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## Supersymmetry and the Nelson-Barr Mechanism<sup>\*</sup>

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### Abstract

One possible solution to the strong CP problem is that CP is an exact symmetry, spontaneously broken at some scale. Some years ago, Nelson and Barr suggested a mechanism for obtaining  $\theta = 0$  at tree level in this framework, and showed that radiative corrections were small in some non-supersymmetric models. Further investigations suggested that the same could be true in supersymmetric theories. In this note, we show that such solutions assume extraordinarily high degrees of degeneracy among squark masses and among other supersymmetry breaking parameters. We argue, using naturalness as well as expectations from string theory, that this is not very plausible.

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# 1. Introduction

Since the strong CP problem was first recognized, three solutions have been suggested. The first scenario has no observable  $\theta$ -parameter, because  $m_u = 0$ .<sup>[1]</sup> The second gives  $\theta = 0$  dynamically, as the result of the existence of a Peccei-Quinn symmetry<sup>[2]</sup> and its resulting axion.<sup>[3]</sup> The third scenario posits that CP is an exact symmetry of the microscopic equations, which is spontaneously broken, in such a way that the effective  $\theta$  turns out to be small.<sup>[5,6,7,8]</sup> To date, there is no definitive experimental evidence on any of these possibilities. In first order chiral perturbation theory, one finds that  $m_u \neq 0$ . However, there is the possibility that higher order corrections may invalidate this conclusion, and there has been much debate in the literature about this possibility.<sup>[1]</sup> The axion solution is tightly constrained by astrophysical and cosmological considerations.<sup>[4]</sup> Within an order of magnitude or so, the lower limit on the axion decay constant,  $f_a$ , seems unassailable. The cosmological limit relies on assumptions about very early cosmology, which might not be correct, but which are extremely plausible. Perhaps the most promising suggestion for implementing spontaneous CP violation with small  $\theta$  is due to Nelson and Barr.<sup>[5,6]</sup> We will review this solution below. Apart from predicting that  $\theta$  is small, however, it is not clear that there are any generic low energy consequences of this picture; the resolution to the strong CP problem is found among some massive states, and the low energy theory is typically just a standard KM model.

In the absence of any definitive experimental answer, it is important to look more closely at how plausible each of these solutions may be from a purely theoretical perspective. Necessarily this requires making certain assumptions. It seems reasonable to assume, first, that any fundamental theory should not contain exact global continuous symmetries. This idea finds support both from string theory, where the only continuous symmetries are gauge symmetries,<sup>[9]</sup> and from considerations of the effects of quantum gravity on low-energy field theories. Discrete symmetries are, however, quite plausible. These do arise in string theory, where in general they appear to be gauge symmetries, and thus protected from violation by wormholes or other phenomena. From this viewpoint, discrete symmetries are almost certainly necessary if one is to implement the axion solution and might also be necessary for the  $m_u = 0$  solution to the strong CP problem. Consider first

$m_u = 0$ . If we assume, for the moment, that there is no new physics between  $m_W$  and  $m_{Pl}$ , all that is required is to suppress one Higgs coupling, and this is easily achieved with a discrete symmetry.<sup>†</sup> Higher dimension operators will make an extremely tiny contribution to, *e.g.*,  $d_n$ . Alternatively, in models in which the quarks obtain their mass from mixing with heavy vector-like isosinglet quarks,<sup>[11]</sup> a massless up quark could arise if there are fewer pairs of isosinglet up quarks than down quarks. In this case there is no need for a discrete symmetry which distinguishes it from the other quarks.

If we cannot impose global symmetries, Peccei-Quinn symmetries must be accidental consequences of gauge and discrete symmetries. In string theory, such Peccei-Quinn symmetries arise automatically, but the decay constants of the associated axions are of order  $m_{Pl}$ .<sup>[12]</sup> It is conceivable, as noted above, that this may be acceptable. If we insist, however, that the axion decay constant be of order  $10^{10} - 10^{11}$  GeV, something further is required. In Refs. 13, it was shown that, within the framework of models with low energy supersymmetry, discrete symmetries can, in principle, lead to such a symmetry. It is necessary, however, to suppress operators up to very high dimension, and this seems to require rather intricate patterns of discrete symmetry — certainly much more intricate than might be required for  $m_u = 0$ .

Finally, we turn to the real subject of this paper: the plausibility of the Nelson-Barr mechanism. The basic idea is to arrange, by a judicious choice of representations and symmetries, that the tree level quark mass matrix, while complex, has real determinant. For definiteness, we will focus in this paper on (supersymmetric) models in which, beyond the usual particles of the (minimal supersymmetric) standard model there are some additional isosinglet quarks with charge  $\pm 1/3$ , denoted  $q$  and  $\bar{q}$ . These particles gain mass of order some scale  $\mu$ . In addition, there are some standard model singlet particles,  $\mathcal{N}^i$ . The superpotential in the  $d$ -quark sector is assumed to take the form:

$$W_d = \lambda_{d,ij} Q^i \bar{d}^j H_1 + \mu q \bar{q} + \gamma_{ij} q \mathcal{N}^i \bar{d}^j \quad (1.1)$$

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<sup>†</sup> Note that this symmetry will suffer from an anomaly. However, in string theory, discrete symmetries with anomalies arise frequently. In the known examples, the discrete anomalies are identical for all groups.<sup>[10]</sup> In the present case, this might imply the masslessness of some neutrino(s), for example.

The parameter  $\mu$  may itself be the expectation value of some scalar field. The singlet vev's and  $\mu$  are typically of order some large intermediate or GUT mass scale. It is assumed that the vev's of the  $\mathcal{N}$  fields spontaneously break CP, while  $\mu$  is real. Then the fermion mass matrix takes the form:

$$m_F = \begin{pmatrix} A & B \\ 0 & \mu \end{pmatrix} \quad (1.2)$$

where the matrices  $A$  and  $\mu$  are real, while  $B$  is complex. This matrix has  $\arg \det m_F = 0$  and so the quark masses do not contribute to  $\theta$  at tree-level. The above is, essentially, the supersymmetrized version of the minimal Nelson-Barr model.<sup>[15]</sup> As we will explain later, one framework for obtaining a mass matrix of this type is provided by  $E_6$  theories, such as those which appear in superstring theories.

In all models of this type, however, there is no symmetry that explains the absence of  $\theta$  at tree level, and so one must ask what is the effect of radiative corrections. In non-supersymmetric theories, Nelson<sup>[5]</sup> showed that radiative corrections can be sufficiently small to give  $\theta < 10^{-9}$ . The supersymmetric case was studied by Barr and Masiero,<sup>[16]</sup> who argued, again, that radiative corrections were small. The main point of the present paper is that the latter analysis, while essentially correct, relied on a set of strong assumptions, which cannot be expected to hold in general: one must require an extremely high level of degeneracy among the squark masses, and one must also demand a very tight proportionality between certain pieces of the squark and quark mass matrices. We will argue that these conditions are quite difficult to satisfy; indeed, the limits from  $K^o$ - $\bar{K}^o$  mixing are already quite problematic in these models, and as we will see the limits from  $\theta$  are orders of magnitude stronger. We are left with the view that the  $m_u = 0$  or axion solutions of the strong CP problem are the most plausible presently known, at least within the framework of supersymmetry.

