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QUANTIZED CONIC SECTIONS; QUANTUM GRAVITY*

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Abstract

Starting from free relativistic particles whose position and velocity can only be measured to a precision $< \Delta r \Delta v > \equiv \pm\kappa/2\ meter^2 sec^{-1}$, we use the relativistic conservation laws to define the relative motion of the coordinate $r = r_1 - r_2$ of two particles of mass m_1, m_2 and relative velocity $v = \beta c = \frac{(k_1-k_2)}{(k_1+k_2)}$ in terms of the conic section equation $v^2 = \Gamma[\frac{2}{r} \pm \frac{1}{a}]$ where "+" corresponds to hyperbolic and "-" to elliptical trajectories. The equation is quantized by expressing Kepler's Second Law as the conservation of angular momentum per unit mass in units of κ. Then the principal quantum number is $n \equiv j + \frac{1}{2}$ with "square" $\frac{A^2}{T^2} = (n-1)n\kappa^2 \equiv \ell_\odot(\ell_\odot + 1)\kappa^2$. Here $\ell_\odot = n - 1$ is the angular momentum quantum number for circular orbits. In a sense, we obtain "spin" from this quantization. Since Γ/a cannot reach c^2 without predicting either circular or asymptotic velocities equal to the limiting velocity for particulate motion, we can *also* quantize velocities in terms of the principle quantum number by defining $\beta_n^2 = \frac{v_n^2}{c^2} = \frac{1}{n^2}(\frac{\Gamma}{c^2 a}) = (\frac{1}{nN_\Gamma})^2$. For the Coulomb case with charges $Z_1 e, Z_2 e$ of the same sign and $\alpha \equiv e^2/m_e\kappa c$, we find that $\Gamma/c^2 a = Z_1 Z_2 \alpha$. The characteristic Coulomb parameter $\eta(n) \equiv Z_1 Z_2 \alpha/\beta_n = Z_1 Z_2 n N_\Gamma$ then specifies the penetration factor $\mathcal{C}^2(\eta) = 2\pi\eta/(e^{2\pi\eta} - 1)$. For unlike charges, with η still taken as positive, $\mathcal{C}^2(-\eta) = 2\pi\eta/(1 - e^{-2\pi\eta})$. For gravitation $\Gamma/c^2 a = m_1 m_2 \alpha_G/m_p^2$ with $\alpha_G = G m_p/\kappa c$.

Relativistic quantum mechanics is recovered if the smallest distance which can be measured electrodynamically is $\Delta l = \hbar/2m_e c$ or $\kappa = \hbar/m_e$ The starting point for quantum electrodynamics is achieved by taking $\alpha = e^2/m_e\kappa c \rightarrow e^2/\hbar c \approx 1/137$. We extend our previous result for the hydrogen spectrum to Coulomb scattering. The starting point for quantum gravity is given by taking $\alpha_G = G m_p/\kappa c \rightarrow G m_p^2/\hbar c \approx 1/(2^{127} + 136)$. Our derivation of "spin" from Kepler's second law allows us to show that the classical tests of General Relativity are met. If we postulate crossing symmetry rather than just CPT, we predict that free anti-matter near the surface of the earth will "fall" up.

"[Mathematics] begins by being a most useful servant when dealing with phenomena of the ordinary scale of magnitude, but ends by dragging us by the scruff of the neck willy nilly into the inside of the electron where it forces us to repeat meaningless gibberish."

— P.W.Bridgman, *The Logic of Modern Physics*, 1927, p. 149.

1. A MODERN OPERATIONAL METAPHYSICS

Segré often used to remark that "You can't measure errors." Philosophers almost always ignore this inconvenient fact, and physicists often do so when they stray from familiar laboratory protocol into unfamiliar subjects such as "quantum mechanical measurement theory".

Errors fall into two rough classes: qualitative and quantitative. Qualitative errors may be due to mistakes in procedure which are undetected at the time but can be given a rational explanation after the fact which suggests testable hypothesis. Or they may be due to some unexpected phenomenon which becomes relevant as further knowledge accumulates, and again can lead to a qualitative explanation. Or, as experimentalists know to their sorrow, and people who talk about Science without understanding it never recognize, errors can remain inexplicable "forever".

Quantitative errors are of two rough types: statistical and bounded. Statistical errors can be attributed to causes so uncorrelated and unstructured that, in the absence of further information, they can be given equal weight in a case count. Elaborate theories can be erected on such a hypothesis, but they are no guard against errors of the first type, which are often called "systematic errors". These have left many a beautiful theory in ruins once they were recognized.

Bounded errors are the type that arise when you can say with reasonable certainty that an experimental value must fall between fixed limits, but when you cannot say much about where they fall in that range. The typical case for us has been called the "counter paradigm". We idealize this as a device with linear macroscopic size Δl measured using some standard laboratory protocol for measuring length in well understood units, and a recording device connected to a clock. If the counter fires, the time read by the clock is recorded, eg by bits on a magnetic tape which count the number of ticks of the clock after some reference tick. If the counter does not fire, no such number is recorded. The time *interval* between two ticks is called Δt, and again is established by standard laboratory protocol. Obviously, without further information, we can assign no meaning to the "time between ticks". Events of this type will be called NO-YES events.

We take the operational position that these quantitative, finite *bounds* on measurement uncertainty have to be taken seriously. We are precluded by our metaphysics from giving *any* meaning to "time between ticks" or "position within a counter" where an unjustifiable extrapolation to shorter distances might seduce us into trying to give meaning to a finer grained "space time event structure". Of course, as we construct more elaborate theories, we may do so indirectly and shrink our Δl and Δt bounds accordingly. But we must always be prepared for surprises when we do so. Plenty have occurred in the history of modern physics.

Bridgman suggested that we construct our theories in such a way that we are prepared for such extrapolative failures. He did not succeed in providing any precise protocol for doing so. Perhaps we now know enough to start doing just that. I suggest that we have learned from relativistic quantum mechanics that our conceptual models cannot measure distance directly to better than $\hbar/2m_e c$ using electrodynamics because we produce electron-positron pairs and the problem becomes non-local. If we try to push our theory to indirect bounds set by the Planck mass, $M^2_{Planck} = \hbar c/G$, we have to stop at $\hbar/M_{Planck}c \approx 10^{-35} meter$ because there is, currently no generally accepted theory of quantum gravity*. We try to implement Bridgman's suggestion that we build limits of measurement into our theory *before* we hit these new and only partially understood phenomena by postulating a technological bound on the accuracy of relativistic space-time measurement, $\Delta l = c\Delta t$, and a bound on the measurement of area per unit time of

⋆ See Appendix 1, letter from C.J.Isham to HPN.

