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ABSTRACT

We show that the transverse polarization asymmetry of the τ-lepton in the decay $t \to b\tau\nu$ is extremely sensitive to CP violating phases arising from the charged Higgs exchange in the Weinberg model of CP non-conservation. Qualitatively, the polarization asymmetries are enhanced over rate or energy asymmetries by a factor of $\approx \frac{m_t}{m_\tau} \approx O(100)$. Thus for optimal values of the parameters the method requires $\approx 10^4$ top pairs to be observable rather than 10^7 needed for rate or energy asymmetries. We also examine τ polarization in b decays via $b \to c\nu\tau$ and find that it can also be very effective in constraining the CP violation parameters of the extended Higgs sector.

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ABSTRACT

We show that the transverse polarization asymmetry of the τ-lepton in the decay $t \to b\tau\nu$ is extremely sensitive to CP violating phases arising from the charged Higgs exchange in the Weinberg model of CP non-conservation. Qualitatively, the polarization asymmetries are enhanced over rate or energy asymmetries by a factor of $\approx \frac{m_t}{m_\tau} \approx O(100)$. Thus for optimal values of the parameters the method requires $\approx 10^4$ top pairs to be observable rather than 10^7 needed for rate or energy asymmetries. We also examine τ polarization in b decays via $b \to c\nu\tau$ and find that it can also be very effective in constraining the CP violation parameters of the extended Higgs sector.

Due to its mass scale the top quark represents a unique probe for addressing to the long-standing issue of CP violation. While the FNAL Tevatron is expected to produce enough top quarks to establish its existence, the hadronic and $e^+ - e^-$ colliders under construction and being proposed should facilitate studies related to CP. In this context, the search for optimal experimental strategies is clearly important and requires extended phenomenological studies.

In the Standard Model (SM) CP violation effects in the top quark are too small to be observable[1]. However, it is difficult to see why the Kobayashi-Maskawa[2] (KM) phase should be the only source for CP non-conservation. Extensions of the SM almost invariably lead to additional CP violating phases. Indeed the observed baryon asymmetry in the universe is often used to argue that additional sources of CP beyond the KM phase are a necessity[3]. We are therefore motivated to continue our investigation of CP non-conservation caused in the production[4] and decays[5,6] of the top quark in one popular extension of the SM[7], namely the Weinberg Model[8] (WM). This model leads to large, possibly observable, dipole moments[9] and rather sizable partially integrated rate asymmetry[5] (PIRA) in the semi-leptonic modes $t \to b\tau\nu_\tau$. In this work we show that the transverse polarization of the τ in these semileptonic transitions is extremely sensitive to CP violation effects due to the extended Higgs sector. Thereby, with optimal values of the parameters in the WM, requiring only a few thousand top quark pairs rather than $\geq 10^7$ needed for the rate, the triple correlation or the energy asymmetries[5,6,10]. Thus the polarization effects become accessible not only to SSC/LHC, where the estimates are for $10^7 - 10^8$ top pairs/year but also to an electron-positron linear collider with an anticipated rate of about 10^4/yr. Clearly this approach can only be used if the top detectors are capable of measuring the τ polarization.

We have also examined the effectiveness of τ polarization as a possible signal of CP violation in semi-leptonic b decays due to the extended Higgs sector. Our finding is that one of the transverse polarization asymmetries is useful in b decays as well as it can lead to stringent constraints on the parameters of the Higgs model with about 10^6 B mesons.

The semi-leptonic decay of the top quark proceeds through W exchange and Higgs exchange graphs (See Fig. 1a and 1b). The CP violating phase is in the Higgs exchange. Since $m_t > m_W$, the dominant contribution of the W-graph is from on shell W-bosons.

Thus Fig 1a possesses both dispersive and absorptive pieces, and indeed both can have significant resonance enhancement, as we shall see below [1,5,6]. The absorptive piece of Fig 1a interferes with the CP violating Higgs phase to contribute to observables (e.g. rate asymmetry) that are CP-odd, T_N even [11]. Here T_N denotes naive time reversal where the spins and momenta of all particles are reversed but initial and final states are not interchanged. On the other hand observables such as momentum triple correlations that are CP-odd, T_N-odd receive contribution from the dispersive part of Fig. 1a.