In the rest of this paper, we will elaborate upon these ideas. In the next section, we will briefly review the limits on degeneracy and proportionality which arise from the neutral kaon system, and then describe the potential one loop contributions to  $\theta$  which give rise to the various constraints. In the third section, we review some general aspects of soft-breaking terms in supersymmetric theories.

We explain why degeneracy is not expected to hold, in general. We use 't Hooft's naturalness criterion to argue for limits on degeneracy and proportionality.<sup>[17]</sup> Indeed, recent results in string-inspired models<sup>[19]</sup> support these limits. In Section 4, we enumerate the requirements on the theory imposed by  $K^o\text{-}\bar{K}^o$  and  $\theta$  in a generic Nelson-Barr model. We will see that they are quite severe. Some of these conditions are analyzed in more detail in Section 5. We will distinguish here two types of models. The most promising are a class of models considered recently by Barr and Segre,<sup>[18]</sup> in which large scales are generated by heavy  $\mathcal{N}$ -type fields which are much more massive than the susy-breaking scale,  $m_{susy}$ . Even for these, however, one needs to make a set of strong assumptions, which are not natural (in the sense of 't Hooft<sup>[17]</sup>). The other type of models we discuss are those in which large intermediate mass scales are generated by light  $\mathcal{N}$ -type fields with masses of order  $m_{susy}$ , as is typical in string-inspired models. We will argue that it is highly unlikely that these constraints can be satisfied in this instance either. In our conclusions, we will comment on possible "ways out."

## 2. One Loop Contributions to $K^o\text{-}\bar{K}^o$ and $\theta$

It is well known that the neutral kaon system requires that the squark mass spectrum exhibit a high degree of degeneracy. The strongest limits come from box diagrams with the exchange of gluinos.<sup>[20]</sup> From the contribution of these diagrams to the real part, the most stringent limit obtained is roughly of the form

$$\frac{\delta m_Q^2}{m_Q^2} \frac{\delta m_d^2}{m_d^2} \frac{1}{m_{susy}^2} \lesssim 10^{-10} \text{GeV}^{-2} \quad (2.1)$$

whereas from the imaginary part one obtains a limit (with a phase) two orders of magnitude stronger. There are also similar bounds involving only left-handed or right-handed squarks which are about an order of magnitude weaker. The quantities  $\delta m_Q^2$ ,  $\delta m_d^2$  are corrections to the left-handed and right handed degenerate down squark mass matrices,  $m_Q^2 \times \mathbf{1}$  and  $m_d^2 \times \mathbf{1}$ , respectively, and  $m_{susy}^2$  is typically of order the squark or gluino masses.\* These limits are already quite striking. How-

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\* In the quark mass eigenstate basis it is the  $ds$  entries which enter into the above constraints, but in the absence of a detailed theory of flavor it is natural to take all  $\delta m^2$  entries of same order.

ever, as we will see, in models with spontaneous CP violation, one loop corrections to  $\theta$  give even stronger constraints on squark degeneracy.

In conventional model-building, as in the minimal supersymmetric standard model, one usually assumes a very simple structure for the soft breaking contributions to the potential for the light fields. First, for the terms of the type  $\phi^*\phi$ , where  $\phi$  is some scalar field, one takes (using the same symbol for the scalar component of a multiplet as for the multiplet itself):

$$V_{\phi\phi^*} = m^2 \sum_i (|Q_i|^2 + |\bar{u}_i|^2 + |\bar{d}_i|^2). \quad (2.2)$$

Clearly this is a strong – perhaps one should say drastic – assumption. It is certainly violated by radiative corrections. Moreover, it is not enforced, in general, by any symmetry and one does not expect it to hold, in general, at tree level in any fundamental theory (it is not the case in string theory, for example<sup>[19]</sup>). One makes a similar, drastic assumption for the cubic terms in the potential: one assumes that they are exactly proportional to the superpotential,<sup>†</sup>

$$V_3 = AW(\phi) + \text{h.c.} \quad (2.3)$$

Again, this will not be respected by radiative corrections and will not hold, generically, at tree level. In the next section, we will consider, in the framework of hidden-sector supergravity models, what these assumptions mean. For now, we simply note that they must hold to a rather good approximation in order to avoid unacceptable flavor-changing neutral currents. Indeed, from the kaon system, the limits we mentioned above imply that these “degeneracy” and “proportionality” conditions must hold to a part in  $10^2$  or  $10^4$ , depending on the value of the supersymmetry-breaking scale and the nature of CP violation. In order that  $d_n$  be small enough, one has restrictions on the gluino and  $A$ -parameter phases as well.

In supersymmetric models of spontaneous CP violation where one has arranged vanishing of  $\theta$  at tree-level, there are a variety of possible contributions to

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<sup>†</sup> We are treating the superpotential here as a homogeneous polynomial. More general cases will be considered below.

$\theta$  at one-loop order. These are given by

$$\delta\theta = Im \text{Tr} [m_u^{-1}\delta m_u + m_d^{-1}\delta m_d] - 3\text{Arg} \tilde{m}_3 . \quad (2.4)$$

$\delta m_{u,d}$  represent the one-loop corrections to the tree level quark mass matrices,  $m_{u,d}$ , and  $\tilde{m}_3$  is the gluino mass including one-loop contributions. In order to analyze these quantities, we first need to make a few more stipulations about the underlying model. To obtain vanishing  $\theta$  at tree level the tree level gluino mass must be real (a non-zero phase represents a contribution to  $\theta$ ). All terms in the Higgs potential must also be real. To accomplish this we assume that supersymmetry breaking dynamics do not spontaneously break CP, so that at some large scale the theory is completely CP-invariant and, in particular, all supersymmetry breaking terms are real.

Nelson-Barr models will contain both light and heavy intermediate or GUT scale quark fields. Let us focus on the light fields and denote the corrections to degeneracy by

$$\delta V_{\phi\phi^*} = Q^*\delta\tilde{m}_Q^2 Q + \bar{d}\delta\tilde{m}_d^2 \bar{d}^* + \bar{u}\delta\tilde{m}_u^2 \bar{u}^* \quad (2.5)$$

and proportionality by

$$\delta V_3 = Q\delta A_d \bar{d} H_1 + Q\delta A_u \bar{u} H_2. \quad (2.6)$$

In general, integrating out the heavy fields leads to contributions to some of the above terms *which will be complex* due to spontaneous CP violation. Bounds on degeneracy from  $K^o-\bar{K}^o$  have been given above. As noted, there are similar bounds on proportionality, the most stringent one for the real part given by<sup>[20]</sup>

$$\frac{\delta A_d \langle H_1 \rangle}{m_{susy}^3} \lesssim 10^{-5} \text{GeV}^{-1}, \quad (2.7)$$

while the limit for the imaginary part again is two orders of magnitude stronger.

At the one-loop level there are a variety of contributions to  $\theta$ , indicated in Figs. 1 and 2, in which the terms of eqs. (2.5) and (2.6) appear as mass insertions. Fig. 1 is the one-loop correction to the gluino mass. This will be complex, for

example, if the corrections to the  $A$  parameters in eq. (2.6) are complex. To satisfy the bound on  $\theta$ , this graph will require that a certain relation hold for the soft-breaking masses of heavy fields to about a part in  $10^7$ .

