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WHICH NATURAL PROCESSES HAVE THE SPECIAL
STATUS OF MEASUREMENTS?*

M. E. Burgos[†]

Stanford Linear Accelerator Center
Stanford University
Stanford, CA 94309

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[†] Permanent address, Departamento de Física, Facultad de Ciencias, Universidad de Los Andes, Mérida 5101, Venezuela; and Centro de Física, Instituto Venezolano de Investigaciones Científicas (IVIC), Apartado 21827, Caracas 1020-A, Venezuela.

ABSTRACT

We assume, in the first place, that two kinds of processes occur in Nature: the strictly continuous and causal ones, which are governed by the Schroedinger equation; and those implying discontinuities, which are ruled by probability laws. In the second place, we adopt a postulate ensuring the statistical sense of conservation laws. These hypotheses allow us to state a rule telling in which situations and to which vectors the system's state must collapse. The way our proposed approach works is illustrated with some examples and with the analysis of a particular measurement problem.

WHICH NATURAL PROCESSES HAVE THE SPECIAL STATUS OF MEASUREMENTS?

1. Our Starting Point

The formulation of quantum theory usually accepted by the partisans of the orthodox interpretation can be summarized in the following way (Jammer, 1974, pp. 5; Cohen-Tannoudji, 1977, pp.215-222):

(A) To every system corresponds a Hilbert space \mathfrak{H} whose vectors (state vectors, wave functions) completely describe the states of the system.

(B) To every dynamical variable A corresponds uniquely a self-adjoint operator A acting in \mathfrak{H} . It has associated the eigenvalue equations

$$A |a_i^v\rangle = a_i |a_i^v\rangle \quad (1)$$

(v is introduced in order to distinguish between the different eigenvectors that may correspond to one eigenvalue a_i), and

$$\sum_{i,v} |a_i^v\rangle \langle a_i^v| = I \quad (2)$$

(where I is the identity operator). If i or v is continuous, the respective sum has to be replaced by an integral.

(C) For a system in the state $|\Phi\rangle$ the probability that the result of a measurement of A lies between a' and a'' is given by $\|\Psi\|^2$, where $\|\Psi\|$ is the norm of

$$|\Psi\rangle = (I_{a''} - I_{a'}) |\Phi\rangle \quad (3)$$

and I_a is the resolution of the identity belonging to A .

(D) Schroedinger Equation: The evolution in time of the state vector $|\Phi\rangle$ is determined by the equation

$$i\hbar d|\Phi\rangle/dt = H |\Phi\rangle, \quad (4)$$

where H represents the Hamiltonian of the system.

(E) Projection Postulate: If a measurement of A yields a result between a' and a'' , then the state of the system immediately after the measurement is an eigenfunction of $I_{a''} - I_{a'}$.

Many physicists, particularly experimenters, think that the state vector refers to an individual system and that its quantum jumps are real (Gisin, 1992). We share this point of view. Nevertheless, quantum theory does not provide a rule to fix unambiguously the precise conditions under which these reductions occur. We consider this the worst flaw this theory confronts (Burgos, 1990a). As Bell (1984) points out, “during ‘measurement’ the linear Schroedinger evolution is suspended and an ill-defined ‘wavefunction collapse’ takes over. There is nothing in the mathematics to tell what is ‘system’ and what is ‘apparatus,’ *nothing to tell which natural processes have the special status of ‘measurements.’* Discretion and good taste, born from experience, allow us to use quantum theory with marvellous success, despite the ambiguity of the concepts named above in quotation marks” (emphasis added). As Bell (1990) does, we also think that “‘apparatus’ should not be separated off from the rest of the world into black boxes, as if it were not made of atoms and not ruled by quantum mechanics.” This is why we assume that measuring devices are just physical systems, and as such they have to be treated on the same footing with every other physical system.

The main object of the present article is to propose a rule that makes clear when the Schroedinger evolution takes place, and when the state vector is projected. To achieve this purpose we are going to assume, in the first place, that postulates (A) and (B) written above are valid. In the second place, we shall claim that two kinds of processes occur in Nature: the strictly continuous and causal ones, which are governed by the Schroedinger equation; and those implying discontinuities, where the wave function changes in a non strictly deterministic way, which are ruled by probability laws. Moreover, we are going to say that both are spontaneous (*i.e.* they happen without the intervention of any observer) and irreducible to one another (Burgos, 1984a, 1984b, 1987a).

