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Introduction to Impedance For Short Relativistic Bunches*

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1. Introduction

The purpose of this paper is to introduce the concept of impedance to calculate the wake field forces left behind by a short bunch which travels at relativistic speed through a structure with discontinuities.[1] We will try to be as intuitive as possible and leave the more rigorous derivations to the second paper on this subject by J. Wang.

2. Representation of Cavity by Equivalent Circuit

We will consider the cavity shown in Fig. (2.1) which has rotational symmetry about the z axis and is excited by the beam current, I_B, passing through the gap. For the time being, we will consider only one mode of excitation for the cavity; namely, the mode where the magnetic field $\vec{B}$ is azimuthal around the beam direction as shown. The current I_L flows in the outer cavity wall in the direction shown to oppose the magnetic field in the cavity excited by the beam current. This current I_L causes a build-up of positive and negative charges on the exit and entrance plane of the gap as shown. This charge build-up produces an increasing electric field in the direction opposite to the direction of the beam current.

We will use Gaussian units and write Maxwell's equation as

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{dt} \tag{2.1}$$

where $\vec{J}$ is the current density, $\vec{B}$ the magnetic field, $\vec{E}$ the electric field, c the speed of light, and t the time. The coefficient $(\frac{4\pi}{c})$ is

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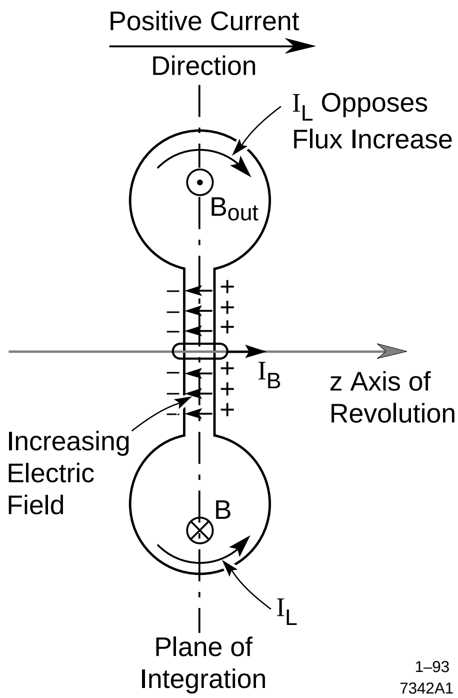

Figure 2.1. Cavity excited by Beam Current I_B.

377 Ω. We integrate both sides of Eq. (2.1) over the surface area of a plane perpendicular to the direction of the beam current as shown in Fig. (2.1). The area of integration includes the walls of the cavity. The term $\int\int(\nabla \times \vec{B}) \cdot d\vec{A} = \oint \vec{B} \cdot d\vec{l} = 0$, since the boundary of the surface is in the walls of the cavity where $B = 0$. The integral of $\vec{J}$ over the area gives the current $I_B + I_L$. The integral of the time variation of the electric field is defined as a displacement current

$$I_C = \frac{1}{4\pi} \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} \tag{2.2}$$

The surface area integral of (2.1) yields Kirchoff's Law

$$I_B + I_L + I_C = 0 \tag{2.3}$$

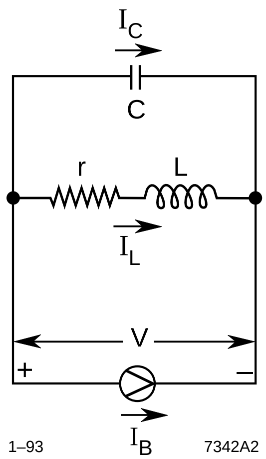

Figure 2.2. Cavity represented by equivalent circuit with wall losses included by series resistance.

We can represent this cavity by an equivalent circuit with I_B a source current as shown in Fig. (2.2). Note that the wall resistance of the cavity has been included as a resistance in series with the inductance. This comes about because the finite conductivity of the wall produces a non-zero electric field parallel to the cavity wall which is proportional to the current I_L. The circuit shown in Fig. (2.2) can be excited to large voltages when the time variation of the exciting current I_B is near the resonant frequency $\omega^2 = (1/LC)$. If, at this frequency, the series resistance is small compared to the inductive reactance, i.e. $r << \sqrt{L/C}$, then the circuit in Fig. (2.2) can be well represented by the circuit shown in Fig. (2.3) with a shunt resistance $R = (L/rC)$. For this circuit Kirchoff's Law becomes

$$C\frac{dV}{dt} + \frac{1}{R}V + \frac{1}{L}\int V dt = -I_B \tag{2.4}$$

Capacitive +	Resistive +	Inductive =	Driving
term	term	term	term

It is quite common to take the time derivative of Eq. (2.4) and use

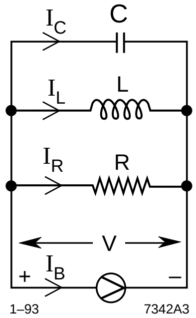

Figure 2.3. Equivalent circuit to approximate cavity with parallel resistance.