Fig. 2 corresponds to corrections to  $\theta$  coming from corrections to the quark masses. In the limit of exact degeneracy and proportionality, these graphs yield contributions to  $\theta$  proportional to  $\text{Tr } m_f m_f^{-1}$ , which are clearly real. ( $m_f$  here denotes the light fermion mass matrices). Insertions of  $\delta V_{\phi\phi^*}$  and  $\delta V_3$  can yield complex corrections. One dangerous possibility arises from two insertions of  $\delta V_{\phi\phi^*}$ . This leads to a correction to the  $d$ -quark mass which is in general complex, and proportional to  $m_b$  and off-diagonal terms in  $\delta\tilde{m}_Q^2$  and  $\delta\tilde{m}_d^2$ . These terms will therefore have to satisfy stringent limits. A simple calculation gives:

$$\frac{(\delta\tilde{m}_Q^2)(\delta\tilde{m}_d^2)}{m_{susy}^4} < 10^{-9} \quad (2.8)$$

where  $m_{susy}$ , again, refers to some typical supersymmetry-breaking mass. The limit is so strong because a factor  $O(m_b/m_d)$  appears in the correction to  $\theta$ .

One might imagine that one would get extremely strong limits on proportionality from this graph as well; in general it will give a contribution to  $\theta$  of order  $\alpha_s/\pi \text{Im Tr } \delta A_d m_f^{-1}$ . It turns out, however, that in Nelson-Barr models this trace is real. Stringent limits do arise on products of  $\delta A$  and  $\delta\tilde{m}^2$  type terms, *e.g.*,

$$\frac{(\delta A_d)(\delta\tilde{m}_d^2)}{m_{susy}^3} < 10^{-9}. \quad (2.9)$$

These are striking constraints, which, in general, are considerably stronger than the  $K^o$ - $\bar{K}^o$  limits discussed above. In the rest of this paper we will ask whether they might plausibly be satisfied.



### 3. Some general aspects of soft-breaking terms in supersymmetric theories.

In this section, we review some aspects of supersymmetric theories, and explain the origin of the non-degenerate and non-proportional terms. For definiteness, we consider the case of  $N = 1$  supergravity theories, with supersymmetry broken in a hidden sector. Such a theory is described, in general, by three functions, the Kähler potential,  $K$ , the superpotential  $W$ , and a function  $f$  which describes the gauge couplings. Here we focus on the form of the scalar potential. Defining a metric on the space of fields by

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^{j*}} \quad (3.1)$$

and defining also

$$d_i = \frac{\partial K}{\partial \phi^i}$$

the general potential is

$$V = e^K \left[ \left( \frac{\partial W}{\partial \phi^i} + d_i W \right) g^{i\bar{j}} \left( \frac{\partial W}{\partial \phi^{\bar{j}}} + d_{\bar{j}} W \right)^* - 3|W|^2 \right]. \quad (3.2)$$

Our assumption that supersymmetry is broken in a hidden sector means that there are two sets of fields:  $z_i$ , responsible for supersymmetry breaking, and the “visible sector fields,”  $y_i$ ;

$$W = g(y) + h(z). \quad (3.3)$$

The scales are such that

$$\left( \frac{\partial h}{\partial z^i} + \frac{\partial K}{\partial z^i} h \right) \sim m_{susy} m_{Pl}. \quad (3.4)$$

As discussed long ago by Hall, Lykken and Weinberg,<sup>[21]</sup> universality is the assumption that there is an approximate  $U(n)$  symmetry of the Kähler potential, where

$n$  is the number of chiral multiplets in the theory. Frequently one takes simply

$$K = \sum \phi_i^* \phi_i. \quad (3.5)$$

Clearly, this is an extremely strong assumption. The Yukawa couplings of the theory exhibit no such symmetry. It does not hold, for example, for a generic superstring compactification, where the symmetry violations are simply  $\mathcal{O}(1)$ .<sup>\*</sup> One can try to invent scenarios to explain some approximate flavor symmetry. However, as we will discuss below, 't Hooft's naturalness condition suggests a limit on how successful any such program can be.

We can characterize the violations of universality quite precisely. For small  $y$ , we can expand  $K$  in powers of  $y$ . Rescaling the fields, we can write<sup>†</sup>

$$K = k(z, z^*) + y_i y_i^* + \ell_{ij}(z, z^*) y_i y_j^* + h_{ij}(z, z^*) y_i y_j + \dots \quad (3.6)$$

There is no reason, in general, why  $\ell_{ij}$  should be proportional to the unit matrix, so the  $zz^*$  components of the metric will contain terms involving  $y_i y_j^*$  which are non-universal. Plugging into eq. (3.2) yields non-universal mass terms for the visible sector fields. In general, there is no symmetry which can forbid these couplings. For example,  $\ell = z^* z$  cannot be eliminated by symmetries.

Violations of proportionality arise in a similar manner. The term  $\ell_{ij}$  in the Kähler potential leads to off-diagonal terms in the  $z^* y_i$  terms in the metric. These in turn, from eq. (3.2), lead to non-universal corrections to the cubic couplings. One might imagine forbidding the dangerous terms by symmetries. In particular, if no hidden-sector fields have Planck scale vev's (as in models of gluino condensation) and if couplings of the type  $zy^*y$  are forbidden by symmetries, then these corrections to the metric would be suppressed. However, any such symmetry would also forbid a coupling of the hidden sector fields to the gauge fields, required in order to obtain a gaugino mass.

Recently, it has been found<sup>[19]</sup> that string theories give a soft supersymmetry-violating sector with degenerate scalars at tree level if the dilaton F-terms dom-

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<sup>\*</sup> For a review of the relevant issues, see Ref. 19.

<sup>†</sup> Requiring that the  $y_i$ 's be canonically normalized gives a condition on  $\langle \ell_{ij} \rangle$ . But  $\ell_{ij} \neq 0$ .

inate over moduli. This is typically not the case when the most important non-perturbative effect is gaugino condensation. However, even if the dilaton does dominate, one expects universality to break down at order  $\alpha_{str}/\pi$ .

So we see that neither degeneracy nor proportionality are assumptions which we would expect to hold generically. The most obvious way to explain such features is to suppose that there is an underlying flavor symmetry. Any such symmetry must be approximate (it may arise, for example, from a gauge symmetry broken at some energy scale; see Ref. 22 for a recent effort to build such models). Even without considering a specific theory with such a flavor symmetry, we can apply 't Hooft's naturalness criterion<sup>[17]</sup> to assess the plausibility of a given degree of degeneracy or proportionality. In particular, we need to ask whether the theory acquires greater symmetry as we assume that some condition on masses holds. Thus, for example, we don't expect degeneracy or proportionality between color-neutral scalars and color-triplet squarks to hold to better than  $\alpha_s/\pi$ , or degeneracy between fields with different  $SU_L(2)$  quantum numbers to hold to better than  $\alpha_W/\pi$ . Similarly, we don't expect degeneracy for different flavors with the same gauge quantum numbers to hold to better than powers of Yukawa couplings.