From this starting point we shall face the problem of finding a rule that tells us (i) in which situations and to which vectors the system’s state must collapse; and (ii) what is the probability for this happen. The part (i) of our proposed rule will be stated in Section 2. For doing so we are going to assume that conservation laws have a statistical sense in every case (Burgos, 1993), including those in which the wave function is reduced. For part (ii) of this rule we shall adopt the following version of Born’s postulate: If according to (i) a system in the normalized state

$$|\Phi\rangle = c_1 |u_1\rangle + c_2 |u_2\rangle \quad (5)$$

(where $\langle u_1 | u_2 \rangle = \delta_{12}$ and $c_{1,2} = \langle u_{1,2} | \Phi \rangle \neq 0$) can jump to the state $|u_1\rangle$, then the probability for this happen is $P = |c_1|^2$. On the contrary, if according to the first part of our rule, *i.e.* (i), the state vector $|\Phi\rangle$ cannot be projected to any $|u\rangle \neq |\Phi\rangle$, then the Schroedinger evolution must follow.

2. The Importance of Conservation Laws

In this article we shall assume that the state of the system can be represented by a vector $|\Phi\rangle$ of the Hilbert space and that it has a Hamiltonian H such that $\partial H / \partial t = 0$.

Let A be a dynamical variable referring to a system (object or thing) S . The corresponding operator A satisfies (1). The *mean* value of A for the system S in the normalized state $|\Phi\rangle$ is

$$\langle A \rangle = \langle \Phi | A | \Phi \rangle. \quad (6)$$

If A fulfills the conditions

$$\partial A / \partial t = 0 \quad (7a)$$

and

$$[A, H] = 0, \quad (7b)$$

and the process is governed by the Schroedinger equation, it is easy to demonstrate that $\langle A \rangle$ does not change over time. It is said that A is a constant of the motion (Cohen-Tannoudji, 1977, pp.247). On the contrary, for processes not ruled by the Schroedinger equation it has not been proven that the validity of conditions (7) implies that $\langle A \rangle$ remains a constant. Moreover, it has been shown that in processes involving projections (traditionally called measurement-like processes), the mean value $\langle A \rangle$ concerning the individual system S (given by (6)) may change even if relations (7) are satisfied (Pearle, 1986; Burgos, 1993). However, the *average* of the changes of $\langle A \rangle$, which is obtained by repeating the process many times, is practically zero (Burgos, 1993). Taking into account these analyses, we shall assume that if conditions (7) are fulfilled, then A must be conserved in a statistical sense, both in cases where the process is ruled by probability laws and in those where it is governed by the Schroedinger equation. It is worth noticing that, since in the latter cases the mean value $\langle A \rangle$ remains a constant in individual processes, *a fortiori* the average of $\langle A \rangle$ is also $\langle A \rangle$.

In order to establish our postulate in a precise way, let us consider a generic orthonormal set $\{|u_i\rangle\}$ ($i = 1, 2, \dots$) such that we can write

$$|\Phi\rangle = \sum_i c_i |u_i\rangle. \quad (8)$$

We have $\langle u_i | u_j \rangle = \delta_{ij}$. The mean value of A in the state $|u_i\rangle$ is $\langle u_i | A | u_i \rangle$. (For reasons we are going to see below we can restrict this treatment to cases in which the set $\{|u_i\rangle\}$ is denumerable.) We now state

Postulate I: If relations (7) are satisfied, the validity of

$$\langle \Phi | A | \Phi \rangle = \sum_i |c_i|^2 \langle u_i | A | u_i \rangle \quad (9)$$

is a necessary condition for projections of the state $|\Phi\rangle$ given by (8) to the vectors of the set $\{|u_i\rangle\}$ to happen, *i.e.* for jumps like

$$|\Phi\rangle \Rightarrow |u_1\rangle \quad (10a)$$

or

$$|\Phi\rangle \Rightarrow |u_2\rangle \quad (10b)$$

etc., to occur.

Let us look at the meaning of this postulate in the following way: if $|\Phi\rangle$ collapses to $|u_i\rangle$, the mean value of A changes from $\langle \Phi | A | \Phi \rangle$ to $\langle u_i | A | u_i \rangle$. The probability of this particular reduction taking place is $|c_i|^2$. Now, if the process is repeated many times starting with the same initial state $|\Phi\rangle$, the average of the different $\langle u_i | A | u_i \rangle$ ($i = 1, 2, \dots$) obtained in the different projections must be close to the sum in (9), and so to the initial mean value $\langle \Phi | A | \Phi \rangle$. This is why we say that the validity of equation (9) ensures the statistical sense of the conservation of A .