the following notation

$$\frac{1}{C} = \frac{\omega_r R}{Q} \text{ and } \omega_r^2 = \frac{1}{LC} \tag{2.5}$$

to arrive at the second order differential equation

$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^2 V = -\frac{\omega_r R}{Q}\dot{I}_B \tag{2.6}$$

So far, we have considered an equivalent cavity with only one possible mode. Most cavities have many modes of excitation. We can represent these cavities by a generalization of the equivalent parallel circuit as shown in Fig. (2.4). We obtain a separate equation for the voltage V_n of each mode n excited by beam current I_B,

$$C_n\ddot{V}_n + \frac{\dot{V}_n}{R_n} + \frac{V_n}{L_n} = -\dot{I}_B \tag{2.7}$$

The total voltage is given by the sum of the voltages over all N modes of the cavity

$$V_t = \sum_{n=1}^{N} V_n \tag{2.8}$$

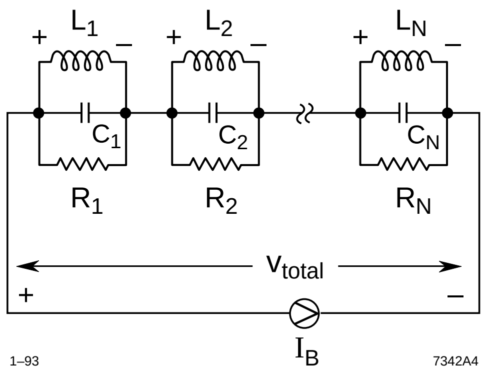

Figure 2.4. Generalization of equivalent circuit for multiple mode cavity.

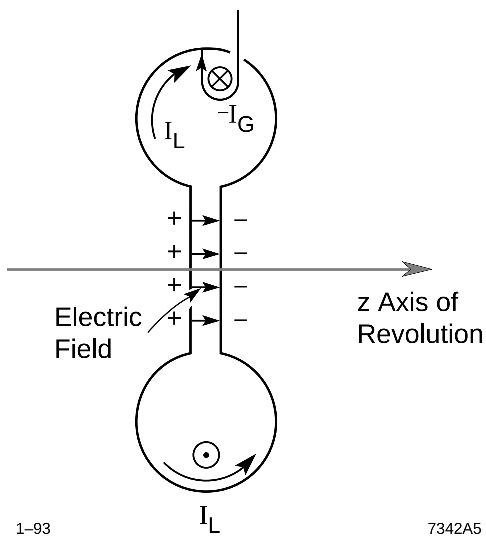

Figure 2.5. Cavity excited by external current.

While the main purpose of this paper is to discuss how the beam excites wake fields in vacuum structures, it is useful, for completeness, to illustrate how a cavity can be driven in its fundamental mode from an external source with a coupling loop as shown in Fig. (2.5). Kirchoff's Law, Eqs. (2.3-2.6), still holds. However, we need to include the generator current $-I_G$ in the loop for the driving term along with the beam current, so

$$C\frac{dV}{dt} + \frac{1}{R}V + \frac{1}{L}\int V\,dt = I_G - I_B \tag{2.9}$$

where we have chosen the direction of the current I_G to produce an accelerating voltage. This circuit equation is used extensively to describe the cavity voltage in a steady state, or in a slowly varying amplitude and phase approximation, and will be discussed in the papers by P. B. Wilson and F. Pedersen. Our purpose is to illustrate their connection to single bunches traveling through different structures in the ring vacuum chamber.

3. Driving Current of a Short Pulse of Charge

We will consider a charge pulse of length $\sigma = vT$, where the center of the pulse passes through the cavity center $z = 0$ at time $t = 0$. The time duration of the pulse is T, and the pulse velocity is v, which we will assume is close to the speed of light. The linear charge density of the pulse can be represented by $\lambda(s)$ with $s = (vt - z)$, the position of the charge in the bunch relative to the center of the bunch. Note that the front of the bunch passes through the cavity at $s < 0$ as shown in Fig. (3.1). This illustrates that bunch density profile is what one would see on an oscilloscope trace. The reader should be careful to note that other authors may display a snapshot of the bunch density profile at a fixed time so that the front of the bunch would be reversed from the convention of this paper. The beam current, which is to be used in the equivalent circuit of the previous section, is given by

$$I_B(t) = v\lambda(vt - z) = v\lambda(s) \tag{3.1}$$

The pulse $\lambda(s)$ can also be represented by its Fourier transform $\tilde{\lambda}(\omega)$ with

$$\lambda(s) = \frac{1}{2\pi}\int \tilde{\lambda}(\omega)e^{-i\omega s/c}d\omega \tag{3.2}$$

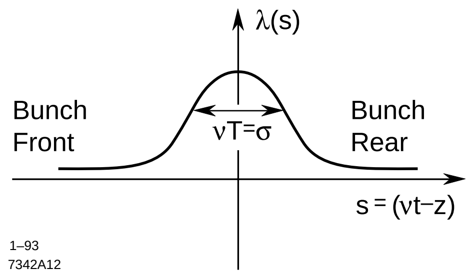

Figure 3.1. Linear density profile of bunch.