#### 4. One loop corrections to $\theta$ in a Generic Model

The supersymmetric Nelson-Barr model we'll consider is the minimal one described in the introduction: in addition to the usual quark and lepton families, we have an additional pair of isosinglet down quark fields,  $q$  and  $\bar{q}$ , as well as some singlet fields,  $\mathcal{N}_i$  and  $\bar{\mathcal{N}}_i$ . It is straightforward to consider models with several  $q$  and  $\bar{q}$  fields, and with additional types of singlets. The terms in the superpotential which give rise to the quark mass matrix are

$$W = \mu q \bar{q} + \gamma^{ij} \mathcal{N}_i q \bar{d}_j + H_1 \lambda_{ij} Q_i \bar{d}_j \quad (4.1)$$

(the terms in the superpotential involving  $u$  quarks and leptons will not be important for our considerations).  $Q\bar{q}$  terms can be forbidden via either gauged  $U(1)$  or discrete symmetries.

One framework for obtaining a mass matrix of this kind is suggested by  $E_6$  models, such as those which appear in superstring theories.<sup>‡</sup> In these models,

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‡ Some of the remarks here have appeared earlier in work of Frampton and Kephart.<sup>[23]</sup>

generations of quarks and leptons arise from the **27** representation. Under  $O(10)$ , the **27** decomposes as a **16**, a **10** and a **1**. The **16** contains an ordinary generation of quarks and leptons, plus a field which we denote by  $\mathcal{N}$ ; the **10** contains an additional  $SU(2)$ -singlet quark and antiquark pair,  $q$  and  $\bar{q}$ . We denote the  $O(10)$ -singlet by  $S$ . Suppose that  $E_6$  is broken at a high scale down to a rank-6 subgroup, such as  $SU(3) \times SU(2) \times U(1)_Y \times U(1)_a \times U(1)_b$ . Then the dimension-four terms in the superpotential include the couplings

$$W_d = Q\lambda\bar{d}^j H_1 + \lambda_S S q \bar{q} + q \mathcal{N} \gamma \bar{d} \quad (4.2)$$

(here we have adopted a matrix notation for the various Yukawa couplings; in general there will be several  $S$ ,  $\mathcal{N}$ ,  $q$  and  $\bar{q}$  fields). If the fields  $S$  have real vev's, while the  $\mathcal{N}$  fields have  $CP$ -violating, complex vev's, the mass matrix has the structure of eq. (1.2). Moreover, plausible mechanisms have been suggested for obtaining such vev's. In particular, in "intermediate scale scenarios," it has been noted that the  $S$  and  $\mathcal{N}$  fields can readily obtain vev's of order  $m_I = \sqrt{m_{3/2} m_{Pl}}$ .<sup>[14]</sup> One can easily check that for a finite range of parameters, the  $\mathcal{N}$  vev's can be complex while the  $S$  vev's are real. In such a model, it is necessary to forbid a variety of other couplings if one is to avoid a tree-level contribution to  $\theta$  and to meet other phenomenological requirements. This can be done using discrete symmetries. We will not present an example here, however, since, as we will see, loop corrections almost inevitably lead to serious problems.

Before estimating  $\theta$ , it is important to first consider the questions of degeneracy and proportionality in models of this kind. We will assume that  $\mu$  is some large scale, such as  $10^{11}$  GeV. Our concern, then, is whether or not the *light* squark mass matrix exhibits degeneracy and proportionality. This requires that we integrate out the fields with mass of order  $\mu$ . Let us first examine the theory at scale  $\mu$ . Because the soft-breaking terms are much smaller than  $\mu$ , it is helpful to consider what the theory looks like in their absence. For both the left-handed ( $q, Q$ ) and right-handed ( $\bar{q}, \bar{d}$ ) sectors there is one state with a mass of order  $\mu$ , and three light states. For the left-handed states, the massive state is simply  $q$ ; for the right-handed states, it is

$$\bar{D} = \left( \frac{1}{m_D^2 + \mu^2} \right)^{1/2} (M_D a^i \bar{d}_i + \mu \bar{q}) \quad (4.3)$$

where

$$M_D^2 = |\gamma^{ji} \mathcal{N}_j|^2 \quad (4.4)$$

and

$$a^i = \frac{1}{M_D} \gamma^{ji} \mathcal{N}_j \quad (4.5)$$

Note that we have defined  $\vec{a}$  so that  $\vec{a}^\dagger \vec{a} = 1$ .

The relevant soft breaking terms at this scale are of two types. Using the same letter for the scalar component of a supermultiplet as for the multiplet itself, the  $\phi\phi^*$  type terms are of the form

$$V_1 = Q^* \tilde{m}_Q^2 Q + \bar{d} \tilde{m}_d^2 \bar{d}^* + \tilde{m}_q^2 q q^* + \tilde{m}_{\bar{q}}^2 \bar{q} \bar{q}^* \quad (4.6)$$

Here,  $\tilde{m}_d^2$ ,  $\tilde{m}_Q^2$ , *etc.*, are assumed to be of order  $m_{susy}^2$ . The  $\phi\phi$  and  $\phi\phi\phi$  terms are of the form:

$$V_2 = Q A_d \bar{d} H_1 + A_\mu \mu q \bar{q} + \mathcal{N} A_\gamma q \bar{d} \quad (4.7)$$

We have defined the  $A$ 's so that they are dimensionful quantities of order  $m_{susy}$ .  $A_d$  and  $A_\gamma$  are matrices. Note our definition of  $A_d$  differs from that of the previous section, where  $A_d$  was defined on the *light states only*.

As we'll see, near degeneracy of the full  $4 \times 4$  right-handed down squark mass matrix and of  $\tilde{m}_Q^2$  will be required, so we write

$$\tilde{m}_d^2 = \tilde{m}_d^2 \times \mathbf{1} + \delta m_d^2; \quad \tilde{m}_{\bar{q}}^2 = \tilde{m}_{\bar{q}}^2 + \delta \tilde{m}_{\bar{q}}^2; \quad \tilde{m}_Q^2 = \tilde{m}_Q^2 \times \mathbf{1} + \delta \tilde{m}_Q^2. \quad (4.8)$$

Near proportionality of  $A_d$  is also necessary and we write

$$A_d \equiv A_d \lambda_d + \delta A_d. \quad (4.9)$$

(Again, the notation  $\lambda_d$  is being used in a different sense than previously.)

Let us now examine the form of the various mass matrices. Calling  $m_d = \lambda_d H_1$ , the fermion mass matrix and its inverse have the form

$$m_F = \begin{pmatrix} m_d & M_D \vec{a} \\ 0 & \mu \end{pmatrix} \quad m_F^{-1} = \begin{pmatrix} m_d^{-1} & \frac{M_D}{\mu} m_d^{-1} \vec{a} \\ 0 & \mu^{-1} \end{pmatrix}. \quad (4.10)$$

The matrix  $m_F$  has the Nelson-Barr form, and its determinant is real.