Let us now qualitatively try to understand why the transverse polarization is much more sensitive compared to energy or rate asymmetries. Recall that PIRA (or energy asymmetry) goes as [12] m_τ^2/m_H^2. It is important to understand the sources for the two powers of m_τ. One of these originates from the Yukawa couplings at the $H\tau\nu_\tau$ vertex. The second power of m_τ originates from the trace over the lepton loop resulting from the interference between the W and the Higgs. This trace goes as:

$$Tr[\gamma^\mu(\not{p}_\tau + m_\tau)(1-\gamma_5)\not{p}_\nu] = 4m_\tau p_\nu^\mu \tag{1}$$

Clearly the m_τ in this trace arises because one is summing over the spins of the final τ. So the m_τ in the above trace can be avoided provided we do not sum over the spins of the τ. Thus we arrive at CP violating transverse polarization asymmetries of the τ that are larger by $\approx m_t/m_\tau \approx 100$ compared to PIRA requiring therefore $\approx 100^2$ fewer top pairs. Thus the transverse polarization asymmetries may well be observable with $\approx 10^4$ $t\bar{t}$ pairs rather than $\approx 10^7$ needed for PIRA.

We now briefly recapitulate some aspects of the Weinberg Model that are relevant to this work [13]. The model consists of three Higgs doublets enabling it to possess a CP violating Higgs sector without the flavor changing neutral currents. It therefore has two charged Higgs states $H_1^\pm$ and $H_2^\pm$. For simplicity we will assume that $H_1^\pm$ has a much higher mass than $H_2^\pm$ (m_H) so that we need to consider effects due to $H_2^\pm$ only. The Lagrangian coupling H_2 to quarks and leptons is given [13] by:

$$\mathbf{L} = \frac{g}{\sqrt{2}} H_2^+ (\bar{u}_i \frac{m_{dj}}{m_W} U_d P_R d_j V_{ij}^{KM} + \bar{u}_i \frac{m_{ui}}{m_W} U_u P_L d_j V_{ij}^{KM} + \bar{\nu}_i \frac{m_{li}}{m_W} U_l P_R l_j \delta_{ij}) + H.C. \tag{2}$$

where

$$U_l = -\frac{c_1 s_2 s_3 + c_2 c_3 e^{i\delta}}{s_1 s_2}$$

$$
\begin{aligned}
U_u =& \frac{c_1 c_2 s_3 - s_2 c_3 e^{i\delta}}{s_1 c_2} \\
U_d =& \frac{s_1 s_3}{c_1}
\end{aligned}
\tag{3}
$$

and V^{KM} is the Kobayashi-Maskawa matrix. We denote $s_i = \sin(\theta_i)$ and $c_i = \cos(\theta_i)$ where θ_i and δ are parameters of the Higgs potential. The CP violation which we consider is proportional to combinations of these couplings such as $V_{ul} \equiv Im(U_u^* \, U_l)$.

In the rest frame of the τ lepton, let us define a reference frame where the momentum of the top quark is in the $-x$ direction, the y direction is defined to be in the decay plane such that the y component of the b momentum is positive and the z axis is defined by the right hand rule. In the limit that the τ mass is small, (WM) can give rise to two kinds of CP violating polarization asymmetries. These are

$$
\begin{aligned}
A_Y =& \frac{\tau^+(\uparrow) \; - \tau^+(\downarrow) \; + \tau^-(\uparrow) \; - \tau^-(\downarrow)}{\tau^+(\uparrow) \; + \tau^+(\downarrow) \; + \tau^-(\uparrow) \; + \tau^-(\downarrow)} \\
A_Z =& \frac{\tau^+(\uparrow) \; - \tau^+(\downarrow) \; - \tau^-(\uparrow) \; + \tau^-(\downarrow)}{\tau^+(\uparrow) \; + \tau^+(\downarrow) \; + \tau^-(\uparrow) \; + \tau^-(\downarrow)}
\end{aligned}
\tag{4}
$$

where for A_Y (A_Z) the arrows indicate the spin up or down in the direction y (z). While both A_Y and A_Z are CP odd, being T_N even, A_Y needs an interaction phase; A_Z is odd under T_N and does not need an interaction phase [14]. Thus $A_Y(A_Z)$ is proportional to absorptive (dispersive) part of Fig. 1a.