A first consequence of Postulate I is that it forbids collapses of the wave function to the vectors of some orthonormal sets and, in particular, of some complete bases (for which (2) is satisfied).

To show that our assertion is right, let us consider a unidimensional harmonic oscillator. The operator H that represents the Hamiltonian fulfills conditions (7) and has associated eigenvalue equations of type (1), which we explicitly write as follows:

$$H |v_i\rangle = E_i |v_i\rangle \quad (11)$$

(where $i = 0, 1, \dots$). In the basis $\{|w_i\rangle\}$ defined by

$$|w_0\rangle = (2/3)^{1/2} |v_0\rangle + (1/3)^{1/2} |v_1\rangle \quad (12a)$$

$$|w_1\rangle = (1/3)^{1/2} |v_0\rangle - (2/3)^{1/2} |v_1\rangle \quad (12b)$$

$$|w_i\rangle = |v_i\rangle \quad (12c)$$

for $i \geq 2$, the state $|v_0\rangle$ is given by

$$|v_0\rangle = \sum_i d_i |w_i\rangle = (2/3)^{1/2} |w_0\rangle + (1/3)^{1/2} |w_1\rangle \quad (13)$$

and the corresponding mean value of the Hamiltonian is $\langle v_0 | H | v_0 \rangle = E_0$. Since

$$\sum_i |d_i|^2 \langle w_i | H | w_i \rangle = (5/9) E_0 + (4/9) E_1, \quad (14)$$

we obtain

$$\langle v_0 | H | v_0 \rangle \neq \sum_i |d_i|^2 \langle w_i | H | w_i \rangle. \quad (15)$$

As a consequence, Postulate I prevents $|v_0\rangle$ jumping to the vectors of the basis $\{|w_i\rangle\}$.

On the contrary, it is evident that the equation

$$\langle \Phi | H | \Phi \rangle = \sum_i |c_i|^2 \langle v_i | H | v_i \rangle \quad (16)$$

is realized for every $|\Phi\rangle = \sum_i c_i |v_i\rangle$, and so Postulate I allows it to be reduced to the vectors of the basis $\{|v_i\rangle\}$.

Now let us consider an operator having a continuous spectrum. For instance, if p_x represents a component of the linear momentum and satisfies (7), collapses to the eigenvectors of p_x are allowed by Postulate I. Nevertheless, since these eigenvectors are not in the Hilbert space, projections to them are forbidden. This is why we are going to exclude every reduction to eigenstates belonging to sets for which i is continuous.

Our next analysis will be restricted to situations in which the system has only three dynamical variables: the Hamiltonian, A and B . We shall assume that the corresponding operators satisfy equations (7) and have discrete spectra. Under these conditions, let us consider the following two cases.

Case (i): If $[A, B] = 0$, the set $\{H, A, B\}$ is a complete set of compatible operators. The vectors of its unique common basis will be denoted by $|E_i a_j b_k\rangle$, where E_i , a_j and b_k are respectively the eigenvalues of H , A and B . As the relations

$$\langle \Phi | H | \Phi \rangle = \sum_{i,j,k} |c_{i,j,k}|^2 \langle E_i a_j b_k | H | E_i a_j b_k \rangle \quad (17a)$$

$$\langle \Phi | A | \Phi \rangle = \sum_{i,j,k} |c_{i,j,k}|^2 \langle E_i a_j b_k | A | E_i a_j b_k \rangle \quad (17b)$$

$$\langle \Phi | B | \Phi \rangle = \sum_{i,j,k} |c_{i,j,k}|^2 \langle E_i a_j b_k | B | E_i a_j b_k \rangle \quad (17c)$$

are satisfied for every state

$$|\Phi\rangle = \sum_{i,j,k} c_{i,j,k} |E_i a_j b_k\rangle, \quad (18)$$

Postulate I does not prohibit projections like $|\Phi\rangle \Rightarrow |E_i a_j b_k\rangle$.

