SLAC–PUB–6055
February 1993
T/E

Excited Heavy Mesons and Kaon Loops in Chiral Perturbation Theory

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We consider the effects of excited states on the $SU(3)$ breaking chiral loop corrections to heavy meson properties. In particular, we compare the size of kaon loops in which an excited heavy meson appears to the size of previously calculated loops with heavy mesons in the ground state. We find that the new effects may indeed be of the same magnitude as the old ones, but that there is a strong dependence on the unknown masses and coupling constants of the new states. As a result, we argue that the ground state loops alone may not be a trustworthy guide to $SU(3)$ corrections, and that the appropriate cutoff for a heavy-light chiral lagrangian which omits excited heavy mesons may be considerably smaller than the naïve expectation of $\Lambda_\chi \approx 1$ GeV.

Submitted to *Physics Letters* B

* Work supported by the Department of Energy under contract DE–AC–76SF00515.

Quantum chromodynamics is known to exhibit new and interesting symmetries both in the chiral limit of zero light quark mass ($m_q \to 0$), and in the opposite limit of infinite heavy quark mass ($m_Q \to \infty$) [1][2]. It has recently been proposed to invoke both of these limits simultaneously to describe the dynamics of systems, such as the B and D mesons, which contain one heavy and one light quark [3]. The resulting "heavy-light chiral lagrangian" is a simultaneous expansion in inverse powers of m_Q and of some low energy cutoff such as the chiral symmetry breaking scale $\Lambda_\chi \approx 1$ GeV. It is the purpose of this note to explore whether the inclusion of excited heavy mesons may affect the appropriate value of this cutoff for $SU(3)$ violating loop effects.

Violations of exact $SU(3)$ flavor symmetry due to the nonzero strange quark mass arise directly at higher order in the chiral lagrangian. However, with the current paucity of precise data on heavy meson systems and their transitions, the inclusion of such terms would introduce enough free parameters into the theory as to preclude predictive power. Instead, what has been done in the past [4]–[6] is to focus on certain "log-enhanced" terms of the form $M_i^2 \ln(M_i^2/\mu^2)$, where M_i is a pseudogoldstone boson mass. Here the $SU(3)$ violation enters indirectly, through the splittings of the masses of bosons which appear in loops (of course, such splittings themselves arise at higher order in the chiral lagrangian). It is hoped that even if such terms are not in fact dominant, or are not dominant enough to yield by themselves an accurate result, at least they may give a useful estimate of the size and sign of the correction of interest. (In this approach, additional uncertainty arises through the appearance of a renormalization scale μ; the μ-dependence is canceled by higher order counterterms which here are neglected.)

In our consideration of the effects of excited heavy mesons, we will adopt the same philosophy. We will not be able to perform computations inherently any more precise than earlier ones, as the same difficulties as before will obtain. Rather our purpose will be to explore whether virtual processes which involve excited heavy mesons in virtual intermediate states give contributions to $SU(3)$ violating effects which are generically as large as those involving just the ground state. To this end we will content ourselves with comparing the "log-enhanced" pieces in each case, to see whether there is a natural suppression of one relative to the other. In fact, what we do here will be somewhat more crude than in the case of ground state mesons, for two reasons. First, the excited states we will consider have not been observed, presumably because they are very broad [7], and in our analysis we will ignore the effects of their unknown widths. Second, there will be certain additional graphs, proportional to new coupling constants, which do not arise when one restricts to the ground state mesons. Hence, in the end we will be able to draw only very rough conclusions about the importance of these excited states to $SU(3)$ splittings. If eventually the necessary inputs are measured, however, our predictions will become more concrete.

A ground state heavy meson has the light degrees of freedom in a spin-parity state $s^P = \frac{1}{2}^-$, corresponding to the usual pseudoscalar-vector meson doublet with $J^P = (0^-, 1^-)$. The first excited state involves a "P-wave excitation", in which the light degrees of freedom have $s^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$. In the second case we have a heavy doublet with $J^P = (1^+, 2^+)$; in fact, such mesons have already been identified in the charm system [7]. However, heavy quark symmetry rules out any one-pion coupling of this doublet to the ground state at lowest order in the chiral expansion [8]; hence we expect the effect of these states to be suppressed and henceforth we shall ignore them. The other excited doublet has $J^P = (0^+, 1^+)$. Neither of these states has yet been observed even in the charm system, presumably because they decay rapidly through S-wave pion emission [7]. However quark models suggest [9] that these states should have masses in the range 2300–2400 MeV, and we will use this estimate in what follows.