For our analysis of top decay, we will ignore the masses of the b quark and the τ lepton (wherever that is a good approximation). Thus let us define the invariants:

$$
s = 2p_\nu \cdot p_\tau \qquad t = 2p_\tau \cdot p_b \qquad u = 2p_b \cdot p_\nu \tag{5}
$$

as well as the quantities $y_W = \Gamma_W^2/m_t^2$, $\lambda = s/m_t^2$ and $x_i = m_i^2/m_t^2$ for $i = b$, τ, H and W. The angle θ is defined to be the angle between the W momentum and the τ momentum in the W rest frame.

Consider now the case of the CP violating polarization in the $x - y$ plane. This gives rise to a polarization asymmetry in the direction

$$
V = \frac{s p_b - t p_\nu}{\sqrt{stu}} \tag{6}
$$

which lies in the $x-y$ plane. In the τ rest frame if ψ_b is the azimuthal angle of the b quark momentum from the x axis and ψ_ν is the angle of the ν then V defined above is a unit vector at angle $\frac{1}{2}(\psi_b+\psi_\nu-\pi)$ in the $x-y$ plane. Note, however that since the τ rest frame is highly boosted with respect to the top quark rest frame, the b and ν are close to the $-x$ direction so V will be very close to the y axis. The differential asymmetry is thus given by (using here and in the relevant formulas to follow $BR(W\to\tau\nu)=1/9$):

$$dA_Y = \frac{9g^2}{32\pi^2}V_{ul}\frac{(1-\lambda)^2\sqrt{x_\tau\lambda y_W x_W}\sin\theta}{(1-x_W)^2(1+2x_W)(x_H-\lambda)((\lambda-x_W)^2+y_W x_W)}d\lambda\, d\cos\theta \qquad (7)$$

Integrating this over θ and λ using the narrow resonance approximation the result for the total asymmetry is

$$A_Y = \frac{9}{64}g^2 V_{ul}\frac{\sqrt{x_\tau x_W}}{(1+2x_W)(x_H-x_W)}. \qquad (8)$$

The CP violating polarization asymmetry in the z-direction is proportional to the real part of the resonant W propagator. The differential asymmetry is:

$$dA_Z = -\frac{9g^2}{32\pi^2}V_{ul}\frac{\sqrt{x_l}\sin\theta(\lambda-x_w)(1-\lambda)\sqrt{\lambda}}{(1-x_W)^2(1+2x_W)((\lambda-x_W)^2+x_W y_W)(x_H-\lambda)}d\lambda\, d\cos\theta. \qquad (9)$$

Integrating this over λ and θ one obtains the total asymmetry

$$A_Z = -\frac{9}{64\pi}g^2 V_{ul}\frac{\sqrt{x_l}}{(1-x_W)^2(1+2x_W)x_H}f(x_W,y_W,x_H) \qquad (10)$$

where f is the integral

$$f(x_W,y_W,x_H) = \int_0^1 \frac{(\lambda-x_W)x_H(1-\lambda)\sqrt{\lambda}}{((\lambda-x_W)^2+x_W y_W)(x_H-\lambda)}d\lambda. \qquad (11)$$

Note that due to the dependence on the real part of the resonance propagator, as s passes through x_W, the net polarization changes sign. There is therefore a partial cancelation of the polarization as defined above. If however the invariant mass of the $\tau\nu$ system can experimentally be determined within a few GeV, this information may be taken into account by weighting events where $s\le x_W$ with -1 and events where $s\ge x_W$ with $+1$. The total asymmetry defined in this way, A'_Z, is thus given by

$$A'_Z = -\frac{9}{64\pi}g^2 V_{ul}\frac{\sqrt{x_l}}{(1-x_W)^2(1+2x_W)x_H}f'(x_W,y_W,x_H) \qquad (12)$$

where f' is the integral as equation (11) except $\lambda - x_W$ is replaced by $|\lambda - x_W|$. Note that whereas f is smooth as $y_W \to 0$, f' grows logarithmically since in this case the resonant enhancement is not cancelled.