Case (ii): If $[A, B] \neq 0$, the operators A and B do not have a common basis. Let us suppose, however, that $\{H, A\}$ and $\{H, B\}$ are two complete sets of compatible operators. In the basis of the former, every state $|\Phi\rangle$ can be written

$$|\Phi\rangle = \sum_{i,j} c_{i,j} |E_i a_j\rangle, \quad (19)$$

and in the basis of the latter we have

$$|\Phi\rangle = \sum_{i,k} d_{i,k} |E_i b_k\rangle. \quad (20)$$

As

$$\begin{aligned} \langle\Phi|B|\Phi\rangle &= \sum_{i,j} |c_{i,j}|^2 \langle E_i a_j | B | E_i a_j \rangle \\ &+ \sum_{i, j \neq j'} c_{i,j}^* c_{i,j'} \langle E_i a_j | B | E_i a_{j'} \rangle \end{aligned} \quad (21)$$

and the second sum is in general non null, in these cases Postulate I prevents collapses to the vectors of the basis $\{|E_i a_j\rangle\}$. A similar argument shows that reductions to the states of $\{|E_i b_k\rangle\}$ are in general also forbidden. Nevertheless, we can write $|\Phi\rangle = \sum_i \alpha_i |t_i\rangle$, where

$$|t_i\rangle = (\sum_j |c_{i,j}|^2)^{-1/2} \sum_j c_{i,j} |E_i a_j\rangle \quad (22)$$

is the normalized projection of $|\Phi\rangle$ into the eigensubspace of the Hamiltonian corresponding to the eigenvalue E_i , and $\alpha_i = (\sum_j |c_{i,j}|^2)^{1/2}$. The orthonormal set $\{|t_i\rangle\}$ is not a basis. (Notice that $|t_i\rangle$ depends on $|\Phi\rangle$ and does not fulfill (2).) Since

$$\langle\Phi|H|\Phi\rangle = \sum_i |\alpha_i|^2 \langle t_i | H | t_i \rangle, \quad (23a)$$

$$\langle\Phi|A|\Phi\rangle = \sum_i |\alpha_i|^2 \langle t_i | A | t_i \rangle \quad (23b)$$

and

$$\langle\Phi|B|\Phi\rangle = \sum_i |\alpha_i|^2 \langle t_i | B | t_i \rangle, \quad (23c)$$

jumps like $|\Phi\rangle \Rightarrow |t_i\rangle$ are allowed by Postulate I.

Our next step will be to assume that $|\Phi\rangle$ has a tendency to collapse to the eigenstates of operators fulfilling conditions (7). Nevertheless, this tendency should not become actualized if projections it induces result in a violation of Postulate I or lead the state vector outside the Hilbert space.

A particular simple situation arises when all of the operators for which (7) are valid belong to the same complete set of commuting operators having discrete spectra, as in the case (i) we dealt with above. Then, if $\{|u_i\rangle\}$ is the unique common basis of all of these operators, equation (9) is necessarily satisfied for each one of them whatever be the state $|\Phi\rangle$. This basis will be called the *preferential basis* of the system. According to our analysis reductions to its vectors are allowed.

On the contrary, if not all of the operators fulfilling (7) commute, or their spectra are at least partially continuous, or the set they constitute is not complete, the system does not have a preferential basis. However, if for the system in the state $|\Phi\rangle$ there is a denumerable set $\{|u_i\rangle\}$ such that all of the operators for which (7) is valid fulfill (9) (as in the case (ii) treated above), we shall say that it is a *preferential set* of the system in the state $|\Phi\rangle$. According to our analysis collapses to its vectors are allowed.

Unlike preferential bases, preferential sets are not unique. In the case (ii) we dealt with above $\{|t_i\rangle\}$ is a preferential set. We shall see that there are others.

Let us write $|\Phi\rangle = \sum_i \beta_i |s_i\rangle$, where $|s_i\rangle = N_i \sum_{j=i,i+1} \alpha_j |t_j\rangle$ ($i = 1, 3, \dots$), the number N_i is a normalizing constant and $\beta_i = \langle s_i | \Phi \rangle$. As

$$\langle \Phi | H | \Phi \rangle = \sum_i |\beta_i|^2 \langle s_i | H | s_i \rangle, \quad (24a)$$

$$\langle \Phi | A | \Phi \rangle = \sum_i |\beta_i|^2 \langle s_i | A | s_i \rangle \quad (24b)$$

and

$$\langle \Phi | B | \Phi \rangle = \sum_i |\beta_i|^2 \langle s_i | B | s_i \rangle, \quad (24c)$$

Postulate I does not prevent reductions as $|\Phi\rangle \Rightarrow |s_i\rangle$.

The sets $\{|t_i\rangle\}$ and $\{|s_i\rangle\}$ are both preferential sets with the following difference: the vectors of the latter are linear superpositions of the vectors of the former, and the converse is false. Projections to the vectors of $\{|t_i\rangle\}$ make actual the tendency the system has to jump to the eigenstates of the operators fulfilling (7) in the highest degree allowed by Postulate I.