$< \Delta r \Delta v > \equiv \pm\kappa/2 \; meter^2 sec^{-1}$. We call "$\kappa$" Kepler's constant.

REMEMBER that we can never be sure that we have eliminated *qualitative* errors.

2. REDUCTION OF PARTICLE DYNAMICS TO INTEGERS

2.1 LABORATORY SCATTERING MEASUREMENT

We reduce the dependence of our analysis of measurement on the concept of mass by replacing it with mass ratios measured relative to some standard particle beam using only space, time, velocity and velocity change measurements. For this purpose we use *counter telescopes* consisting of two *counters* with thickness Δl connected to recording clocks having the time between ticks Δt. We pick our units such that $\Delta l = c\Delta t = 1$, making all measurable distances and times *integers.* This commits us to insuring that we never talk about fractional space and time intervals as *measurable.* If the spacial interval between the counters is L and the time interval between two sequential counter firings is T we attribute the counter firings to the passage of a particle with velocity $V = L/T$. All data discussed here will be collected at a slow enough rates so that the interval between the passage of particles allows this measurement to be unambiguous.

We now consider the arrangement of four counter telescopes pictured in figure 1. The arrangement is set up to measure the process $(1,2) \rightarrow (3,4)$. S_1 is a source of particles (eg an accelerator) of known type and S_2 a second source of particles of the same or a different type. D_3 and D_4 are detectors which may or may not be included in the setup, and if included are used to identify what types of particles emerge through the counter telescopes 3 and 4. After a sufficient number of calibration experiments, we can use this setup to measure mass ratios of all types of particles we encounter relative to some one source taken as providing particles with a standard *mass.*

The entrance counters into the scattering chamber for the incident particles (1' and 2') and the exit counters for the scattered or produced particles (3 and 4) lie on a circle of macroscopic radius d, and all four telescopes point to the center of this circle. This "center" is not a point but a region specified somewhat inaccurately by the geometrical arrangement. The recording clocks in each of the eight counters are synchronized to a single time sequence by using the Einstein synchronization procedure, so that the lengths of the counter telescopes can be given as "radar distances", i.e. as half the time it takes a light signal to go from one position to the other and be reflected back. Thus the length of each telescope L_i and the time T_i it takes a particle of velocity $v_i = \beta_i c$ to traverse each telescope can be specified in terms of two time parameters ($t'_i > t_i$, $i \in 1,2,3,4$) on this synchronized scale by

$$L_i = N(t'_i - t_i);\ T_i = N(t'_i + t_i);\ \beta_i = L_i/T_i;\ i \in 1,2,3,4 \tag{2.1}$$

Often we cannot measure velocities directly by "time of flight" as this paradigm implicitly assumes. In that case, we need to make a more complicated analysis to arrive at the uncertainty $< \Delta v >$. We make a start in that direction by introducing a factor N, as is done in Eq. 2.1, in such a way that $t'_i \pm t_i$ *are* known to the nearest integer, but L_i and T_i may be known less accurately.

We consider two correlated sequences of events interpreted as caused by a single scattering event, namely

$$t_1 < t'_1 < t_3 < t'_3$$

and

$$t_2 < t'_2 < t_4 < t'_4$$

which we simplify by using the standard notation 11'33' and 22'44'. If the scattering event is presumed to take place at the center of the chamber at time t on the same time scale, the correlation implied by this assumption is that

$$t = t_i + t'_i + d/\beta_i c \tag{2.2}$$

to the accuracy of our knowledge of the eight times and the geometry. For sufficiently weak beams, the association of a single t with the eight counter firings will be unambiguous.

2.2 Measurement of mass ratios

Consider the special case when all four of the counter telescopes (four *pairs* of counters) record the *same* speed, $\beta_i c = \beta c = v$. We assume that the lines 1'3 of length b and 2'4 of length b' are parallel and are bisected by a line perpendicular to them through X, as illustrated in figure 2. That is we have two isosceles triangles with a common vertex and parallel bases b, b'. Calling their inferred altitudes a, a' and angles at the vertices opposite the bases $\pi - \theta$, $\pi - \theta'$ we have, as a first approximation,

$$a^2 = d^2 - \frac{1}{4}b^2 = 2d^2 sin^2\frac{\theta}{2} = a^2[(\frac{d}{b})^2 - \frac{1}{4}] \tag{2.3}$$

and similarly for a', θ'. We take the coordinate direction j parallel to the altitudes a, a', take the coordinate direction k parallel to the bases b, b', and assign coordinates (x_j, x_k) as follows:

$$1' : (0,0); \quad X : (a, b/2); \quad 3 : (0, b) \tag{2.4}$$

We now relate this geometry to the common velocity v registered by all four counter telescopes and the fact that we can only measure times to an accuracy $\Delta T = t_1 + t'_1$. Explicitly

$$1'X = X3 = d = 2v\Delta T; \quad 1'3 = b = (\frac{b}{d})2v\Delta T \tag{2.5}$$

and the velocity components are

$$v_j^{1'\to X} = (\frac{a}{d})v; \;\; v_j^{X\to 3} = -(\frac{a}{d})v; \;\; v_k^{1'\to 3} = (\frac{b}{2d})v; \;\; v_j^2 + v_k^2 = v^2 \qquad (2.6)$$

We attribute this confluence of events to the scattering of one particle from each beam within the region X. Note that this is a *fiction*. We have know direct way of knowing what happens inside the scattering chamber, let alone that it happened at X. The approach to short distances through scattering is necessarily *non-local.* However, we can use these non-local measurements to *define* the ratio of the mass m' of the particles in the second beam to the mass m of the particles in the first beam by the equality

$$m'b' = mb \qquad (2.7)$$

It is a matter of experience that the *scale invariant* equality $m'/m = b/b'$ is independent of the common measured velocity v for all known pairs of particles which can be compared in this way and hence defines a velocity invariant mass ratio scale for all particles relative to any one type. Note that we make the comparison at the *same velocity* to avoid the complications of relativistic kinematics.

