both the horizontal and vertical planes. This calibrated model for SPEAR now gives us excellent agreement between simulated orbits and the measured orbit data, and accurately computes the Twiss parameters at every element in SPEAR.

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V. REFERENCES

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