In view of our remarks in Section 2, it is the form of the scalar mass matrices which particularly concerns us. Consider first the  $\phi\phi^*$  type terms. For the squarks in the  $\mathbf{3}$  representation of  $SU(3)$ , these take the form, on the full  $4 \times 4$  set of states:

$$\mathcal{M}_{LL}^2 = \begin{pmatrix} \tilde{m}_d^2 + m_d^T m_d & M_D m_d^T \vec{a} \\ \vec{a}^\dagger m_d M_D & \tilde{m}_q^2 + M_D^2 + \mu^2 \end{pmatrix}. \quad (4.11)$$

Similarly, for the  $\bar{\mathbf{3}}$  squarks (“right-handed” squarks), we have:

$$\mathcal{M}_{RR}^2 = \begin{pmatrix} \tilde{m}_{\bar{d}}^2 + m_d m_d^T + M_D^2 \vec{a} \vec{a}^\dagger & \mu M_D \vec{a} \\ \mu M_D \vec{a}^\dagger & \tilde{m}_{\bar{q}}^2 + \mu^2 \end{pmatrix}. \quad (4.12)$$

Finally, for the  $\phi\phi$ -type matrix, which connects the  $\mathbf{3}$  and  $\bar{\mathbf{3}}$  squarks, we have

$$\mathcal{M}_{RL}^2 = \begin{pmatrix} A_d \langle H_1 \rangle + \mu_H \frac{H_2}{H_1} m_d & M_5^2 \vec{b} \\ 0 & A_{\mu\mu} \end{pmatrix} \quad (4.13)$$

where  $\mu_H$  is the coefficient of the  $H_1 H_2$  term in the superpotential, and  $M_5$  and  $\vec{b}$  are defined by

$$M_5^2 b^i = A_\gamma^{ji} \mathcal{N}_j + \left( \frac{\partial W}{\partial \mathcal{N}_j} \right)^* \gamma^{ji} \quad ; \quad \vec{b}^\dagger \vec{b} = 1 \quad (4.14)$$

Note, in general,  $\vec{a}$  is not proportional to  $\vec{b}$  (see Section 5).

We are now in a position to integrate out the heavy fields to obtain an effective lagrangian for the light fields. The first question we should address is that of degeneracy and proportionality. Even before considering  $\theta$ , degeneracy is necessary to understand the properties of neutral kaons, and proportionality is required to suppress other contributions to  $d_n$ . To construct the mass matrices for the light fields and examine these questions, it is convenient to introduce projection operators onto the light and heavy states, in the supersymmetric limit (*i.e.*, ignoring corrections of order  $m_{susy}$  or  $\langle H \rangle$ ). The projector onto the heavy “left-handed”

states is just

$$\mathbf{P}_L^h = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (4.15)$$

The projector onto the light states is simply:

$$\mathbf{P}_L^l = \mathbf{1} - \mathbf{P}_L^h. \quad (4.16)$$

The projectors onto the right-handed states are slightly more complicated. Onto the massive state, it is

$$\mathbf{P}_R^h = \left( \frac{1}{M_D^2 + \mu^2} \right) \begin{pmatrix} M_D^2 \vec{a} \vec{a}^\dagger & \mu M_D \vec{a} \\ \mu M_D \vec{a}^\dagger & \mu^2 \end{pmatrix}. \quad (4.17)$$

The light projector is then just  $\mathbf{P}_R^l = \mathbf{1} - \mathbf{P}_R^h$ .

With these we can immediately project the fermion mass matrix onto the light states:

$$m_F^l = \mathbf{P}_R^l m_F \mathbf{P}_L^l = \begin{pmatrix} \left( \mathbf{1} - \frac{M_D^2}{M_D^2 + \mu^2} \vec{a} \vec{a}^\dagger \right) m_d & 0 \\ -\frac{\mu M_D}{M_D^2 + \mu^2} \vec{a}^\dagger m_d & 0 \end{pmatrix}. \quad (4.18)$$

To work out the scalar matrices requires somewhat greater care. In addition to performing a projection of the type described above, there are couplings of heavy to light states. Taking these into account, and integrating out the heavy states, as in Fig. 3, yields additional corrections to the light squark mass matrices. Consider, first, the result of projecting the matrices onto the light states. For the left-handed squarks, one has

$$\mathcal{M}_{LL}^l = \begin{pmatrix} \tilde{m}_d^2 + m_d^T m_d & 0 \\ 0 & 0 \end{pmatrix} \quad (4.19)$$

If  $\tilde{m}_d^2$  is proportional to the unit matrix, ignoring the terms proportional to fermion masses, this expression is proportional to  $\mathbf{P}_L^l$ , *i.e.*, for degeneracy in the left-handed sector, it is sufficient that the  $3 \times 3$  matrix  $\tilde{m}_d^2$  be degenerate.

The situation for the right-handed squarks is more complicated. Here one has

$$\mathcal{M}_{RR}^{l,2} = \mathbf{P}_R^l \mathcal{M}_{RR}^2 \mathbf{P}_R^l. \quad (4.20)$$

Rather than write down this expression in detail, let us simply consider a particular case as an example; suppose in eq. (4.7) we set  $\delta\tilde{m}_{\bar{d}}^2 = 0$ . Then, ignoring terms proportional to quark masses,

$$\mathcal{M}_{RR}^{l,2} = \tilde{m}_{\bar{d}}^2 \mathbf{P}_R^l + \delta\tilde{m}_{\bar{q}}^2 \frac{M_D^2}{(M_D^2 + \mu^2)^2} \begin{pmatrix} \mu^2 \vec{a} \vec{a}^\dagger & -\mu M_D \vec{a} \\ -\mu M_D \vec{a}^\dagger & M_D^2 \end{pmatrix}. \quad (4.21)$$

$\delta m_{\bar{q}}^2 \approx 0$ , then, is required in order to obtain degeneracy, *i.e.*, we need  $4 \times 4$  degeneracy if the light  $\bar{d}$ -squark matrix is to be degenerate. In terms of the naturalness criteria we developed in the preceding section, this means that the  $\bar{q}$  and  $\bar{d}$  fields should have the same gauge quantum numbers; this will certainly not be the case in models which use gauge symmetries to obtain the Nelson-Barr structure. Moreover, all of the Yukawa couplings involving the  $\bar{q}$  fields must be small.

This is not the whole story, however, since, in the basis we have chosen, there are in general couplings of the light squarks to the heavy squarks of order  $m_{susy}\mu$ . Thus integrating out heavy squarks will give additional contributions to the light squark mass matrices of order  $m_{susy}^2$ . A simple calculation shows that the only light-heavy couplings of order  $\mu$  arise from the matrix  $\mathcal{M}_{RL}^2$  and involve the couplings of the massive  $q$  to the light  $\bar{d}$  fields. The required coupling is described by the matrix:

$$\mathcal{M}_{lh}^2 = \mathbf{P}_R^l \mathcal{M}_{RL}^2 \mathbf{P}_L^h = \begin{pmatrix} 0 & M_5^2 \left( \mathbf{1} - \frac{M_D^2}{M_D^2 + \mu^2} \vec{a} \vec{a}^\dagger \right) \vec{b} - \frac{\mu^2 A_\mu M_D}{M_D^2 + \mu^2} \vec{a} \\ 0 & -\frac{\mu M_D M_5^2}{M_D^2 + \mu^2} \vec{a}^\dagger \vec{b} + \frac{\mu A_\mu M_D^2}{M_D^2 + \mu^2} \end{pmatrix}. \quad (4.22)$$