If there is a unique preferential set $\{|u_i\rangle\}$ such that the vectors of every other preferential set can be written as linear superpositions of the $|u_i\rangle$, the set $\{|u_i\rangle\}$ will be called the *maximal preferential set* of the system in the state $|\Phi\rangle$. A preferential basis is, thus, a

maximal preferential set whose vectors do not depend on $|\Phi\rangle$ (and so fulfill the closure relation (2)).

We shall say that the vectors of the preferential basis or of the maximal preferential set are *preferential states*. It is worth noticing that they are stationary states.

Now we present the rule announced in Section 1 (for cases which satisfy the restrictions imposed at the beginning of Section 2).

Rule I: The state

$$|\Phi\rangle = \sum_i c_i |u_i\rangle \quad (25)$$

can be projected to $|u_i\rangle$ iff $|u_i\rangle$ is a preferential state; the probability of this happening is $|c_i|^2$. If the system in the state $|\Phi\rangle$ has neither a preferential basis nor a maximal preferential set, the Schrodinger evolution must follow.

Before concluding this section let us make the following comments.

- (a) Rule I tells us in which situations natural processes having the special status of measurements are going to result. Nevertheless, it does not say anything about the instant the system will jump into a preferential state. Concerning this point we confront the same problem faced in the traditional treatment of the measurement problem. In our approach, the vector $|\Phi\rangle$ given by (25) may be considered an unstable state that eventually decays to one of the stable states $|u_i\rangle$, so the probability that the system survives in the unstable state should decrease with time according to an exponential law (Cohen-Tannoudji, 1977, pp.338). The details will be analysed elsewhere.
- (b) An ensemble of systems initially in the same state $|\Phi\rangle$ given by (25) will finally be distributed in the different preferential vectors $|u_i\rangle$. The corresponding processes are ruled by probability laws, so they are irreversible and entail an increase of entropy. On the contrary, processes governed by the Schrodinger equation are reversible and do not entail any change of entropy. In our view, time irreversibility has its roots in quantum jumps (Burgos, 1990b). In this sense it could be said that the increase of entropy is the macroscopic result of quantum mechanical laws (Landau, 1958, pp.30-31). In other words: collapses build

time's arrow up since they fix the past and leave uncertain the future. A similar idea was first proposed by Phipps (1973) and developed by Noyes (1975).

- (c) Those who think that every quantum process is determined by the Schroedinger equation face the puzzle of the entanglement. According to Ghirardi et al. (1988), "if pushed to its extreme consequences, it leads to the conception of the universe as an unbroken whole whose parts have lost any individual entity." This is why, in their opinion, "quantum entanglement is the enemy to be defeated." In our approach quantum jumps break entanglements when the preferential vectors are factorized states. However, it is important to notice that if this condition is not realized, collapses produce entanglements.

3. Some Examples

In order to show how our approach works, let us consider the following simple examples.

- (i) *The free particle*: The operators H , p_x , p_y , p_z , L_x , L_y and L_z , which respectively represent the Hamiltonian and the components of the linear and angular momenta, fulfill conditions (7). However, since the operators H , p_x , p_y and p_z have continuous spectra, projections to their eigenstates are forbidden; and since L_x , L_y and L_z do not commute, Postulate I prohibits collapses to their eigenvectors. The free particle does not have a preferential basis and does not seem to have a maximal preferential set for the states it is normally supposed to be in. Its evolution should be determined by the Schroedinger equation.

- (ii) *A spin in a homogeneous magnetic field $\mathbf{B} = B_z \mathbf{k}$* : The operators that interest here are H , and S_x , S_y and S_z (which represent the three components of the spin). We have $H = -\gamma B_z S_z$, where γ is the gyromagnetic ratio. The operators S_x and S_y do not satisfy (7), but S_z does. The spectra of H and S_z are discrete. The preferential basis of this system is $\{|+\rangle, |-\rangle\}$, the basis of the eigenstates of S_z . So a spin initially in the state

$$|\Phi\rangle = c_+ |+\rangle + c_- |-\rangle \quad (26)$$

has probability $|c_+|^2$ of jumping to $|+\rangle$ and probability $|c_-|^2$ of jumping to $|-\rangle$.

