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CONSERVATION LAWS VERSUS THE ORTHODOX VERSION OF QUANTUM MECHANICS*

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ABSTRACT

In this article it is shown that the orthodox version of quantum mechanics contradicts the idea that conservation laws are valid in individual processes.

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I. INTRODUCTION

The Schroedinger evolution of a system leads, in some circumstances, to coherent superpositions of macroscopically distinct states. This is dramatically illustrated in Schroedinger's cat paradox, and constitutes the great puzzle of quantum measurements.

To explain this fact, several hypotheses have been proposed. The best known is the projection postulate, an ingredient of the so-called orthodox interpretation of quantum mechanics (due to von Neumann), which is at present almost the only version taught. The projection postulate establishes that

◊ when a measurement is performed, the system's state jumps to an eigenstate of the operator representing the dynamical variable measured, and

◊ the pointer of the measuring device is led to a definite position; *i.e.,* it breaks down the coherent superposition of macroscopically distinct states [1].

This postulate has been criticized on several grounds:

◊ it introduces a subjective element into the theory [2,3],

◊ it conflicts with the Schroedinger equation [4,5], and

◊ it implies a kind of action-at-a-distance [4,6].

The traditionally opposed approach faces the conceptual difficulties of the measurement problem by assuming that the state function $|\phi_S\rangle$ is no more than a tool to calculate probabilities. Differing from the orthodox version, in this view $|\phi_S\rangle$ is not an attribute of an individual system S but of an ensemble; hence a process state reduction is not required [7]. Nevertheless, many physicists think that $|\phi_S\rangle$ refers to an individual system, so the ensemble interpretation of $|\phi_S\rangle$ that allows rejection of the projection postulate is, paradoxically, the main reason that this approach is frequently discarded.

Other theories close to, but different from, quantum mechanics have recently been proposed. Most of them seem to be motivated by the desire to find a solution to the measurement problem that is compatible with the individual interpretation of $|\phi_S\rangle$. In general, they modify the Schroedinger equation in a way that leads to spontaneous collapses. Ballentine has demonstrated [7] that the theories developed by Ghiradi, Rimini and Weber [8], Diosi [9], and Joos and Zeh [10] violate energy conservation and are incompatible with the existence of stationary states.

The role of conservation laws in quantum measurements has been studied by several authors [11–15]. It has been shown that the presence of an additive conserved quantity imposes restrictions on the measurement of dynamical variables incompatible with this quantity. The main object of the present article is to point out an even deeper conflict between conservation laws and the orthodox version of quantum mechanics: if the individual interpretation of $|\phi_S\rangle$ and the projection postulate are taken as valid, then conservation laws cannot be satisfied in measurement processes, except in cases where the initial state of S is an eigenstate of the operator representing the quantity to be measured.

II. CONSERVATION LAWS IN SPONTANEOUS AND MEASUREMENT PROCESSES

Let A_S be a dynamical variable referred to the individual system S. It is represented by the operator A_S whose eigenvalue equations are

$$A_S\,|a_i\rangle = a_i\,|a_i\rangle\ . \tag{1}$$

(For simplicity we treat the discrete and non-degenerate case.)

If the individual interpretation of $|\phi_S\rangle$ is adopted, it can be said that A_S has the value a_i in cases $|\phi_S\rangle = |a_i\rangle$. But if

$$|\phi_S\rangle = \sum_j c_j |a_j\rangle \ , \tag{2}$$

it is not possible to attribute a definite value to A_S. In contrast, the mean value

$$\langle A_S\rangle = \langle\phi_S|A_S|\phi_S\rangle \tag{3}$$

is sharp. It is important to stress that in the framework of the interpretation we are analyzing both $|\phi_S\rangle$ and A_S refer to the individual system S. As a consequence, the mean value $\langle A_S\rangle$ refers also to this individual system.