and

$$\tilde{\lambda}(\omega) = \frac{1}{v} \int \lambda(s) e^{i\omega s/c} ds \tag{3.3}$$

One of the most common pulse distributions considered is the Gaussian distribution

$$\lambda(s) = \frac{Q}{\sqrt{2\pi}\sigma} e^{-s^2/2\sigma^2} \tag{3.4}$$

where Q is the total charge in the bunch. This Gaussian distribution has a Fourier transform

$$\tilde{\lambda}(\omega) = Q e^{-\omega^2 \sigma^2 / c^2} \tag{3.5}$$

The spectrum for $\tilde{\lambda}(\omega)$ falls off rapidly for frequencies $\omega > c/\sigma$ as shown in Fig. (3.2). Of course, when the density distribution is influenced by the wake fields, the assumption that the distribution is Gaussian is suspect.

It is instructive to consider the case when the time variation of the current or the time of interest for the cavity voltage is small compared to the resonant period of the cavity modes, i.e. when the bunch is short enough or the mode frequencies of the cavity low enough that $\omega_n T << 1$. For this case, we can ignore the second and third terms on the left hand side of Eq. (2.7) and approximate the

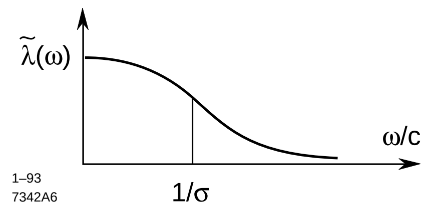

Figure 3.2. Fourier spectrum of Gaussian pulse.

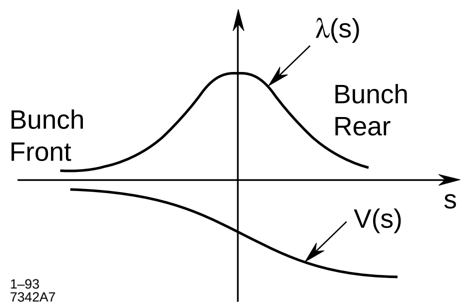

Figure 3.3. Wake voltage for short bunch (capacitive).

wake voltage in the cavity as

$$V(t) = \sum_{n=1}^{N} V_n = -\sum_{n=1}^{N} \frac{1}{C_n} \int I_B(t)dt \tag{3.6}$$

We denote this form of the wake field voltage as seen by the charge in the bunch as a *Capacitive Wake*. This is shown in Fig. (3.3).

Next we consider the case when the time variation of the current or the time of interest for the cavity voltage (equal to the duration of the bunch passage through the cavity) is large compared to the resonant period of the cavity modes, i.e. $\omega_n T >> 1$. We also assume that the Q of the cavity is sufficiently high that the fields do not decay appreciably during the passage of the bunch through the cavity. For this case, the third term is the dominant term on the left side of

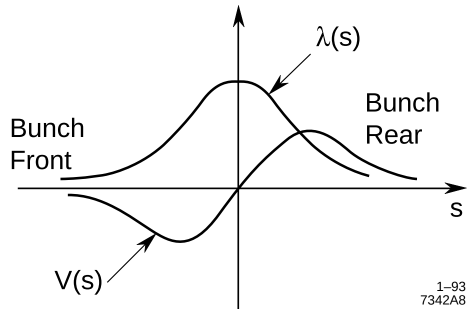

Figure 3.4. Wake voltage for long bunch (inductive).

Eq. (2.7), and we can approximate the wake voltage in the cavity as

$$V(t) = \sum_{n=1}^{N} V_n = -\sum_{n=1}^{N} L_n \frac{dI_B}{dt} \tag{3.7}$$

We call this form of the wake field voltage as seen by the charge in the bunch an *Inductive Wake*. This type of wake is shown in Fig. (3.4), where we see that the energy lost by the front of the bunch is gained by the rear of the bunch, so that, for a pure inductive wake, the net energy lost by the bunch is zero.

There is one other case which we should consider for completeness; namely, the case of a very long bunch or a very lossy cavity where the fields decay in a time much shorter than the time it takes for the bunch to pass through the cavity. In this case, $\omega_n T >> Q$, and we can approximate the cavity voltage as

$$V(t) = \sum_{n=1}^{N} V_n = -\sum_{n=1}^{N} \frac{1}{R_n} I_B \tag{3.8}$$

This is called a *Resistive Wake* and is shown in Fig. (3.5), where we see that the cavity voltage and bunch density are exactly in phase.

For the case of high Q cavities and intermediate bunch lengths, the total wake field is the sum of the capacitive and inductive parts of the wake as shown in Fig. (3.6). Note that even for a zero shunt resistance (i.e. an infinite Q), there is a net loss of energy for the

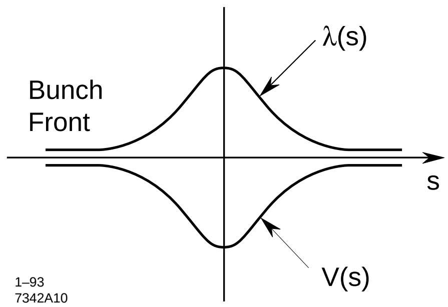

Figure 3.5. Wake voltage for very long bunch (resistive).