The heavy-light chiral lagrangian contains both heavy meson fields and pseudogoldstone bosons, coupled together in an $SU(3)_L \times SU(3)_R$ invariant way. To implement the heavy quark symmetries, the heavy meson doublets are represented by 4×4 Dirac matrices, transforming as antitriplets under the unbroken flavor $SU(3)$. The ground state $J^P = (0^-, 1^-)$ doublet $(M_a, M_a^{*\mu})$ is assembled into the "superfield" H_a, while the excited $J^P = (0^+, 1^+)$ doublet $(M_{0a}^*, M_{1a}'^{\mu})$ is represented by the object S_a [8][10]:

$$
\begin{aligned}
H_a(v) &= \frac{1+\not{v}}{2\sqrt{2}} \left[M_a^{*\mu}\gamma_\mu - M_a\gamma^5 \right] , \\
S_a(v) &= \frac{1+\not{v}}{2\sqrt{2}} \left[M_{1a}'^{\mu}\gamma_\mu\gamma^5 - M_{0a}^* \right] .
\end{aligned} \tag{1}
$$

Here v^μ is the fixed four-velocity of the heavy meson. Because we have absorbed mass factors $\sqrt{2M}$ into the fields, they have dimension 3/2; to recover the correct relativistic normalization, we will multiply amplitudes by $\sqrt{2M}$ for each external meson. The matrix of pseudogoldstone bosons appears in the usual exponentiated form $\xi = \exp(i\mathcal{M}/f_\pi)$, where

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix} \tag{2}$$

and $f_\pi \approx 135$ MeV (we will ignore the difference between f_π and f_K). The bosons couple to the heavy fields through the covariant derivative and axial vector field,

$$\begin{aligned} D^\mu_{ab} &= \delta_{ab}\partial^\mu + V^\mu_{ab} = \delta_{ab}\partial^\mu + \tfrac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)_{ab}, \\ A^\mu_{ab} &= \tfrac{i}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)_{ab} = -\frac{1}{f_\pi}\partial_\mu\mathcal{M}_{ab} + \mathcal{O}(\mathcal{M}^3). \end{aligned} \tag{3}$$

Lower case roman indices correspond to flavor $SU(3)$. Under chiral $SU(3)_L \times SU(3)_R$, the pseudogoldstone bosons and heavy meson fields transform as $\xi \to L\xi U^\dagger = U\xi R^\dagger$, $A^\mu \to UA^\mu U^\dagger$, $H \to HU^\dagger$ and $(D^\mu H) \to (D^\mu H)U^\dagger$, where the matrix U_{ab} is a nonlinear function of the pseudogoldstone boson matrix $\mathcal{M}$.

The chiral lagrangian is an expansion in derivatives and pion fields, as well as in inverse powers of the heavy quark mass. The kinetic energy terms take the form [3][8]

$$\mathcal{L}_{\text{kin}} = \frac{1}{8}f_\pi^2\,\partial^\mu\Sigma_{ab}\,\partial_\mu\Sigma^\dagger_{ba} - \mathrm{Tr}\left[\overline{H}_a(v)iv\cdot D_{ba}H_b(v)\right] + \mathrm{Tr}\left[\overline{S}_a(v)(iv\cdot D_{ba} - \Delta\delta_{ba})S_b(v)\right], \tag{4}$$

where $\Sigma_{ab} = \xi^2$, and Δ is the mass splitting of the excited doublet S_a from the ground state H_a. The leading interaction terms are of dimension four. There are couplings of the pions to the mesons within a given doublet,

$$g\,\mathrm{Tr}\left[\overline{H}_a(v)H_b(v)\,\not{A}_{ba}\gamma^5\right] + g'\,\mathrm{Tr}\left[\overline{S}_a(v)S_b(v)\,\not{A}_{ba}\gamma^5\right], \tag{5}$$

as well as a coupling which links the doublets together,[1]

$$h\,\mathrm{Tr}\left[\overline{H}_a(v)S_b(v)\,\not{A}_{ba}\gamma^5\right] + \text{h.c.}. \tag{6}$$

Note that the leading contribution of each of these terms is a Feynman rule with a single pion. Naïve power counting indicates that the couplings g, g' and h should be of order one.

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1 In ref. [8], the coupling h was denoted f''.

We now turn to two simple calculations in which $SU(3)$ splitting effects arise at one loop order in chiral perturbation theory. In each case we will compute only the nonanalytic "log-enhanced" pieces, first only including ground state heavy mesons and then with excited states as well. While the calculations for the ground states are already in the literature [4]–[6], we will present them here, both for completeness and because we will include additional contributions which heretofore have been neglected.