2.3 MASS QUANTIZATION;LORENTZ BOOSTS IN ENERGY-MOMENTUM SPACE

Assume that there is a smallest measurable mass, or energy change, which we call Δm. However we arrive at our mass quantization, *no* current empirical information prohibits us from assuming that energy, momentum and velocity can be parameterized — using units such that $c = 1$ — in terms of two *integers* k_1, k_2 by the equations

$$E = (k_1 + k_2)\Delta m; \; P = (k_1 - k_2)\Delta m; \; \beta(k_1, k_2) = \frac{P}{E} = \frac{k_1 - k_2}{k_1 + k_2} \qquad (2.8)$$

In order to calculate Lorentz boosts in 1+1 dimensions, and in particular in 1+1 energy-momentum space, it suffices to know the velocity addition law

$$\beta" = \frac{\beta + \beta'}{1 + \beta\beta'} \tag{2.9}$$

For our rational fraction velocities, this gives us the transformation law

$$\beta(k_1", k_2") = \beta" = \frac{k_1" - k_2"}{k_1" + k_2"} = \frac{k_1 k_1' - k_2 k_2'}{k_1 k_1' + k_2 k_2'} \tag{2.10}$$

For reasons which come from the way conservation laws are used in high energy physics, and *also* because of crossing symmetry, it is useful to consider the special case $k_1' \to k_2$, $k_2' \to k_3$. This reduces the transformation law to

$$\beta" = \frac{k_1 - k_3}{k_1 + k_3} \tag{2.11}$$

Clearly, for rational fraction velocities, we can always use the definition and useful symmetric identity

$$p_{ij} \equiv k_i - k_j \Rightarrow p_{ij} + p_{jk} + p_{ki} = 0 \tag{2.12}$$

together with

$$e_{ij} = k_i + k_j; \quad e_{ij}\beta_{ij} = p_{ij} \tag{2.13}$$

for *any* three integers k_i, k_j, k_k. These definitions then guarantee that the velocity addition law *never* takes us out of the space of rational fraction velocities.

We now note that for three fixed integers defining the velocities, the invariants $E^2 - P^2 \equiv \Sigma^2$ can be scaled *up* from the unobserved quantum of mass Δm by any integer $N < 10^{19}$ as follows

$$E_{ij} = N(k_i + k_j)\Delta m = e_{ij} N \Delta m$$

$$P_{ij} = N(k_i - k_j)\Delta m = \beta_{ij} E_{ij} = p_{ij} N \Delta m \tag{2.14}$$

$$S_{ij} = s_{ij} N^2 \Delta m^2 = \Sigma_{ij}^2 = (N\sigma_{ij}\Delta m)^2 = 4N^2 k_i k_j \Delta m^2 \tag{2.15}$$

In classical relativistic particle mechanics the invariant Σ for a free particle is just the "rest mass" of the particle, but in quantum mechanics, or for a system with internal energy, we must often consider the more general case $\Sigma \neq m$. This is sometimes called "off mass shell". It is analagous to the "off energy shell" situation in non-relativistic quantum mechanics which arises from the "energy-time uncertainty principle."

We can now define the standard 1+1 Lorentz transformations in terms of the usual dilation factor γ, with $\beta(k_1, k_2)$ given by (2.10) as

$$P' = \gamma(P + \beta E); \; E' = \gamma(E + \beta P) \tag{2.16}$$

where

$$\gamma^2 = \frac{(k_1 + k_2)^2}{4k_1 k_2}; \; \gamma^2\beta^2 = \gamma^2 - 1 = \frac{(k_1 - k_2)^2}{4k_1 k_2} \tag{2.17}$$

We assert that the implied square root in this standard form need never be taken.

2.4. THE VELOCITY CONSTRAINT

Clearly, if we use counter telescopes to measure space-time and velocity coordinates in the manner discussed in the section on mass-ratio measurement (Sec. 2.2), we have two different integer ratio velocities, $v/c = (t_i' - t_i)/(t_i + t_i')$ defined by the counter telescope and what we called $v_k^{13'} = (b/2d)v$. This is characteristic of a situation in which we make both position-velocity and energy-momentum measurements on the same particle. We generalize it by the following definitions

$$R = N(t'-t)\Delta l; \; T = N(t+t')\Delta t; \; P = N(k'-k)c\Delta m; \; E = N(k+k')c^2\Delta m \tag{2.18}$$

In the simplest situation, the two velocities are the same and

$$\frac{t'-t}{t'+t} = \frac{k'-k}{k'+k} \Rightarrow t'k = tk' \tag{2.19}$$

Then, for t, t', k, k'integer, we have a minimum period T/N and minimum energy E/N bounded from below by our time and mass quantization, but which can be scaled up by any factor N without destroying any integer relationships we establish at short distance and small momentum. Thus we can give a general description of any free particle in terms of four integers respecting the relativistic conservation laws down to some lower bound set by technology — or, eventually down to the constraint set by being able to measure Planck's constant in some unambiguous way.

2.5 THE SIXTEEN DEGREES OF FREEDOM

The standard kinematic relations[1] used in relating theoretical predictions to experimental observations (cross sections, which are Lorentz invariant probabilities) in the practice of high energy particle physics are given in Ref.1, p 162, et. seq. They express both Lorentz invariance and the conservation of energy and momentum in scattering experiments (particle reactions) such as $(1,2) \rightarrow (3,4)$. We have seen above that we can use four integers to define the position, time, energy momentum of any free particle, and hence 16 integers for any such scattering experiment. Once we have related these sixteen integers to the conventional Mandelstam variables in a *scale invariant* way, the assumption that these sixteen numbers can be expressed as integers on some length and time scale compatible with the *integer definition* (Ref 1, p. 2) of the limiting speed for information transfer

$$c \equiv 299\ 792\ 458\ m\ sec^{-1} \tag{2.20}$$

amounts to defining a minimum distance and time resolution bounded from below by technology. The step to relativistic quantum mechanics is then trivial once we have some well defined laboratory phenomenon dependent on Planck's constant h or $\hbar = h/2\pi$, such as deBroglie wave interference, the hydrogen spectrum, Compton scattering, black body radiation, electron-positron pair creation, ... It is

$$\Delta l \rightarrow \hbar/m_e c; \ \Delta t \rightarrow \hbar/m_e c^2 \tag{2.21}$$

Our task then becomes to deduce all the well known quantum effects, — a task which is still "work in progress".

Returning to Figure 1, we note that we have already defined eight of the sixteen degrees of freedom by the eight times t_i, t'_i. Four more parameters relate the whole apparatus to the walls of the laboratory and the laboratory clock. We can choose the remaining four parameters to be the chords connecting the two entrance and two exit counters: $B_{1'3}, B_{34}, B_{42'}, B_{2'1'}$. This particular choice corresponds to going around the circle representing the scattering chamber in a clockwise sense. The remaining two distances, $B_{1'4}$ and $B_{2'3}$ are fixed if, as we have implicitly assumed so far, all four counters lie in a plane. In the general tetrahedral situation given in figure 3 taken from an earlier attempt to discuss bit-string scattering theory[2] these two additional numbers provide useful information. They turn out to be fixed by the conservation laws when we have made enough measurements, and hence provide the kind of "redundant" information that experimentalists use to estimate systematic and reduce statistical uncertainties.