Note that this coupling vanishes if  $\vec{a} = \vec{b}$  and  $M_5^2 = A_\mu M_D$ . If these conditions do not hold, integrating out the massive field then leads to a shift in  $\mathcal{M}_{RR}^{l,2}$ ,

$$\delta \mathcal{M}_R^{l,2} = \mathcal{M}_{lh}^2 \frac{1}{M_D^2 + \mu^2} \mathcal{M}_{lh}^{2\dagger}. \quad (4.23)$$

The resulting expression is rather involved. The main point, however, is that the expression is not proportional to  $\mathbf{P}_R^l$ , so these terms are not degenerate. In fact,



generically, the resulting non-degeneracy would exceed the bounds from  $K^o\text{-}\bar{K}^o$ . One can attempt to suppress  $\delta\mathcal{M}_R^{l,2}$  by a judicious choice of parameters. For example, for  $\vec{b} = \vec{a}$  and  $\mu \ll M_D$ , these couplings are suppressed by  $\mu^2/M_D^2$ . The ratio  $\mu^2/M_D^2$  can be sufficiently small, *e.g.*, of order  $10^{-4}$ , and still permit a realistic light quark mass matrix. A hierarchy of entries in  $\vec{b}$  reflecting the hierarchy of entries in  $m_d$ , *e.g.*,  $b_{1,2} \ll b_3$ , can also suppress the dangerous  $sd$ -entries in  $\delta\mathcal{M}_R^{l,2}$ . As we will see shortly, there is also a suppression for large  $\mu/M_D$ ; however, in this limit, it is easy to see that the KM phase is likewise suppressed.<sup>★</sup>

In string-inspired models with low energy generation of intermediate mass scales, the coupling of eq. (4.22) does not vanish. Recently, Barr and Segre<sup>[18]</sup> have argued that the conditions  $\vec{a} = \vec{b}$  and  $M_5^2 = A_\mu M_D$  will hold provided that the  $\mathcal{N}$  fields are much more massive than  $m_{susy}$ , given certain assumptions about proportionality. However, as we will see in the next section, these assumptions violate the naturalness criteria we have set forth; one expects that this condition cannot hold to better than  $\alpha_s/\pi$ .

In order to insure near proportionality for the light states to satisfy the limits from  $K^o\text{-}\bar{K}^o$  it is sufficient that the  $3 \times 3$  block of  $\mathcal{M}_{RL}^2$  be approximately proportional, as in eq. (4.9). In this case, the light left-right squark mass matrix has the form

$$\mathcal{M}_{RL}^{l,2} = \mathbf{P}_R^l \mathcal{M}_{RL}^2 \mathbf{P}_L^l = (A_d + \mu_H \frac{H_2}{H_1}) m_F^l + \delta A_d^l \langle H_1 \rangle, \quad (4.24)$$

where  $\delta A_d^l$  is obtained from eq. (4.18) by replacing  $m_d$  with  $\delta A_d$ . So approximate proportionality holds for the light quark and squark states. Integrating out the heavy fields at tree level yields only contributions suppressed by powers of  $m_{susy}/\mu$ .

So, even before worrying about  $\theta$ , we see that in supersymmetrized Nelson-Barr models there is the potential for severe violations of degeneracy, even making the assumptions typical of supergravity models. In addition to assuming that the various  $3 \times 3$  parts of the squark mass matrices are nearly degenerate and proportional at the high energy scale, we need that the  $\bar{d}$ -squark mass matrix have a  $4 \times 4$  degeneracy, to about a part in  $10^3$ , and that dangerous entries in the matrix  $\delta\mathcal{M}_R^{l,2}$  be very small. These conditions are quite disturbing. Even if there is exact

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★ If the KM phase is suppressed, one is lead to consider models where susy box diagrams dominate  $\epsilon$ .

degeneracy of the  $4 \times 4$  squark mass matrix at tree level, loop corrections will violate it. These will typically be enhanced by large logarithms. In order that one-loop corrections not be too large, the Yukawa couplings,  $\gamma^{ij}$ , must be small, about  $10^{-2}$  or smaller. If the  $\bar{q}$  and  $\bar{d}$  fields carry different gauge quantum numbers, as would be typical in simple string models, one expects corrections of order  $\frac{\alpha}{\pi} \ln(m_{Pl}^2/\mu^2)$ . In the following, we will simply assume that this condition is somehow satisfied, *e.g.*, through small couplings and suitable quantum numbers. But already we view this as unattractive.

Let us now ask how severe are the constraints arising from the smallness of  $\theta$ . First consider the gluino mass diagram. This receives contributions not only from light states but from heavy states as well. The result is easy to work out in terms of the projectors above, and it is complex in general if  $\vec{a} \neq \vec{b}$ :

$$\text{Im } \delta m_\lambda \sim \frac{\alpha_s}{4\pi} \frac{M_D^2}{(M_D^2 + \mu^2)} \frac{M_5^2}{M_D} \text{Im } \vec{a}^\dagger \vec{b}. \quad (4.25)$$

Indeed, this diagram leads to the requirement that the phases of these vectors line up to about one part in  $10^{-7}$ . Certainly the simplest way to satisfy this is  $\vec{b} = \vec{a}$ ; we will assume this to be the case in the remainder of this section. In the next section, we will investigate this condition and argue that it is not natural. The light fermion contributions to the gluino mass lead to a weaker limit on proportionality. If  $A_d$  is not proportional to the unit matrix, one will obtain a complex result, in general. This will give a limit suppressed by powers of the  $b$ -quark mass over the susy-breaking scale:

$$\frac{\text{Im } \vec{a}^\dagger \delta A_d m_d^T \vec{a}}{m_{susy}^3} \frac{\langle H_1 \rangle M_D^2}{M_D^2 + \mu^2} \lesssim 10^{-7}. \quad (4.26)$$

More significant limits arise from the graphs of Fig. 2. From one proportionality violating insertion and one degeneracy violating insertion we obtain:

$$\frac{\text{Im } \vec{a}^\dagger \delta A_d \lambda_d^{-1} \left( \delta \tilde{m}_d^2 - \delta \tilde{m}_q^2 \times \mathbf{1} \right) \vec{a}}{m_{susy}^3} \frac{M_D^2}{M_D^2 + \mu^2} \lesssim 10^{-6}, \quad (4.27)$$

and

$$\frac{\text{Im } \vec{a}^\dagger \delta A_d \lambda_d^{-1} \vec{a}}{m_{susy}^3} \frac{(M_5^2 - A_\mu M_D)^2 \mu^2}{(M_D^2 + \mu^2)^2} \lesssim 10^{-6}. \quad (4.28)$$

The second equation is essentially due to the contribution of  $\delta \mathcal{M}_R^{l,2}$ , *i.e.*, of integrating out heavy fields, to right-handed squark non-degeneracy.