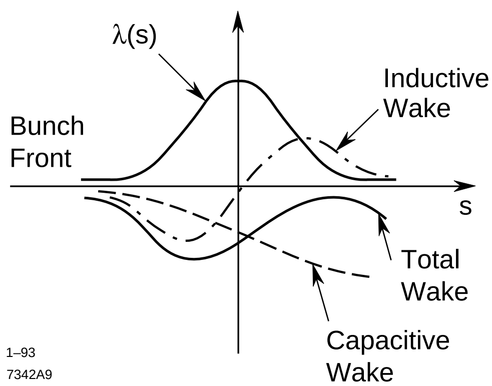

Figure 3.6. Total wake field for intermediate bunch lengths.

bunch, just as in the case when a resistive term is present. In addition to the bunch length, both the transverse distance to the outer wall and the length of the gap determine whether the cavity is capacitive or inductive. Three examples of cavity shapes and bunch lengths are shown in Fig. (3.7), one illustrates a capacitive wake, and the others inductive wakes. For the first example, shown in Fig. (3.7), we see that if

$$(g + l/2)l < 2(b - a)^2 \tag{3.9}$$

with g the gap length, l the bunch length, a the radius of the inner wall, and b the radius of the outer wall, the fields produced by the

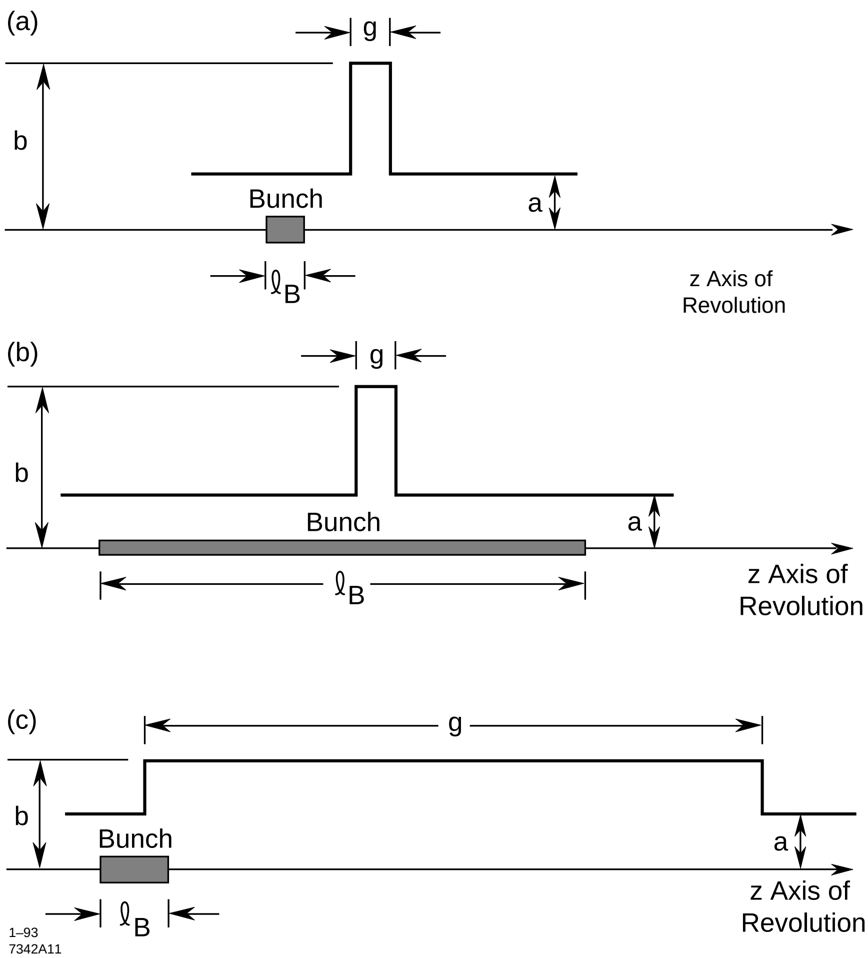

Figure 3.7. Three examples of beam cavity interactions: a) capacitive wake b) & c) inductive wake.

front of the bunch do not have time to propagate to the outer cavity wall and back before the bunch leaves the cavity. The cavity modal frequency, ω_n, is of the order of c/b. The time duration T of the pulse is given by l/c so that the condition $\omega_n T << 1$ satisfies Eq. (3.9), and the wake field of this example is mainly capacitive. On the other hand, if the field produced by the front of the bunch can propagate to the outer wall and back in time to effect most of the particles in the bunch, the wake field is mainly inductive. This is shown by the last two examples in Fig. (3.7).