We will relate these 16 parameters to bit-string kinematics on another occasion. Here we show instead how to make use of scale invariant quantization in the context of bound state orbits and scattering trajectories traditionally explained in terms of an inverse square law "action at a distance". This is important because it shows that these concepts are still useful in a relativistic theory where measurement accuracy is bounded from below but otherwise is scale invariant. This in turn allows us to find a well defined "correspondence limit" for relativistic quantum mechanics in classical relativistic particle mechanics with e/m fixed and classical electromagnetic and gravitational fields, as we rough out in Appendix 2.

2.6 Crossing Symmetry and Anti-Particles

One reason for expressing relativistic particle kinematics in the succinct Mandelstam form goes beyond its simplification of structure. Feynman proved long ago that if one represents the matrix elements of relativistic quantum field theory order by order in terms of some coupling constant by "Feynman Diagrams", the diagrams obtained by reversing the velocity of a particle represent a quantum number and energy-momentum conserving process in which the particle has been replaced by its antiparticle. Since reversing a velocity is equivalent to reversing the time in the derivative which defines the velocity, this gave rise to the intuitive picture of an anti-particle as a particle "moving backward in time".

The more inclusive symmetry consistent with this is that if the quantum numbers of *all particles*, called C for generalized *Charge conjugation*, are reversed *and* the coordinates are mirrored, called P for *Parity*, *and* the velocities are reversed, called T for *Time reversal* the theory must predict identical probabilities for the process so described. This is called the CPT theorem. Simply changing one or more (but not all) the particles for anti-particles is a more restrictive symmetry called *crossing symmetry*. The consequence is also supposed to lead to a process for which the theory gives a unique answer, but since the kinematics change, one needs a detailed theory to make any predictions. Although Chew's S-matrix theory took crossing symmetry as a postulate, it has so far proved impossible to cast the theory in a general form which *both* conserves probability (i.e. is unitary) *and* is "crossing symmetric". To show that bit-string physics might meet this challenge is a task for the future.

Making use of the fact that CPT invariance requires anti-particle masses $\bar{m}_i$ to be identical to particle masses m_i, we simply note that the kinematics for the anelastic process $(1,2) \to (3,4)$ and four-vectors $p_i \equiv (m_i; \vec{p}_i)$, with inner products $p_i p_j = E_i E_j - \vec{p}_i \cdot \vec{p}_j$ and the 4-vector conservation law

$$p_1 + p_2 = p_3 + p_4 \tag{2.22}$$

defines the physical ranges for the Mandelstam scalar invariants

$$s = (p_1 + p_2)^2; \; t = (p_1 - p_3)^2; \; u = (p_1 - p_4)^2$$

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 \tag{2.23}$$

These are $s \geq (m_1 + m_2)^2$ and t in the negative range given by (E.5, Ref 1). Then crossing symmetry asserts that the dynamics for the anelastic process $(1,2) \to$

(3,4) must also describe the process $(1,3) \rightarrow (2,4)$ when $t \geq (m_1+m_3)^2$ and s and u are in the negative ranges analagous to (E.5) appropriate to serve as momentum transfer variables. This is called the "t-channel". Further, it must also describe the process $(1,\bar{4}) \rightarrow (2,\bar{3})$ when $u \geq (m_1+m_4)^2$ and s and t are in the appropriate negative ranges. The arbitrariness in how we choose to designate the *labels* 1,2,3,4 for the particles implies additional discrete symmetries which are particularly important when two or more of the particles are *indistinguishable.*

3. QUANTIZED CONIC SECTIONS

3.1 KEPLER'S SECOND LAW: SPIN

Biedenharn[3] has attacked the problem of why Sommerfeld was able to calculate the fine structure of the hydrogen atomic spectrum correctly *without* having heard of "spin" in the Pauli or Dirac sense. Here we provide another explanation based on lower bounds on measurement of angular momentum using Kepler's second law and our definition of "Kepler's Constant" $\kappa =< \Delta l \Delta v >$.

Consider a particle moving past a center which moves a distance $v\Delta T$ in time ΔT in a direction perpendicular to a line from the center bisecting the line connecting its initial and final positions. Assume that the initial and final positions are the same distance r from the center, forming an isosceles triangle with apex at the center, sides r, base $v\Delta T$ and hence altitude $a^2 = r^2 - \frac{1}{4}(v\Delta T)^2$. In the absence of further information, Kepler' second law states that the area A swept out by the line r in time ΔT is a constant of the motion. Using the formula that the square of the area of a triangle with sides a, b, c is given by

$$16A^2 = (a+b+c)(a+b-c)(b+c-a)(c+a-b) \tag{3.1}$$

we find that

$$(\frac{A}{\Delta T})^2 = \frac{1}{4}v^4\Delta T^2[(\frac{r}{v\Delta T})^2 - \frac{1}{4}] \tag{3.2}$$

This formula gives us two choices for the ratio $r/v\Delta T$, which differ by ± 1.

According to our philosophy, the optimum measurement of either of these distances cannot be smaller than Δl. If we consider the "line" between the two positions from which the motion starts to be a rectangle with side $v\Delta t$ in the direction of motion and width Δl, this defines two values for r, drawn to the two nearer or the two farther corners respectively. For the case of "circular" motion with radius $R = r$ and period T, the two lines forming the long sides of the rectangle correspond to the chord and the tangent to the circle respectively. If we define the half-integer $j = \ell_{\odot} + \frac{1}{2}$ and require the polygon formed by the straight line segments to equal the radius of the circle and close, we have that for the chords $2\pi R = \ell_{\odot} vT$, and that for the tangents that $2\pi R = (\ell_{\odot} + 1)v\Delta T$. Calling the area per unit time K, we find that the square of this constant of the motion is given by

$$K^2 = (\frac{\pi R^2}{T})^2 \equiv \ell_{\odot}(\ell_{\odot} + 1)\kappa^2 \tag{3.3}$$

Here we have taken the obvious step of using this as the paradigmatic case for defining the minimum unit for Kepler' constant κ. If our smallest relevant mass is m_e, the two cases correspond to a minimum angular momentum $\frac{1}{2}m_e\kappa$ and hence to $\frac{1}{2}\hbar$ in a conventional quantum theory. This is consistent with all the known quantum phenomena, as we will explain in detail elsewhere. In a classical theory bounded from below by technology, this limit might be set, for example, by the smallest dipole magnetic moment which could be measured in the multipole expansion of the electromagnetic field due to a self-consistently specified system of moving charges with a fixed and measured charge to mass ratio.