According to the orthodox version of quantum mechanics there are two kinds of processes: the spontaneous processes that are governed by the Schroedinger equation, and the measurement processes that are ruled by the projection postulate. For cases in which the evolution of the state follows the Schroedinger equation, the conditions

$$\partial A_S/\partial t = 0 \tag{4a}$$

and

$$[A_S, H_S] = 0 \ , \tag{4b}$$

where H_S is the Hamiltonian of S, ensure that

$$\langle A_S\rangle (t) = \langle\phi_S(t)|A_S|\phi_S(t)\rangle \tag{5}$$

remains a constant in time [16]. Thus it can be said that conservation laws are valid for this kind of process.

The ideal measurement scheme [1] assumes that for a measuring device M of A_S initially in the state $|m_0\rangle$ the interaction between S and M produces the transition

$$|a_i\rangle|m_0\rangle \Rightarrow |a_i\rangle|m_i\rangle \ , \tag{6}$$

with a probability of one. (Here $|m_i\rangle$ are orthonornal states of M when the measurement process is over.) This scheme is supposed to be valid in cases where the measured quantity is compatible with every conserved quantity referred to S + M [11–15].

Let A represent a conserved quantity referred to S + M, and H be its Hamiltonian. We can write

$$H = H_S + H_M + H_i \ , \tag{7}$$

The conditions

$$\partial A/\partial t = 0 \tag{8a}$$

and

$$[A, H] = 0 \tag{8b}$$

are fulfilled.

To ensure that measurements of A_S can be performed according to the ideal scheme, we assume that A_S commutes with every operator representing another conserved quantity referred to S + M; and, since transition (6) has a probability of one that it will happen, there is no inconvenience in assuming that it is a result of the Schroedinger evolution.

If at t_0 (when the interaction between S and M starts) and at t_f (when this interaction is over) it is possible to write

$$A = A_S + A_M \tag{9}$$

(where A_M refers to M), we have

$$\langle A\rangle_i(t_0) = a_i + \langle m_0|A_M|m_0\rangle \tag{10}$$

and

$$\langle A\rangle_i(t_f) = a_i + \langle m_i|A_M|m_i\rangle \ . \tag{11}$$

The validity of (8) thus implies that $\langle A\rangle_i(t_0) = \langle A\rangle_i(t_f)$, and hence

$$\langle m_0|A_M|m_0\rangle = \langle m_i|A_M|m_i\rangle \ . \tag{12}$$

This relation must necessarily be fulfilled in the ideal measurement scheme, so it can be said that in cases for which the initial state of S is an eigenstate of the operator representing the quantity to be measured, the corresponding conservation law is valid. This result can also be seen as a natural consequence of the hypothesis that the process described by (6) is ruled by the Schroedinger equation.

Now, if the initial state of S is given by (2) and the Schroedinger equation rules the measurement process, then the Hamiltonian H (referred to S + M) induces the evolution

$$\sum_j c_j |a_j\rangle |m_0\rangle \Rightarrow \sum_j c_j |a_j\rangle |m_{j0}\rangle \ . \qquad (13)$$

It is easy to show that in this case the validity of (8) implies that $\langle A\rangle(t_0) = \langle A\rangle(t_f)$; *i.e.*, that the corresponding conservation law is satisfied. Nevertheless, the linear superposition in the r.h. of (13) is of the type mentioned in Section I, that constitutes the great puzzle of quantum mesurements.

On the contrary, the projection postulate states that in measurement processes, coherent superpositions break down. According to this postulate, the evolution of S + M is not given by (13), and the transition

$$\sum_j c_j |a_j\rangle |m_0\rangle \Rightarrow |a_i\rangle |m_i\rangle \qquad (14)$$

has a probability $|c_i|^2$ to happen. In this last case we have

$$\langle A\rangle(t_0) = \sum_j |c_j|^2 a_j + \langle m_0 | A_M | m_0\rangle \qquad (15)$$

and

$$\langle A\rangle_i(t_f) = a_i + \langle m_i | A_M | m_i\rangle \ . \qquad (16)$$

Hence, taking into account (11), we can write

$$\langle A\rangle_i(t_f) = a_i + \langle m_0 | A_M | m_0\rangle \ , \qquad (17)$$

and we obtain

$$\langle A\rangle(t_0) \neq \langle A\rangle_i(t_f) \tag{18}$$

for every i, even though conditions (8) are fulfilled.