From Fig. 2, with two degeneracy-violating insertions, we find contributions to  $\theta$  of order:

$$10^{-1} \frac{\alpha_s}{4\pi} \frac{A_d \text{Im } \vec{a}^\dagger m_d \delta \tilde{m}_Q^2 m_d^{-1} (\delta \tilde{m}_d^2 - \delta \tilde{m}_{\bar{q}}^2 \times \mathbf{1}) \vec{a}}{m_{susy}^5} \frac{M_D^2}{M_D^2 + \mu^2}, \quad (4.29)$$

which lead to the constraints

$$\frac{(\delta \tilde{m}_{\bar{q}}^2, \delta \tilde{m}_d^2)}{m_{susy}^2} \frac{(\delta \tilde{m}_Q^2)}{m_{susy}^2} \lesssim 10^{-9}. \quad (4.30)$$

Non-degeneracy due to  $\delta \mathcal{M}_R^{l,2}$  leads to a contribution of order

$$10^{-1} \frac{\alpha_s}{4\pi} \frac{\text{Im } \vec{a}^\dagger m_d \delta \tilde{m}_d^2 m_d^{-1} \vec{a} (M_5^2 - A_\mu M_D) M_D}{m_{susy}^3 (M_D^2 + \mu^2)}. \quad (4.31)$$

How plausible is it that one can satisfy these constraints? The gluino diagram constraint is satisfied by  $\vec{b} = \vec{a}$ ; we will explore in the next section the meaning of this condition. The degeneracy and proportionality constraints require satisfying limits on  $\delta \tilde{m}_d^2$ ,  $\delta \tilde{m}_{\bar{d}}^2$ ,  $\delta \tilde{m}_{\bar{q}}^2$  and  $\delta A_d$ , which are considerably more stringent than those obtained from  $K^o$ - $\bar{K}^o$  mixing. It is hard to comprehend how they could be satisfied in the absence of a detailed theory of flavor. These constraints require as well a condition on  $M_5$ , or perhaps some other condition on parameters. For example, the equality  $M_5^2 = M_D A_\mu$  would eliminate light-heavy squark couplings, so that contributions to  $\theta$  which arise from integrating out heavy fields would vanish, see eqs. (4.27) and (4.30). However, this condition, as we will see in the next section, requires exact degeneracy (at the Planck scale); again, naturalness arguments preclude this possibility. Alternatively, one could try to exploit the fact

that  $M_D \ll \mu$  would suppress all of the above contributions to  $\theta$ . Unfortunately, this strategy is limited by the fact that the induced KM phase or the phase entering the susy box graph would be of order  $\frac{M_D^2}{\mu^2}$  and so this ratio must be  $\gtrsim 10^{-2}$  in order to generate large enough  $\epsilon$ .

## 5. The $\vec{b} = \vec{a}$ and $M_5^2 = M_D A_\mu$ constraints

We would now like to comment on whether or not one is likely to find  $\vec{a} = \vec{b}$  in a given model. In light of some recent observations of Barr and Segre,<sup>[18]</sup> we will distinguish two cases: one, suggested by string-inspired models, where the  $\mathcal{N}$  fields have masses of order  $m_{susy}$  yet generate large intermediate mass scales by exploiting (approximate) F-flat and D-flat directions, and one in which the  $\mathcal{N}$  fields have much larger mass. The potential for trouble exists because both  $\phi\phi\phi$  soft breaking terms and F-terms contribute to off-diagonal components of the squark mass matrix. The calculation of Barr and Segre is particularly simple, and we will describe it first. Suppose that the fields,  $\mathcal{N}$ , couple to some other fields,  $Y$ , and that all of these fields have mass (at the minimum of the potential) much greater than  $m_{susy}$ . Motivated by supergravity models, these authors suppose that the holomorphic soft breaking terms in the potential have the form

$$V_{soft} = a m_{susy} W + b m_{susy} \sum \phi_i \frac{\partial W}{\partial \phi_i} + \text{h.c.} \quad (5.1)$$

Now, because the masses of the  $\mathcal{N}$  and  $Y$  fields are assumed to be large, it follows that

$$m_{susy} \frac{\partial W}{\partial \phi_i} \ll \frac{\partial^2 W}{\partial \phi^2} \quad (5.2)$$

Similarly, in obtaining the minimum, one can neglect the soft-breaking terms of the type  $|\mathcal{N}|^2$ , *etc.* As a result, at the minimum, the potential satisfies

$$m_{susy} b \phi_j \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} = - \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \frac{\partial W^*}{\partial \phi_j^*} \quad (5.3)$$

But this gives immediately that

$$\frac{\partial W^*}{\partial \phi_i^*} = -b m_{susy} \phi_i \quad (5.4)$$

From this it follows from eq. (4.14) that  $\vec{a} = \vec{b}$ .

Similarly, with these assumptions, it follows that  $m_{\xi}^2 = A_{\mu}M_D$ . To see this, note first that  $A_{\mu} = a + 2b$ . Also,  $A_{\gamma} = a + 3b$  in eq. (4.14). So substituting (5.4) in eq. (4.14), we immediately obtain the desired equality. It is crucial here that the parameter  $b$  in eq. (5.1) is common to all terms in the superpotential.

However, from our earlier discussions of proportionality, we see that this is not natural in the sense of 't Hooft. For example, the  $a$  and  $b$  parameters for the  $q$  and  $\mathcal{N}$  fields, *i.e.*, for  $A_{\gamma}$ ,  $A_{\mu}$  and for the purely singlet scalar potential, would be expected to differ by amounts at least of order  $\frac{\alpha_s}{\pi}$ , since the  $\mathcal{N}$  fields do not carry color. This, in turn, leads to violations of the  $M_{\xi}^2 = A_{\mu}M_D$  condition of this order. This leads to unacceptably large  $\theta$ ; one is forced to try and suppress  $\theta$  by further choice of parameters (*e.g.*, by taking  $M_D/\mu$  very small, as discussed in the last section). This prospect does not appear to be feasible.

In models in which the  $\mathcal{N}$  fields have masses of order  $m_{susy}$ , the situation is no better. In intermediate scale models, in addition to the  $\mathcal{N}$  fields, one adds  $\bar{\mathcal{N}}$  fields with opposite gauge quantum numbers. Intermediate mass scales are generated along the resulting (approximate) F- and D-flat directions of the scalar potential. As before, from eq. (4.14), the condition  $\vec{a} = \vec{b}$  will automatically be satisfied if, at the minimum of the potential, we have:

$$\frac{\partial W}{\partial \mathcal{N}_a} = \alpha m_{susy} \mathcal{N}_a^* \quad (5.5)$$

for some (real)  $\alpha$ . This condition is not completely implausible but we will see that it relies on a certain form of the superpotential and soft breaking terms. Denote the part of the superpotential depending on the fields  $\mathcal{N}$  and  $\bar{\mathcal{N}}$  as  $g(\mathcal{N}, \bar{\mathcal{N}})$ . In the string-inspired intermediate scale scenario,  $g(\mathcal{N}, \bar{\mathcal{N}})$  is of dimension five, *e.g.*, containing terms of the form  $\frac{\mathcal{N}^2 \bar{\mathcal{N}}^2}{m_{Pl}}$ . Extremization of the potential gives:

$$\frac{\partial^2 g}{\partial \mathcal{N}_a \partial \mathcal{N}_b} \frac{\partial g^*}{\partial \mathcal{N}_b^*} + \frac{\partial^2 g}{\partial \mathcal{N}_a \partial \bar{\mathcal{N}}_b} \frac{\partial g^*}{\partial \bar{\mathcal{N}}_b^*} + \frac{\partial V_{SSB}}{\partial \mathcal{N}_a} + \sum_i e_{\mathcal{N}}^{(i)} D^{(i)} \mathcal{N}_a^* = 0 \quad (5.6)$$