- (iii) *A spin in an inhomogeneous magnetic field*

$$\mathbf{B}(\mathbf{r}) = B_x(\mathbf{r}) \mathbf{i} + B_y(\mathbf{r}) \mathbf{j} + B_z(\mathbf{r}) \mathbf{k}, \quad (27)$$

where \mathbf{r} is the position: the components of the field vary with \mathbf{r} , and now

$$H(\mathbf{r}) = -\gamma[B_x(\mathbf{r}) S_x + B_y(\mathbf{r}) S_y + B_z(\mathbf{r}) S_z]. \quad (28)$$

As the operators S_x , S_y and S_z do not fulfill conditions (7), their eigenvectors are not preferential states, and collapses to them are forbidden. In particular, a spin in the magnetic field of a Stern-Gerlach device (whose strongest component is B_z) cannot be projected to the eigenstates of S_z . The study of H , which depends on \mathbf{r} , lies beyond the scope of this article.

(iv) *The benzene molecule*: In an idealized model $|\psi_1\rangle$ and $|\psi_2\rangle$ represent the only two states of the molecule which correspond to the two possible positions of the three double bonds. Since $|\psi_1\rangle$ and $|\psi_2\rangle$ are not eigenvectors of H , they are not preferential states. On the contrary, the basis $\{|u_1\rangle, |u_2\rangle\}$ where H is diagonal and $\langle u_1|H|u_1\rangle \neq \langle u_2|H|u_2\rangle$, is the preferential basis of the molecule. An ensemble of them will finally be distributed in the states $|u_1\rangle$ and $|u_2\rangle$. The fact that the lowest level of energy is

$$\langle u_1|H|u_1\rangle < \langle \psi_1|H|\psi_1\rangle = \langle \psi_2|H|\psi_2\rangle, \quad (29)$$

is supposed to explain that the benzene molecule is more stable than expected (Cohen-Tannoudji, 1977, pp.411).

4. The Measurement Problem

We have not imposed on the system (or object) S the restriction of being isolated (as the free particle or the benzene molecule analyzed in Sec. 3), or in interaction with other objects through, for instance, a magnetic field. We have not said that S has to be microscopic or macroscopic. Like Einstein, we do not believe in micro and macro laws, but in laws of general validity. His principal objection to quantum mechanics was to the subjective character of the theory. He held that basic physical theories should represent the physical world itself, not merely connections between human observations (Pauli, 1971). Our approach would not be subject to Einstein's objection and, if it is right, should be of some help in the study of many of the problems fulfilling the conditions specified at the beginning of Section 2. So our next analysis will be of a more complicated case than those treated in Section 3.

Let S_1 be a system in the initial state $|\psi_i\rangle$ ($i = 1, 2, \dots$), where $\{|\psi_i\rangle\}$ is a basis in the Hilbert space of S_1 ; and let S_2 be a system in the initial state $|\chi\rangle$. We shall assume that if the initial state of $S = S_1 + S_2$ is $|\Phi_i\rangle = |\psi_i\rangle \otimes |\chi\rangle$ ($i = 1, 2, \dots$), the interaction between S_1 , S_2 and the environment leads the state of S to $|u_i\rangle$, *i. e.* we have

$$|\Phi_i\rangle \Rightarrow |u_i\rangle \quad (30)$$

for every i . Now, if the initial state of S_1 is $|\psi\rangle = \sum_i c_i |\psi_i\rangle$, and so S is in the initial state

$$|\Phi\rangle = [\sum_i c_i |\psi_i\rangle] \otimes |\chi\rangle, \quad (31)$$

according to our approach it can happen (i) that the state $|\Phi\rangle$ evolves guided by the Schroedinger equation, and then

$$|\Phi\rangle \Rightarrow \sum_i c_i |u_i\rangle; \quad (32)$$

or (ii) that $|\Phi\rangle$ is projected. If we do not know at least some details of the problem we want to analyze, it is completely impossible to tell which of these two alternatives will be realized. Nevertheless, as quantum jumps can lead $|\Phi\rangle$ only to the preferential states of S , taking into account (30) we can say that, if S has a preferential basis or a maximal preferential set, it is $\{|u_i\rangle\}$. As a consequence, in that case collapses like

$$|\Phi\rangle \Rightarrow |u_1\rangle, \quad (33a)$$

or

$$|\Phi\rangle \Rightarrow |u_2\rangle, \quad (33b)$$

etc., must occur.