It is worth noticing that inequalities (18) result from assuming that evolution (13) does not occur, but transitions (14) are realized. In other words: if the individual interpretation of the state vector and the projection postulate are taken as valid, then we are forced to conclude that for S in the initial state (2), the corresponding conservation law is violated. In particular, if A is the Hamiltonian H (for which (9) is valid at t_0 and at t_f), then equation (18) implies that the energy of S + M is not conserved in the process of measuring the energy of S.

III. AN EXPERIMENT WITH POLARIZED LIGHT

In order to illustrate the problem referred to in the previous section, we are going to analyze the following experiment.

Let S be a photon and M be a filter that transmits right circularly polarized light and absorbs left circularly polarized light. We shall assume that M can freely rotate only about the optical axis z. Following Dirac [17], we shall say that this filter is a measuring device of J_S, the photon's angular momentum about z.

We shall call J_M the operator representing the filter's angular momemtum about z, and J the operator corresponding to the z–component of total angular momentum (referred to S + M). At t_0 (when the interaction between s and m starts) and at t_f (when this interaction is over), we can write

$$J = J_S + J_M \ . \tag{19}$$

Since there is not an external torque about z, the conditions

$$\partial J/\partial t = 0 \tag{20a}$$

and

$$[J,H] = 0 \tag{20b}$$

are fulfilled (here H is the Hamiltonian of S + M that includes the interaction between S and M). Notice that this is not true for the other components of the total angular momentum.

In the orthodox version of quantum mechanics, the individual interpretation of the state vector is adopted. Hence, we are going to attribute a state of polarization to individual photons.

We saw in Section II that if conditions (8) are fulfilled, the corresponding conservation laws are valid in processes ruled by the Schroedinger equation. We also saw that this is the case of measurement processes, if the initial state of S is an eigenstate of the operator representing the quantity to be measured.

In our example, $|R\rangle$ (circularly polarized to the right) and $|L\rangle$ (circularly polarized to the left) are eigenstates of J_S, and J fulfils conditions (20). So the conservation of $\langle J\rangle$ must necessarily follow if the initial state of S is either $|R\rangle$ or $|L\rangle$. Let us deal with this point in more detail.

If the initial state of S is $|R\rangle$ and that of M is $|m_0\rangle$, the interaction between S and M produces the transition

$$|R\rangle|m_0\rangle \Rightarrow |R\rangle|m_0\rangle \ . \tag{21}$$

We have

$$\langle J\rangle_R \langle t_0\rangle = \langle J\rangle_R \langle t_f\rangle = \hbar + \langle m_0|J_M|m_0\rangle \ , \tag{22}$$

where $\langle J\rangle_R\langle t_0\rangle$ and $\langle J\rangle_R\langle t_f\rangle$ are respectively the initial and final mean value of the total angular momentum about z for S in the initial state $|R\rangle$.

If the initial state of S is $|L\rangle$, the interaction between S and M produces the transition

$$|L\rangle\,|m_0\rangle \Rightarrow |\emptyset\rangle\,|m_L\rangle \qquad (23)$$

(here $|\emptyset\rangle$ represents the state void for S, and $|m_L\rangle$ the filter's state after absorption of a photon in the state $|L\rangle$. We obtain

$$\langle J\rangle_L\,\langle t_0\rangle = \langle J\rangle_L\,\langle t_f\rangle = -\hbar + \langle m_0|J_M|m_0\rangle\ , \qquad (24)$$

where $\langle J\rangle_L\langle t_0\rangle$ and $\langle J\rangle_L\langle t_f\rangle$ are respectively the initial and final mean value of the total angular momentum about z for S in the initial state $|L\rangle$.