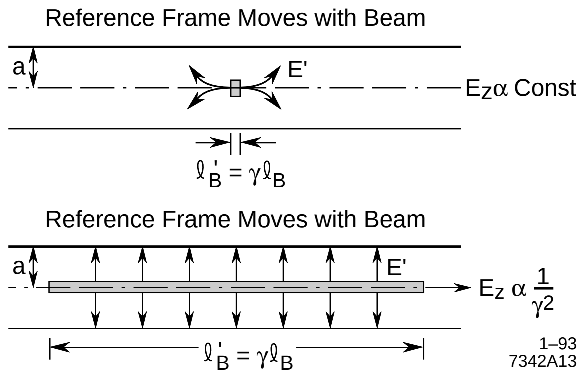

Figure 3.8. Beam density profile in reference of the beam a)"very short" bunch $(a/\gamma) >> l_B$ b) "short" bunch $(a/\gamma) << l_B << a$.

In order to properly define a "short" bunch it is necessary to consider the relativistic energy parameter of the beam γ which is defined as the ratio of the particle energy to the rest mass energy. We assume that the particles in the bunch travel near the speed of light, and we can substitute c for v except when the difference is required, and then we can substitute $(c - v)$ by $c/(2\gamma^2)$. As discussed above, the bunch is short when the cavity gap and the bunch length are much smaller than the transverse size of the cavity.

However, even in this case there are two regimens of interest which are shown in Fig. (3.8). In the first regime, which we refer to as the "very short" bunch regime, we find that $(a/\gamma) >> l_B$, even for $\gamma >> 1$. The second regime, which we refer to as the "short" bunch regime is where $(a/\gamma) << l_B << a$. The reference frame used in Fig. (3.8), which we designate by a prime on the bunch length, is a reference frame moving at the velocity of the bunch (i.e. the rest frame of the beam). In the "very short" bunch regime the length of the bunch in the rest frame of the particles is still much less than the transverse size of the chamber, but in the "short" bunch regime, the length of the bunch in the rest frame of the particles is much greater than the transverse size of the chamber. The first regime is where many Free Electron Lasers operate, while the second regime is where the high energy colliders considered in this school operate. In the following, we will consider only the "short" bunch regime, so that only wake fields which are due either to the finite conductivity or to the discontinuities of the vacuum chamber are considered.

4. Connection Between Wake Potential and Impedance

In the previous section, we found that the wake voltage V could be written as a function of the driving current (or its equivalent the linear charge density of the bunch). We write

$$V(s) = -\int_{-\infty}^{\infty} W_{\|0}\left(s - s'\right) \lambda(s') ds' \tag{4.1}$$

where we have chosen V positive for an accelerating voltage.[2] For the capacitive, resistive and inductive wakes discussed in the previous section, the potential kernels are given by the following expressions:

$$W_{\|0}\left(s - s'\right) = \sum_{n=1}^{N} \frac{1}{C_n} H\left(s - s'\right) \tag{4.2a}$$

$$W_{\|0}\left(s - s'\right) = c \sum_{n=1}^{N} \frac{1}{R_n} \delta\left(s - s'\right) \tag{4.2b}$$

and

$$W_{\|0}\left(s - s'\right) = c^2 \sum_{n=1}^{N} L_n \frac{d\delta\left(s - s'\right)}{ds} \tag{4.2c}$$

where $H(s)$ is the unit step function equal to one for $s > 0$ and equal to zero for $s < 0$, while $\delta(s)$ is the Dirac Delta function. When Eqs. (4.2a-4.2c) are substituted into Eq. (4.1), one obtains Eqs. (3.6), (3.7) and (3.8). It is useful to define the longitudinal impedance by the Fourier transform of the wake function $W_{\|0}(s)$

$$Z_{\|}\left(\omega\right) = \frac{1}{c} \int W_{\|0}(s) e^{i\omega s/c} ds \tag{4.3a}$$

and

$$W_{\parallel 0}(s) = \frac{1}{2\pi} \int Z_{\parallel}(\omega) e^{-i\omega s/c} d\omega \tag{4.3b}$$

where the impedance can be written as the sum of the real part (resistance) and imaginary part (reactance)

$$Z_{\parallel}(\omega) = R_{\parallel}(\omega) + iX_{\parallel}(\omega) \tag{4.4}$$

From causality, $W(s)$ is zero for $s < 0$, so that $R_{\parallel}$ is an even function, and $X_{\parallel}$ an odd function of ω. The amount of energy lost by the bunch, which travels through a cavity, divided by the charge in the bunch is called the loss factor and is given by

$$k_{\parallel} = -\frac{1}{Q} \int \lambda(s) V(s) ds \tag{4.5a}$$

or in terms of the impedance

$$k_{\parallel} = \frac{1}{Q} \int \widetilde{\lambda}^2(\omega) R_{\parallel}(\omega) dw \tag{4.5b}$$

where we have used the fact that the term $X_{\parallel}(\omega)$ integrates to zero.