We begin by considering the one loop contribution to the ratio of charmed meson decay constants f_{D_s}/f_D. The pseudoscalar decay constants are defined by the matrix element of the weak current

$$\langle 0|\,\overline{\mathrm{q}}_a\gamma^\mu(1-\gamma^5)Q\,|D_a(p)\rangle = -\mathrm{i}f_{D_a}p^\mu\,, \tag{7}$$

and they are related to those for the vector mesons by heavy quark symmetry. The dimension three operator in the chiral lagrangian to which the left-handed current $\overline{\mathrm{q}}_a\gamma^\mu(1-\gamma^5)Q$ corresponds is [3]

$$\frac{\mathrm{i}}{2}f_D^{(0)}\sqrt{2M}\,\mathrm{Tr}\left[\gamma^\mu(1-\gamma^5)H_b(v)\xi^\dagger_{ba}\right]+\ldots\,, \tag{8}$$

where the $SU(3)$ invariant coefficient is fixed at this order by matching the matrix element (7) in QCD and the effective theory. $SU(3)$ violating chiral loop effects induce corrections to the lowest order relation $f_{D_s}/f_D = 1$ [4][6]. The leading contributions come from the renormalization of the vertex (8), as in fig. 1a, and from the wavefunction renormalization of the meson fields in fig. 1b. Unlike in refs. [4][6], we include the effects of the various mass splittings $\Delta_{D^*D}=M_{D^*}-M_D$, $\Delta_{D^*D_s}=M_{D^*}-M_{D_s}$, $\Delta_{D_s^*D_s}=M_{D_s^*}-M_{D_s}$, and $\Delta_{D_s^*D}=M_{D_s^*}-M_D$. While the splittings Δ_i arise at order $1/m_Q$ in the heavy quark expansion, we find that terms of the form $\Delta_i^2\ln(\Delta_i^2/\mu^2)$ are as large as those proportional to pseudogoldstone boson masses. The diagrams in fig. 1 renormalize the bare value $f_D^{(0)}$

of the decay constant differently for the D_s and D mesons, with the result

$$
\begin{aligned}
f_{D_s} = f_D^{(0)}\Big\{1 &- \frac{1}{16\pi^2 f_\pi^2}\left[M_K^2\ln(M_K^2/\mu^2) + \tfrac{1}{3}M_\eta^2\ln(M_\eta^2/\mu^2)\right] \\
&- \frac{g^2}{16\pi^2 f_\pi^2}\Big[3M_K^2\ln(M_K^2/\mu^2) - 6\Delta_{D^*D_s}^2\ln(\Delta_{D^*D_s}^2/\mu^2) \\
&\qquad + M_\eta^2\ln(M_\eta^2/\mu^2) - 2\Delta_{D_s^*D_s}^2\ln(\Delta_{D_s^*D_s}^2/\mu^2)\Big]\Big\} + \dots\,, \\
f_D = f_D^{(0)}\Big\{1 &- \frac{1}{16\pi^2 f_\pi^2}\left[\tfrac{3}{4}M_\pi^2\ln(M_\pi^2/\mu^2) + \tfrac{1}{2}M_K^2\ln(M_K^2/\mu^2) + \tfrac{1}{12}M_\eta^2\ln(M_\eta^2/\mu^2)\right] \\
&- \frac{g^2}{16\pi^2 f_\pi^2}\Big[\tfrac{9}{4}M_\pi^2\ln(M_\pi^2/\mu^2) - \tfrac{9}{2}\Delta_{D^*D}^2\ln(\Delta_{D^*D}^2/\mu^2) \\
&\qquad + \tfrac{3}{2}M_K^2\ln(M_K^2/\mu^2) - 3\Delta_{D_s^*D}^2\ln(\Delta_{D_s^*D}^2/\mu^2) \\
&\qquad + \tfrac{1}{4}M_\eta^2\ln(M_\eta^2/\mu^2) - \tfrac{1}{2}\Delta_{D^*D}^2\ln(\Delta_{D^*D}^2/\mu^2)\Big]\Big\} + \dots\,.
\end{aligned}
\tag{9}
$$

In principle, the μ-dependence in these expressions cancels against a higher-order counterterm in the effective lagrangian, which we do not include. The value $g^2 \approx 0.5$ is suggested by the upper limit on the total rate for the decay $D^* \to D\pi$ and by the quark model [3]. Taking $g^2 = 0.5$, $\mu = 1$ GeV, $\Delta_{D^*D} = \Delta_{D_s^*D_s} = 140$ MeV, $\Delta_{D^*D_s} = 40$ MeV and $\Delta_{D_s^*D} = 240$ MeV, we find

$$
\frac{f_{D_s}}{f_D} = 1 + 0.07 + (0.11 + 0.12) + \dots\,, \tag{10}
$$

where the first piece comes from the g-independent vertex renormalization in fig. 1a, the second is due to the pseudogoldstone boson masses (including the pions) in fig. 1b, and the third arises from the meson splittings Δ_i in the same diagram. Note that this final term, which had been omitted in previous analyses, is not negligible.