Of course the polarization of the τ is not directly observable and must be inferred from the decay distributions of the τ. Let us define the sensitivity S of a method of measuring the polarization of the τ so that given N_τ τ leptons, the error in the measurement of the polarization, ΔP, is given by $\Delta P = (S\sqrt{N})^{-1}$. In a study [15] the decay modes $\pi\nu$, $2\pi\nu$, $3\pi\nu$, $e\nu\bar{\nu}$ and $\mu\nu\bar{\nu}$ are considered as polarization analyzers. The sensitivity which could be obtained in an ideal detector is found to be $S = 0.25$ if one considers only the mode $2\pi\nu$ while if one combines information from all four decay modes, $S = 0.35$. Thus, the error in measuring a polarization asymmetry A, given N_t top quarks, is given by

$$\Delta A = (S\sqrt{NBr(t \to \tau\nu b)})^{-1}. \tag{13}$$

In [5] the measurement of CP violation in this model is considered in $t \to \tau\nu b$ without using polarization information from the τ. CP violation is manifested by a difference in the distribution of τ leptons in $\cos\theta$ leading to an asymmetry in the partially integrated rate defined through the quantity α_+ as

$$\alpha_+ = \frac{\Gamma_f(t \to b\tau^+\nu_\tau) - \Gamma_f(\bar{t} \to b\tau^-\nu_\tau)}{\Gamma_f(t \to b\tau^+\nu_\tau) + \Gamma_f(\bar{t} \to b\tau^-\nu_\tau)} \tag{14}$$

where Γ_f is the rate for the decay into a state where the angle between the t momentum and the τ momentum in the W rest frame is greater than $\frac{\pi}{2}$. The value of this asymmetry is:

$$\alpha_+ = \frac{9\sqrt{2}}{4\pi}\frac{G_F m_\tau^2 r_{WH} V_{ul}}{(2 + r_{Wt})(1 - r_{WH})} \tag{15}$$

where $r_{WH} = m_W^2/m_H^2$ and $r_{Wt} = m_W^2/m_H^2$.

All these quantities are proportional to V_{ul}. As in [5] we now consider what the maximum value of V_{ul} is that is consistent with experimental observations. It turns out that the one constraint comes from $\tau \to l\nu_l\nu_\tau$ decays where a 2% violation of lepton universality is not excluded by experiment [16]. We therefore impose

$$\frac{\Gamma(\tau \to \mu\nu_\mu\nu_\tau)_{W+H} - \Gamma(\tau \to \mu\nu_\mu\nu_\tau)_W}{\Gamma(\tau \to \mu\nu_\mu\nu_\tau)_W} \leq 0.02 \tag{16}$$

where the subscript $H+W$ means the total rate including both H and W exchange while the subscript W means only the W exchange (ie the SM). This gives rise to the condition

$$\frac{m_\mu^2|U_u|^2|U_l|^2}{M_H^2}\left|\frac{m_\tau^2|U_u|^2|U_l|^2}{4m_H^2}-2\right|\leq 0.02. \qquad (17)$$

Another constraint comes from the non-observation of $b\rightarrow s\gamma$. This decay in the WM model is calculated in [17] . Following this paper we take $BR(b\rightarrow s\gamma)\leq 8.5\times 10^{-4}$. In addition perturbative limits on the the charged higgs fermion couplings imply:

$$|U_u|m_t,\quad |U_d|m_b,\quad |U_l|m_\tau\leq\frac{4\pi m_W}{\sqrt{2}g} \qquad (18)$$

The result thus obtained is $V_{ul}=10^3$. Using this value of V_{ul} Table 1 shows (see the second column) the value of each of the asymmetries α_+, A_Y, A_Z and A_Z' for $m_t=150GeV$ and $m_H=200$ and $300GeV$. We then show how many top quarks are needed to observe these asymmetry where we take the sensitivity to τ polarization $S=0.25$ which is obtained using only the $2\pi\nu$ mode. We also calculate the restriction which may be placed on V_{ul} using 10^6 top quarks (see the last column). We see that the polarization asymmetries place much more stringent restrictions than α_+ being one to two orders of magnitude more effective.