with similar equations for  $\bar{\mathcal{N}}$ . (Here  $V_{SSB}$  is the susy-breaking potential and the last term arises from the  $D^2$  terms in the potential, with  $e_{\mathcal{N}}^{(i)}$  the coresponding charges.) All terms in this equation are of order  $m_{susy}^2 m_I$ . Now if  $g(\mathcal{N}, \bar{\mathcal{N}})$  is

homogeneous in  $\mathcal{N}$  and  $\bar{\mathcal{N}}$ , as is typical in intermediate scale scenarios, then eqs. (5.6), supplemented by eq. (5.5) (with a similar equation for  $\bar{\mathcal{N}}$ , with  $\alpha$  replaced by  $\beta$ ), are equivalent to

$$(\alpha^2 + 2\alpha\beta + A\alpha)\mathcal{N}_a + \tilde{m}_{\mathcal{N},ab}^2\mathcal{N}_b = 0 \quad (5.7)$$

where  $\tilde{m}_{\mathcal{N},ab}^2$  is the soft-SUSY-breaking mass matrix for  $\mathcal{N}$ . Unless  $\tilde{m}_{\mathcal{N},ab}^2$  is proportional to the unit matrix (*i.e.* highly degenerate), eqs. (5.5) and (5.7) are an overdetermined set of equations for  $\mathcal{N}_a$ . Thus we see that, with assumptions on the form of the superpotential, eq. (5.5) can be understood as a degeneracy problem in the  $\mathcal{N}$ -sector. By the naturalness criteria we enunciated earlier, the level of degeneracy we can expect in this sector is presumably of order the associated Yukawa couplings. Worse, in string-inspired models, the  $\bar{q}$ ,  $\bar{d}$ , and  $\mathcal{N}$  fields typically carry different quantum numbers under a gauged  $U(1)$ , and thus the corresponding  $A$  terms would be expected to differ by amounts of order  $\alpha$ .

In models of this type, it seems even harder to understand the  $M_5^2 = A_\mu M_D$  condition, even approximately. Thus again we are forced into some special region of parameters, if these models are to be viable at all.

To conclude this discussion, we mention that one might hold out hope that, given a complete theory of flavor, sufficient degeneracy could be obtained. In fact, let us, contrary to our naturalness arguments and stringy expectations, assume exact degeneracy at the Planck scale. By studying the renormalization group equations, one finds that most contributions to  $\theta$  may be kept small enough given sufficient suppression of the couplings  $\gamma_{ij}$ . However, the contribution of eq. (4.29) is problematic if there is a gauged  $U(1)$  which distinguishes  $\bar{q}$  and  $\bar{d}^*$ . The (LL) part receives a renormalization proportional to the up-quark Yukawa couplings while the (RR) factor is renormalized by gauge couplings. It is well-known that such renormalizations are not problematic for FCNC's but the resulting  $\theta$  in Nelson-Barr models is much too large.

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★ This is commonly the case in string-inspired models.

## 6. Conclusions

We have found that the constraints on degeneracy and proportionality of the squark mass matrices in Nelson-Barr scenarios are much stronger than those obtained from data on flavor changing neutral currents. Moreover, based on naturalness as well as expectations from string theory, we have argued that these constraints are not likely to be satisfied.

We are not able, of course, to say with certainty that a model cannot be constructed which satisfies the constraints on degeneracy between light scalar masses and proportionality of scalar versus fermionic couplings of the light states which have been enumerated here in a natural way. We have remarked already that one can construct models in which horizontal symmetries give rise to modest squark degeneracies. Perhaps more ingenious constructions can give the higher degrees of degeneracy required. Those couplings which might be expected (by 't Hooft criterion) to differ by  $\alpha_s/\pi$  conceivably could be nearly equal in some unified framework. Another possibility might be scenarios incorporating dynamical supersymmetry breaking, in which large degeneracies are expected since supersymmetry is broken by gauge interactions which are flavor blind.<sup>[24]</sup> It is also interesting that in such schemes, scalar trilinear couplings are expected to be greatly suppressed, which would help with the various proportionality constraints, including  $\vec{b} = \vec{a}$ .

From all of this, we are left with the feeling that the  $m_u = 0$  and axion solutions to the strong  $CP$  problem (in the framework of supersymmetry) are the most plausible. Successfully implementing the Nelson-Barr scheme requires a much more ingenious understanding of flavor physics than has been offered to date.

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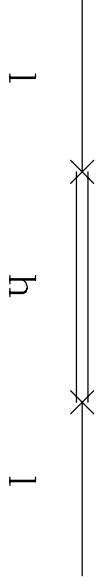
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## FIGURE CAPTIONS

- 1) One loop diagram which can contribute a phase to the gluino mass. Matter fields may be light or heavy.
- 2) One loop diagrams which will contribute a phase to the quark mass matrix, if strict degeneracy or proportionality do not hold.
- 3) Simple diagram which describes integrating out the massive fields to obtain corrections to the squark mass matrices.



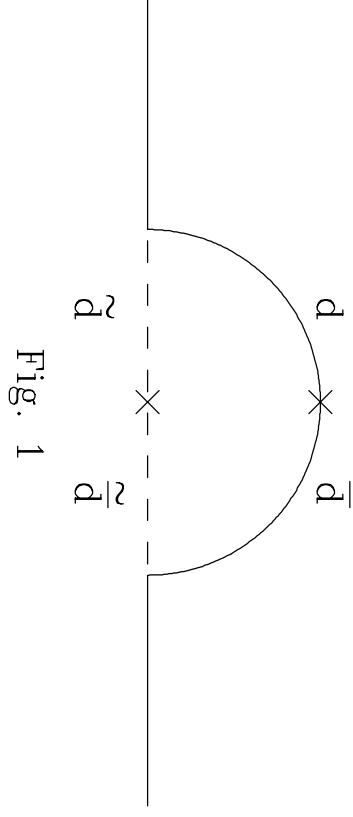


Fig. 1

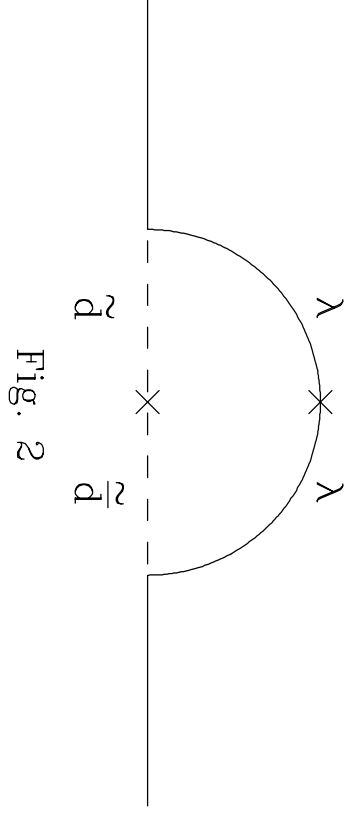


Fig. 2