We now extend this result by including the analogous diagram in which the virtual heavy meson is in an excited state, as in fig. 1c. This graph will depend on the splittings $\Delta_{D_0^*D} = M_{D_0^*} - M_D$, $\Delta_{D_0^*D_s} = M_{D_0^*} - M_{D_s}$, $\Delta_{D_{0s}^*D_s} = M_{D_{0s}^*} - M_{D_s}$ and $\Delta_{D_{0s}^*D} =$

$M_{D^*_{0s}} - M_D$, and on the new coupling h. We find

$$f_{D_s} = f_D^{(0)}\Big\{1 - \frac{h^2}{16\pi^2 f_\pi^2}\big[M_K^2\ln(M_K^2/\mu^2) - 6\Delta^2_{D_0^* D_s}\ln(\Delta^2_{D_0^* D_s}/\mu^2)$$
$$+ \tfrac{1}{3}M_\eta^2\ln(M_\eta^2/\mu^2) - 2\Delta^2_{D^*_{0s} D_s}\ln(\Delta^2_{D^*_{0s} D_s}/\mu^2)\big]\Big\}\,,$$

$$f_D = f_D^{(0)}\Big\{1 - \frac{h^2}{16\pi^2 f_\pi^2}\big[\tfrac{3}{4}M_\pi^2\ln(M_\pi^2/\mu^2) - \tfrac{9}{2}\Delta^2_{D_0^* D}\ln(\Delta^2_{D_0^* D}/\mu^2)$$
$$+ \tfrac{1}{2}M_K^2\ln(M_K^2/\mu^2) - 3\Delta^2_{D^*_{0s} D}\ln(\Delta^2_{D^*_{0s} D}/\mu^2)$$
$$+ \tfrac{1}{12}M_\eta^2\ln(M_\eta^2/\mu^2) - \tfrac{1}{2}\Delta^2_{D_0^* D}\ln(\Delta^2_{D_0^* D}/\mu^2)\big]\Big\}\,. \qquad (11)$$

To estimate the magnitude of the result, we take two estimates for the unknown masses of the excited states, $M_{D_0^*} = 2300$ MeV and 2400 MeV. In all cases we take the strange mesons to be heavier than the nonstrange ones by 100 MeV. Then we find a total correction

$$\frac{f_{D_s}}{f_D} = 1 + 0.07 + 0.23 + \frac{h^2}{0.5}\begin{pmatrix} 0.04 + 0.09 \\ 0.04 + 0.04 \end{pmatrix} + \dots\,. \qquad (12)$$

Here the first two terms are respectively the g-independent and g^2 terms of eq. (10), and in the parentheses the upper numbers refer to $M_{D_0^*} = 2300$ MeV and the lower to 2400 MeV, the first to the contributions from pseudogoldstone boson masses and the second from the mass splittings Δ_i. We will discuss the likely value of h^2 below; for now we simply observe that unless it is much smaller than g^2, the effects of intermediate excited states are indeed not negligible.

Finally, we would like to include the diagram in fig. 1d (the analogous graph with a virtual ground state meson vanishes [4]). This graph depends on the unknown decay constant $f_{D_0^*}$ of the excited charmed meson, defined by

$$\langle 0|\,\overline{q}\gamma^\mu(1-\gamma^5)Q\,|D_0^*(p)\rangle = \mathrm{i}f_{D_0^*}p^\mu\,. \qquad (13)$$

It is consistent at this order to neglect $SU(3)$ splittings in $f_{D_0^*}$ itself. The corresponding operator in the heavy-light chiral lagrangian is

$$-\frac{i}{2} f_{D_0^*} \sqrt{2M}\, \mathrm{Tr}\left[\gamma^\mu (1-\gamma^5) S_b(v) \xi^\dagger_{ba}\right] + \dots\,, \tag{14}$$

which generates the one pion Feynman rule contributing to fig. 1d. We then find an additional contribution to the ratio f_{D_s}/f_D, given by

$$\frac{f_{D_s}}{f_D} = 1 + \frac{h}{0.5}\frac{f_{D_0^*}}{f_D^{(0)}}\begin{pmatrix} 0.07 + 0.06 \\ 0.07 + 0.03 \end{pmatrix} + \dots\,, \tag{15}$$

where the terms in parentheses are to be interpreted as in eq. (12). Unfortunately, even less is known about $f_{D_0^*}$ than about f_D, although the quark model would suggest a decay constant of the P-wave excited state somewhat smaller than that of the ground state. Hence there is little we can say about the relative size (or sign) of this contribution, but barring an odd and fortuitous cancellation against the graph in fig. 1c, it should not affect the substance of our results.