A more general expression for the impedance may be given by expanding in powers of $(1/\sqrt{\omega})$ (Eq. 3),

$$\begin{aligned} Z(\omega) = -i\omega L + B\sqrt{\omega} + R \\ + \frac{Z_1}{\sqrt{\omega}} + \frac{1}{\omega C} + \cdots \end{aligned} \tag{4.6}$$

The first term is the inductive term caused by bellows, slots, ports, etc. in the vacuum chamber, and is only valid for the lower frequencies below cutoff. This term is discussed in the previous section. The second term, proportional to $\sqrt{\omega}$, is due to the finite conductivity of the chamber wall and is present even in a smooth chamber. This term is valid up to frequencies, where the displacement current in the wall becomes comparable to the ohmic current. The third term is a resistive term with R the usual dc resistance, also discussed in the previous section. The fourth term, proportional to $1/\sqrt{\omega}$, is due to the diffraction of the electromagnetic field at sharp discontinuities in the chamber. This term is valid for the frequencies driven by the beam in the "short bunch" regime. The last term is the usual capacitive term which is higher order in $1/\sqrt{\omega}$ and can be ignored for high frequencies. Both the resistive wall term and the diffraction term are discussed below.

5. Resistive Wall Impedance

In order to understand physically the wake field, which is responsible for the resistive wall impedance, consider the following argument.[4] As the beam passes by a given point in the vacuum wall, a surface current is induced on the wall. Subsequently, if the conductivity of the wall is finite, this current diffuses into the metal giving rise to wake fields. This process can best be illustrated by a simple example. We will examine the currents in the wall for the case of a charge particle pulse traveling parallel to an infinite metallic plane as shown in Fig. (5.1c). Imagine that the pulse of particles is made up of two semi-infinite beams, one positive and one negative, as shown in Figs. 5.1a and 5.1b. The image charges and currents are also shown for the case of a perfectly conducting wall. Because the wall conductivity is infinite, no current can exist inside the metal, and the induced currents stay on the surface of the wall. The wall currents and charges due to the (+) and (-) beams have the same magnitudes but opposite signs; by superposition, they cancel each other in the region behind the pulse as shown in Fig. (5.1c). Hence, no current is left in the wall for the case of a perfectly conducting wall. However, if the wall conductivity is finite, the surface currents can diffuse toward the inside of the metal. The diffusion of the image currents of the (+) and (-) beams is shown in Figs. (5.2a) and (5.2b). Because these image currents are turned on at different times, the wall current corresponding to the (+) beam has diffused farther into the metal than that of the (-) beam at the same point along the wall. This gives rise to currents in the wall in the region behind the pulse as shown in Fig. (5.2c). Near the wall surface, the currents are positive, and inside the metal the currents are negative. Hence, in the presence of

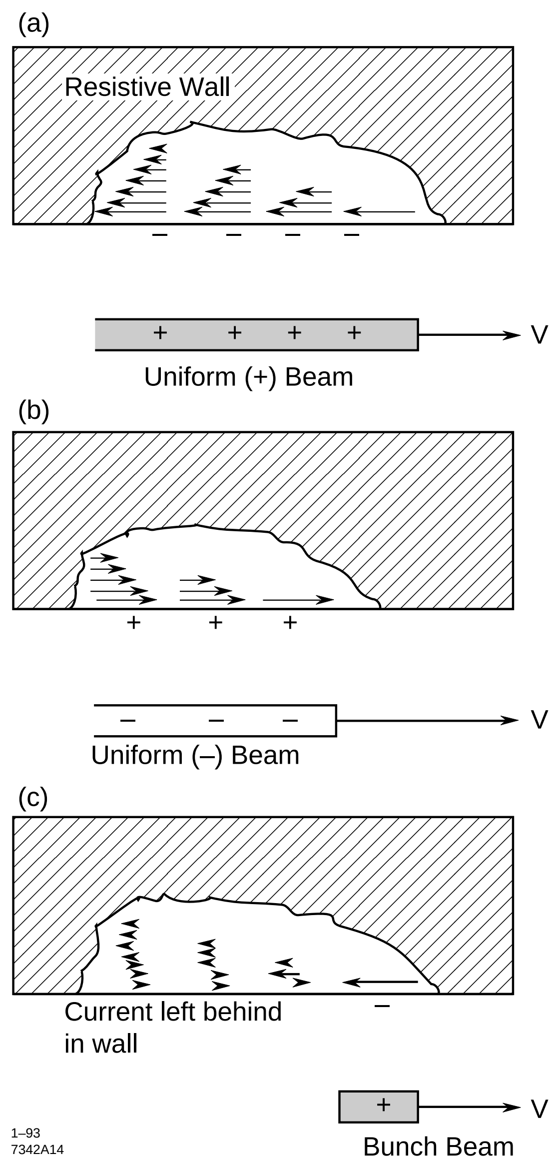

Figure 5.1. An illustration of resistive wall effects for a perfectly conducting wall: a) a semi-infinite (+) beam, b) a semi-infinite (-) beam, and c) a bunched (+) beam.