We can immediately extend our circular model to elliptical orbits with semi-major axis a and semi-minor axis b, by identifying the principle quantum number n and orbital angular quantum number ℓ as

$$n = \ell_{\odot} + 1; \ j = \ell + \frac{1}{2}; \ 0 \le \ell \le \ell_{\odot} \tag{3.4}$$

Then the extension to parabolic and hyperbolic orbits is immediate if we use the standard gravitational equation[4]

$$v^2 = \Gamma[\frac{2}{r} \pm \frac{1}{a}] \tag{3.5}$$

where r is now the relative distance between the two gravitating objects, "+" corresponds to hyperbolic (scattering) trajectories, "-" to elliptical and circular orbits, and the unique case in between, i.e $v^2 = 2\Gamma/r$, to the parabolic or escape trajectory. Since we have relativistic 3-momentum conservation in our formalism, this "one particle" formula actually applies to two systems in the relativistically defined zero momentum frame with an invariant energy also relativistically defined and required to reduce to $m_1 m_2/(m_1 + m_2)$ in the naive treatment.

As we have already noted, until new phenomena are included which show that $\kappa = \hbar/m_e$ rather than simply being bounded from below by some larger number, the actual quantization is "unobservable". It is imposed by our refusal to extrapolate beyond what we can "measure" unambiguously to specified accuracy. It is an *a priori* or "metaphysical" quantization rather than a physical theory which can be tested.

3.2 "Classical" QED and Quantum Gravity

It will seem somewhat bizarre to model a "classical" Bohr-Sommerfeld hydrogenic system without using Planck's constant. Yet we trust that what we have done above gives sufficient clues as to how we intend to do so. Although we have not given all the details, we have been at some pains to show that we are dealing with a finite and discrete "relativistic" formalism with the usual *empirical* relations between mass, energy, momentum, time dilation and so on down to our fixed measurement accuracy. Consequently, if we replace the usual continuum quantization for the hydrogen atom by a lowest orbital frequency, or lowest measurable energy change, or some such postulate, we can construct such "atoms" which can make only a finite number of energy changes. All we need do is replace $\hbar$ by $m_e\kappa$ in the usual treatments and then check that we have not introduced phenomena that could be observed *without* measuring Planck's constant.

In our lowest order treatment of the hydrogen atom[5] McGoveran and I started with the "deBroglie quantization" $2\pi r = j\lambda = jh/p$ for circular orbits in the hydrogen atom, and arrived at the relativistic Bohr formula. Clearly we can get the same result in our "classical" theory because we have derived the same relation in the last section without using Planck's constant. However, it is useful to quote a more recent discussion.[6]

"We consider the bound system with mass μ interacting with a larger system which can have a maximum mass-energy $M_x = Nm$ where N is an integer to be fixed by the intent of the modeling exercise. Our first assumption is that the bound state is *stable* against spontaneous decay. However, in a "second quantized" theory[7] virtual transitions up to this maximum can occur. Assuming that "externally" μ is at rest, these fluctuations can be interpreted as massless radiation whose energy and hence whose momentum is $p = N\mu$. However, since $m_1 + m_2$ must have the same non-spacial conserved quantum numbers as μ, a fluctuation leading to this radiation and a system of Nm masses will have energy $E = Nm$ But for the overall system $E^2 - p^2$ is invariant and equal to the square of the rest energy of the bound state with which we started:

$$\mu^2 = (Nm)^2 - (N\mu)^2 \tag{3.6}$$

"To recover our previous result, we rewrite this as

$$\mu^2 + (\frac{\mu}{N})^2 = m^2 \tag{3.7}$$

or

$$(\frac{m-\epsilon}{m})^2[1 + (\frac{1}{N})^2] = 1 \tag{3.8}$$

If we take $N = 137n$, with $n = \ell + 1$ the principal quantum number and 137 an approximation for $\hbar c/e^2$, this is Bohr's relativistic generalization of his model for the energy levels of the hydrogen atom.[8] Non-relativistically, or to order $\alpha = e^2/\hbar c \approx 1/137$, the energy levels given by this formula are $\epsilon(n) \approx \frac{me^4}{2\hbar^2 n^2}$. The orbital velocity is $\beta_n = 1/137n = 1/N$ and the radii of the orbits are $R_n = n\hbar^2/me^2 = n\hbar/\alpha mc$."

Thus we see that the "quantization" of bound, relativistic systems amounts to relativistic 3-momentum conservation with all the degrees of freedom corresponding to electromagnetic radiation which can be emitted and absorbed by such systems. Further, we now think that the results claimed in our abstract follow. The basic point is that neither orbital nor close encounter nor exterior velocities can reach c. This allows us to set an energy scale enforcing this restriction, which reduces to the relativistic Bohr atom in the restricted case already considered, and allows us to quantize velocities in terms of the principle quantum number by defining $\beta_n^2 = \frac{v_n^2}{c^2} = \frac{1}{n^2}(\frac{\Gamma}{c^2 a}) = (\frac{1}{nN_\Gamma})^2$. For the Coulomb case with charges $Z_1 e, Z_2 e$ of the same sign and $\alpha \equiv e^2/m_e \kappa c$, we find that $\Gamma/c^2 a = Z_1 Z_2 \alpha$. The characteristic Coulomb parameter $\eta(n) \equiv Z_1 Z_2 \alpha/\beta_n = Z_1 Z_2 n N_\Gamma$ then specifies the penetration factor $C^2(\eta) = 2\pi\eta/(e^{2\pi\eta} - 1)$. For unlike charges, with η still taken as positive, $C^2(-\eta) = 2\pi\eta/(1 - e^{-2\pi\eta})$. For gravitation $\Gamma/c^2 a = m_1 m_2 \alpha_G/m_p^2$ with $\alpha_G = G m_p/\kappa c$.

In both cases, our classical limit shows us how to quantize hyperbolic trajectories simply by using the smallest energy interval allowed in the bound state case, rather than introducing the "continuous spectrum" which leads to infrared divergences. We hope that this way of looking at "radiative corrections" to high energy charged particle scattering will eventually lead to practical applications in data analysis.