Of course, the size of the new effect found in eq. (12) depends on what one takes for the low energy parameter h^2. In particular, is it possible that there is a significant suppression of h^2 relative to $g^2 \approx 0.5$? In the nonrelativistic quark model, the values of g and h depend in part on the overlap of light quark wavefunctions, and we may expect this overlap to be larger for mesons in the same doublet (g) than for mesons in different doublets (h). However, such an overlap also governs the decays of the $J^P = (1^+, 2^+)$ doublet into $D^{(*)}\pi$, mediated by a dimension five operator in the chiral lagrangian [8]. To fit the observed decay rates, the coefficient of this operator must be approximately 0.5 GeV^{-1}; if one assumes the usual power-counting denominator of $\Lambda_\chi \approx 1$ GeV, then the dimensionless overlap factor is of order one. In addition, the width of the excited $J^P = (0^+, 1^+)$ doublet is proportional to h^2. For example, for $M_{D_0^*} = 2400$ MeV, $\Gamma(D_0^* \to D\pi) = h^2 \times 1500$ MeV, while for $M_{D_0^*} = 2300$ MeV, $\Gamma(D_0^* \to D\pi) = h^2 \times 900$ MeV (the corresponding width $\Gamma(D_1' \to D^*\pi)$ is smaller by $2 \sim 4$ because of phase space.) Given that we believe that these states have not been identified because they are very wide, a suppression by an order of magnitude of h^2 relative to g^2 is again indicated against. Hence we expect that the value $h^2 \approx 0.5$ taken in eq. (12) is not unreasonable. Finally we note that within the flux tube model of ref. [11], one actually deduces a much larger value, $h^2 \approx \mathcal{O}(10)$. While such a model may well not be trustworthy, it provides a tantalizing hint that possibly these excited states are quite important indeed.

In the same spirit, we now consider the contribution of excited states to the ratio of Isgur-Wise functions $\xi(v\cdot v')$ for strange and non-strange charmed mesons. The Isgur-Wise function is the single function which in the heavy quark limit parameterizes all semileptonic decays $B_a \to D_a^{(*)}\ell\overline{\nu}_\ell$ [2]. In the heavy-light chiral lagrangian, the operator responsible for this weak decay is

$$-\beta(w)\,\mathrm{Tr}\left[\overline{H}_a^{(\mathrm{c})}(v')\gamma^\mu(1-\gamma^5)H_a^{(\mathrm{b})}(v)\right]+\dots\,, \tag{16}$$

where $w = v\cdot v'$ and to this order $\beta(w) = \xi(w)$. An $SU(3)$ splitting in the ratio $\xi_{\mathrm{s}}(w)/\xi_{\mathrm{u,d}}(w)$ arises from one loop corrections [5][6]. For the contributions from intermediate ground state mesons, this comes from the diagram in fig. 2a, along with the wave-function renormalization in fig. 1b. So as not to confuse $1/M_Q$ effects with $SU(3)$ splittings, we will consider the strict heavy quark limit $M_{\mathrm{c}}, M_{\mathrm{b}} \to \infty$, in which $\Delta_{D^*D} = \Delta_{B^*B} = 0$. Keeping again only the "log-enhanced" pieces, the ratio then takes the simple form

$$\begin{aligned}\frac{\xi_{\mathrm{s}}(w)}{\xi_{\mathrm{u,d}}(w)} &= 1-\frac{g^2}{16\pi^2 f_\pi^2}\,(r(w)-1)\left[3M_\pi^2\ln(M_\pi^2/\mu^2)-2M_K^2\ln(M_K^2/\mu^2)-M_\eta^2\ln(M_\eta^2/\mu^2)\right]\\ &= 1+0.05\,(w-1)+\dots\,,\end{aligned} \tag{17}$$

where

$$r(w) = \frac{1}{\sqrt{w^2-1}}\ln\left(w+\sqrt{w^2-1}\right)\,. \tag{18}$$

By way of comparison, we would now like include the analogous diagram with an excited intermediate meson state, as in fig. 2b. The result will be moderately more complicated, since it will involve the nonzero mass splitting $\Delta = M_{D_0^*} - M_D = M_{B_0^*} - M_B$ (equal in the heavy quark limit), as well $\Delta \pm 100$ MeV when strange mesons appear in the loop. The dimensionally regularized graph involves the integral

$$I(\Delta, M^2, w) = \int \frac{\mathrm{d}^{4-\epsilon}p}{(2\pi)^{4-\epsilon}}\frac{(p\cdot v)(p\cdot v')}{(p\cdot v-\Delta)(p\cdot v'-\Delta)(p^2-M^2)}\,, \tag{19}$$

but since we need only the logarithmically divergent pieces proportional to Δ^2 and M^2, it is sufficient to consider the simpler quantity

$$J(\Delta, M^2, w) = \frac{1}{2}\Delta^2 \frac{\partial^2 I}{\partial \Delta^2}\bigg|_{\Delta=M^2=0} + M^2 \frac{\partial I}{\partial M^2}\bigg|_{\Delta=M^2=0} . \tag{20}$$

However, we notice immediately that the term in J proportional to M^2 is independent of the velocity variable w. Hence the wavefunction renormalization contributions in fig. 1c, which by heavy quark symmetry cancel the correction to the vertex at the zero recoil point $w = 1$, in fact cancel the correction for all w. We find that the only "log-enhanced" term with w-dependence is that proportional to Δ^2. This cancellation of the $M^2 \ln(M^2/\mu^2)$ terms suppresses considerably the contributions of excited states to the ratio $\xi_s(w)/\xi_{u,d}(w)$.