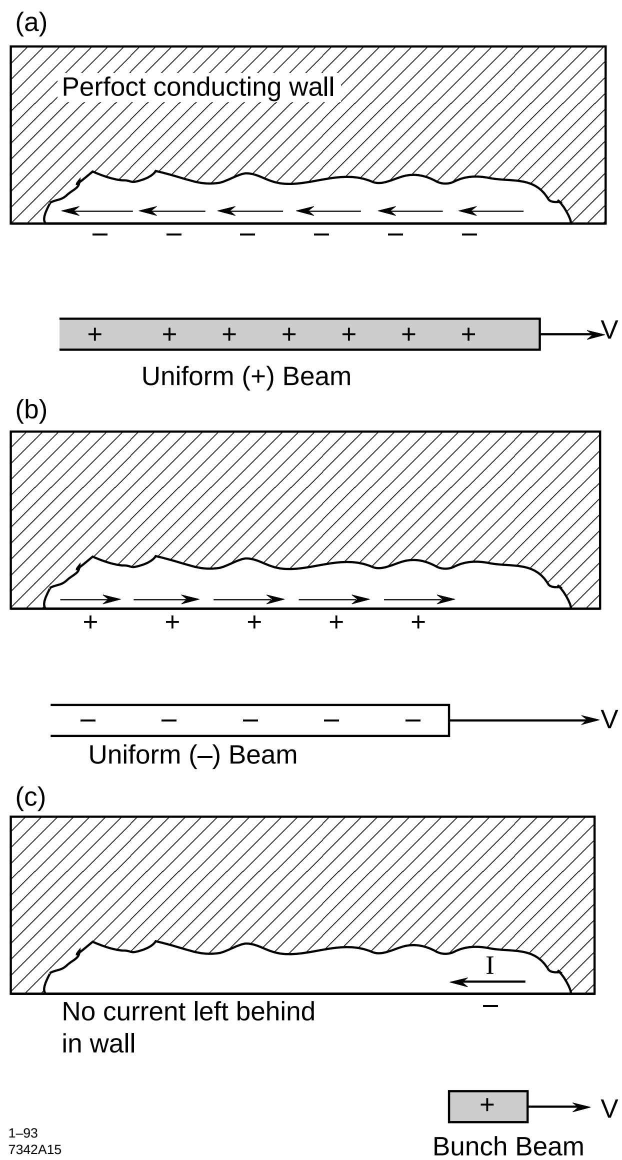

Figure 5.2. An illustration of resistive wall effects for a lossy wall: a) a semi-infinite (+) beam, b) a semi-infinite (-) beam, and c) a bunched (+) beam.

wall resistance, there are wall currents left behind a pulse of charged particles. These currents provide the source for the wake fields.

The diffusion distance of the current into the wall is given by the skin depth

$$\delta = c/\sqrt{2\pi\omega\sigma_c}$$

where ω is a frequency given by $\omega\ell_B \sim c$, see Fig. (3.2). The conductivity of the wall is denoted by σ_c which for copper is 0.5×10^{18}/sec. The current density at the wall of radius a is $J \sim I/a\delta$. This current density corresponds to a longitudinal electric field component $E_z = V/\ell_w = J/\sigma_c$, where ℓ_w is the total length of the resistive wall. Hence, the impedance $Z(\omega) = V/I \propto \sqrt{\omega}$. Because of causality, $Z(\omega) = Z^*(-\omega)$, and the actual value of B in Eq. (4.6) is given by

$$B = \frac{(1-i)\ell_w Z_0}{2a\sqrt{2\pi\sigma_c}}$$

with $Z_0 = 4\pi/c$ the impedance of free space equal to 377 Ω.

6. Diffraction Impedance

The diffraction term was originally derived by J. D. Lawson in the "very short bunch" regime to obtain the γ dependence for the energy lost by a zero length bunch passing through a cavity.[5] It should be pointed out that the dependence of $1/\sqrt{\omega}$ for the impedance of the diffraction term is quite subtle, and its derivation and range of validity were the subject of a whole special issue of Particle Accelerators devoted to a workshop attended by the "experts" in the field of impedances for short bunches. A beautiful physical argument to derive this result has been given by R. B. Palmer.[6] With his kind permission, we have repeated his treatment here.

We consider the cavity excited by a short bunch as shown in Fig. (6.1). When the bunch enters the cavity, the electromagnetic field which had been contained within the pipe of radius a starts to diffract away from the cavity edge as shown. When the rear of the bunch is

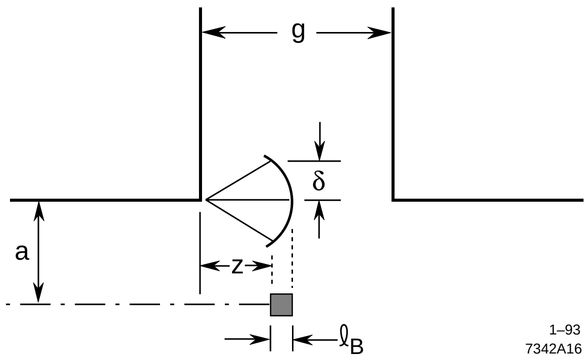

Figure 6.1. Diffraction of electromagnetic energy of beam passing through a short gap.