CP violating τ-lepton polarization asymmetries may also be considered in semileptonic decays of B mesons to τ via: $B\rightarrow\tau+\nu+X$. Now the signal will be largely proportional to V_{ld} [18] . Since for semileptonic B decays the spectator model works quite well it is sufficient for our purpose to study the free quark decay, $b\rightarrow\tau\nu c$. In this case, the W is far off shell so that the asymmetries which depend on interaction phases are higher order in the coupling compared to asymmetries which do not require such a phase. Thus, polarization asymmetries in the $x-y$ plane and energy asymmetries of the τ lepton are expected to be much smaller than the polarization asymmetry in the z direction. The value of this asymmetry may be readily calculated as in the case of the top quark giving

$$A_Z=\frac{8\pi}{35}(V_{ld}-\frac{m_c^2}{m_b^2}V_{ul})\frac{\sqrt{u_\tau}}{u_H}\frac{J(u_\tau,u_c)}{I(u_\tau,u_c)} \qquad (19).$$

where $u_i=m_i^2/m_b^2$ for $i=\tau$, c and H, I and J are the kinematic integrals:

$$I(u_c,u_\tau)=12\int_{r_0}^1(r-u_c-u_\tau)(1-r)^2\lambda^{\frac{1}{2}}(r,u_\tau,u_c)\frac{dr}{r}$$

$$J(u_c,u_\tau)=\frac{105}{16}\int_{r_0}^1 r^{-\frac{3}{2}}(1-r)^2\lambda(r,u_\tau,u_c)\;dr \qquad (20)$$

Here

$$r_0 = (u_c^{\frac{1}{2}} + u_\tau^{\frac{1}{2}})^2; \qquad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca. \tag{21}$$

Evaluating the above expression we find that $A_Z = 1.2 \times 10^{-4} V_{ld}$ for $m_H = 200 GeV$ and $A_Z = 5.2 \times 10^{-5}(V_{ld} - \frac{m_c^2}{m_b^2} V_{ul})$ for $m_H = 300 GeV$. We thus note that asymmetries from b decay will primarily put restrictions on the quantity V_{ld} because the factor $\frac{m_c^2}{m_b^2}$ suppresses the contribution of the term with V_{ul} by about an order of magnitude. Returning to the discussion of the existing experimental bounds on the Weinberg model, we find that there is little restriction on V_{lb} from existing experiments; in fact it may be as large as 1.3×10^4. Even if V_{lb} is an order of magnitude less it would require about 10^5 B's (assuming the same efficiency of 0.25 for detecting the τ polarization) to show up as a non-vanishing effect. At a B-factory, given 10^8 b quarks, under ideal conditions (using just the $2\pi\nu$ decay mode to determine the polarization of the τ), at 1σ one may put a restriction of $V_{ld} \leq 16$ if $m_H = 200 GeV$ and $V_{ld} \leq 37$ for $m_H = 300 GeV$. We thus see that it takes about 100 times more $B's$ than top quarks to attain a similar reach on the CP parameters, i.e. $V's$, of the extended Higgs model.

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Table 1

Here the value for the asymmetries (second column) above are the maximum consistent with the restrictions described above (i.e. using $V_{ul} = 10^3$). N_t is the number of top quarks required to see the asymmetry, given in the second column, taking into account the sensitivity to τ polarization using only the $2\pi\nu$ decay mode. V_{ul}^{max} is the restriction on V_{ul} that may be attainable with 10^6 top quarks. In all cases $m_t = 150GeV$, the upper number in each case is for $m_H = 200GeV$ and the lower is for $m_H = 300GeV$.

Asymmetry	Value(10^{-3})	N_t	V_{ul}^{max}
α_+	2.8	3.2×10^6	1800
	1.4	13×10^6	3600
A_Y	161	5.6×10^3	75
	65	3.4×10^4	184
A_Z	96	1.6×10^4	126
	29	1.7×10^5	412
A'_Z	540	4.9×10^2	22
	220	3.0×10^3	55

References

1. G. Eilam, J. Hewett and A. Soni, Phys. Rev. Lett. 67, 1979 (1991) and 68, 2103(1992) (Comment).

2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652(1973).