Unlike the diagram in fig. 2a, the contribution from intermediate excited states in fig. 2b is not proportional to the original form factor $\beta(w)$. Instead, it depends on the analogous function $\zeta(w)$ for transitions of the excited doublet. Expanding about $w = 1$, the contribution to the Isgur-Wise function of this process takes the form

$$\begin{aligned} &\frac{h^2}{16\pi^2 f_\pi^2} \sum_i C_i \Big\{ [\beta(w) - \zeta(w)] \big[- M_i^2 \ln(M_i^2/\mu^2) + 6\Delta_i^2 \ln(\Delta_i^2/\mu^2) \big] \\ &\qquad\qquad + (w-1) \big[\tfrac{2}{3} \Delta_i^2 \ln(\Delta_i^2/\mu^2) \big] + \ldots \Big\} \\ &= \frac{h^2}{16\pi^2 f_\pi^2} (w-1) \sum_i C_i \Big\{ [\beta'(1) - \zeta'(1)] \big[- M_i^2 \ln(M_i^2/\mu^2) + 6\Delta_i^2 \ln(\Delta_i^2/\mu^2) \big] \\ &\qquad\qquad + \tfrac{2}{3} \Delta_i^2 \ln(\Delta_i^2/\mu^2) + \ldots \Big\} , \end{aligned} \tag{21}$$

where the sum runs over the pseudogoldstone bosons which appear in the loop. The positive constants C_i are products of coefficients in the boson matrix $\mathcal{M}$, and they depend on the $SU(3)$ flavor of the decaying meson. Note that because both form factors are normalized at $w = 1$, $\beta(1) = \zeta(1) = 1$, all corrections to the vertex vanish at zero recoil as required by heavy quark symmetry. Making the same estimates as before for the masses of the excited states, we obtain

$$\frac{\xi_s(w)}{\xi_{u,d}(w)} = 1 + (w-1) \left\{ 0.05 + \frac{h^2}{0.5} \begin{pmatrix} 0.02 + 0.25\,[\beta'(1) - \zeta'(1)] \\ 0.01 + 0.16\,[\beta'(1) - \zeta'(1)] \end{pmatrix} \right\} + \ldots , \tag{22}$$

where again the upper numbers are for $M_{D_0^*} = 2300$ MeV and the lower for 2400 MeV. The dominant corrections seem to be those proportional to the difference of charge radii

$\beta'(1) - \zeta'(1)$. Unfortunately, little is known about the function $\zeta(w)$, although the quark model would suggest that the charge radius of the excited P-wave doublet is larger than that of the ground state S-wave. Hence the quantity $\beta'(1) - \zeta'(1)$ is most probably positive and of order one or smaller. (Recall that the derivatives $\beta'(1)$ and $\zeta'(1)$ are negative.) Finally, we note that we are neglecting diagrams such as in fig. 2c, which, like the one in fig. 1d, depend on additional new and unknown form factors.

We see that, depending on the values of h^2 and $\beta'(1) - \zeta'(1)$, the contributions of excited heavy mesons to $SU(3)$ splitting effects may indeed be important. Perhaps we should not be so surprised at this, as there is no symmetry to enforce a large mass splitting of the excited states from the ground state. In ordinary chiral perturbation theory, chiral symmetry suppresses the masses of the π, K and η relative to those of the nearest excited octet of ρ, K^* and ϕ, with the result that a low energy theory in which only the pseudogoldstone bosons are included may be sensible up to the order of the chiral symmetry breaking scale. By contrast, no such mechanism applies to the heavy-light chiral lagrangian. The excited states are nearby and may be easy to produce, and we may expect that loops sufficiently off shell to include kaons should include excited heavy mesons as well.

Unfortunately, the relative sizes of these effects for the charm system depend crucially on the unknown properties of the excited D_0^* and D_1' mesons. When in the future these states are positively identified and studied, we will know more firmly whether their contributions invalidate the usual estimates of $SU(3)$ splittings based solely on the D and D^*. If so, an alternative interpretation of our results is that one should include neither kaons nor excited heavy mesons in the heavy-light chiral lagrangian, instead computing only pion loops in an $SU(2)$ theory with a cutoff of a few hundred MeV, the mass splitting of the first excited state. We note that it has been argued elsewhere [12] that the appropriate cutoff for the heavy-light chiral lagrangian may be significantly smaller than $\Lambda_\chi \approx 1$ GeV. While the reasoning here is logically independent, both results may be pointing us to the same conclusion.