at position z from the edge of the cavity, the electromagnetic field at radius a, which started at the front of the bunch, has diffracted transversely a distance δ such that the field at $r = a+\delta$ and $r = a-\delta$ is at a distance ℓ_B behind the field at $r = a$. We will consider the case where $\delta << a$ and $z >> \ell_B$. The transverse distance δ is related to the longitudinal distance z by

$$\sqrt{z^2 + \delta^2} = z + \ell_B$$

or (6.1)

$$\delta \approx \sqrt{2\ell_B z}$$

When the distance z equals the gap width g, the beam enters the pipe, and the portion of the field that has diffracted by the amount $\delta \approx \sqrt{2\ell_B g}$ is retarded such that it can not catch up with the beam. This portion of the energy is lost. In the "very short bunch" regime, we must substitute $\ell_B = a/\gamma$, while in the "short" bunch regime, ℓ_b is the length of the bunch. The electromagnetic field at radius a in the pipe is given by

$$E_r(a) = B_\theta(a) = \frac{2Q}{a\ell_b} \tag{6.2}$$

The amount of electromagnetic energy loss in the ring at $r = a$ with thickness 2δ is approximately equal to

$$\Delta U \approx \frac{1}{8\pi}\left[\left(\frac{E_r}{2}\right)^2 + \left(\frac{B_\theta}{2}\right)^2\right][2\pi a\delta\ell_B] \tag{6.3}$$

which for the "very short bunch" is

$$\Delta U = \frac{Q^2 g^{1/2}\gamma^{1/2}}{\sqrt{2}a^{3/2}} \tag{6.4}$$

While for the "short bunch" regime

$$\Delta U = \frac{Q^2 g^{1/2}}{\sqrt{2}a\ell_b^{1/2}} \tag{6.5}$$

The result for the "very short bunch" originally derived by Lawson shows the $\sqrt{\gamma}$ dependence on the energy loss. We, of course, are only interested in the "short bunch," so we will only consider Eq. (6.5). We use the definitions for the voltage and current $V = \Delta U/Q$ and $I = Qc/\ell_B$ to obtain the impedance at the frequency $\omega = c/\ell_B$ given by

$$Z \approx \sqrt{\frac{gc}{2\omega}}\frac{Z_0}{4\pi a} \tag{6.6}$$

Again we must use causality with $Z(\omega) = Z^*(-\omega)$. The value of the term in Eq. (4.6) for a Gaussian bunch is given by

$$Z_1 = \frac{(1+i)Z_0}{2a}\sqrt{\frac{cg}{\pi^3}}$$

7. Summary

The main purpose of this paper is to give an introduction to the jargon used in discussing wake fields excited by a short bunch of particles passing through a structure. Often, terms such as the inductive or resistive part of the wake are used to describe the characteristics of the field or voltage which acts on the particles in the bunch. We have tried to illustrate how some of these terms relate to the common parallel circuit of a cavity. Many people will refer to a resistive portion of the total wake voltage shown in Fig. (3.6) when a portion of the wake voltage is in phase with the beam density profile. This is because the wake voltage for a resistive impedance is in phase with the beam density as shown in Fig. (3.2). Of course, we know that this portion of the wake in Fig. (3.6) comes from the capacitive part of the impedance. To the particles in the beam, however, this portion of the impedance has a resistive effect, since it produces a net energy loss. If, after reading this paper, the student has a more physical feel for the wake field in different regimes and finds the more advanced papers easier to read, then this paper has fulfilled its purpose.

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References

1. Much of the original work on wake fields and their influence on the particle beam dynamics was done by L. J. Laslett, V. K. Neil, and A. M. Sessler, see for example Rev. Sci. Instr. Vol 32, No. 3, March, 1961, p. 256-279. For example, on page 279, Eq. (4.8) gives the famous criteria for the energy spread in a beam, as a function of the impedance, necessary to stabilize the microwave instability. The first use of the term wake field

to describe fields left behind a pulse of charge was in a paper on resistive wall by P. L. Morton, V. K. Neil and A. M. Sessler J. Appl. Phys. Vol. 37, No. 10, Sept. 1966. The use of the equivalent circuit to describe the beam-cavity interaction was introduced by K. W. Robinson in a CEA report CEAL-1010, Feb. 1964 to describe the beam-cavity stability. A more recent report, on a workshop held at LBL on Impedance Beyond Cutoff, Part. Acc. Vol. 25, No. 2-4, contains some of the most up-to-date results on this subject.

2. The notation used in this section has been chosen to agree with that of K. Bane and M. Sands in the workshop on Impedance Beyond Cutoff, Ref. (1), p. 73.

3. S. Heifets, Broad Band Impedance of the B-factory ABC-60, Note SLAC, 1991.

4. Original work on this subject was by P. L. Morton, V. K. Neil, and A. M. Sessler, Ref. (1), and K. W. Robinson SLAC Report No. 49, 1965, p. 32. More recent papers discussing this subject are in the 1982 SLAC summer school on Physics of High Energy Particle Accelerators by A. W. Chao, AIP Conf. No. 105, 1982, p. 361 and in the 1983 BNL/SUNY summer school on Physics of High Energy Particle Accelerators by K. Bane and P. Wilson, AIP Conf. No. 127, p. 903.

5. J. D. Lawson, workshop on Impedance Beyond Cutoff, Ref. (1), p. 107.

6. R. B. Palmer, workshop on Impedance Beyond Cutoff, Ref. (1), p. 97.