In the traditional treatment of the measurement problem it is supposed that the apparatus S_2 measures the dynamical variable A corresponding to the system S_1 , which has the eigenvalue equation

$$A |\psi_i\rangle = a_i |\psi_i\rangle. \quad (34)$$

Transitions (30) do not present any difficulty. (Since the probability that they happen is $P = 1$, they may be attributed to the Schroedinger evolution.) The same is valid for (32). Now, *if it is assumed that the behaviour of $|\Phi\rangle$ must always be determined by the Schroedinger equation, transitions (33), i.e. the transitions "observed," cannot be explained.* This is the great puzzle of the measurement problem. In our approach there is no such puzzle: if the $|u_i\rangle$ are the preferential states of S in $|\Phi\rangle$, the evolution (32) should not occur, and transitions (33) must take place.

We do not claim, however, that the above remarks solve every measurement problem. This is true, first, because quantum mechanics (like good old classical physics!) does not have just one measurement problem but as many as there are dynamical variables corresponding to different systems worth measuring, with as many different methods as it is possible to imagine, and, second, because finding the preferential basis or sets of an object is not always easy. In particular, a macroscopic system seldom fulfills the restrictions specified at the beginning of Section 2, which in our treatment are necessary conditions for the concept of preferential states to make sense. For these reasons it seems dubious that our approach will prove to be a useful tool to deal with particular measurement problems. It could, perhaps, be of some help in the study of processes taking place at the microscopic level.

Being aware of these difficulties, let us try to analyze a very simple measurement process: the determination of a particle's position with a detector.

We shall assume that a particle S_1 arrives at a detector S_2 that counts particles entering it, and whose window covers the interval (x_1, x_2) . (To make things easier we are going to treat this problem as if it were unidimensional.) Although we consider S_2 to be a quantum system, if it is macroscopic enough, its window's boundary is well-defined, as in the classical case (Burgos, 1988), and so x_1 and x_2 have sharp values (Burgos, 1984b).

As we do not know the Hamiltonian and the other dynamical variables referred to $S = S_1 + S_2$, we cannot tell whether it has a preferential basis (set) or not; and if it has, which one is it. Hence, we are forced to state an ad-hoc hypothesis:

- (a) if the normalized state of S_1 is $|\psi_a\rangle$, where $\psi_a(x) = \langle x|\psi_a\rangle$ and $\psi_a(x)$ has non null values only in the interval (x_1, x_2) , then the probability that the particle will be detected is $P_a = 1$;
- (b) if the normalized state of S_1 is $|\psi_b\rangle$, where $\psi_b(x) = \langle x|\psi_b\rangle$ and $\psi_b(x)$ is null in the interval (x_1, x_2) , then the probability that the particle will be detected is $P_b = 0$; and
- (c) if the normalized state of S_1 is

$$|\psi\rangle = c_a |\psi_a\rangle + c_b |\psi_b\rangle, \quad (35)$$

the particle can be either detected or not detected, and there is not a third possibility.

Let us assume that the detector has $M + P$ orthogonal states, that in an idealized model its states $|\chi_i\rangle$ ($i = 1, 2, \dots, M$) corresponding to cases in which the particle is captured can be written in a brief notation as $|\chi_a\rangle$, and that the states $|\chi_i\rangle$ ($i = M + 1, \dots, M + P$) corresponding to cases in which the particle is not in the detector can be written as $|\chi_b\rangle$.

Now, if the initial state of S is $|\Phi\rangle = |\psi_a\rangle \otimes |\chi_b\rangle$, according to part (a) of our ad-hoc hypothesis, the probability of the transition

$$|\Phi\rangle \Rightarrow |\Phi_a\rangle = |\psi_v\rangle \otimes |\chi_a\rangle \quad (36)$$

is $P = 1$ (here $|\psi_v\rangle$ represents the state void for S_1); if the initial state of S is $|\Phi\rangle = |\psi_b\rangle \otimes |\chi_b\rangle$, according to part (b) of our ad-hoc hypothesis, the probability of the transition

$$|\Phi\rangle \Rightarrow |\Phi_b\rangle = |\psi_b\rangle \otimes |\chi_b\rangle \quad (37)$$

is $P = 1$; and if the initial state of S is

$$|\Phi\rangle = (c_a |\psi_a\rangle + c_b |\psi_b\rangle) \otimes |\chi_b\rangle, \quad (38)$$

according to part (c) of our ad-hoc hypothesis, it could be that

$$|\Phi\rangle \Rightarrow |\Phi_a\rangle \quad (39a)$$

or that

$$|\Phi\rangle \Rightarrow |\Phi_b\rangle. \quad (39b)$$

There is no other possibility. As the evolution of $|\Phi\rangle$ is not guided by a deterministic law, we must conclude that the process we are studying involves quantum jumps; and since these can lead S only to its preferential states, we can say that (in our idealized model) these are $|\Phi_a\rangle$ and $|\Phi_b\rangle$.