It is a pleasure to thank Lisa Randall and Mark Wise for helpful conversations.

References

[1] S. Nussinov and W. Wetzel, Phys. Rev. 36 (1987) 130;
M. B. Voloshin and M. A. Shifman, Sov. J. Phys. 47 (1988) 511.

[2] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113;
N. Isgur and M.B. Wise, Phys. Lett. B237 (1990) 527.

[3] M.B. Wise, Phys. Rev. D45 (1992) 2188;
G. Burdman and J. Donoghue, Phys. Lett. B280 (1992) 287;
T. M. Yan *et al.*, Phys. Rev. D46 (1992) 1148.

[4] B. Grinstein *et al.*, Nucl. Phys. B380 (1992) 369.

[5] E. Jenkins and M. Savage, Phys. Lett. B281 (1992) 331.

[6] P. Cho, Harvard University preprint HUTP-92-A039 (1992).

[7] N. Isgur and M. B. Wise, Phys. Rev. Lett. 66 (1991) 1130.

[8] A. F. Falk and M. Luke, Phys. Lett. B292 (1992) 119.

[9] S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189;
S. Godfrey and R. Kokoski, Phys. Rev. D43 (1991) 1679.

[10] A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B343 (1990) 1;
J. D. Bjorken, in *Proceedings of the 4th Rencontres de la Physique de la Valle d'Aoste*, La Thuile, Italy, 1990, edited by M. Greco (Editions Frontières, Gif-sur-Yvette, France, 1990);
A. F. Falk, Nucl. Phys. B378 (1992) 79.

[11] R. Kokoski and N. Isgur, Phys. Rev. D35 (1987) 907.

[12] L. Randall and E. Sather, MIT preprint MIT–CTP–2167 (1992).

Figure Captions

Fig. 1. One loop diagrams contributing to the renormalization of the decay constant of the ground state meson. The square denotes the weak current and the dashed lines signify pseudogoldstone bosons. Figure (a) is g-independent. In (b) the virtual meson is in the ground state doublet, while in (c) and (d) it is excited.

Fig. 2. One loop diagrams contributing to the Isgur-Wise function. The circle denotes the flavor-changing weak current. In (a) the virtual meson is in the ground state doublet, while in (b) it is in an excited state. Diagrams such as (c) will not be included.