As already noted, relativistic quantum mechanics is recovered if the smallest distance which can be measured electrodynamically is $\Delta l = \hbar/2m_e c$ or $\kappa = \hbar/m_e$ As in earlier work, we take our first approximations for the fine structure constant from the combinatorial result

$$\alpha = e^2/m_e \kappa c \to e^2/\hbar c \approx 1/137$$

Similarly, the combinatorial starting point for quantum gravity is

$$\alpha_G = Gm_p/\kappa c \rightarrow Gm_p^2/\hbar c \approx 1/(2^{127} + 136)$$

Our recent derivation of the Maxwell equations[9] and Ref. 6 show that we have "spin 1" photons in addition to the attractive and repulsive Coulombic cases discussed above. As we have discussed elsewhere, all that is needed to get the classical predictions of General Relativity is the Newtonian case we already developed above and "travelling gravitons" with spin 2. Since for gravitation like "charges" attract, we cannot use spin 1 for gravitation and must go to spin 2 as the next simplest hypothesis. As is well known, this "weak quantum gravity" is in agreement with experiment. Our treatment provides it with a proper correspondence limit. But the question of what happens under crossing symmetry remains open.

3.3 General Relativity vs. Crossing Symmetry: Anti-Gravity★

We have already given a brief discussion of crossing symmetry above. Applied to our version of electrodynamics it provides all the usual Coulombic and spin effects for electrically charged particles and anti-particles achieved by more conventional methods. In our bit-string theory, this crossing symmetry derives from the fact that, if we make the proper identification between quantum numbers and kinematic variables derived from bit-strings, interchanging 0's and 1's in a bit-string corresponds to interchanging particle and antiparticle. In particular, this is true of our representation of the standard model of quarks and leptons using strings of 16 bits, although the published demonstrations of this statement are incomplete. If we interchange the 0's and 1's in *all* the strings for a theory in which the combinatorial hierarchy construction has closed, we produce a dual theory which is formally distinct but which is indistinguishable so far as *all* physical predictions go. I have called this Amson invariance. In conventional theories

★ Quoted, in part, from Ref.9.

this is the CPT theorem: changing all particles to anti-particles, reversing their velocities ($P_i \rightarrow -P_i$), and making a mirror reflection across three perpendicular planes ($J_i \rightarrow -J_i$) can have no observable consequences. In particular, this theorem requires particles and anti-particles to have identical *inertial* masses. But in the absence of an accepted theory of quantum gravity, gravitational mass (or better "gravitational charge") could either reverse or stay the same.

It is important to realize that crossing symmetry is more restrictive than CPT invariance. For instance, since we know that protons fall toward the earth, all CPT says is that anti-protons fall toward an anti-earth. This is not helpful for constructing an *experiment crusis*! But crossing symmetry applied to the coulomb problem tells us that anti-particles have opposite electric charge to particles and hence that if a particle is attracted toward a center, an anti-particle will be repelled by it. This follows immediately from the conic section formalism we have developed. But for gravitation, the definition of inertial mass remains the same as for coulomb attraction, and the same crossing symmetry applies. Hence, since particles are known to attract each other gravitationally, a particle and its anti-particle should repel each other. *Our prediction of anti-gravity is that simple.* From our empirical point of view, the ultimate decision must rest with the outcome of anti-proton and anti-neutron experiments which are already being vigorously pursued.

4. Appendix 1: Lack of consensus on how to formulate "quantum gravity".

29 July 1992

Dear Pierre:

It is my genuine belief that there exist basic conceptual incompatibilities between quantum theory and general relativity; or, more precisely, between the categorical schemes in which both are normally presented. However, it must be said at once that this is a complex issue since, at the very least, it requires agreement on precisely what those 'categorical schemes' are, and a universally held position on this is not easy to find. But, for example, I do not think it is hard to argue for an incompatibility between the so-called 'Copenhagen interpretation' of quantum theory (with its assignment of an *a priori* status to the notions of measurement and a classical realm) and any view that a theory of quantum gravity should maintain the conventional spacetime picture of general relativity. Some of the reasons for this are discussed in my Schladming notes; others are spelt out in more detail in a review paper that I have almost finished on the problem of time in quantum gravity. (This paper is based on a course of lectures I gave last month in Spain: I shall send you a copy when the final version is completed.)

However, it is clear that to judge whether or not any real incompatibility exists between quantum theory and general relativity one needs first to decide how large the domain of each of the two theories actually is and, in particular, if the two domains really overlap at all. If they do not, *i.e.* if one suspects that the standard technical or conceptual structures of quantum theory and/or general relativity must be modified in some way before they come into meaningful contact with each other, then the problem of quantum gravity looks very different, and it is rather misleading to talk about an 'incompatibility' between then.

To my mind, it is still unclear what superstring theory has to say about all this. It may well be that superstrings, or some related idea, will provide a coherent theory of quantum gravity, but I do not see that this has yet been shown conclusively at all. The main claim of superstring theory is that graviton scattering amplitudes are free of ultra-violet divergences, and that is undeniably a very important result. The problem is that all such calculations are made from within a framework of some sort of fixed background spacetime structure within which the gravitons propagate, which is precisely what most people who work in "traditional" quantum gravity have always tried to avoid. What is needed is a new non-perturbative approach to string theory in which the notion of a graviton is not so basic. Several have been suggested along that line (e.g. the Friedan-Shenkar ideas, string field theory) but activity in that directions seems to have petered out in recent years.

As I said in an earlier letter, in some respects the situation is not dissimilar to that which would have applied had the graviton calculations of naive quantum gravity turned out by some miracle to be finite: that would have been the *beginning* of the really exciting part of the quantum gravity programme, not the end. In so far as the string graviton amplitudes are finite, there is a *prima facie* case for saying that starting with a superstring approach is a better bet than starting with conventional GR, but a lot more work is needed to show that this view is justified. The considerable interest shown in the Ashtekar programme demonstrates that the debate is very far from being settled, at least in the eyes of those who work in 'conventional' quantum gravity.

Which brings us to the sociological issues. Like you, I get annoyed by the over-hyping that is sometimes given to superstring theory, particularly when it is made by people who are quite ignorant (or at the best, only partly aware) of the enormous literature on quantum gravity and, in particular, of what those working in that subject deem to be the real problems of their subject. Unfortunately, this issue frequently generates great emotions, and so it is difficult to arrive at a rational resolution. The more aggressive superstring affectionados present the quantum gravity community as being made up largely of 'has-beens' who are simply incapable of understanding superstring theory, and conversely unflattering views emerge from us about the general ignorance of the, largely particle-physics trained, superstring community. The whole business is very stupid and childish but it is deeply ingrained. For example, when giving talks about the Ashtekar approach to quantum gravity I frequently get the reaction from superstring activists that "it must be wrong because (i) quantum gravity is non-renormalisable, and (ii) superstring theory is right"; which ignores the fact that (i) the whole point of the Ashtekar programme is the claim that the normal perturbative results are irrelevant and, (ii) the scientific enterprise in general is not based on the deliverance of *ex cathedra* pronouncements that a certain theory is correct, although this is precisely what is happening in this case.