Moreover, we can say that if (39a) takes place, the particle just disappears, and if (39b) happens, the state of S is projected to $|\Phi_b\rangle$. According to (37) the particle's state jumps to $|\psi_b\rangle$. In other words, it has probability $|c_b|^2$ of losing its components in the interval (x_1, x_2) as if they were cut off. As a consequence, if we repeat the experiment with N particles in the initial state $|\psi\rangle$ given by (35), a number $N |c_a|^2$ of them will be detected and $N |c_b|^2$ will be led to the state $|\psi_b\rangle$.

5. Concluding Remarks

In our view, the tremendous success of quantum mechanics suggests that this theory reflects certain aspects of Nature, that it is more than merely a man-made tool for calculating expectations. In its orthodox version, which is at present *the* version exposed in classical books and practically the only one taught, it includes the Projection Postulate and the concept of measurement (see Section 1), even though nobody has been able to tell in a precise way what a measurement is. This is perhaps one of the reasons why so many people are afraid of projections. “Mention collapse of the wave function, and you are likely to encounter vague uneasiness or, in extreme cases, real discomfort. This uneasiness can usually be traced to a feeling that wave-function collapse lies ‘outside’ quantum mechanics: the real quantum mechanics is said to be the unitary Schroedinger evolution; wave-function collapse is regarded as an ugly duckling of questionable status, dragged in to interrupt the beautiful flow of Schroedinger evolution” (Caves, 1986).

On the contrary, we are not afraid of projections. Moreover, as Heisenberg once said, we are of the opinion that discontinuities are the most interesting things in quantum theory, and that one can never stress them enough (Hendry, 1985). As a matter of fact, our approach is of the objective Heisenberg reduction type (Stapp, 1992). We think that even if it is difficult to accept that quantum jumps spontaneously occur in Nature, the adoption of this point of view has some advantages. Two of these are that it unifies the treatment of micro and macro objects, and so the traditional measurement problem just disappears; and further, that it allows quantum mechanics to be made compatible with philosophical realism (Burgos, 1983, 1987*b*), a doctrine in which there is no room for observers and superobservers.

Bell (1990) says that “however legitimate and necessary in application, [the following words] have no place in a formulation with any pretention to physical precision: system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, information, measurement... on this list of bad words... the worst of all is ‘measurement’.” The idea that projections are a kind of natural processes and that conservation laws should have a statistical sense in every case (including those in which collapses take place), led us to work out an approach where, except for “system,” it is not necessary to use these bad words or the corresponding concepts. None of them appear in Postulate I or Rule I (see

Section 2), just in their applications. (Concerning the word system, it could be replaced with object or thing, but they represent the same idea, which is central in our treatment.) On the contrary, authors who do not accept the concept of projections seem to be doomed to use this complete list of bad words, over and over again.

Bell (1990) also observes that “it would seem that [quantum mechanics] is exclusively concerned about ‘results of measurements,’ and has nothing to say about anything else. What exactly qualifies some physical system to play the role of ‘measurer’? Was the wavefunction of the world waiting to jump for [billions] of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a Ph D? If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on all the time, more or less everywhere? Do we have jumping all the time?” Our answer to the two last questions is yes. Concerning the first question, Rule I provides an answer, at least in principle, for cases fulfilling the restrictions imposed at the beginning of Section 2.

Other approaches aiming to solve the measurement problem are close to, but different from quantum mechanics. In contrast, ours does not modify the theory. So, if it were right, we would claim that it complements the orthodox version of quantum mechanics (as summarized in Section 1) and renders it a quite acceptable theory that, for the moment, does not seem to need any fundamental change (at least at the non-relativistic limit). Nevertheless, we have to recognize that it is too early to pass a favorable judgement on the present approach, whatever its success may have been in the analysis of the few examples we dealt with in this article.

To conclude, let us say that we are aware that, as Maxwell once said, there is always more than one way of looking at things. In our view the state vector refers to an individual system and quantum jumps are real. On the contrary, according to Bohr it is wrong to think that the task of physics is to find out how Nature is, since physics concerns only what we can say about Nature; and nowadays it is frequently considered that “quantum theory, in a strict sense, is nothing more than a set of rules whereby physicists compute probabilities for the outcome of macroscopic tests” (Peres, 1990). Since, independently of the point of view adopted, everybody faces the question “what is

a measurement?" (Peres, 1990), we hope that this work will contribute in something new to the answer.

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