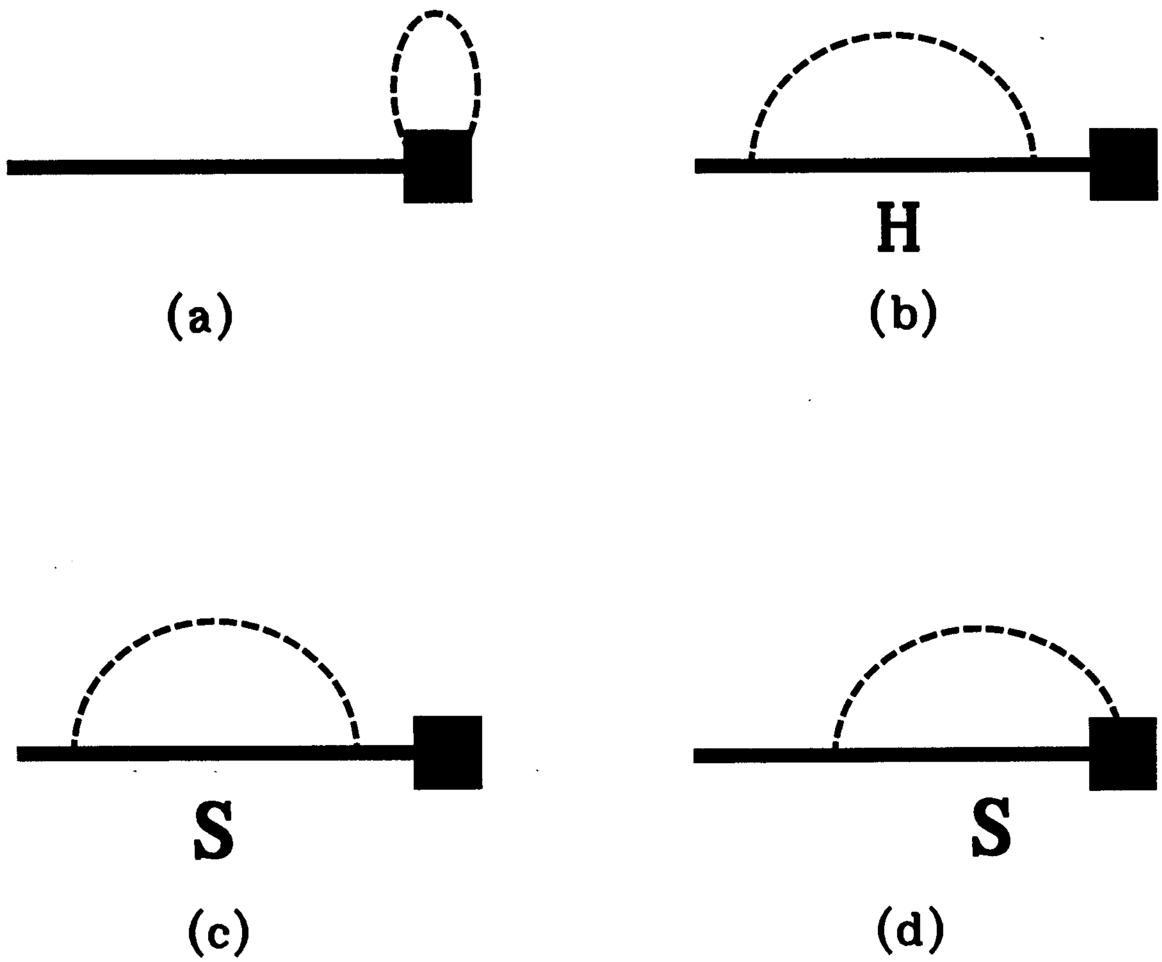

Fig. 1

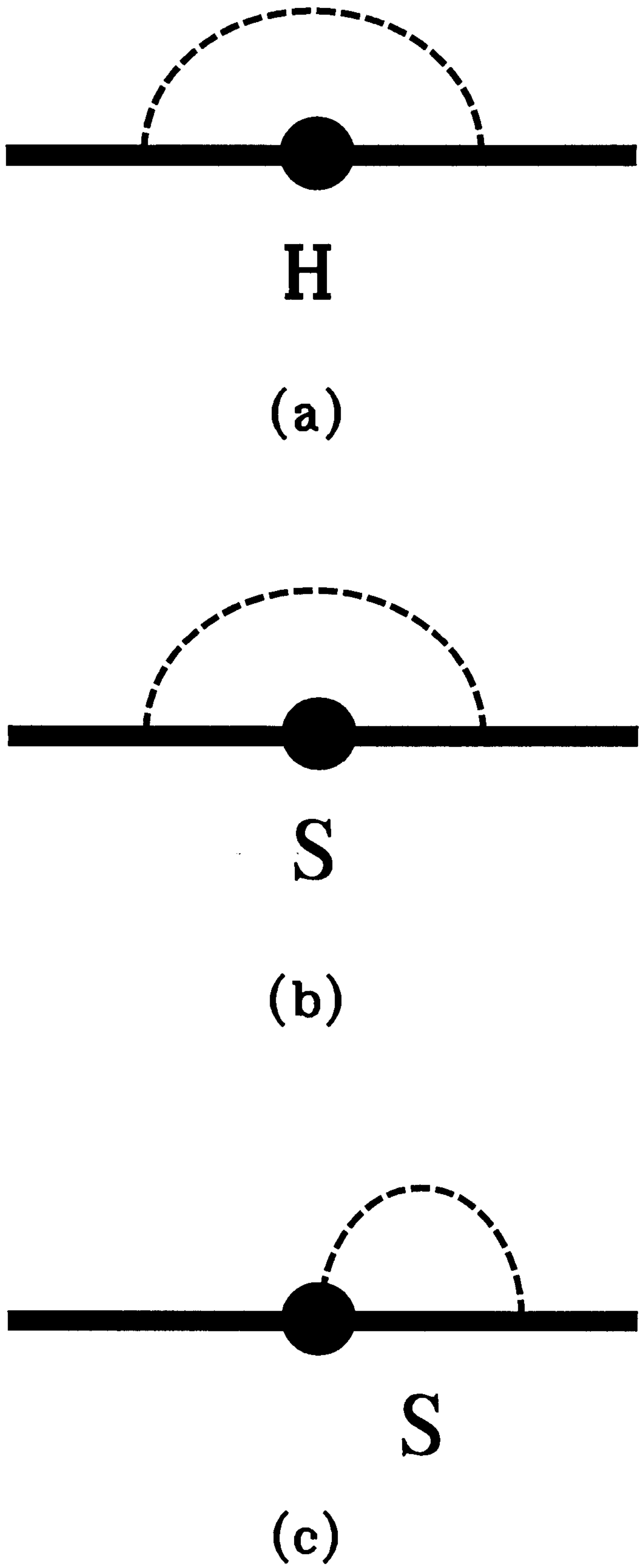

Fig. 2