However, I fear these remarks of mine will not help you much. If you pass them on to one of the strident superstring people he will merely tell you that "Chris Isham is one of yesterday's men", and it is not easy to refute that type of attitude. I always find this particularly irritating because I was actively supporting (*i.e.* pushing the award of SERC grants, etc.) the work of, for example, Mike Green long before it became popular. And I still give my full support to requests for funding in these areas. But there you are, it is an unfair world!

All good wishes
Chris Isham

5. Appendix 2: ON THE CORRESPONDENCE LIMIT OF RELATIVISTIC QUANTUM MECHANICS*

One of the many problems of second quantized relativistic field theory is that the "correspondence limit" either in non-relativistic quantum mechanics for atomic systems or in non-relativistic quantum mechanics for strongly interacting nuclear systems or in classical relativistic particle mechanics is not well specified. In this paper we argue that by using a fully finite and discrete approach to relativistic quantum mechanics we can arrive at a theory which does not have these defects, yet reproduces many of the same *empirical* results which are conventionally accounted for by elementary particle physics and the related physical cosmology.

To illustrate one difficulty with the conventional approach, consider Weinberg's discussion of the low momentum limit of massless fields whose quanta carry spin j.[10] He shows that as the momentum p carried by the field quanta approaches zero, Lorentz invariance requires the interaction to vanish like p^j. This would be a disaster for the conventional theory since only scalar quanta could survive, ruling out photons and gravitons! He saves the second quantized relativistic field theory by showing that if one *also* requires gauge invariance, a well specified limit exists. The resulting theory then predicts that the force between two identical particles mediated by a field whose quanta have spin 1 is repulsive while the force between particle and anti-particle is attractive. For spin 0 and spin 2 the predicted force is always attractive. Both predictions are consistent with currently available experimental information.

This result appears to be a triumph for the theory, particularly since subsequent developments have singled out the class of non-abelian gauge theories as appropriate for describing the observations cited in support of weak-electromagnetic unification and quantum chromodynamics. But this success has a price. It requires in some sense the reification of the concept of "potential" at a fundamental level, in contrast the classical situation where potentials have no objective significance. Although the Aharanov-Bohm effect might seem to support this point of view, there is no consensus. For instance, topological explanations which invoke only forces rather than the electromagnetic vector potential have been put forward; unfortunately they do not compel acceptance.

An alternative fundamental theory,[11–13] (see also Ref. 5) which has been discussed at the three previous conferences in this series,[14] offers a route by which

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all these problems can be avoided. Define *particles* as the *conceptual* carriers of conserved quantum numbers between events and *events* as regions across which quantum numbers are conserved. Take as the basic paradigm for two events the sequential firing of two counters separated by distance L and time interval T, where the clocks recording the firings are synchronized using the Einstein convention. Define the velocity of the "particle" connecting these two events as $v = \beta c = L/T$ where c is the limiting velocity for the transfer of *information.* Given a beam of particles of this velocity selected by a collimator and counter telescope incident on two slits a distance w apart we find a double slit interference pattern at a detector array a distance D behind the slits whose maxima are separated by a distance s. Define the *deBroglie wavelength* $\lambda = ws/D$ using laboratory units of length. If a different source producing particles with the same velocity incident on the same arrangement gives a fringe spacing s', define the mass ratio $m'/m = s/s'$ Introduce Planck's constant h by the definition $\lambda = h/p$ where $\beta = pc/E$, $E^2 - p^2c^2 = m^2c^4$. *Postulate* that two events mediated by a particle of mass m and velocity βc can, but need not, take place *only* when they are separated by an integer number of deBroglie wavelengths.

Consider a particle bound to a center a distance r away which receives an impulsive force toward the center each time it has moved a deBroglie wavelength. Assume that the area swept out per unit time by the radial distance to the particle is constant for each step (Kepler's Second Law) and that the polygon closes after j steps. If we take $2\pi r = j\lambda$, and compute the square of the quantized angular momentum consistent with this correspondence limit we find it equal to $(j^2 - \frac{1}{4})\hbar^2 = \ell(\ell+1)\hbar^2$ where we have defined $\ell = j - \frac{1}{2}$. Assuming that the probability of the impulsive force occurring after one *Compton* wavelength is 1/137 we obtain[11] Bohr's relativistic formula $(\frac{m-\epsilon_\ell}{m})^2[1+(\frac{1}{137(\ell+1)})^2] = 1$ for the levels of the hydrogen atom(Ref. 8) in the approximation $e^2/\hbar c \approx 1/137$, and hence *his* correspondence limit. Adding a second degree of freedom gives us the Sommerfeld formula and an improvement of three significant figures[11] in our value for $e^2/\hbar c$. After deriving the commutation relations, we can invoke Feynman's proof of the Maxwell Equations (Ref.3) to show that we also have the correct classical fields in the appropriate correspondence limit. For gravitational orbits about a center containing N particles of mass m, orbital velocity reaches c when $\ell = 0$ and $N = M_{Planck}/m$ where $M_{Planck} = (\frac{\hbar c}{G})^{\frac{1}{2}}$ is the Planck mass. Consequently the shortest distance (between *two* events!) in the theory is the Planck length $h/M_{Planck}c$. Thanks to the fact that our Lorentz-invariant (for finite and discrete boosts and rotations!) theory predicts both the (quantized) Newtonian interaction and spin 2 gravitons, it meets the three classical tests of general relativity. Other successes of the theory will be reported.

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FIGURE CAPTIONS

1) Laboratory measurement of Mandelstam variables (Sec. 2.3) using counter telescopes for the process $(1,2) \to (3,4)$.

2) Geometrical arrangement for measuring mass ratios when all four telescopes record the same velocity.

3) The eight counter paradigm.