3. See e.g. A. E. Nelson, D. B. Kaplan and A. G. Cohen, Nucl. Phys. B373, 453 (1992) and references therein.

4. D. Atwood, A. Aeppli and A. Soni, Phys. Rev. Lett. 69, 2754 (1992).

5. D. Atwood, G. Eilam, A. Soni, R. Mendel and R. Migneron, BNL-48162 (1992), to appear in Phys. Rev. Lett.

6. R. Cruz, B. Grzadkowski and J. F. Gunion, Phys. Lett. B 289, 440(1992).

7. G. Kane, G. Ladinsky and C.P. Yuan, Phys. Rev. D45, 124 (1991); C. Schmidt and M. Peskin, Phys. Rev. Lett. 69, 410 (1992); B. Grzadkowski and J.F. Gunion, Phys. Lett. 287, 237 (1992); W. Bernreuther, O. Nachtman, P. Overmann and T. Schröder, preprint HD-THEO-92-14; N.G. Deshpande, B. Margolis and H.D. Trottier, Phys. Rev. D45, 178 (1992); C.R. Schmidt, SLAC-PUB-5878.

8. S. Weinberg, Phys. Rev. Lett. 37, 657(1976); see also T. D. Lee, Phys. Rev. D8, 1226 (1973).

9. A. Soni and R. M. Xu, Phys. Rev. Lett. 69, 33(1992).

10. See also M. Peskin and C. Schmidt, ibid.

11. For simplicity we are assuming $m_H > m_t$, thereby Fig. 1b is purely dispersive.

12. See Atwood et. al. Ref. [5] in particular eqns. 11 and 12.

13. C. H. Albright, J. Smith and S-H. H. Tye, Phys. Rev. D21, 711 (1980).

14. Note that a polarization asymmetry similar to A_Z was considered in the context of $K_{\mu 3}$ decays in R. Garisto and G. Kane, Phys. Rev. D44, 2038(1991).

15. A. Rouge, Workshop on Tau Lepton Physics, Orsay France (1990) p 213. Note also that tau polarizations have actually been measured at LEP see ALEPH Collaboration, Phys. Lett. B265, 430 (1991) and L3 Collaboration Phys. Lett. B294, 466 (1992).

16. W. J. Marciano, Phys. Rev. D45, R721 (1992).

17. P. Krawczyk and S. Pokorski, Nuc. Phys. B364, 10 (1991).

18. CP violation in B-decays in the Weinberg model has previously been considered by J. F. Donoghue and E. Golowich, Phys. Rev. D37, 2542 (1988). However, their effects are driven by V_{du} so our work complements theirs.

Figure Caption

Figure 1a: The Feynman diagram for $t \to b\tau^+\nu_\tau$ through W^+ exchange (i.e. the SM mechanism). Figure 1b: The Feynman diagram for $t \to b\tau^+\nu_\tau$ through H^+ exchange. For b decays via $b \to c\tau\nu$, replace $t \to b$ with $b \to c$ in the two diagrams.

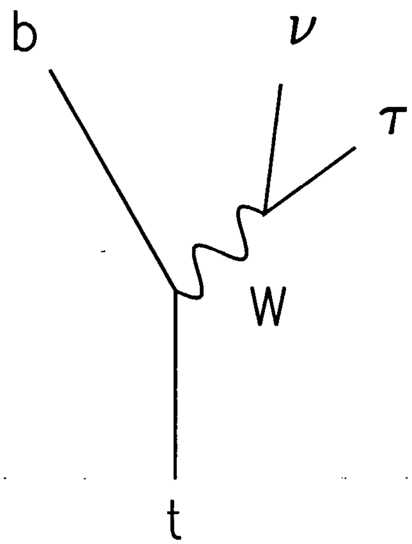

Figure 1a

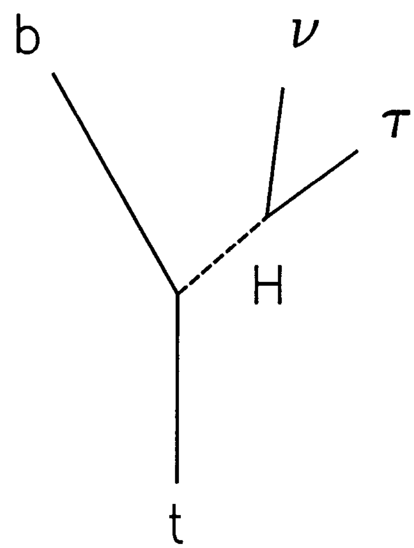

Figure 1b