Now, if S is initially plane polarized, its state is not an eigenstate of J_S but can be written

$$|P\rangle = \frac{1}{\sqrt{2}}\,|R\rangle + \frac{1}{\sqrt{2}}\,|L\rangle\ . \qquad (25)$$

Supposing that the interaction between S and M can be described by the Schroedinger equation, we must conclude that the evolution

$$|P\rangle\,|m_0\rangle \Rightarrow \frac{1}{\sqrt{2}}\,|R\rangle\,|m_0\rangle + \frac{1}{\sqrt{2}}\,|L\rangle\,|mL_0\rangle \qquad (26)$$

occurs. This hypothesis allows us to ensure that $\langle J\rangle$ remains a constant, but now we cannot say that the photon is either transmitted or absorbed. This is the way the great puzzle of quantum mechanics becomes apparent in our example.

If, on the contrary, we assume that measurement processes are not ruled by the Schroedinger equation, but by the projection postulate, we are led to conclude that the evolution (26) does not happen, and the transition

$$|P\rangle\,|m_0\rangle \Rightarrow |R\rangle\,|m_0\rangle\ . \qquad (27a)$$

or

$$|P\rangle\,|m_0\rangle \Rightarrow |\emptyset\rangle\,|m_L\rangle \qquad (27b)$$

results, with a probability of 1/2.

Transition (27a) corresponds to the case in which the photon is transmitted in the state $|R\rangle$. Since the r.h. of (27a) and that of (21) are identical, the final $\langle J\rangle$ is given by (22). Transition (27b) corresponds to the case in which the photon is absorbed. Since the r.h. of (27b) and (23) are identical, the final $\langle J\rangle$ is given by (22). Transition (27b) corresponds to the case in which the photon is absorbed. Since the r.h. of (27b) and (23) are identical, the final $\langle J\rangle$ is given by (24).

On the other hand, the l.h. of (27a) and (27b) represent the same state. Hence, the initial $\langle J\rangle$ is also the same. It is easy to show that

$$\langle J\rangle_P \langle t_0\rangle = \langle m_0 | J_M | m_0\rangle \ , \tag{28}$$

where $\langle J\rangle_P \langle t_0\rangle$ is the initial $\langle J\rangle$ for S in the state $|P\rangle$. As a consequence, we obtain for transition (27a)

$$\langle J\rangle_P \langle t_0\rangle \neq \langle J\rangle_R \langle t_f\rangle \ , \tag{29a}$$

and for transition (27b)

$$\langle J\rangle_P \langle t_0\rangle \neq \langle J\rangle_R \langle t_f\rangle \ . \tag{29b}$$

In other words: the z–component of the mean value $\langle J\rangle$ referred to S + M is not conserved either in the process of transmission nor in that of absorption of a photon initially plane polarized, even if there is not an external torque about z acting on S + M.

IV. CONCLUDING REMARKS

We have seen that during Schroedinger evolutions the validity of (8) ensures that $\langle A\rangle$ remains a constant in time. Now, *if the rule governing the process is replaced with a law different from the Schroedinger equation,* a priori *the validity of conservation laws cannot be guaranteed.* Ballentine points out [7] that some theories which modify this equation in order to include spontaneous state reductions lead to the result that the energy is not conserved. Our study shows that projections induced by measurements, as they are considered in the framework of orthodox quantum mechanics, lead to a conflict with conservation laws.

These two analyses make apparent, however, a difference worth noticing. In the theories Ballentine refers to, energy is continuously gained, although its magnitude is too small to be detected [7]. In collapses occurring in the framework of orthodox quantum mechanics, the change $\langle A\rangle_i(t_f) - \langle A\rangle(t_0)$ is not necessarily small and has a probability $|c_i|^2$ to happen. This implies that the average of these changes obtained when the processes of measurement of A_S is repeated many times should be close to

$$\sum_i |c_i|^2 (a_i + \langle m_0|A_M|m_0\rangle) - \langle A\rangle(t_0) = 0 \ . \qquad (30)$$

In our view there is nothing sacred about conservation laws. Like every other scientific law, they could be false. The same is true of the orthodox interpretation of quantum mechancics. The intent of our contribution is to show that there is a contradiction between these two ideas, both of which are adopted, perhaps, by the majority of physicists.

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