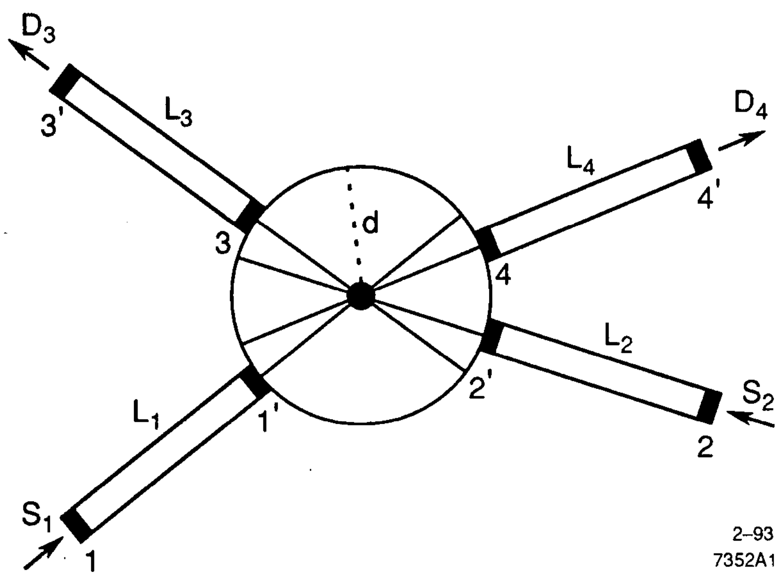

Fig. 1

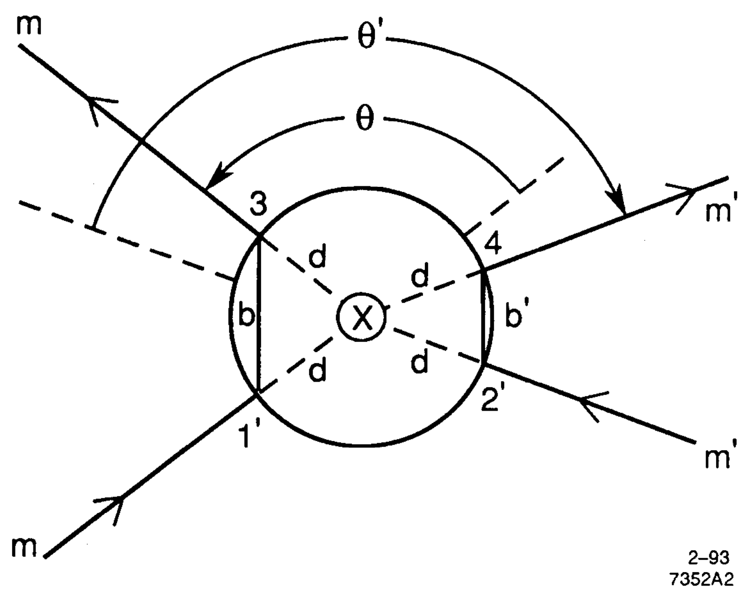

Fig. 2

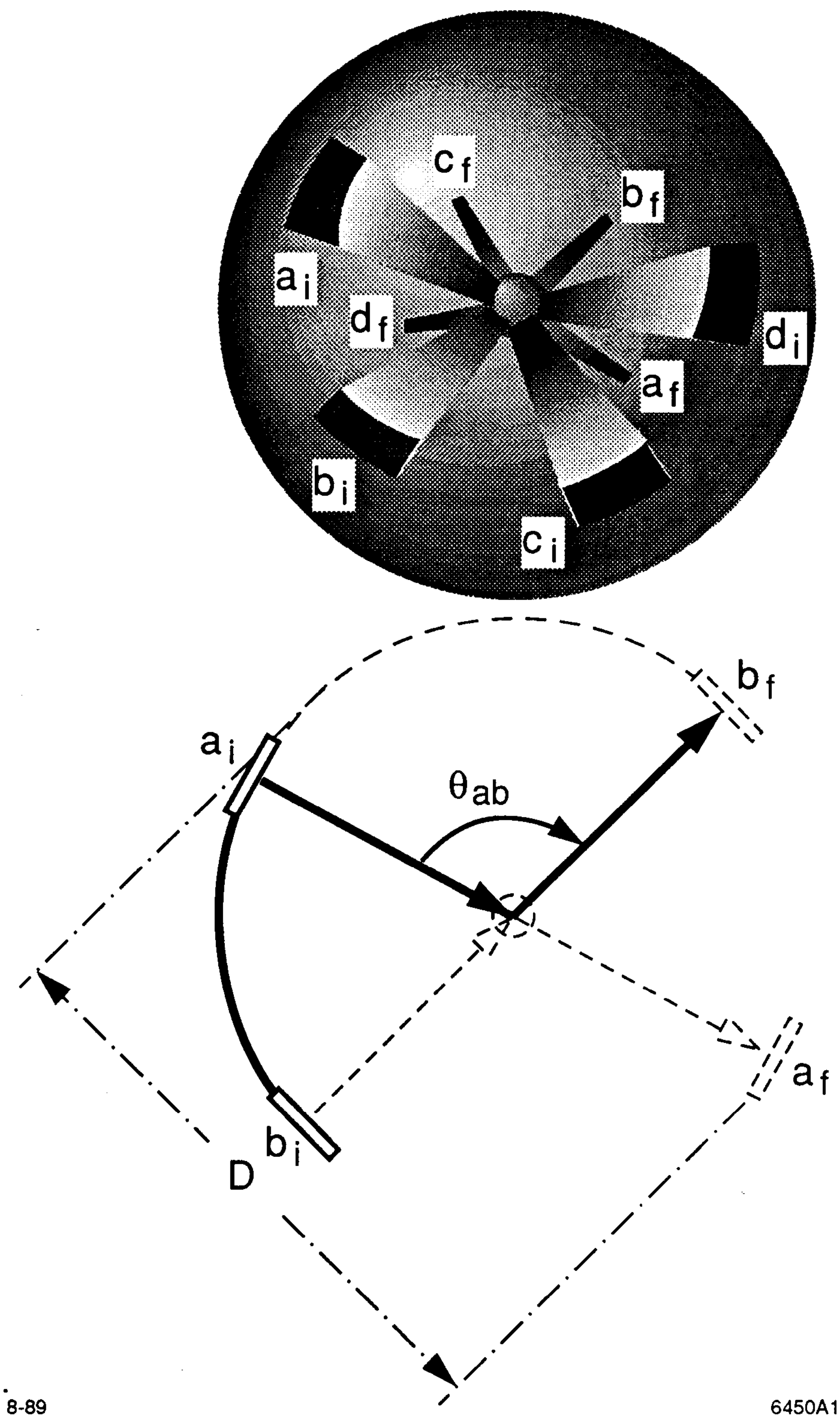

Fig. 3